PHYSICAL REVIEW D **86,** 033012 (2012)

Getting the best out of T2K and NOνA

Suprabh Prakash,[1,*] Sushant K. Raut,[1,†] and S. Uma Sankar[1,2,‡]
[1]*Department of Physics, Indian Institute of Technology Bombay, Mumbai 400076, India*
[2]*Department of Theoretical Physics, Tata Institute of Fundamental Research, Mumbai 400005, India*
(Received 20 March 2012; published 27 August 2012)

We explore the combined physics potential of T2K and NOνA in light of the moderately large measured value of θ_{13}. For $\sin^2 2\theta_{13} = 0.1$, which is close to the best fit value, a 90% C.L. evidence for the hierarchy can be obtained only for the combinations (Normal hierarchy, $-170° \le \delta_{CP} \le 0°$) and (Inverted hierarchy, $0° \le \delta_{CP} \le 170°$), with the currently planned runs of NOνA and T2K. However, the hierarchy can essentially be determined for any value of δ_{CP}, if the statistics of NOνA are increased by 50% and those of T2K are doubled. Such an increase will also give an allowed region of δ_{CP} around its true value, except for the *CP* conserving cases $\delta_{CP} = 0$ or $\pm 180°$. We demonstrate that any measurement of δ_{CP} is not possible without first determining the hierarchy. We find that comparable data from a shorter baseline ($L \sim 130$ km) experiment will not lead to any significant improvement.

DOI: 10.1103/PhysRevD.86.033012 PACS numbers: 14.60.Pq, 13.15.+g, 14.60.Lm

I. INTRODUCTION

Neutrino physics has entered a phase of precision measurements. During the past few years, the following precise measurements of neutrino parameters have been made with high intensity sources:

(i) The smaller mass-squared difference $\Delta_{21} = m_2^2 - m_1^2$ is measured by KamLAND [1] while the precision on θ_{12} is controlled by the solar experiments [2]. Global analysis of all the data, in the three flavor oscillation framework, gives $\Delta_{21} = (7.6 \pm 0.2) \times 10^{-5}$ eV^2 and $\sin^2\theta_{12} = 0.312 \pm 0.016$ [3].

(ii) MINOS [4] experiment has measured the magnitude of the mass-squared difference in the ν_μ survival probability. The precision on θ_{23} is controlled by atmospheric neutrino data [5]. Global analysis gives two distinct values of Δ_{31} depending on whether it is positive [which is the case for normal hierarchy (NH)] or negative [which is the case for inverted hierarchy (IH)]. The ranges are $\Delta_{31}(\mathrm{NH}) = (2.45 \pm 0.09) \times 10^{-3}$ eV^2 and $\Delta_{31}(\mathrm{IH}) = (-2.31 \pm 0.09) \times 10^{-3}$ eV^2 with $\sin^2\theta_{23} = 0.51 \pm 0.06$ for both cases [3].

(iii) The global fits to data from the accelerator experiments T2K [6] and MINOS [7] and the reactor experiments DChooz [8], Daya Bay [9] and RENO [10] have determined θ_{13} to be nonzero at 5σ level, with the best fit very close to $\sin^2 2\theta_{13} \simeq 0.1$ [11,12].

We expect the following improvements in precision during the next few years:

(i) Very high statistics data from T2K [13] and MINOS [4] experiments will improve the precision on $|\Delta_{31}|$ and $\sin^2 2\theta_{23}$ to a few percent level.

(ii) Reactor experiments are taking further data [14–17]. The survival probability at these reactor experiments is sensitive only to the mixing angle θ_{13} and hence they can measure this angle unambiguously. By the time they finish running (around 2016), we estimate that they should be able to measure $\sin^2 2\theta_{13}$ to a precision of about 0.005.

In light of these current and expected near future measurements, the next goals of neutrino oscillation experiments are the determination of neutrino mass hierarchy, detection of *CP* violation in the leptonic sector and measurement of δ_{CP}. These goals can be achieved by high statistics accelerator experiments measuring $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillation probabilities. Among such experiments, T2K is presently taking data and NOνA is under construction and is expected to start taking data around 2014. All other experiments, capable of making these measurements, are far off in the future. In this paper, we study the combined ability of T2K and NOνA to achieve the above goals.

In the above discussion, we have two different magnitudes for Δ_{31} for the two hierarchies because the mass-squared difference measured in ν_μ survival probability is not Δ_{31} but is an effective one defined by [18,19]

$$\Delta m^2_{\mu\mu} = \Delta_{31} - (\cos^2\theta_{12} - \cos\delta_{CP}\sin\theta_{13}\sin 2\theta_{12}\tan\theta_{23})\Delta_{21}. \quad (1)$$

Accelerator experiments, such as MINOS and T2K, measure the magnitude of the above quantity. But the magnitudes of Δ_{31} will turn out to be different for Δ_{31} positive (NH) and Δ_{31} negative (IH).

*suprabh@phy.iitb.ac.in
†sushant@phy.iitb.ac.in
‡uma@phy.iitb.ac.in

II. SIMULATION DETAILS

Before discussing various physics issues, we discuss the details of our simulation. We do this because we will illustrate various points through the means of simulation.

We use the software GLoBES [20,21] for simulating the data of T2K, NOνA and an envisaged short baseline experiment from CERN to Fréjus (C2F), which is a scaled down version of MEMPHYS [13,22–32]. Various details of these experiments and their characteristics, especially the signal and background acceptances, are given in Table I. The basic properties of NOνA are taken from Ref. [24] and for T2K taken from Ref. [13]. The efficiencies for each of the experiments are taken from GLoBES [20,21]. The background errors consist of errors in flux normalization (norm) and in spectrum (tilt).

We have kept the solar parameters Δ_{21} and θ_{12} fixed at their best fit values throughout the calculation. We have taken the central values of $|\Delta_{31}|$ and θ_{23} to be their best fit values. We took $\sigma(\sin^2 2\theta_{23}) = 0.02$ and $\sigma(|\Delta_{31}|) = 0.03 \times (|\Delta_{31}|)$, because of the precision expected from T2K. We have done computations for various different values of $\sin^2 2\theta_{13}$ in the range 0.05–0.2 [11,12]. We took $\sigma(\sin^2 2\theta_{13}) = 0.005$ which is the final precision we can hope for from the reactor experiments. The value of the CP-violating phase δ_{CP} is varied over its entire range $-180°$ to $180°$.

We compute statistical $\chi^2_{\rm st}$ as

$$\chi^2_{\rm st} = \sum_i \frac{(N_i^{\rm true} - N_i^{\rm test})^2}{N_i^{\rm true}}, \qquad (2)$$

where $N_i^{\rm true}$ is the event distribution for true hierarchy and some fixed true value of δ_{CP}. $N_i^{\rm test}$ is the event distribution with the test hierarchy either true or wrong and a varying test value of δ_{CP} as inputs. The index i runs over the number of energy bins. The final χ^2 is computed including the systematic errors, described in Table I, and the priors on $|\Delta_{31}|$, $\sin^2 2\theta_{23}$ and $\sin^2 2\theta_{13}$. The prior on $\sin^2 2\theta_{13}$ effectively takes into account the data due to reactor neutrino experiments

In the following we consider two kinds of plots both of which are shown as contours in the $\sin^2 2\theta_{13}$-δ_{CP} plane.

(i) Hierarchy exclusion plots: These are plotted in the plane of true values of $\sin^2 2\theta_{13}$-δ_{CP}. The contours in these plots define the line $\chi^2 = 2.71$. In computing this χ^2, we have marginalized over the parameter ranges described above. For all sets of parameter values to the right of the contour, the wrong hierarchy can be ruled out at 90% C.L.

(ii) Allowed region plots: These are plotted in the plane of test values of $\sin^2 2\theta_{13}$-δ_{CP}. The contours in these plots are defined by $\chi^2 = 4.61$. The region enclosed by them is the set of allowed values of $\sin^2 2\theta_{13}$-δ_{CP} at 90% C.L. for a given set of neutrino parameters.

Throughout this paper, the phrase "hierarchy determination" implies 90% C.L. evidence for hierarchy.

III. HIERARCHY DETERMINATION WITH $P_{\mu e}$

The $\nu_\mu \to \nu_e$ channel is sensitive to a number of neutrino parameters and hence is the most sought after in

TABLE I. Properties of various long baseline experiments considered in this paper.

Characteristic	NOνA	T2K	C2F (assumed)
Baseline	812 km	295 km	130 km
Location	Fermilab—Ash River	J-PARC—Kamioka	CERN—Fréjus
Beam	NuMI beam 0.8° off—axis	JHF beam 2.5° off—axis	SPL superbeam
Beam power	0.7 MW	0.75 MW	0.75 MW
Flux peaks at	2 GeV	0.6 GeV	0.35 GeV
$P_{\mu e}$ 1st oscillation maximum	1.5 GeV	0.55 GeV	0.25 GeV
Detector	TASD, 15 kton	Water Čerenkov, 22.5 kton	Water Čerenkov, 22.5 kton
Runtime (years)	3 in ν + 3 in $\bar{\nu}$	5 in ν	3 in ν + 3 in $\bar{\nu}$
Signal 1 (acceptance)	ν_e appearance(26%)	ν_e appearance(87%)	ν_e appearance(71%)
Signal 1 error (norm.,tilt)	5%, 2.5%	2%, 1%	2%, 0.01%
Background 1 (acceptance)	mis—id muons/anti—muons(0.13%), NC events (0.28%), Beam $\nu_e/\bar{\nu}_e$ (16%)	mis—id muons/anti—muons, NC events, Beam $\nu_e/\bar{\nu}_e$ (binned events from GLoBES [20,21])	mis—id muons/anti—muons (0.054%), NC events (0.065%), Beam $\nu_e/\bar{\nu}_e$(70%)
Background 1 error (norm.,tilt)	10%, 2.5%	20%, 5%	2%, 0.01%
Signal 2 (acceptance)	$\bar{\nu}_e$ appearance (41%)	$\bar{\nu}_e$ appearance(87%)	$\bar{\nu}_e$ appearance(68%)
Signal 2 error (norm.,tilt)	5%, 2.5%	2%, 1%	2%, 0.01%
Background 2 (acceptance)	mis—id muons/anti—muons (0.13%), NC events (0.88%), Beam $\nu_e/\bar{\nu}_e$(33.6%)	mis—id muons/anti—muons, NC events, Beam $\nu_e/\bar{\nu}_e$ (binned events from GLoBES [20,21])	mis—id muons/anti—muons (0.054%), NC events (0.25%), Beam $\nu_e/\bar{\nu}_e$ (70%)
Background 2 error (norm.,tilt)	10%, 2.5%	20%, 5%	2%, 0.01%

the study of neutrino oscillation physics using long baseline experiments. In the presence of matter, the $\nu_\mu \to \nu_e$ oscillation probability, expanded perturbatively in the small mass-squared difference, Δ_{21} is given by [33–35]

$$\begin{aligned} P(\nu_\mu \to \nu_e) &= P_{\mu e} \\ &= \sin^2 2\theta_{13} \sin^2\theta_{23} \frac{\sin^2 \hat{\Delta}(1-\hat{A})}{(1-\hat{A})^2} \\ &\quad + \alpha \cos\theta_{13} \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23} \\ &\quad \times \cos(\hat{\Delta} + \delta_{CP}) \frac{\sin \hat{\Delta}\hat{A}}{\hat{A}} \frac{\sin\hat{\Delta}(1-\hat{A})}{1-\hat{A}} \\ &\quad + \alpha^2 \sin^2 2\theta_{12} \cos^2\theta_{13} \cos^2\theta_{23} \frac{\sin^2\hat{\Delta}\hat{A}}{\hat{A}^2}, \end{aligned} \tag{3}$$

where $\hat{\Delta} = \Delta_{31}L/4E$, $\hat{A} = A/\Delta_{31}$, $\alpha = \Delta_{21}/\Delta_{31}$. A is the Wolfenstein matter term [36] and is given by $A(\mathrm{eV}^2) = 0.76 \times 10^{-4} \rho(\mathrm{gm/cc}) E(\mathrm{GeV})$.

For NH Δ_{31} is positive and for IH Δ_{31} is negative. The matter term A is positive for neutrinos and is negative for anti-neutrinos. Hence, in neutrino oscillation probability, $\hat{A}$ is positive for NH and is negative for IH. For antineutrinos, $\hat{A}$ is negative for NH and positive for IH and the sign of δ_{CP} is reversed. The presence of the term $\hat{A}$ in $P_{\mu e}$ and in $P_{\bar{\mu}\bar{e}}$ makes them sensitive to hierarchy. The longer the baseline of an experiment, the greater is the sensitivity to hierarchy because $P_{\mu e}$ peaks at a higher energy for longer baseline and the matter term is larger for higher energies.

As can be seen from Eq. (3), $P_{\mu e}$ is dependent on θ_{13}, hierarchy and δ_{CP} in addition to other well-determined parameters. A measurement of this quantity will not give us a unique solution of neutrino parameters but instead will lead to a number of degenerate solutions [37–40]. Since θ_{13} is measured unambiguously and precisely [8–10], degeneracies involving this parameter are no longer relevant. Only hierarchy-δ_{CP} degeneracy has to be considered. This degeneracy prevents any one experiment from determining hierarchy and δ_{CP}, leading to the need for data from two or more long baseline experiments [22,41–44].

A. Hierarchy-δ_{CP} degeneracy for NOνA

First we consider the hierarchy determination capacity of NOνA alone because the matter term and the hierarchy dependence is the largest for this experiment, due to the flux peaking at higher energy. In Fig. 1 (left panel), we have plotted $P_{\mu e}$ vs E for both NH and for IH for NOνA baseline of 812 km. The bands correspond to the variation of δ_{CP} from $-180°$ to $+180°$. The values of $P_{\mu e}$ are, in general, higher for NH and lower for IH. This is a straightforward consequence of the $\hat{A}$ dependence of $P_{\mu e}$. Further, we note that for both NH and IH, the value of $\delta_{CP} = +90°$ gives the lowest curve in the band and the value of $\delta_{CP} = -90°$ gives the highest curve in the band. This behavior can also be easily understood from Eq. (3). At the oscillation maximum, $\hat{\Delta} \simeq 90°$. Hence $\cos(\hat{\Delta} + \delta_{CP})$ is $+1$ for $\delta_{CP} = -90°$ and is -1 for $\delta_{CP} = +90°$. As can be seen from the figure, there is an overlap of the bands for (NH, $\delta_{CP} \simeq +90°$) and (IH, $\delta_{CP} \simeq -90°$). Hence, if the measured probability comes to be these values, then we have two degenerate solutions. In Fig. 1 (right panel), we have plotted the corresponding antineutrino probabilities. $P_{\bar{\mu}\bar{e}}$ is higher for IH and lower for NH as a consequence of the reversal of $\hat{A}$ sign. Since δ_{CP} sign is reversed for antineutrinos, here $\delta_{CP} = +90°$ defines the upper curves and $\delta_{CP} = -90°$ defines the lower curves. Here again there is an overlap between (NH, $\delta_{CP} \simeq +90°$) and (IH, $\delta_{CP} \simeq -90°$) so we get the same degenerate solutions as the neutrino case.

From Fig. 1, we can define the concept of favorable half plane for each hierarchy. Suppose NH is the true hierarchy. If δ_{CP} is in the lower half-plane (LHP) ($-180° \le \delta_{CP} \le 0°$, LHP) then all the curves for $P_{\mu e}(\mathrm{NH}, \delta_{CP})$ lie much above the set of curves for $P_{\mu e}(\mathrm{IH}, \delta_{CP})$. In the case of antineutrinos, $P_{\bar{\mu}\bar{e}}(\mathrm{NH}, \delta_{CP})$ will be much below $P_{\bar{\mu}\bar{e}}(\mathrm{IH}, \delta_{CP})$. In such a situation, the data from NOνA

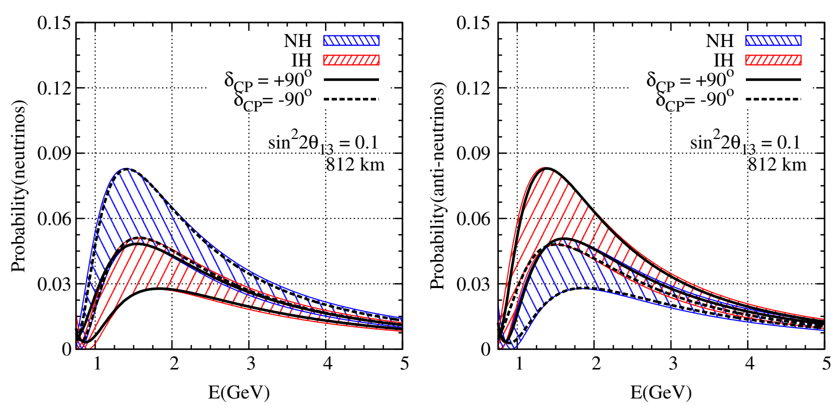

FIG. 1 (color online). $P_{\mu e}$ (left panel) and $P_{\bar{\mu}\bar{e}}$ (right panel) bands for NOνA for $\sin^2 2\theta_{13} = 0.1$

alone can determine the hierarchy. Therefore we call the LHP to be the favorable half-plane for NH. Similar arguments hold if IH is true hierarchy and δ_{CP} is in the upper half-plane (UHP). So UHP is the favorable half-plane for IH. Thus, if nature chooses one of the following two combinations (NH, LHP) or (IH, UHP), then NOνA, by itself, can determine the hierarchy.

The separation between the set of curves $P_{\mu e}(\text{NH}, \delta_{CP})$ and $P_{\mu e}(\text{IH}, \delta_{CP})$ also depends on θ_{13}. The two sets have more overlap for smaller values of θ_{13} but become more separated for larger values of θ_{13}. This is illustrated in Fig. 2, showing $P_{\mu e}$ vs E, for a lower and higher value of $\sin^2 2\theta_{13}$. It is easier to determine the hierarchy if the separation between the curves is larger, that is if θ_{13} is larger.

The favorable and unfavorable half-planes for a particular hierarchy can also be defined from Eq. (3), where the δ_{CP} dependence occurs purely in the form $\cos(\hat{\Delta} + \delta_{CP})$. If NH is the true hierarchy, $\hat{\Delta} \approx 90°$ around the probability maximum. Then, the δ_{CP} dependent term increases $P_{\mu e}$ if δ_{CP} is in the LHP and decreases it if δ_{CP} is in the UHP. Hence a cleaner separation from $P_{\mu e}(\text{IH}, \delta_{CP})$ can be obtained only if δ_{CP} is in the LHP. If IH is the true hierarchy, $\hat{\Delta} \approx -90°$. Then $P_{\mu e}$ is reduced and moved away from $P_{\mu e}(\text{NH}, \delta_{CP})$ if δ_{CP} is in the UHP. Thus UHP forms the favorable half-plane for IH, whereas LHP is the favorable half-plane for NH. Even if we use the antineutrino oscillation probabilities, the same considerations will hold. Therefore, the same relation between hierarchy and half-plane holds for both neutrino and antineutrino data.

We plot the hierarchy discrimination ability of NOνA in Fig. 3. We see that, for $\sin^2 2\theta_{13} = 0.1$, the hierarchy can be determined at 90% C.L. for the following two combinations: (NH, $-170° \leq \delta_{CP} \leq -10°$) or (IH, $10° \leq \delta_{CP} \leq 170°$). The statistics for the experiment are not quite enough to determine the hierarchy for the whole favorable half-plane for this value of θ_{13}. If $\sin^2 2\theta_{13} = 0.12$, then the hierarchy can be determined for the whole favored half-plane. It was shown in Ref. [45] that NOνA can determine the hierarchy for 45% of the δ_{CP} range for $\sin^2 2\theta_{13} = 0.1$.

For smaller values of $\sin^2 2\theta_{13}$, one needs larger statistics to determine the hierarchy for the whole favorable

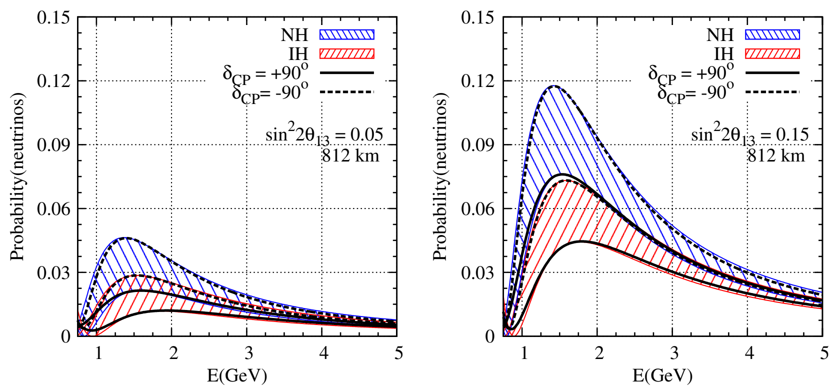

FIG. 2 (color online). $P_{\mu e}$ bands for NOνA for $\sin^2 2\theta_{13} = 0.05$ (left panel) and 0.15 (right panel)

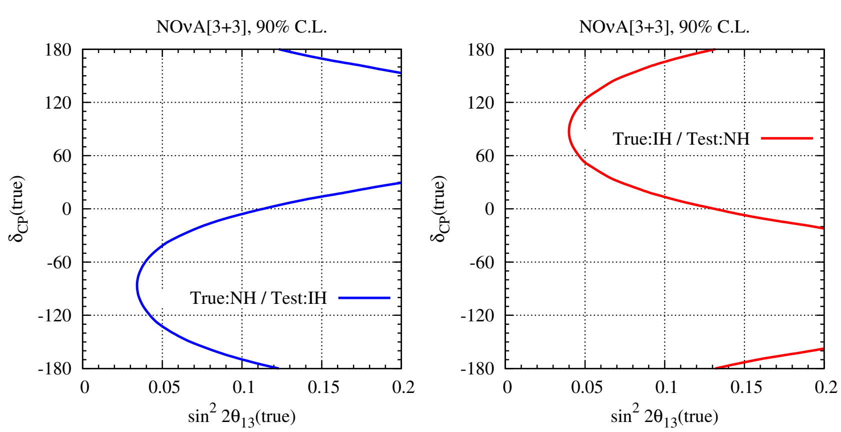

FIG. 3 (color online). Hierarchy exclusion plots for NOνA for $3\nu + 3\bar{\nu}$ running when NH is true (left panel) and when IH is true (right panel)

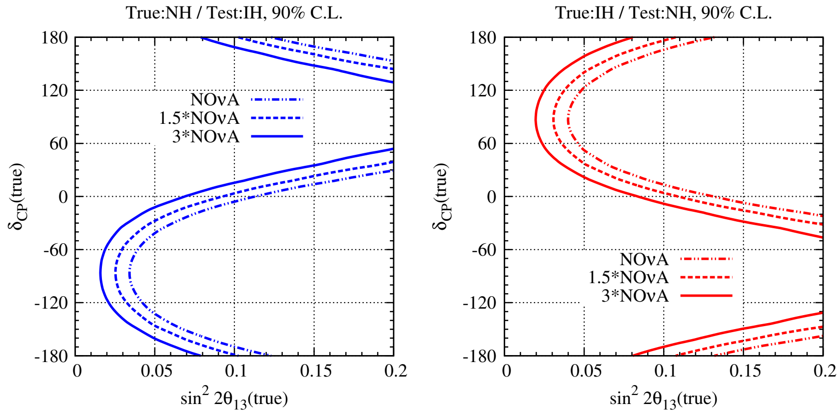

FIG. 4 (color online). Hierarchy exclusion plots for NOνA with boosted statistics for $3\nu + 3\bar{\nu}$ running when NH is true (left panel) and when IH is true (right panel).

half-plane. This is illustrated in Fig. 4. With 1.5 times the presently projected statistics of NOνA, one can determine the hierarchy for the whole of the respective favorable half-planes, for both NH and IH, for $\sin^2 2\theta_{13} = 0.1$. Similar conclusions were obtained earlier in Ref. [46]. If δ_{CP} happens to be in the unfavorable half-plane, even tripling of statistics leads to hierarchy determination only for a very small range of δ_{CP}.

B. Resolving the hierarchy-δ_{CP} degeneracy with T2K

As we demonstrated in the previous subsection, NOνA alone cannot determine the hierarchy if nature chooses one of the unfavorable combinations (NH, UHP) or (IH, LHP). In this subsection, we explore how data from T2K can help in resolving this problem. Since the baseline of T2K is smaller, the probability peaks at a lower energy and hence the flux is designed to peak at a lower energy. Therefore the matter term A is much smaller for T2K.

In Fig. 5, we plot the combined hierarchy exclusion capability of NOνA and T2K. From this figure we see that, for $\sin^2 2\theta_{13} \leq 0.1$, hierarchy determination is not possible for any δ_{CP} in the unfavorable half-plane, Hence, in our example, we assume that the statistics of NOνA are 50% more than the nominal value and those of T2K are twice the nominal value.

We illustrate the effect of T2K data on hierarchy determination by a set of examples. First we assume that NH is the true hierarchy and the true value of $\delta_{CP} = 90°$, i.e., in the unfavorable half-plane. In such a situation, NOνA data gives two degenerate solutions in the form of (NH, $\delta_{CP} \approx 90°$) and (IH, δ_{CP} in LHP), as shown in Fig. 6 (left panel).

But, the addition of T2K data almost rules out the (IH, LHP) solution, seen in the right panel of Fig. 6. It is true that a very small part of the allowed region is left behind. But, comparing the two panels of Fig. 6, we see that the addition of T2K data reduces the allowed NH region only by a small amount whereas the allowed IH region is drastically reduced. This gives a very strong indication of which hierarchy is correct. Thus the data of NOνA in conjunction with that of T2K can effectively discriminate

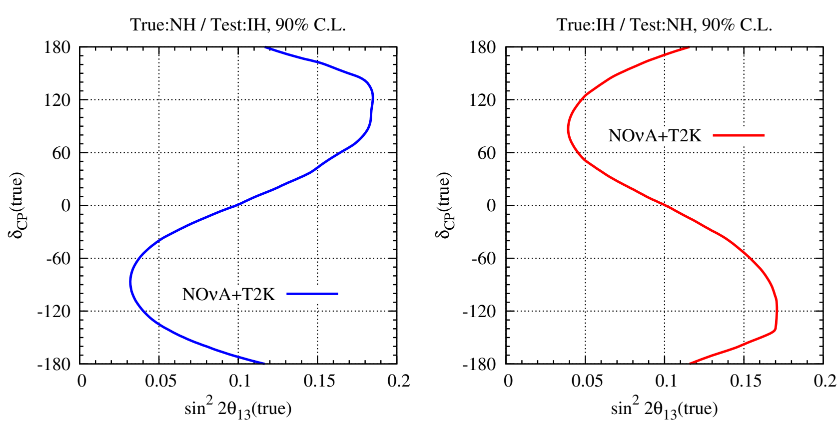

FIG. 5 (color online). Hierarchy exclusion plots for NOνA + T2K with nominal statistics when NH is true (left panel) and when IH is true (right panel).

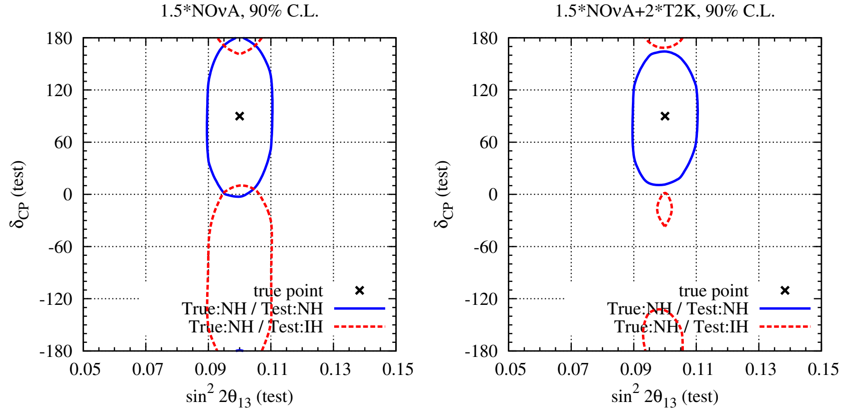

FIG. 6 (color online). Allowed $\sin^2 2\theta_{13}$-δ_{CP} plots for 1.5*$NO\nu A$ (left panel) and 1.5*NOνA + 2*T2K (right panel) with true $\sin^2 2\theta_{13} = 0.1$ and true $\delta_{CP} = 90°$.

against the wrong hierarchy. This holds true for the case of IH being the true hierarchy with δ_{CP} in LHP, illustrated in Fig. 7. Figures 6 and 7 are similar to Figs. 2 and 3 of Ref. [45], which are done for the same δ_{CP} values. Those figures also show the large shrinkage of the wrong hierarchy solution, with the addition of T2K data. In the following, we will demonstrate that this feature occurs for all values of δ_{CP} (true) in the unfavorable half-plane.

A theoretical analysis of the hierarchy-δ_{CP} degeneracy resolution, with data from NOνA and T2K, was done in Ref. [47]. To keep the arguments simple, first it was assumed that θ_{23} is maximal and that $\sin^2 2\theta_{13}$ is measured accurately by the reactor experiments. In such a situation, given a probability measurement, there exist two degenerate solutions: (correct hierarchy, correct δ_{CP}) and (wrong hierarchy, wrong δ_{CP}). In Ref. [47], it was shown that, for a given experiment, $[\sin(\text{correct}\delta_{CP}) - \sin(\text{wrong}\delta_{CP})]$ is proportional to the matter term A for that experiment. For T2K, this difference is small and is about 0.7 for $\sin^2 2\theta_{13} = 0.1$. For NO$\nu$A it is 3 times larger. Therefore, the wrong δ_{CP} values for T2K data and for NOνA data are widely different. A combined analysis of data from T2K and NOνA will pick out the correct hierarchy and a range of δ_{CP} around the correct value, provided the statistics from each experiment are large enough.

The above idea is illustrated below in Figs. 8 and 9. In Fig. 8, we have plotted χ^2 vs δ_{CP} (test) for various true values of δ_{CP} for NOνA experiment. In the left panel, the true values of δ_{CP} are all in LHP which is the favorable half-plane for NH. We find that, except for the *CP* conserving case of $\delta_{CP} = -180°$, all the χ^2 are above 9. Hence the wrong hierarchy can be excluded for most of the values of δ_{CP} in the favorable half-plane. In the right panel, the true values of δ_{CP} are all in UHP, which is the unfavorable half-plane for NH. And we find that in all cases, the χ^2 becomes nearly zero (except for $\delta_{CP} = 0$) for $-120° \le \delta_{CP}(\text{test}) \le -60°$. These are the degenerate (wrong hierarchy, wrong δ_{CP}) solutions mentioned above. Hence it is impossible to rule out the wrong hierarchy if true δ_{CP} is in the unfavorable half-plane.

In Fig. 9, we have plotted χ^2 vs δ_{CP} (test) for various true values of δ_{CP} for T2K experiment. Once again, the left panel contains plots for δ_{CP} in LHP and the right panel the plots for δ_{CP} in UHP. From the left panel, we see that T2K

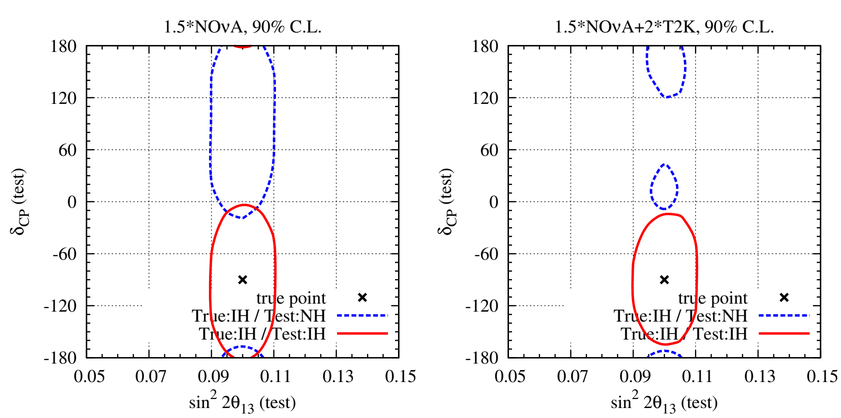

FIG. 7 (color online). Allowed $\sin^2 2\theta_{13}$-δ_{CP} plots for 1.5*NOνA (left panel) and 1.5*NOνA + 2*T2K (right panel) with true $\sin^2 2\theta_{13} = 0.1$ and true $\delta_{CP} = -90°$.

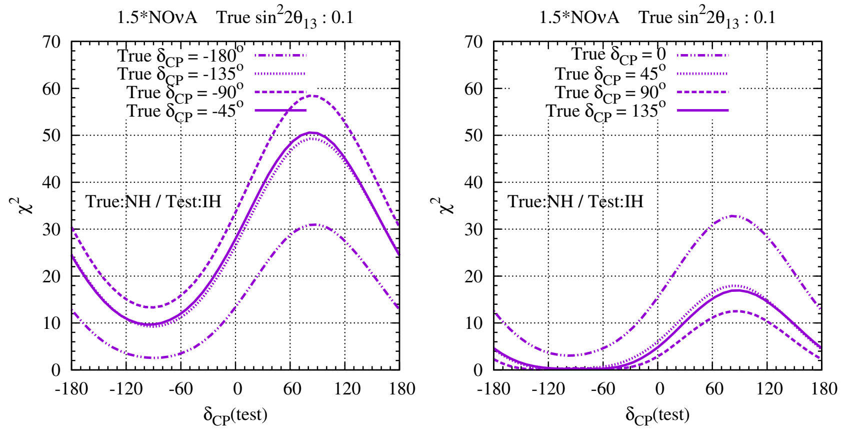

FIG. 8 (color online). χ^2 vs test δ_{CP} for 1.5*NOνA. Here true and test $\sin^2 2\theta_{13} = 0.1$. NH is true and IH is test. Different curves correspond to various true δ_{CP} in lower half-plane (left panel) and upper half-plane (right panel).

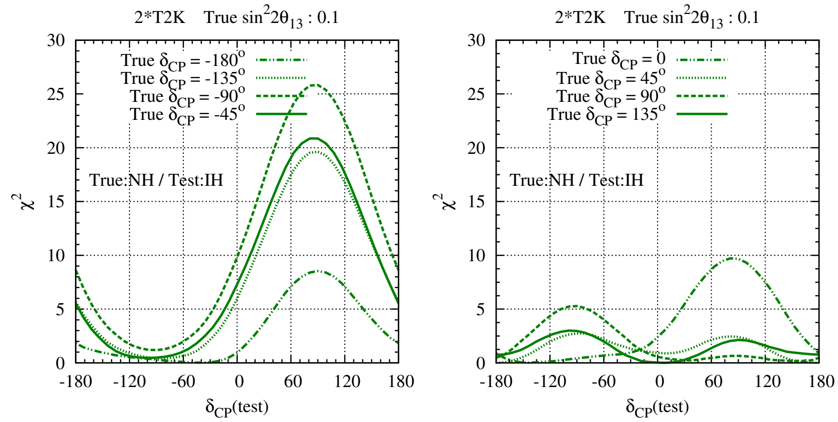

FIG. 9 (color online). χ^2 vs test δ_{CP} for 2*T2K. Here true and test $\sin^2 2\theta_{13} = 0.1$. NH is true and IH is test. Different curves correspond to various true δ_{CP} in lower half-plane (left panel) and upper half-plane (right panel).

cannot rule the wrong hierarchy. This is to be contrasted with the NOνA case, where the wrong hierarchy is ruled out by NOνA alone, if δ_{CP} is in the favorable half-plane. But, as we see below, T2K data is crucial for hierarchy discrimination, if δ_{CP} is in the unfavorable half-plane.

From the right panel, we see that the degenerate (wrong hierarchy, wrong δ_{CP}) solution for T2K occurs for δ_{CP} (test) around 0 or $\pm 180°$. And in the range $-120° \leq \delta_{CP}(\text{test}) \leq -60°$, where the degenerate wrong hierarchy solution for NOνA occurred, the χ^2 for T2K is quite large. Because of this wide difference between the δ_{CP} values of the degenerate (wrong hierarchy, wrong δ_{CP}) solutions of NOνA data and T2K data, together they rule out the wrong hierarchy.

We illustrate this hierarchy discriminating power for a few cases where true value of δ_{CP} is in the unfavorable half-plane. Figures 10–12, show the χ^2 plots for $\delta_{CP} = 90°$, $45°$ and 0, respectively, with NH as the true hierarchy. The left panel shows χ^2 for 1.5*NOνA alone whereas the right panel shows the χ^2 for 1.5*NOνA + 2*T2K. These plots show χ^2 for the two cases where the true and test hierarchies are the same and are opposite. In these plots, we have marginalized over $\sin^2 2\theta_{13}$. In the left panel of Fig. 10, there is a large allowed region of δ_{CP} (test) in the wrong half-plane, if the test hierarchy is the wrong hierarchy. In the right panel, this region is almost completely ruled out, with the addition of T2K data. There is a just a small region, centered around δ_{CP} (test) $\approx 180°$, where the χ^2 dips just below 2.71, the cut off for 90% C.L. We see very similar features for true $\delta_{CP} = 45°$ in Fig. 11 and for true $\delta_{CP} = 0$ in Fig. 12. Essentially identical features are seen for the case where IH is true hierarchy in Fig. 13 with true $\delta_{CP} = -90°$, Fig. 14 with true $\delta_{CP} = -45°$ and Fig. 15 with true $\delta_{CP} = 0$.

Finally we consider how hierarchy sensitivity improves with increasing statistics. We consider three scenarios:

(i) T2K will have a 5-yr neutrino run with its design luminosity and NOνA will run according to its present plan.

(ii) T2K will have twice the above statistics and NOνA will have 1.5 times its designed statistics.

(iii) T2K will have 4 times the above statistics and NOνA will have thrice its designed statistics.

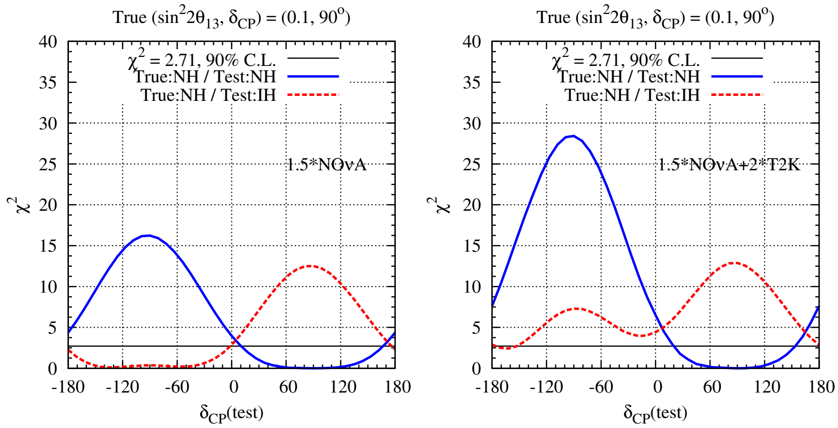

FIG. 10 (color online). χ^2 vs δ_{CP} (test) plots for 1.5*NOνA (left panel) and 1.5*NOνA + 2*T2K (right panel) with true $\sin^2 2\theta_{13} = 0.1$ and true $\delta_{CP} = 90°$.

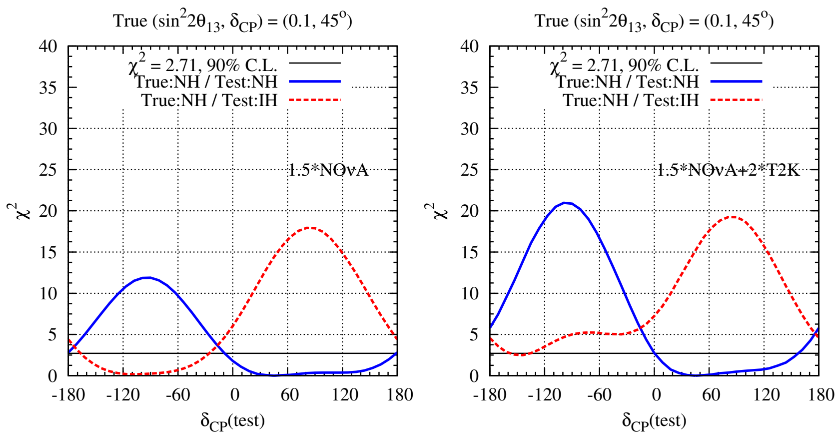

FIG. 11 (color online). χ^2 vs δ_{CP} (test) plots for 1.5*NOνA (left panel) and 1.5*NOνA + 2*T2K (right panel) with true $\sin^2 2\theta_{13} = 0.1$ and true $\delta_{CP} = 45°$.

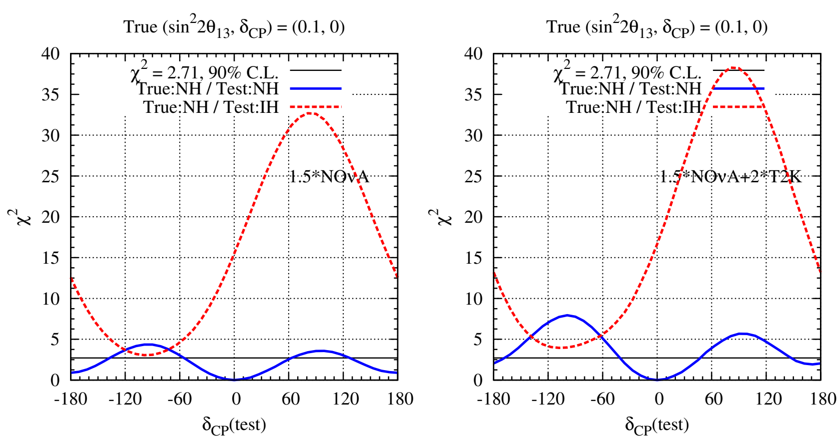

FIG. 12 (color online). χ^2 vs δ_{CP} (test) plots for 1.5*NOνA (left panel) and 1.5*NOνA + 2*T2K (right panel) with true $\sin^2 2\theta_{13} = 0.08$ and true $\delta_{CP} = 0$ (systematics included).

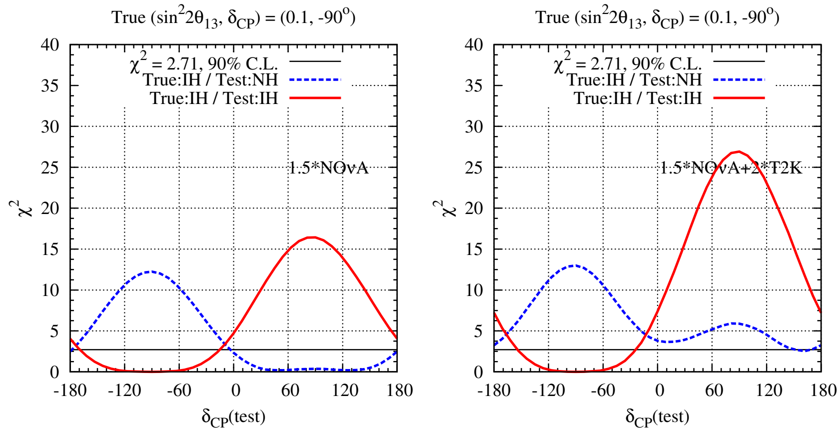

FIG. 13 (color online). χ^2 vs δ_{CP} (test) plots for 1.5*NOνA (left panel) and 1.5*NOνA + 2*T2K (right panel) with true $\sin^2 2\theta_{13} = 0.1$ and true $\delta_{CP} = -90°$.

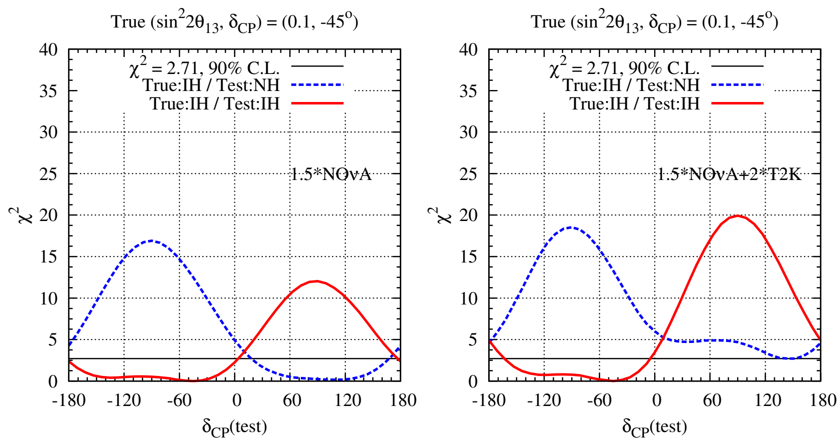

FIG. 14 (color online). χ^2 vs δ_{CP} (test) plots for 1.5*NOνA (left panel) and 1.5*NOνA + 2*T2K (right panel) with true $\sin^2 2\theta_{13} = 0.1$ and true $\delta_{CP} = -45°$.

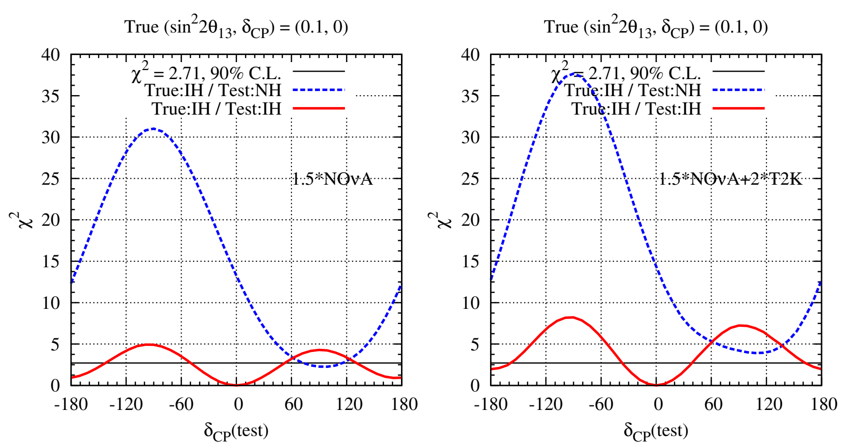

FIG. 15 (color online). χ^2 vs δ_{CP} (test) plots for 1.5*NOνA (left panel) and 1.5*NOνA + 2*T2K (right panel) with true $\sin^2 2\theta_{13} = 0.1$ and true $\delta_{CP} = 0$.

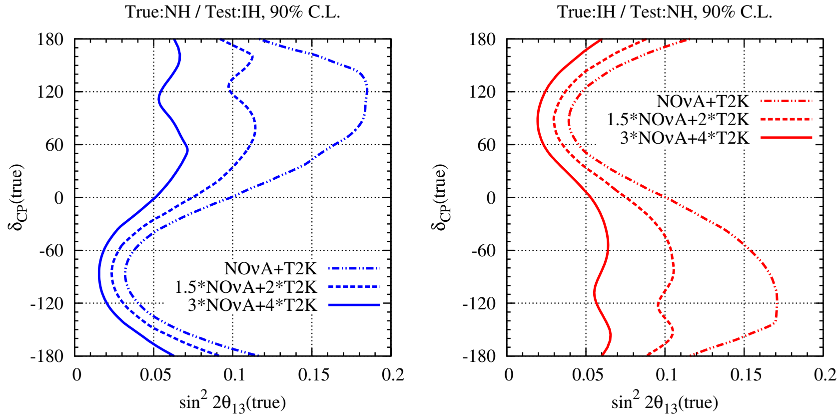

FIG. 16 (color online). Hierarchy exclusion plots for combined data from NOνA and T2K with various boosts in statistics when NH is true (left panel) and when IH is true (right panel).

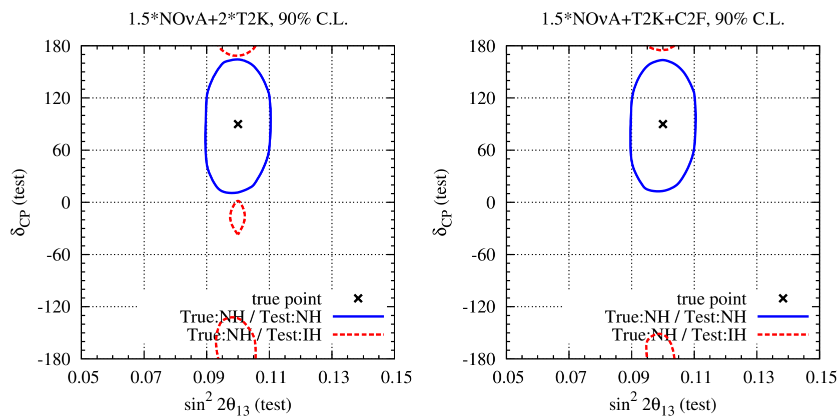

FIG. 17 (color online). Allowed $\sin^2 2\theta_{13}$-δ_{CP} plots for 1.5*NOνA + 2*T2K (left panel) and 1.5*NOνA + T2K + C2F (right panel) with true $\sin^2 2\theta_{13} = 0.1$ and true $\delta_{CP} = 90°$.

The exclusion plots are given in Fig. 16. For all points to the right of the contours, the wrong hierarchy can be ruled out.

In the left panel we assumed NH is the true hierarchy and in the right panel we assumed IH is the true hierarchy. We see that increasing the statistics from nominal values to 1.5*NOνA +2*T2K dramatically improves the ability to rule out the wrong hierarchy, if δ_{CP} (true) is in the unfavorable half-plane. Further improvement occurs if the statistics are increased even more. In particular, if $\sin^2 2\theta_{13} = 0.1$ [11,12] the hierarchy can be essentially established at 90% C.L., for any true value of δ_{CP}, with 1.5 times the designed statistics of NOνA and twice the designed statistics of T2K. This point was noted previously in Ref. [46].

It is evident now that an experiment that can exclude the wrong δ_{CP} plane effectively can be of great help in determining hierarchy when run in conjunction with NOνA. We saw that T2K, with a short baseline and smaller matter effects, has such properties. We now inquire whether having an experiment with a baseline shorter than T2K, such as the C2F, which is 130 km long, can help. For such a short baseline, $P_{\mu e}$ is maximum at $E = 0.25$ GeV. At such energies, the matter term A is very small.

To make a just comparison in terms of cost, we assume C2F to have the same beam power and detector size as that of T2K and 3 yrs each of ν and $\bar{\nu}$ running. We consider two scenarios. NOνA with 1.5 times its designed statistics and T2K with twice its designed statistics (scenario A) against NOνA with 1.5 times its designed statistics and T2K and C2F with their nominal designed statistics (scenario B). In Fig. 17, we compare the ability of scenario A (left panel) and scenario B (right panel) to exclude the wrong hierarchy—wrong δ_{CP} region. The two panels are essentially identical. We found that scenarios A and B give the same allowed regions for all true values of δ_{CP} in the unfavorable half-plane. Therefore, a shorter baseline experiment ($L \sim 130$ km) will not help in hierarchy determination.

IV. MEASURING δ_{CP} WITH $P_{\mu e}$

A. δ_{CP} measurement with T2K alone

In the previous section, we discussed the capability of NOνA and T2K to determine the mass hierarchy. We now turn our attention to the measurement of δ_{CP}. Because of the hierarchy-δ_{CP} degeneracy, the determination of these two quantities go hand in hand. Matter effects, which are hierarchy-dependent, induce a *CP*-like change in the oscillation probabilities. Therefore, it is expected that the effects of these two parameters can be disentangled by choosing baselines and energies where matter effects are small. Thus, a natural choice for accurate measurement of δ_{CP} seems to be an experiment with a short baseline and low energy, like T2K or C2F. But, here we demonstrate that δ_{CP} *cannot* be measured in such experiments without first determining the hierarchy. For the purpose of this demonstration, in this subsection alone, we will assume that T2K will have *equal 3-yr runs* in neutrino and antineutrino modes. This is done because such runs have the best capability to determine δ_{CP}. However, even in such a case, δ_{CP} cannot be determined without first determining the hierarchy.

In the following, we present "allowed δ_{CP}" graphs. In generating these, we have kept $\sin^2 2\theta_{13}$ fixed at 0.1. The graphs are plotted in the true δ_{CP}-test δ_{CP} plane. For every true value of δ_{CP}, we indicate the range in test δ_{CP} that can be excluded at 90% C.L. The plots have been shown for both true and wrong hierarchies. The dotted range, defined by $\chi^2 \le 2.71$, shows the values of test δ_{CP} that are compatible with the data, generated with δ_{CP} (true) as input. For a given true value of δ_{CP}, the error in measuring δ_{CP} is indicated by the spread of the dotted range along that δ_{CP} (true) vertical line.

Figure 18 shows the allowed δ_{CP} plot for T2K. The points on the thick, dashed line in this figure correspond to the values of δ_{CP} (test) for which χ^2 is minimum. If the test hierarchy is the same as the true hierarchy, then the χ^2 minimum occurs for $\delta_{CP}(\text{test}) = \delta_{CP}(\text{true})$ and the allowed range of test δ_{CP} is around true δ_{CP}. But, if the test hierarchy is the wrong hierarchy, then the minimum of χ^2 occurs for $\delta_{CP}(\text{test}) \neq \delta_{CP}(\text{true})$ and, in general, these two points are widely separated. This already gives a hint that an accurate measurement of δ_{CP} is not possible without first determining the hierarchy. This point is made more dramatic, when we consider the situation with more data from T2K. Figure 19 shows the allowed δ_{CP} plot for 10 times the statistics of T2K. For δ_{CP} (true) in the middle of the favorable half plane ($-140° \le \delta_{CP} \le -40°$), the wrong hierarchy solution is ruled out. Thus both the hierarchy and the correct range of δ_{CP} are simultaneously determined. For all other values of δ_{CP} (true), we get a wrong value of δ_{CP}, if we assume the wrong hierarchy. For example, we see from the right panel of Fig. 19, for true $\delta_{CP} = -30°$, we find that $-130° \le \delta_{CP}(\text{test}) \le -70°$, when the test hierarchy is the wrong hierarchy. Similarly for true $\delta_{CP} = +60°$, we find $140° \le \delta_{CP}(\text{test}) \le 200°(= -160°)$. In particular, if true δ_{CP} is $-10°$, close to the *CP* conserving value 0, we have $-150° \le \delta_{CP}(\text{test}) \le -50°$, encompassing maximal *CP* violation. The situation is similar for true $\delta_{CP} = -170°$. Conversely, for true $\delta_{CP} = 90°$, we have two allowed regions between 0 to 40° and 140° to 180°, both of which are close to *CP* conservation. This figure makes it clear that it is impossible to have a measurement of δ_{CP} if we do not know the correct hierarchy. In fact, we are likely to get a completely misleading estimate of δ_{CP} if we assume the wrong hierarchy. The corresponding figures for C2F experiment show similar features.

B. δ_{CP} measurement with T2K and NOνA

In this subsection, we consider the δ_{CP} measuring capability of NOνA and T2K together. Here we revert back to the original assumption that T2K will run in neutrino mode only for 5 yrs. Figure 20 shows the allowed δ_{CP} plot of NOνA, assuming NH is true. If the test hierarchy is the true hierarchy, the allowed range of δ_{CP} will surround true δ_{CP}. If the test hierarchy is the wrong hierarchy we obtain a large allowed range with δ_{CP} far from the true value.

Figure 21 shows the allowed δ_{CP} plot for NOνA and T2K together. In the left panel, the allowed range of δ_{CP} for

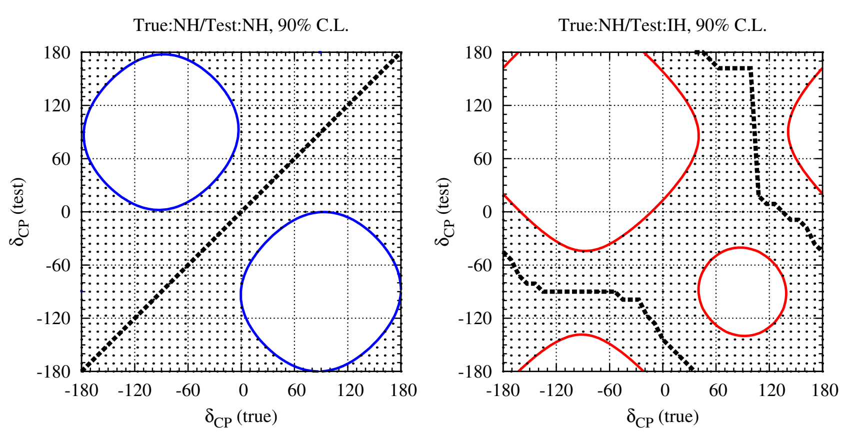

FIG. 18 (color online). Allowed δ_{CP} plots for T2K. Here NH is true. True and test $\sin^2 2\theta_{13} = 0.1$. Test hierarchy is normal (left panel) and inverted (right panel).

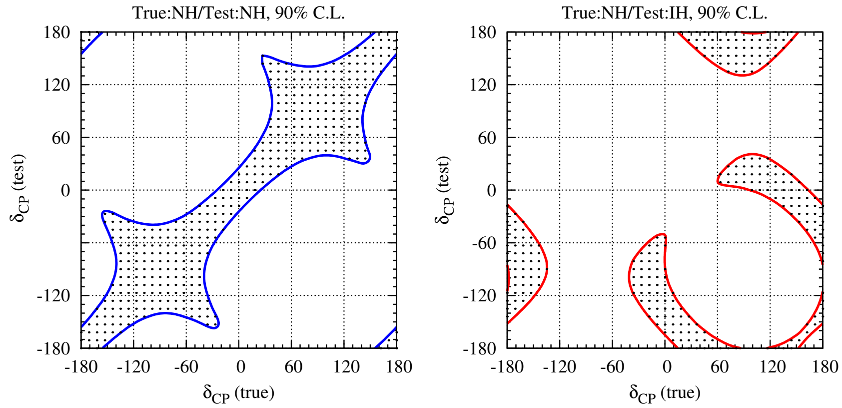

FIG. 19 (color online). Allowed δ_{CP} plots for 10*T2K. Here NH is true. True and test $\sin^2 2\theta_{13} = 0.1$. Test hierarchy is normal (left panel) and inverted (right panel).

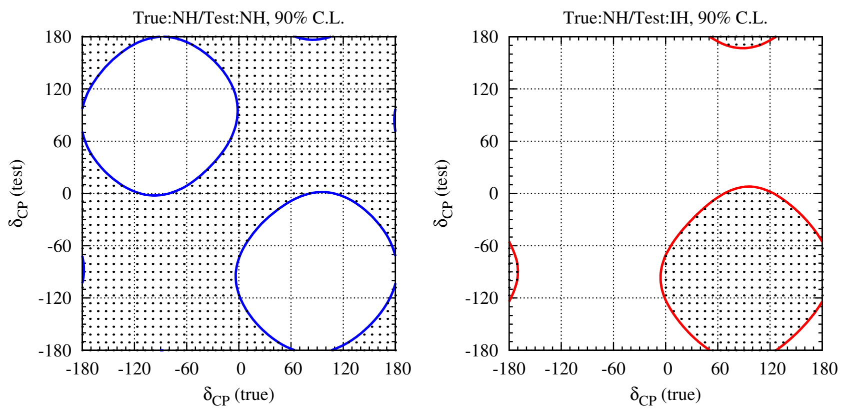

FIG. 20 (color online). Allowed δ_{CP} plots for NOνA. Here NH is true. True and test $\sin^2 2\theta_{13} = 0.1$. Test hierarchy is normal (left panel) and inverted (right panel).

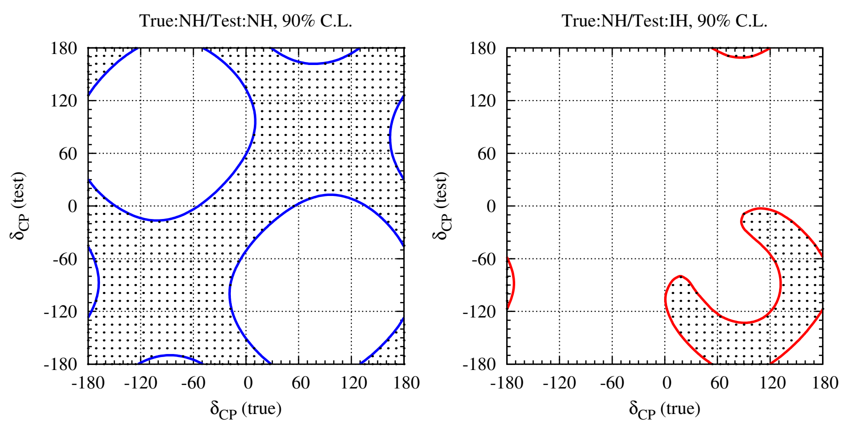

FIG. 21 (color online). Allowed δ_{CP} plots for NOνA + T2K. Here NH is true. True and test $\sin^2 2\theta_{13} = 0.1$. Test hierarchy is normal (left panel) and inverted (right panel).

the true hierarchy is shown. We see that this range is mostly in the correct half-plane. For wrong hierarchy, shown in the right panel, the large allowed region in wrong half-plane is reduced, but a substantial region is still allowed. For the case where IH is the true hierarchy, similar features occur.

If the statistics are increased to 1.5*NOνA + 2*T2K, as seen in Fig. 22, then most of the wrong hierarchy allowed region is ruled out as already noted in Sec. III. For the true hierarchy, the allowed region is centered around true δ_{CP} and is mostly in the correct half-plane. For the *CP*

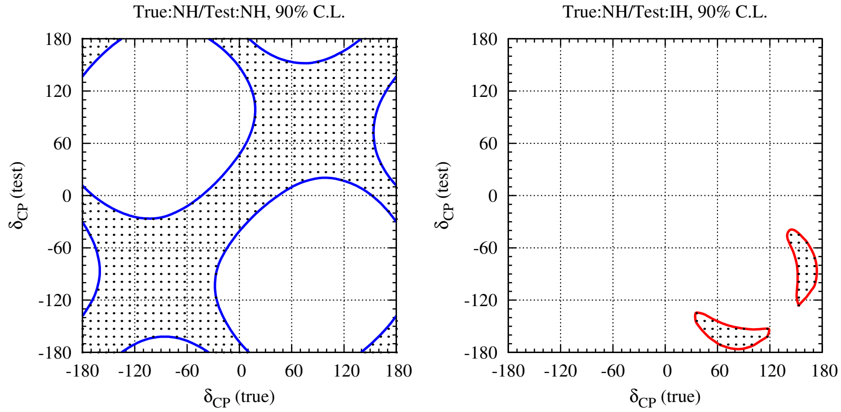

FIG. 22 (color online). Allowed δ_{CP} plots for 1.5*NOνA + 2*T2K. Here NH is true. True and test $\sin^2 2\theta_{13} = 0.1$. Test hierarchy is normal (left panel) and inverted (right panel).

conserving case $\delta_{CP} = 0$ ($\delta_{CP} = \pm 180°$), there is a small additional allowed region around $\delta_{CP} = \pm 180°$ ($\delta_{CP} = 0$) but for which χ^2 is higher. If we limit our attention to the regions around χ^2_{min}, then 1.5*NOνA + 2*T2K can measure δ_{CP} with an accuracy of $\pm 40°$ for true $\delta_{CP} = 0$ and $\pm 60°$ for true $\delta_{CP} = \pm 90°$.

It is curious that the *CP* conserving values of δ_{CP} can be measured with better accuracy than large *CP*-violating values. However, this point can be understood very simply in terms of Eq. (3). δ_{CP} occurs in this equation as $\cos(\hat{\Delta} + \delta_{CP})$. Any experiment is designed such that the flux peaks at the energy where $\hat{\Delta} \approx 90°$. Thus the δ_{CP} term is approximately $-\sin\delta_{CP}$. The slope of $\sin x$ is large at $x \approx 0$ or $180°$ and is very small at $x \approx \pm 90°$. Therefore the uncertainty in δ_{CP} is small near 0 or 180° and is large when δ_{CP} is close to $\pm 90°$.

Thus we are led to the following important conclusion: 1.5*NOνA + 2*T2K can essentially determine the hierarchy and also give an allowed region of δ_{CP} centered around its true value. Doubling of statistics will not lead to too much improvement in the allowed range of δ_{CP}. Further strategies are needed to measure δ_{CP} to a good accuracy.

A recent paper [48] envisaged some future very long baseline superbeam experiments. They found that the early data from these will determine hierarchy, and additional data is needed to measure δ_{CP}. We find that in the current scenario also, these considerations hold true.

V. SUMMARY

In this paper we explored the hierarchy—δ_{CP} degeneracy of $P_{\mu e}$ of medium long baseline experiments. This degeneracy severely limits the ability of any single experiment to determine these quantities. The observed moderately large value of θ_{13} is certainly very good news for the upcoming NOνA, as it will lie in the region where NOνA has appreciable reach for hierarchy determination if the value of δ_{CP} happens to be favorable. We define the concept of favorable half-plane of δ_{CP} and show that the LHP(UHP) is the favorable(unfavorable) half-plane for NH and vice-versa for IH. We also show that NOνA by itself can determine the hierarchy if δ_{CP} is in the favorable half-plane and $\sin^2 2\theta_{13} \geq 0.12$. When δ_{CP} is in the unfavorable half-plane, the data from NOνA and T2K beautifully complement each other to rule out the wrong hierarchy. We explore the underlying physics in detail and deduce the statistics needed for hierarchy determination. Given the current best fit of $\sin^2 2\theta_{13} \simeq 0.1$, the combined data from NOνA and T2K can *essentially* resolve mass hierarchy for the entire δ_{CP} range if the statistics for NOνA and T2K are boosted by factors 1.5 and 2, respectively. A baseline of ~130 km will not be a bonus, over and above T2K, unless supplemented by huge statistics.

In the last section we estimate the δ_{CP} reach of NOνA and T2K. We demonstrate that without knowing the hierarchy, measuring δ_{CP} would be impossible. With 1.5*NOνA + 2*T2K, the allowed region of δ_{CP} is centered around its true value and is mostly in the correct half-plane. Here also, a short baseline of ~130 km will not provide better information than T2K with the same statistics.

ACKNOWLEDGMENTS

We thank Thomas Schwetz-Mangold for a comment on an earlier version, which led to an improvement of the manuscript.

[1] K. Eguchi *et al.* (KamLAND), Phys. Rev. Lett. **92**, 071301 (2004).
[2] Q. R. Ahmad *et al.* (SNO), Phys. Rev. Lett. **87**, 071301 (2001).
[3] T. Schwetz, M. Tortola, and J. Valle, New J. Phys. **13**, 109401 (2011).
[4] D. G. Michael *et al.* (MINOS), Phys. Rev. Lett. **97**, 191801 (2006).
[5] R. Wendell *et al.* (Super-Kamiokande), Phys. Rev. D **81**, 092004 (2010).
[6] K. Abe *et al.* (T2K), Phys. Rev. Lett. **107**, 041801 (2011).
[7] P. Adamson *et al.* (MINOS Collaboration), Phys. Rev. Lett. **107**, 181802 (2011).
[8] Y. Abe *et al.* (Double Chooz), Phys. Rev. Lett. **108**, 131801 (2012).
[9] F. An *et al.* (Daya-Bay Collaboration), Phys. Rev. Lett. **108**, 171803 (2012).
[10] J. Ahn *et al.* (RENO collaboration), Phys. Rev. Lett. **108**, 191802 (2012).
[11] P. A. N. Machado, H. Minakata, H. Nunokawa, and R. Funchal, J. High Energy Phys. 05 (2012) 023.
[12] G. Fogli, E. Lisi, A. Marrone, D. Montanino, A. Palazzo *et al.*, arXiv:1205.5254.
[13] Y. Itow *et al.* (T2K), arXiv:hep-ex/0106019.
[14] F. Ardellier *et al.* (Double Chooz), arXiv:hep-ex/0405032.
[15] F. Ardellier *et al.* (Double Chooz), arXiv:hep-ex/0606025.
[16] X. Guo *et al.* (Daya-Bay), arXiv:hep-ex/0701029.
[17] S.-B. Kim (RENO), AIP Conf. Proc. **981**, 205 (2008).
[18] H. Nunokawa, S. J. Parke, and R. Zukanovich Funchal, Phys. Rev. D **72**, 013009 (2005).
[19] A. de Gouvea, J. Jenkins, and B. Kayser, Phys. Rev. D **71**, 113009 (2005).
[20] P. Huber, M. Lindner, and W. Winter, Comput. Phys. Commun. **167**, 195 (2005).
[21] P. Huber, J. Kopp, M. Lindner, M. Rolinec, and W. Winter, Comput. Phys. Commun. **177**, 432 (2007).
[22] M. Ishitsuka, T. Kajita, H. Minakata, and H. Nunokawa, Phys. Rev. D **72**, 033003 (2005).
[23] P. Huber, M. Lindner, and W. Winter, Nucl. Phys. **B645**, 3 (2002).
[24] D. Ayres *et al.* (NOνA Collaboration), arXiv:hep-ex/0503053.
[25] T. Yang and S. Wojcicki (NOνA) (2004), Off-Axis-Note-SIM-30, nova-docdb.fnal.gov/0003/000330/001/nova0030.pdf.
[26] J.-E. Campagne, M. Maltoni, M. Mezzetto, and T. Schwetz, J. High Energy Phys. 04 (2007) 003.
[27] J. E. Campagne and A. Cazes, Eur. Phys. J. C **45**, 643 (2005).
[28] M. Mezzetto, J. Phys. G **29**, 1781 (2003).
[29] M. D. Messier, Ph.D. thesis, University of Michigan UMI-99-23965 (1999).
[30] E. Paschos and J. Yu, Phys. Rev. D **65**, 033002 (2002).
[31] M. Fechner, DAPNIA-06-01-T, May 2006.
[32] I. Kato (T2K Collaboration), J. Phys. Conf. Ser. **136**, 022018 (2008).
[33] A. Cervera, A. Donini, M. B. Gavela, J. J. Gomez Cádenas, P. Hernández, O. Mena, and S. Rigolin, Nucl. Phys. **B579**, 17 (2000).
[34] E. K. Akhmedov, R. Johansson, M. Lindner, T. Ohlsson, and T. Schwetz, J. High Energy Phys. 04 (2004) 078.
[35] M. Freund, Phys. Rev. D **64**, 053003 (2001).
[36] L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978).
[37] V. Barger, D. Marfatia, and K. Whisnant, Phys. Rev. D **66**, 053007 (2002).
[38] H. Minakata and H. Nunokawa, J. High Energy Phys. 10 (2001) 001.
[39] J. Burguet-Castell, M. Gavela, J. Gomez-Cadenas, P. Hernandez, and O. Mena, Nucl. Phys. **B646**, 301 (2002).
[40] H. Minakata, H. Nunokawa, and S. J. Parke, Phys. Rev. D **66**, 093012 (2002).
[41] T. Kajita, H. Minakata, S. Nakayama, and H. Nunokawa, Phys. Rev. D **75**, 013006 (2007).
[42] K. Hagiwara, N. Okamura, and K. ichi Senda, Phys. Lett. B **637**, 266 (2006).
[43] O. Mena Requejo, S. Palomares-Ruiz, and S. Pascoli, Phys. Rev. D **72**, 053002 (2005).
[44] O. Mena, S. Palomares-Ruiz, and S. Pascoli, Phys. Rev. D **73**, 073007 (2006).
[45] P. Huber, M. Lindner, T. Schwetz, and W. Winter, J. High Energy Phys. 11 (2009) 044.
[46] D. Ayres *et al.* (NOνA Collaboration), Report No. fERMILAB-DESIGN-2007-01, 2007.
[47] O. Mena and S. J. Parke, Phys. Rev. D **70**, 093011 (2004).
[48] S. K. Agarwalla, T. Li, and A. Rubbia, J. High Energy Phys. 05 (2012) 154.