

Next-to-leading order non-Fermi-liquid corrections to the neutrino emissivity and cooling of the neutron star

Souvik Priyam Adhya,^{*} Pradip K. Roy,[†] and Abhee K. Dutt-Mazumder[‡]

High Energy Nuclear and Particle Physics Division, Saha Institute of Nuclear Physics,

1/AF Bidhannagar, Kolkata-700 064, India

(Received 18 April 2012; published 13 August 2012)

In this work we derive the expressions of the neutrino mean free path (MFP) and emissivity with non-Fermi-liquid corrections up to next-to-leading order (NLO) in degenerate quark matter. The calculation has been performed for both the absorption and the scattering processes. Subsequently the role of these NLO corrections on the cooling of the neutron star has been demonstrated. The cooling curve shows moderate enhancement compared to the leading order non-Fermi-liquid result. Although the overall correction to the MFP and emissivity are larger compared to the free Fermi gas, the cooling behavior is not altered significantly.

DOI: [10.1103/PhysRevD.86.034012](https://doi.org/10.1103/PhysRevD.86.034012)

PACS numbers: 12.38.Mh, 12.38.Cy, 97.60.Jd

I. INTRODUCTION

It has been known for quite sometime now that the degenerate Fermi gas at low or zero temperature gives rise to a phenomenon which is different from the normal Fermi-liquid (FL) behavior once the magnetic interactions are included. This interesting feature is characterized by the appearance of an anomalous logarithmic term in the expressions for various physical quantities like specific heat, entropy etc. [1]. It has also been revealed recently that at strictly zero temperature, there is a logarithmic singularity in the inverse group velocity, which leads to the breakdown of the usual Fermi-liquid picture in the presence of the magnetic interactions. Historically, such a deviation from the normal Fermi-liquid behavior was exposed for the first time in [1], where, the specific heat of a degenerate gas due to the current-current interactions was calculated and the result contained the $T \ln T^{-1}$ term which emanates from the unscreened magnetic interactions. It is to be mentioned here that the non-Fermi-liquid behavior of highly dense color superconducting QCD plasma has been studied in great detail [2,3].

For nonrelativistic systems, the magnetic interaction is suppressed in powers of $(v/c)^2$ and therefore might not be of much quantitative importance. However, for a dense plasma where the constituents like quarks or electrons are moving with a velocity close to the velocity of light, the magnetic interactions cannot be neglected. In fact, it has been revealed recently that in many contexts, the transverse interactions, due to its infrared sensitivity, may become more important than its longitudinal counterpart in this kinematic regime. For example, while calculating the fermion damping rate and energy loss, it has been shown in Refs. [4,5] that the leading order (LO) contributions come

from the magnetic interaction while the longitudinal interactions contribute only at the subleading order. In fact, it has been seen that the first two leading order contributions in the expressions for the fermion damping rate in ultradegenerate plasma come from the transverse sector alone. A similar behavior has also been reported in [6,7] where the authors have studied the non-Fermi-liquid behavior (NFL) of the drag and diffusion coefficient in degenerate plasma. A more elaborate discussion on the NFL aspects of the cold and dense QED and QCD plasmas has been presented in [8].

This recently discovered phenomenon of non-Fermi-liquid behavior, which relates itself to the modified quark dispersion relation for excitations close to the Fermi surface, also finds important application in astrophysics. For example, it has been shown that the NFL corrections to the quark self-energy enhance the neutrino emissivity of ungapped quark matter which may exist in the core of neutron stars [9,10]. Like emissivity, in dense quark matter, the neutrino mean free path (MFP) also receives significant NFL corrections as has been demonstrated in [11]. It might be mentioned here that in all these calculations the evaluation of the quark self-energy was restricted to the leading logarithmic order. In [12], on the other hand, the authors determined the quark dispersion relations in ultradegenerate relativistic plasmas beyond LO, which, at zero temperature, is characterized by the appearance of the fractional higher powers in the energy variable. In another work, the specific heat of normal degenerate quark matter has also been calculated where in the higher order terms the fractional powers also show up [13].

In view of these contemporary investigations, here we plan to evaluate the neutrino MFP and corresponding emissivity in normal degenerate quark matter beyond leading logarithmic order and compare it with the LO results. Here LO refers to the anomalous logarithmic term $T \log(1/T)$ that occurs as the first term in the non-Fermi-liquid contribution to the fermion self-energy.

^{*}souvikpriyam.adhya@saha.ac.in

[†]pradipk.roy@saha.ac.in

[‡]abhee.dm@saha.ac.in

Quantities such as mean free path and emissivity calculated with this term are called the LO corrections. Next-to-leading order (NLO) terms include all other terms beyond the LO that contain the fractional powers of T and up to the $(T^3) \log(1/T)$ that occur in the expression of the fermion self-energy. Similarly, quantities calculated with this correction are labeled as NLO corrections [12]. Equipped with these results and knowing the specific heat of dense quark matter up to the order concerned, we investigate the cooling behavior of the neutron star with a dense quark core.

The outline of the paper is as follows. In Sec. II, we describe the formalism, where we start with the quark dispersion relations and the modifications due to NFL effects followed by evaluation of the MFP for the degenerate and nondegenerate neutrinos. These are followed by the calculation of emissivity of the neutrinos. Section III is devoted to the study of the cooling process via neutrino emission. Finally, the results are summarized in Sec. IV followed by our conclusions in Sec. V.

II. FORMALISM

A. Quark dispersion relation

To calculate the quark dispersion relation in degenerate plasma one needs to evaluate the quark self-energy. For this, we consider Fig. 1, where the solid line represents the fermion propagator and the blob implies that the gluon propagator used here is a hard dense loop corrected propagator [14]. Mathematically, the quark self-energy can be written as [3,5,12,13,15]

$$\Sigma(P) = -g^2 C_F T \sum_s \int \frac{d^3 q}{(2\pi)^3} \gamma_\mu S_f(i(\omega_n - \omega_s), \mathbf{p} - \mathbf{q}) \times \gamma_\nu \Delta_{\mu\nu}(i\omega_s, \mathbf{q}), \quad (1)$$

where, $P^\mu = (p^0, \mathbf{p})$, $p_0 = i\omega_n + \mu$, $q_0 = i\omega_s$, $\omega_n = (2n + 1)\pi T$ and $\omega_s = 2\pi s T$ are the Matsubara frequencies for the fermion and the boson, respectively, with integers n and s . After performing the sum over Matsubara frequency in Eq. (1), $i\omega_n + \mu$ is analytically continued to the Minkowski space so that $P^\mu = (E, \mathbf{p})$ [12]. $\Sigma(P)$ can be written as a combination of quasiparticle and antiquasiparticle self-energies as [12]

$$\Sigma(P) = \gamma_0 \Lambda_{\mathbf{p}}^+ \Sigma_+(P) - \gamma_0 \Lambda_{\mathbf{p}}^- \Sigma_-(P), \quad (2)$$

where

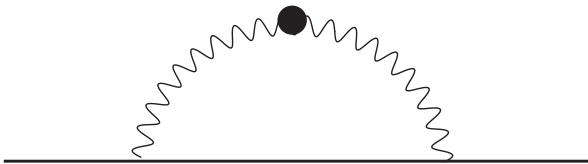


FIG. 1. Fermion self-energy with resummed gluon propagator.

$$\begin{aligned} \gamma_0 S^{-1} &= S_{\pm}^{-1} \Lambda_{\mathbf{p}}^{\pm} + S_{\pm}^{-1} \Lambda_{\mathbf{p}}^{-}, \\ S_{\pm}^{-1} &= -[p^0 \mp (|\mathbf{p}| + \Sigma_{\pm})], \end{aligned} \quad (3)$$

with the energy projection operators given by

$$\Lambda_{\mathbf{p}}^{\pm} = \frac{1}{2}(1 \pm \gamma_0 \gamma^i \hat{p}^i). \quad (4)$$

Interactions within the medium severely modify the on-shell self-energy of the quarks which is manifested in the slope of the dispersion relation for the relativistic degenerate plasma. For quasiparticles with momenta close to the Fermi momentum $p_f(i)$ where i denotes the quark flavor, the one-loop self-energy is dominated by the soft gluon exchanges [5,6]. For the calculation of the MFP and emissivity one needs to know the modified dispersion relation which is determined as [14–16]

$$\omega_{\pm} = \pm(E_{p(\omega_{\pm})} + \text{Re}\Sigma_{\pm}(\omega_{\pm}, p(\omega_{\pm}))), \quad (5)$$

where ω is the quasiparticle/antiquasiparticle energy which is a solution of the dispersion relation and $E_{p(\omega)} = \sqrt{p^2(\omega) + m_q^2}$ is the kinetic energy. As we are considering only quasiparticles, we will consider only ω_+ and denote ω_+ by ω . The above expression will be used to obtain $dp/d\omega$ needed for the phase space evaluation of the mean free path. The authors of [12] have already calculated the fermion self-energy with terms beyond LO. Here we quote the low temperature expansion of the on-shell fermion self-energy for $|\mathbf{p}| = E$ (ultrarelativistic case) [5,12] and notice that no explicit dependence on the spatial momentum \mathbf{p} occurs [15]:

$$\begin{aligned} \Sigma_+(\omega) &= \frac{M_{\infty}^2}{2E} - g^2 C_F m \left\{ \frac{\varepsilon}{12\pi^2 m} \left[\log\left(\frac{4\sqrt{2}m}{\pi\varepsilon}\right) + 1 \right] \right. \\ &\quad + \frac{i\varepsilon}{24\pi m} + \frac{2^{1/3}\sqrt{3}}{45\pi^{7/3}} \left(\frac{\varepsilon}{m}\right)^{5/3} (\text{sgn}(\varepsilon) - \sqrt{3}i) \\ &\quad + \frac{i}{64\sqrt{2}} \left(\frac{\varepsilon}{m}\right)^2 - 20 \frac{2^{2/3}\sqrt{3}}{189\pi^{11/3}} \left(\frac{\varepsilon}{m}\right)^{7/3} (\text{sgn}(\varepsilon) + \sqrt{3}i) \\ &\quad - \frac{6144 - 256\pi^2 + 36\pi^4 - 9\pi^6}{864\pi^6} \left(\frac{\varepsilon}{m}\right)^3 \\ &\quad \left. \times \left[\log\left(\frac{0.928m}{\varepsilon}\right) - \frac{i\pi \text{sgn}(\varepsilon)}{2} \right] + \mathcal{O}\left(\left(\frac{\varepsilon}{m}\right)^{11/3}\right) \right\}, \end{aligned} \quad (6)$$

where $\varepsilon = (\omega - \mu) \sim T$ where NFL effects dominate. The scale of the last logarithm was determined by resumming infrared enhanced contributions. The first term in the above expression gives the hard part contribution to the self-energy where $M_{\infty}^2 = g^2 C_F \mu^2 / (4\pi^2)$ and m is given by $m^2 = N_f g^2 \mu^2 / (4\pi^2)$ and is related to the Debye mass by $m^2 = m_D^2 / 2$. It is interesting to note here that at higher order, fractional powers in ε appear. This can be attributed

to the dynamical screening for the transverse exchange of gauge bosons.

B. MFP of the degenerate neutrinos

The degenerate neutrinos refer to the case where the neutrino chemical potential (μ_ν) is much larger than the temperature. In the interior of a neutron star, there are two distinct phenomena for which the neutrino mean free path is calculated, one is absorption and the other involves scattering of neutrinos [17]. To calculate the MFP for the absorption process we consider the simplest β decay reactions; i.e. the absorption process and its inverse [17–19],

$$d + \nu_e \rightarrow u + e^-, \quad (7)$$

$$u + e^- \rightarrow d + \nu_e. \quad (8)$$

The corresponding mean free paths are denoted by $l_{\text{mean}}^{\text{abs}}$ and $l_{\text{mean}}^{\text{scatt}}$.

The neutrino MFP is related to the total interaction rate due to neutrino emission averaged over the initial quark spins and summed over the final state phase space and spins. It is given by [17]

$$\begin{aligned} \frac{1}{l_{\text{mean}}^{\text{abs}}(E_\nu, T)} &= \frac{g'}{2E_\nu} \int \frac{d^3 p_d}{(2\pi)^3} \frac{1}{2E_d} \int \frac{d^3 p_u}{(2\pi)^3} \frac{1}{2E_u} \\ &\times \int \frac{d^3 p_e}{(2\pi)^3} \frac{1}{2E_e} (2\pi)^4 \\ &\times \delta^4(P_d + P_\nu - P_u - P_e) |M|^2 \\ &\times \{n(p_d)[1 - n(p_u)][1 - n(p_e)] \\ &+ n(p_u)n(p_e)[1 - n(p_d)]\}, \end{aligned} \quad (9)$$

where, g' is the spin and color degeneracy, taken to be 6, and E , p and n_p are the energy, momentum and distribution functions for the corresponding particle. $|M|^2$ is the squared invariant amplitude and is given by $|M|^2 = 64G^2 \cos^2 \theta_c (P_d \cdot P_\nu)(P_u \cdot P_e)$. Here $G \simeq 1.435 \times 10^{-49} \text{ erg-cm}^3$ is the weak coupling constant. Here, we work with the two flavor system as the interaction involving strange quark is Cabibbo suppressed. We now consider the case of degenerate neutrinos i.e. when $\mu_\nu \gg T$. So in this case both the direct Eq. (7) and the inverse Eq. (8) processes can occur. Consequently, the β equilibrium condition becomes $\mu_d + \mu_\nu = \mu_u + \mu_e$. Now, to carry out the momentum integration, $d^3 p_d$ and $d^3 p_u$ can be evaluated as

$$\begin{aligned} d^3 p_d &= 2\pi \frac{p_f(d)}{p_f(\nu)} p d p \frac{d p_d}{d\omega} d\omega; \\ d^3 p_u &= 2\pi \frac{p_f(u)p_f(e)}{p} dE_e \frac{d p_u}{d\omega} d\omega, \end{aligned} \quad (10)$$

where we define $p \equiv |p_d + p_\nu| = |p_u + p_e|$. $dp(\omega)/d\omega$ can be evaluated from the modified dispersion relation as follows [15]:

$$\begin{aligned} \frac{d\omega}{dp(\omega)} &\simeq \frac{dE_{p(\omega)}}{dp(\omega)} + \frac{\partial \text{Re}\Sigma_+(\omega)}{\partial \omega} \frac{d\omega}{dp(\omega)}, \\ \frac{dp(\omega)}{d\omega} &= \left(1 - \frac{\partial \text{Re}\Sigma_+(\omega)}{\partial \omega}\right) \frac{E_{p(\omega)}}{p(\omega)}, \end{aligned} \quad (11)$$

where $\partial \text{Re}\Sigma_+/\partial p \simeq 0$, since p does not appear explicitly in the expression for $\Sigma_+(\omega)$. Neglecting the quark-quark interactions, the leading order result is obtained as

$$\begin{aligned} \frac{1}{l_{\text{mean}}^{\text{abs},D}} \Big|_{\text{LO}} &\simeq \frac{2}{3\pi^5} G_F^2 C_F \cos^2 \theta_c \frac{\mu_e^3}{\mu_\nu^2} \left[1 + \frac{1}{2} \left(\frac{\mu_e}{\mu}\right) + \frac{1}{10} \left(\frac{\mu_e}{\mu}\right)^2\right] \\ &\times [(E_\nu - \mu_\nu)^2 + \pi^2 T^2] (g\mu)^2 \log\left(\frac{4g\mu}{\pi^2 T}\right). \end{aligned} \quad (12)$$

The NLO result is evaluated as

$$\begin{aligned} \frac{1}{l_{\text{mean}}^{\text{abs},D}} \Big|_{\text{NLO}} &\simeq \frac{8}{\pi^3} G_F^2 C_F \cos^2 \theta_c \frac{\mu_e^3}{\mu_\nu^2} \left[1 + \frac{1}{2} \left(\frac{\mu_e}{\mu}\right) + \frac{1}{10} \left(\frac{\mu_e}{\mu}\right)^2\right] \\ &\times [(E_\nu - \mu_\nu)^2 + \pi^2 T^2] \\ &\times \left[a_1 T^{2/3} (g\mu)^{4/3} + a_2 T^{4/3} (g\mu)^{2/3} \right. \\ &\left. + a_3 \left[1 - 3 \log\left(\frac{0.209g\mu}{T}\right)\right] T^2 \right], \end{aligned} \quad (13)$$

where the constants are

$$a_1 = \frac{2^{2/3}}{9\sqrt{3}\pi^{5/3}}, \quad a_2 = -\frac{140 \times 2^{4/3}}{189\sqrt{3}\pi^{7/3}}, \quad (14)$$

and

$$a_3 = \frac{6144 - 256\pi^2 + 36\pi^4 - 9\pi^6}{432\pi^4}. \quad (15)$$

To arrive at the Fermi-liquid result, one can use the free-dispersion relation to arrive at

$$\begin{aligned} \frac{1}{l_{\text{mean}}^{\text{abs},D}} \Big|_{\text{FL}} &= \frac{4}{\pi^3} G_F^2 \cos^2 \theta_c \frac{\mu^2 \mu_e^3}{\mu_\nu^2} \left[1 + \frac{1}{2} \left(\frac{\mu_e}{\mu}\right) + \frac{1}{10} \left(\frac{\mu_e}{\mu}\right)^2\right] \\ &\times [(E_\nu - \mu_\nu)^2 + \pi^2 T^2]. \end{aligned} \quad (16)$$

Since quarks and electrons are assumed to be massless, the chemical equilibrium condition gives $p_f(u) + p_f(e) = p_f(d) + p_f(\nu)$, which we use to derive Eq. (15). We have further assumed that $\mu_d \sim \mu_u = \mu$. Next we calculate the MFP for the quark-neutrino scattering process,

$$q_i + \nu_e(\bar{\nu}_e) \rightarrow q_i + \nu_e(\bar{\nu}_e), \quad (17)$$

for each quark component of flavor i ($= u$ or d). Including the NFL corrections through the phase space and assuming $m_{q_i}/p_{f_i} \ll 1$, we obtain

$$\frac{1}{l_{\text{mean}}^{\text{scatt},D}} \Big|_{\text{FL}} = \frac{3}{4\pi} n_{q_i} G_F^2 \times [(E_\nu - \mu_\nu)^2 + \pi^2 T^2] \Lambda(x_i), \quad (18)$$

$$\frac{1}{l_{\text{mean}}^{\text{scatt},D}} \Big|_{\text{LO}} \simeq \frac{1}{8\pi^3} n_{q_i} C_F G_F^2 [(E_\nu - \mu_\nu)^2 + \pi^2 T^2] \times \Lambda(x_i) g^2 \log\left(\frac{4g\mu}{\pi^2 T}\right), \quad (19)$$

$$\frac{1}{l_{\text{mean}}^{\text{scatt},D}} \Big|_{\text{NLO}} \simeq \frac{3}{2\pi} n_{q_i} C_F G_F^2 [(E_\nu - \mu_\nu)^2 + \pi^2 T^2] \Lambda(x_i) \times \left[a_1 g^{4/3} \left(\frac{T}{\mu}\right)^{2/3} + a_2 g^{2/3} \left(\frac{T}{\mu}\right)^{4/3} + a_3 \left\{ 1 - 3 \log\left(\frac{0.209g\mu}{T}\right) \right\} \left(\frac{T}{\mu}\right)^2 \right], \quad (20)$$

where n_{q_i} is the number density of quark of flavor i , given by

$$n_{q_i} = 6 \int \frac{d^3 p}{(2\pi)^3} \frac{1}{e^{\beta(E_{q_i} - \mu_{q_i})}}, \quad (21)$$

where 6 is the quark degeneracy factor. m_{q_i} is the mass of the quark, $\sigma_0 \equiv 4G_F^2 m_e^2 / \pi$ [20,21] and $\Lambda(x_i)$ is defined in [11] where $x_i = \mu_\nu / \mu_{q_i}$ if $\mu_\nu < \mu_{q_i}$ and $x_i = \mu_{q_i} / \mu_\nu$ if $\mu_\nu > \mu_{q_i}$. The contributions from the Fermi liquid, LO and NLO are added to obtain the MFP for the corresponding process. Further, one can combine $l_{\text{mean}}^{\text{scatt}}$ with $l_{\text{mean}}^{\text{abs}}$ to define total mean free path as [22]

$$\frac{1}{l_{\text{mean}}^{\text{total}}} = \frac{1}{l_{\text{mean}}^{\text{abs}}} + \frac{1}{l_{\text{mean}}^{\text{scatt}}}. \quad (22)$$

In addition to the inclusion of the NLO terms, the Fermi-liquid term and the LO term agree with that of [11].

C. Mean free path of nondegenerate neutrinos

We now derive MFP for nondegenerate neutrinos i.e. when $\mu_\nu \ll T$ beyond the Fermi-liquid contribution. For nondegenerate neutrinos the inverse process (8) is dropped. We are considering only depopulation of neutrinos as in the nondegenerate (untrapped) case of repopulation or when the reverse reaction is assumed to be zero. Hence, we neglect the second term in the curly braces of Eq. (9) [17,18]. For free quarks, the matrix element vanishes [17,23,24], since u , d quarks and electrons are collinear in momenta. The inclusion of strong interactions between quarks relaxes these kinematic restrictions resulting in a nonvanishing squared matrix amplitude. We can neglect the neutrino momentum in an energy conserving relation due to the thermal production of the neutrinos [17]. Following the procedure described in [15,25,26] the MFP for the Fermi-liquid case, LO and NLO are obtained as

$$\frac{1}{l_{\text{mean}}^{\text{abs},ND}} \Big|_{\text{FL}} = \frac{3C_F \alpha_s}{\pi^4} G_F^2 \cos^2 \theta_c \mu_d \mu_u \mu_e \frac{(E_\nu^2 + \pi^2 T^2)}{(1 + e^{-\beta E_\nu})}, \quad (23)$$

$$\frac{1}{l_{\text{mean}}^{\text{abs},ND}} \Big|_{\text{LO}} \simeq \frac{C_F^2 \alpha_s}{2\pi^6} G_F^2 \cos^2 \theta_c \mu_e \frac{(E_\nu^2 + \pi^2 T^2)}{(1 + e^{-\beta E_\nu})} (g\mu)^2 \times \log\left(\frac{4g\mu}{\pi^2 T}\right), \quad (24)$$

$$\frac{1}{l_{\text{mean}}^{\text{abs},ND}} \Big|_{\text{NLO}} \simeq \frac{3C_F^2 \alpha_s}{\pi^4} G_F^2 \cos^2 \theta_c \mu^2 \mu_e \frac{(E_\nu^2 + \pi^2 T^2)}{(1 + e^{-\beta E_\nu})} \times \left[b_1 g^{4/3} \left(\frac{T}{\mu}\right)^{2/3} + b_2 g^{2/3} \left(\frac{T}{\mu}\right)^{4/3} + b_3 \left\{ 1 - 3 \log\left(\frac{0.209g\mu}{T}\right) \right\} \left(\frac{T}{\mu}\right)^2 \right], \quad (25)$$

where the constants are evaluated as

$$b_1 = \frac{2^{5/3}}{9\sqrt{3}\pi^{5/3}}; \quad b_2 = -\frac{280 \times 2^{4/3}}{189\sqrt{3}\pi^{7/3}} \quad (26)$$

and

$$b_3 = \frac{6144 - 256\pi^2 + 36\pi^4 - 9\pi^6}{216\pi^4}. \quad (27)$$

Similarly, for the scattering of nondegenerate neutrinos in quark matter with appropriate phase space corrections, we obtain

$$\frac{1}{l_{\text{mean}}^{\text{scatt},ND}} \Big|_{\text{FL}} = \frac{C_{V_i}^2 + C_{A_i}^2}{5\pi} n_{q_i} G_F^2 \frac{E_\nu^3}{\mu}, \quad (28)$$

$$\frac{1}{l_{\text{mean}}^{\text{scatt},ND}} \Big|_{\text{LO}} \simeq \frac{C_{V_i}^2 + C_{A_i}^2}{30\pi^3} n_{q_i} G_F^2 C_F \frac{E_\nu^3}{\mu} g^2 \log\left(\frac{4g\mu}{\pi^2 T}\right), \quad (29)$$

$$\frac{1}{l_{\text{mean}}^{\text{scatt},ND}} \Big|_{\text{NLO}} \simeq (C_{V_i}^2 + C_{A_i}^2) n_{q_i} G_F^2 C_F \times \left[b'_1 \frac{T^{2/3} g^{4/3}}{\mu^{5/3}} + b'_2 \frac{T^{4/3} g^{2/3}}{\mu^{7/3}} + b'_3 \left\{ 1 - 3 \log\left(\frac{0.209g\mu}{T}\right) \right\} \left(\frac{T^2}{\mu^3}\right) \right], \quad (30)$$

where the constants are

$$b'_1 = \frac{2^{5/3}}{45\sqrt{3}\pi^{8/3}}; \quad b'_2 = -\frac{56 \times 2^{4/3}}{189\sqrt{3}\pi^{10/3}}, \quad (31)$$

and

$$b'_3 = \frac{6144 - 256\pi^2 + 36\pi^4 - 9\pi^6}{1080\pi^5}. \quad (32)$$

Here, we have assumed $m_{q_i} / p_{f_i} \ll 1$. Thus, the total MFP for nondegenerate neutrinos is obtained by summing up the contributions from the absorption and scattering parts to get the expression of the MFP of the nondegenerate neutrinos up to the NLO terms. So, we get the expression of the MFP of the nondegenerate neutrinos up to the NLO terms.

D. Emissivity of nondegenerate neutrinos

The total emissivity of the nondegenerate neutrinos is obtained by multiplying the neutrino energy with the inverse of the MFP with appropriate factors and integrating over the neutrino momentum. The relation between neutrino emissivity and the neutrino mean free path is thus obtained as [27]

$$\varepsilon = \int \frac{d^3 p_\nu}{(2\pi)^3} E_\nu \frac{1}{l(-E_\nu, T)}. \quad (33)$$

Using the mean free path for the nondegenerate neutrinos, we obtain

$$\varepsilon - \varepsilon_0 = \varepsilon_{\text{LO}} + \varepsilon_{\text{NLO}}, \quad (34)$$

where

$$\varepsilon_0 \simeq \frac{457}{630} G_F^2 \cos^2 \theta_c \alpha_s \mu_e T^6 \mu^2 \quad (35)$$

is the usual Fermi-liquid contribution which agrees with the result presented in Ref. [17]. At the LO we have obtained

$$\varepsilon_{\text{LO}} \simeq \frac{457}{3780} G_F^2 \cos^2 \theta_c C_F \alpha_s \mu_e T^6 \frac{(g\mu)^2}{\pi^2} \ln\left(\frac{4g\mu}{\pi^2 T}\right), \quad (36)$$

which is in agreement with the result quoted in Ref. [9]. Now, following the procedure in [9,11], we obtain the NLO contribution to the neutrino emissivity as

$$\varepsilon_{\text{NLO}} \simeq \frac{457}{315} G_F^2 \cos^2 \theta_c C_F \alpha_s \mu_e T^6 \left[c_1 T^2 + c_2 T^{2/3} (g\mu)^{4/3} - c_3 T^{4/3} (g\mu)^{2/3} - c_4 T^2 \ln\left(\frac{0.656g\mu}{\pi T}\right) \right], \quad (37)$$

where the constants are evaluated as

$$\begin{aligned} c_1 &= -0.0036\pi^2; & c_2 &= \frac{2^{2/3}}{9\sqrt{3}\pi^{5/3}}; \\ c_3 &= \frac{40 \times 2^{1/3}}{27\sqrt{3}\pi^{7/3}}; \end{aligned} \quad (38)$$

$$\begin{aligned} \frac{C_v - C_v^0}{N_g} &= \frac{g_{\text{eff}}^2 \mu^2 T}{36\pi^2} \left(\ln\left(\frac{4g_{\text{eff}}\mu}{\pi^2 T}\right) + \gamma_E - \frac{6}{\pi^2} \zeta'(2) - 3 \right) - 40 \frac{2^{2/3} \Gamma(\frac{8}{3}) \zeta(\frac{8}{3})}{27\sqrt{3}\pi^{11/3}} T^{5/3} (g_{\text{eff}}\mu)^{4/3} \\ &+ 560 \frac{2^{1/3} \Gamma(\frac{10}{3}) \zeta(\frac{10}{3})}{81\sqrt{3}\pi^{13/3}} T^{7/3} (g_{\text{eff}}\mu)^{2/3} + \frac{2048 - 256\pi^2 - 36\pi^4 + 3\pi^6}{180\pi^2} T^3 \left[\ln\left(\frac{g_{\text{eff}}\mu}{T}\right) + \bar{c} - \frac{7}{12} \right] \\ &+ O(T^{11/3} / (g_{\text{eff}}\mu)^{2/3}) + O(g^4 \mu^2 T \ln T), \end{aligned} \quad (41)$$

where the coupling constant g is related to g_{eff} as

$$g^2 = \frac{2g_{\text{eff}}^2}{N_f}, \quad (42)$$

and N_f is the number of quark flavors. The contribution from the free part is given as

and

$$c_4 = \frac{6144 - 256\pi^2 + 36\pi^4 - 9\pi^6}{144\pi^4}. \quad (39)$$

The NFL correction only appears in the phase space integral of the MFP [11] and subsequently in the expression of the emissivity [9]. It is actually related to the unscreened transverse interaction [5]. The factor of T^6 can be understood easily. Naively, we can see that one power of T is obtained from the phase space integral of a degenerate fermion. Further, we obtain a T^3 from the phase space integral of the neutrino. One power of T from the energy conserving δ function is cancelled by a power from the emitted neutrino energy. It is to be noted that for excitations near the Fermi surface, the angular integrals give no temperature dependence. Now putting the standard values [9,11] for the parameters, the corrections can be compared with the results given in [9].

III. COOLING PROCESS VIA NEUTRINO EMISSION

The temperature of the neutron star with a quark matter core shows a dependency with time. To analyze the cooling of the star [28–30], the specific heat capacity of the quark matter core needs to be taken into consideration along with the emissivity via the cooling process [17,31],

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial T} \frac{\partial T}{\partial t} = c_v(T) \frac{\partial T}{\partial t} = -\varepsilon(T), \quad (40)$$

where u is the internal energy, t is time and we have assumed that there is no surface emission. The NFL effects on the specific heat capacity of the degenerate quark matter were calculated recently in [13]. The specific heat capacity for a non-color-superconducting degenerate quark matter is given as [13]

$$C_v^0 = NN_f \frac{\mu^2 T}{3}. \quad (43)$$

Using the above expression for specific heat and the emissivity expression up to NLO, we analyze the cooling behavior of the neutron star.

IV. RESULTS

An estimation of the MFP of neutrinos with the temperature is been presented in this section. For this purpose, we have assumed the quark chemical potential to be 500 MeV. This is in good agreement with the high density

$\sim 6\rho_0$ (ρ_0 being nuclear matter saturation density) at the core. The chemical potential may well have a time dependence. This would gain significance for terms beyond order T^3 since $T/(g\mu) \ll 1$. We have also taken $\mu_e = 15$ MeV and $\alpha_s = 0.1$. In the left-hand panel of Fig. 2 we note there is a considerable decrease in MFP of degenerate neutrinos

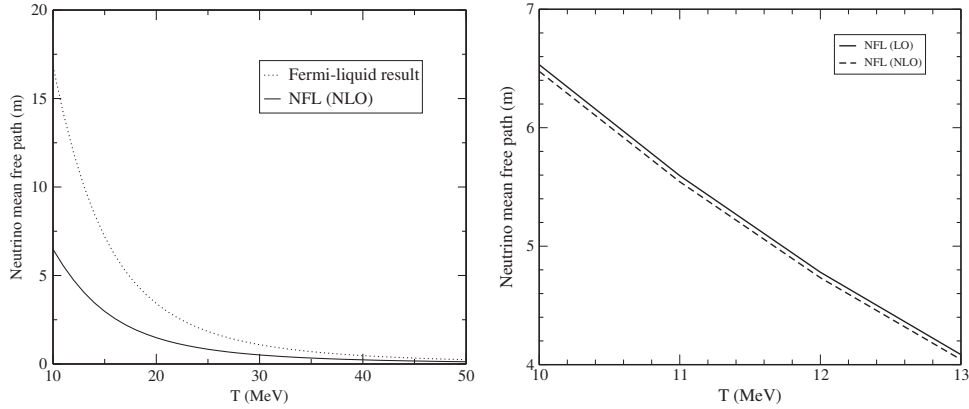


FIG. 2. Mean free path of degenerate neutrinos. The left-hand panel shows a comparison between the Fermi-liquid result and NLO corrections for the non-Fermi-liquid effects. The right-hand panel shows the reduction of the MFP due to NLO corrections.

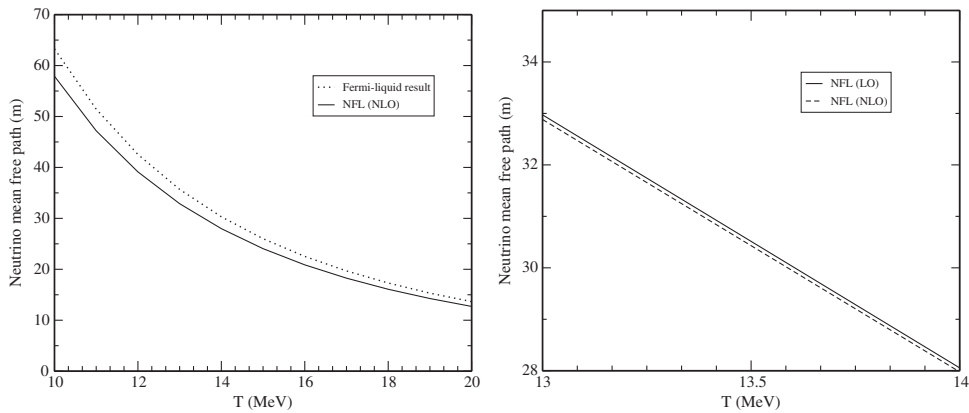


FIG. 3. Same as Fig. 2 for nondegenerate neutrinos.

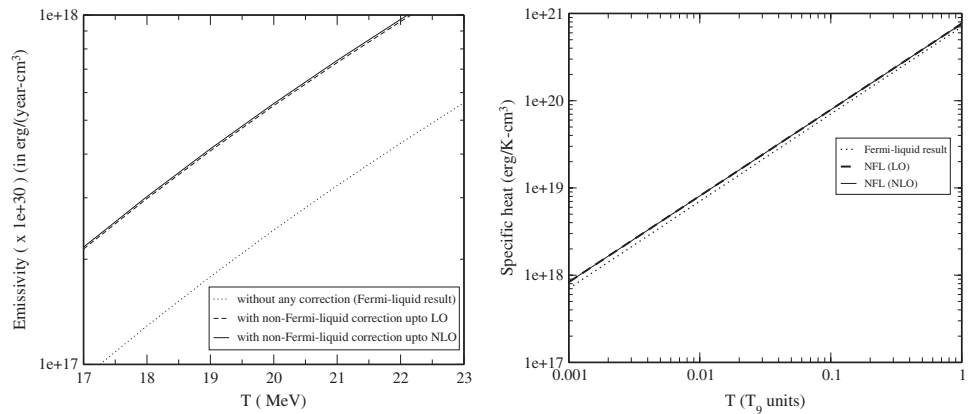


FIG. 4. The left-hand panel shows the emissivity of the neutrinos with temperature in degenerate quark matter. The right-hand panel shows the behavior of the specific heat of the degenerate quark matter with temperature (T_9 in units of 10^9 K).

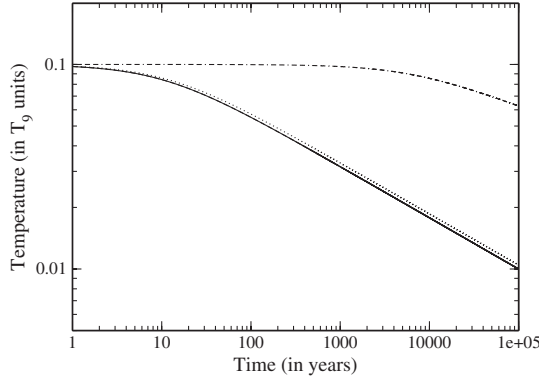


FIG. 5. The cooling behavior of a neutron star with core as neutron matter and degenerate quark matter. The dotted line represents the Fermi-liquid result, the solid line represents the non-Fermi NLO correction. The dash-dotted line gives the cooling behavior of the neutron star core made up of purely neutron matter.

due to NLO corrections over the Fermi-liquid result. In the right-hand panel of Fig. 2 the LO correction is compared with the NLO correction and it is seen that the MFP with NLO correction is reduced marginally as compared to LO correction. Similar features have been observed in the case of the MFP of nondegenerate neutrinos as displayed in Fig. 3. This marginal difference in the MFP between NFL LO and NLO corrections also leads to a marginal difference in the emissivity for the two cases. This is shown in Fig. 4. These small reductions are reflected in the marginally enhanced emissivity of the nondegenerate neutrinos which has been shown in the left-hand panel of Fig. 4. We find that there is a modest increase in the emissivity of the

neutrinos. The right-hand panel of Fig. 4 gives a comparison of the NFL corrections to the specific heat already reported in [13]. The complicated cooling equation cannot be solved analytically and we have resorted to numerical calculation. We observe that the cooling of a neutron star is marginally faster in the case of NFL (NLO) as compared to the Fermi-liquid result (shown in Fig. 5).

V. SUMMARY AND DISCUSSIONS

In this work, we calculated the MFP of degenerate and nondegenerate neutrinos for both the scattering and the absorption processes. We then found the expression for neutrino emissivity for nondegenerate neutrinos with NLO corrections. It is seen that both MFP and emissivity contain terms at the higher order which involve fractional powers in (T/μ) . We have found that there is a decrease in the MFP due to NLO corrections. We reconfirm that the leading order correction to the quantities like MFP or emissivity are significant compared to the Fermi-liquid results. The NLO corrections, which we derive here, have, however, been found to be numerically close to the LO results. We have also examined the cooling behavior of a neutron star by incorporating NLO correction to the specific heat and emissivity which affect the results considerably compared to the simple Fermi-liquid case.

ACKNOWLEDGMENTS

One of the authors (S.P.A.) would like to thank Sreemoyee Sarkar and Mahatsab Mandal for useful discussions regarding different aspects of the paper and would like to thank UGC, India (Serial No. 2120951147) for providing the fellowship during the tenure of this work.

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