

Seesaw model in $SO(10)$ with an upper limit on right-handed neutrino masses

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In the framework of $SO(10)$ gauge unification and the seesaw mechanism, we show that the upper bound on the mass of the heaviest right-handed neutrino $M_{R_3} < 3 \times 10^{11}$ GeV, given by the Pati-Salam intermediate scale of $B-L$ spontaneous symmetry breaking, constrains the observables related to the left-handed light neutrino mass matrix. We assume such an upper limit on the masses of right-handed neutrinos and, as a first approximation, a Cabibbo form for the matrix V^L that diagonalizes the Dirac neutrino matrix m_D . Using the inverse seesaw formula, we show that our hypotheses imply a triangular relation in the complex plane of the light neutrino masses with the Majorana phases. We obtain normal hierarchy with an absolute scale for the light neutrino spectrum. Two regions are allowed for the lightest neutrino mass m_1 and for the Majorana phases, implying predictions for the neutrino mass measured in Tritium decay and for the double beta decay effective mass $|\langle m_{ee} \rangle|$.

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I. INTRODUCTION

The present status of neutrino oscillations, conceived many years ago by Pontecorvo [1], provides the following approximate values for the square mass differences and the mixing angles of the PMNS matrix [2,3]:

$$\Delta m_s^2 = |m_2|^2 - |m_1|^2 \simeq 8 \times 10^{-5} \text{ eV}^2, \quad (1)$$

$$\tan^2 \theta_s \simeq 0.4, \quad (2)$$

$$\begin{aligned} \Delta m_a^2 &= |m_3|^2 - \cos^2 \theta_s |m_2|^2 - \sin^2 \theta_s |m_1|^2 \\ &\simeq 2.5 \times 10^{-3} \text{ eV}^2, \end{aligned} \quad (3)$$

$$\tan^2 \theta_a \simeq 1. \quad (4)$$

The following experimental limits constrain the effective mass matrix of the left-handed neutrinos:

$$2.6 \times 10^{-3} \text{ eV} < m_{\nu_e} < 2.2 \text{ eV}, \quad (5)$$

$$|\langle m_{ee} \rangle| < 0.4 \text{ eV}, \quad (6)$$

$$0.06 \text{ eV} < \sum_i m_i < 0.6 \text{ eV}. \quad (7)$$

These upper limits are, respectively, obtained from the high energy spectrum of the electron in nuclear beta decay, from the upper limit on the rate in neutrinoless double beta decay (for Majorana neutrinos) and from cosmology.

The lower limits on m_{ν_e} and $\sum_i m_i$ are, respectively, obtained from the bounds

$$m_{\nu_e} > \Delta m_s \sin^2 \theta_s, \quad (8)$$

$$\sum_i m_i > \Delta m_s + \Delta m_a. \quad (9)$$

An upper limit has also been found for the component of the ν_{eL} along the third mass eigenstate, supposedly the heaviest, i.e. the one that is not involved in solar neutrino oscillations:

$$\sin^2 \theta_{13} < 0.03. \quad (10)$$

It is generally recognized that $SO(10)$ unified gauge theories [4] provide a very natural framework for the seesaw model [5], accounting naturally for the fact that left-handed neutrinos have masses several orders of magnitude smaller than the charged fundamental fermions. Indeed, the **16** representation of $SO(10)$ contains, besides the **10** and **5** of $SU(5)$, a singlet that can get a large mass, unrelated to the electroweak symmetry breaking scale.

Moreover, in the most appealing gauge unified $SO(10)$ model, the one with $SU(4) \times SU(2) \times SU(2)$ Pati-Salam [6] intermediate symmetry, $B-L$ is broken around 3×10^{11} GeV [7–9], providing the scale for right-handed neutrino masses by the $\Delta L = 2$ vacuum expectation value (VEV) of the **126** representation.

In $SO(10)$ one expects a spectrum for the eigenvalues of the Dirac neutrino mass matrix that is similar to the masses of the quarks with charge $\frac{2}{3}$, apart from some scale factor due to the different scale dependence of quark and lepton masses.

It is also very reasonable to assume that the matrix V^L appearing in the biunitary transformation that diagonalizes the Dirac neutrino mass matrix m_D has the same structure as the Cabibbo-Kobayashi-Maskawa matrix V_{CKM} [10], namely, a hierarchical structure, the mixing angle between the first two generations being larger than the other angles.

This statement is strictly correct within the simplifying hypothesis of assuming that the Higgs bosons providing the Dirac masses and mixing belong to **10** representations.

II. THE INVERSE SEESAW

In this paper we intend to deduce the consequences of two main hypothesis:

- (i) We assume an upper limit for the right-handed neutrino masses.
- (ii) Within the $SO(10)$ gauge unification scheme, the Dirac mass matrix (eigenvalues and mixing) has the same structure as the up quark mass matrix (eigenvalues and mixing).

More quantitatively, we shall assume for the eigenvalues of the Dirac neutrino mass matrix the same values as in [11], namely,

$$\begin{aligned} m_{D_1} &= 10^{-3} \text{ GeV} & m_{D_2} &= 0.4 \text{ GeV} \\ m_{D_3} &= 100 \text{ GeV}. \end{aligned} \quad (11)$$

Moreover we shall take for V^L a matrix that, to begin with, has the Cabibbo form with only θ_{12} different from zero,

$$V^L = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (12)$$

which was a very instructive approximation [12]. The rest of the angles are considered as perturbations relative to the simple ansatz (12) and, as shown in [12], even the quantitative features of the light left-handed neutrino spectrum are correctly described.

Let us consider the inverse seesaw formula:

$$M_R = -m_D^t m_L^{-1} m_D. \quad (13)$$

Diagonalizing the neutrino Dirac mass matrix m_D by

$$m_D = V^{L\dagger} m_D^{\text{diag}} V^R \quad (14)$$

one gets the relation

$$\begin{aligned} M_R &= -V^{Rt} m_D^{\text{diag}} V^{L*} m_L^{-1} V^{L\dagger} m_D^{\text{diag}} V^R \\ &= -V^{Rt} m_D^{\text{diag}} A^L m_D^{\text{diag}} V^R \end{aligned} \quad (15)$$

where the matrix A^L is defined by [13]

$$m_L^{-1} = \begin{pmatrix} \frac{c_s^2}{m_1} + \frac{s_s^2}{e^{-2i\alpha} m_2} & -\frac{c_s s_s}{\sqrt{2}} \left(\frac{1}{m_1} - \frac{1}{e^{-2i\alpha} m_2} \right) & \frac{c_s s_s}{\sqrt{2}} \left(\frac{1}{m_1} - \frac{1}{e^{-2i\alpha} m_2} \right) \\ -\frac{c_s s_s}{\sqrt{2}} \left(\frac{1}{m_1} - \frac{1}{e^{-2i\alpha} m_2} \right) & \frac{1}{2} \left(\frac{s_s^2}{m_1} + \frac{c_s^2}{e^{-2i\alpha} m_2} + \frac{1}{e^{-2i\beta} m_3} \right) & -\frac{1}{2} \left(\frac{s_s^2}{m_1} + \frac{c_s^2}{e^{-2i\alpha} m_2} - \frac{1}{e^{-2i\beta} m_3} \right) \\ \frac{c_s s_s}{\sqrt{2}} \left(\frac{1}{m_1} - \frac{1}{e^{-2i\alpha} m_2} \right) & -\frac{1}{2} \left(\frac{s_s^2}{m_1} + \frac{c_s^2}{e^{-2i\alpha} m_2} - \frac{1}{e^{-2i\beta} m_3} \right) & \frac{1}{2} \left(\frac{s_s^2}{m_1} + \frac{c_s^2}{e^{-2i\alpha} m_2} + \frac{1}{e^{-2i\beta} m_3} \right) \end{pmatrix}. \quad (23)$$

Therefore, m_L^{-1} being symmetric and V^L unitary, the matrix A^L is also symmetric.

$$A^L = V^{L*} m_L^{-1} V^{L\dagger}. \quad (16)$$

Moreover, within $SO(10)$, with the electroweak Higgs boson belonging to the **10** and **126** representations, and *no component along the 120 representation*, the mass matrices are symmetric. As a consequence, the unitary matrices V^R and V^L that diagonalize Dirac neutrino matrix (14) are related:

$$V^R = V^{L*} \quad (17)$$

and the matrix M_R (15) becomes

$$M_R = -V^{L\dagger} m_D^{\text{diag}} A^L m_D^{\text{diag}} V^{L*}. \quad (18)$$

The Cabibbo limit (12) taken by us would be a good approximation of V^L in the limit of quark-lepton symmetry, with only components along the **10** representations for the electroweak Higgs, where V^L should be equal to V_{CKM} .

The neutrino mass matrix m_L is diagonalized by the PMNS unitary neutrino mixing matrix, which reads

$$U \simeq \begin{pmatrix} c_s & s_s & 0 \\ -\frac{s_s}{\sqrt{2}} & \frac{c_s}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s_s}{\sqrt{2}} & -\frac{c_s}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \cdot \text{diag}(1, e^{i\alpha}, e^{i\beta}) \quad (19)$$

in the approximation that we will consider here for the angle (10)

$$\sin\theta_{13} \simeq 0. \quad (20)$$

In writing (19) we have taken the maximal mixing angle for atmospheric neutrino oscillation and $s_s \equiv \sin\theta_s$ ($c_s \equiv \cos\theta_s$) and the angles α and β are the Majorana phases. We use in (15) the notation of Davidson *et al.* [14] for the Majorana phases, that have the ranges $0 \leq \alpha \leq \pi$, $0 \leq \beta \leq \pi$. In the PDG Tables [15] they are defined as $\alpha_{21}/2$ and $\alpha_{31}/2$, with $0 \leq \alpha_{21} \leq 2\pi$, $0 \leq \alpha_{31} \leq 2\pi$.

Then, the left-handed neutrino light mass matrix reads

$$m_L = U^* m_L^{\text{diag}} U^\dagger \quad m_L^{-1} = U(m_L^{\text{diag}})^{-1} U^\dagger \quad (21)$$

where

$$m_L^{\text{diag}} = \text{diag}(m_1, m_2, m_3) \quad m_i \geq 0 \quad (i = 1, 2, 3) \quad (22)$$

are the light neutrino masses, real positive parameters, since the Majorana phases have been factorized out, as it should.

For the inverse m_L^{-1} of the matrix (21) we will have

Of interest for our discussion will be the consideration of the matrix $m_D^{\text{diag}} A^L m_D^{\text{diag}}$ that enters in the r.h.s. of the expression (11):

$$m_D^{\text{diag}} A^L m_D^{\text{diag}} = \begin{pmatrix} A_{11}^L m_{D1}^2 & A_{12}^L m_{D1} m_{D2} & A_{13}^L m_{D1} m_{D3} \\ A_{12}^L m_{D1} m_{D2} & A_{22}^L m_{D2}^2 & A_{23}^L m_{D2} m_{D3} \\ A_{13}^L m_{D1} m_{D3} & A_{23}^L m_{D2} m_{D3} & A_{33}^L m_{D3}^2 \end{pmatrix}. \quad (24)$$

The coefficient A_{33}^L of the square of the highest Dirac eigenvalue (11), $m_{D3}^2 = (100 \text{ GeV})^2$, within the simplifying hypotheses of a Cabibbo form for V^L (19) and $s_{13} = 0$ (16), is [12,16]

$$A_{33}^L = (m_L^{-1})_{33} = \frac{1}{2} \left(\frac{s_s^2}{m_1} + \frac{c_s^2}{e^{-2i\alpha} m_2} + \frac{1}{e^{-2i\beta} m_3} \right) \quad (25)$$

and in the limit $m_{D1}, m_{D2} \ll m_{D3}$ (11) one has roughly

$$M_{R3} \sim |A_{33}^L| m_{D3}^2 = \frac{1}{2} \left| \frac{s_s^2}{m_1} + \frac{c_s^2}{e^{-2i\alpha} m_2} + \frac{1}{e^{-2i\beta} m_3} \right| m_{D3}^2. \quad (26)$$

The expression (25) found for A_{33}^L follows from the assumption (12) for the matrix V^L . Let us notice that in all generality it will also depend on the square of the mixing angle between the third generation and the other two lighter ones, that is assumed to be small.

Let us first remark that a rather conservative upper limit on the mass of the heaviest right-handed neutrino of the order

$$M_{R3} \leq 10^{15} \text{ GeV} \quad (27)$$

implies a lower limit for the mass of the lightest left-handed neutrino, since in the small m_1 region, when the first term in (25) dominates, one should have, with the value (11) for m_{D3} ,

$$\frac{1}{2} \frac{s_s^2}{m_1} \times 10^4 \text{ GeV}^2 \leq 10^{15} \text{ GeV} \quad (28)$$

which implies

$$m_1 \geq 1.4 \times 10^{-3} \text{ eV}. \quad (29)$$

Since m_2 and m_3 , according to (1) and (3), are monotonically increasing functions of m_1 , one has

$$|\det(m_L)| \geq 1.4 \times 10^{-3} \Delta m_s \Delta m_a = 6.43 \times 10^{-7} \text{ eV}^3 \quad (30)$$

and for the Majorana mass matrix of right-handed neutrinos one has

$$|\det(M_R)| \leq 2.5 \times 10^{30} \text{ GeV}^3. \quad (31)$$

III. IMPOSING AN UPPER BOUND ON THE HEAVIEST M_{ν_R} EIGENVALUE

Let us stress that large cancellations are required in (26) if we impose on the masses of the right-handed neutrinos the more stringent limit

$$M_{R3} \leq 3 \times 10^{11} \text{ GeV} \quad (32)$$

i.e. the scale of $B-L$ spontaneous symmetry breaking in the $SO(10)$ unified gauge theory with Pati-Salam intermediate symmetry.

The trivial bound

$$|A_{33}^L| \leq \frac{1}{2} \left(\frac{s_s^2}{m_1} + \frac{c_s^2}{m_2} + \frac{1}{m_3} \right) \quad (33)$$

would be effective to constrain M_R to be smaller than 10^{11} GeV only in a region of *unrealistically large* neutrino masses. In fact $|A_{33}^L|$ should be about 2 orders of magnitude smaller than $\frac{1}{2m_3}$, taking into account the upper limit on $\sum_i m_i$ (7).

From (11) and (26) we see that (32) implies

$$|A_{33}^L| < 3 \times 10^{-2} \text{ eV}^{-1} \ll 2.5 \text{ eV}^{-1} \leq \frac{1}{2m_3}. \quad (34)$$

More precisely, only the third term in the r.h.s. of (33) would give rise at least, by assuming the largest value for m_3 consistent with the square mass differences fixed by the oscillations (3) and the rather conservative cosmological limit (7) on the sum of their masses, 0.6 eV, to a mass around $M_{R3} \simeq 2.5 \times 10^{13}$ GeV, 2 orders of magnitude larger than the value expected in the ordinary $SO(10)$ unified model with Pati-Salam intermediate symmetry.

Therefore, we underline again that one needs a strong cancellation between the three terms in (25), which have moduli related by the square mass differences implied by neutrino oscillations.

Notice the very important point that this is already a hint for large relative Majorana phases. In this respect, it is interesting to look for the implications for the neutrinoless double beta decay effective mass (6):

$$\langle m_{ee} \rangle = c_s^2 m_1 + s_s^2 e^{-2i\alpha} m_2. \quad (35)$$

Owing to (25) $\langle m_{ee} \rangle$ can be exactly expressed in terms of $m_i (i = 1, 2, 3)$ and A_{33}^L by the formula

$$\langle m_{ee} \rangle = -e^{-2i(\alpha-\beta)} \frac{m_1 m_2}{m_3} + 2A_{33}^L e^{-2i\alpha} m_1 m_2. \quad (36)$$

Taking into account (34), one can neglect the second term in the r.h.s. of (36) and we obtain, just from the imposed upper limit on M_{R3} (32), the simple expression for $\langle m_{ee} \rangle$:

$$\langle m_{ee} \rangle = -e^{-2i(\alpha-\beta)} \frac{m_1 m_2}{m_3}. \quad (37)$$

In the following we shall take

$$A_{33}^L = \frac{1}{2} \left(\frac{s_s^2}{m_1} + \frac{c_s^2}{e^{-2i\alpha} m_2} + \frac{1}{e^{-2i\beta} m_3} \right) = 0 \quad (38)$$

since the second term in the r.h.s. of (36) is at most 1% of the first one. Notice that relation (38) follows from the fact that A_{33}^L , due to Eq. (24), is affected by the square of the largest mass m_{D_3} and has nothing to do with the values of the other eigenvalues in (11).

IV. A TRIANGLE IN THE COMPLEX PLANE OF LIGHT NEUTRINO MASSES AND MAJORANA PHASES

Let us now examine carefully the consequences of the condition (38). This cancellation condition defines a *triangle in the complex plane*:

$$\frac{s_s^2}{m_1} + \frac{c_s^2}{e^{-2i\alpha} m_2} + \frac{1}{e^{-2i\beta} m_3} = 0 \quad (39)$$

which we have drawn in Fig. 1.

Equations (1) and (3) give m_2 and m_3 in terms of m_1 and (39) may be satisfied if one has the inequality

$$\left| \frac{s_s^2}{m_1} - \frac{c_s^2}{m_2} \right| \leq \frac{1}{m_3} \leq \frac{s_s^2}{m_1} + \frac{c_s^2}{m_2} \quad (40)$$

which is violated for $m_1 < 2.9926 \times 10^{-3}$ eV or in the range $(6.2194 \times 10^{-3}$ eV, 1.9861×10^{-2} eV).

We thus get two regions where the triangular relation holds:

$$\text{Region I} \quad r_1 \leq m_1 \leq r_2$$

$$(r_1 = 2.9926 \times 10^{-3} \text{ eV}, r_2 = 6.2194 \times 10^{-3} \text{ eV}), \quad (41)$$

$$\text{Region II} \quad m_1 \geq r_3 \quad (r_3 = 1.9861 \times 10^{-2} \text{ eV}). \quad (42)$$

On the boundaries of both regions $m_1 = r_1, r_2, r_3$ one has $\sin(2\alpha) = \sin(2\beta) = 0$, but these two quantities can be reasonably large in their interior.

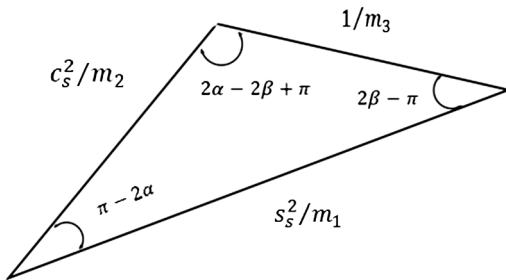


FIG. 1. The triangle in the complex plane (39). The sides are given in terms of the inverses of the light neutrino masses and the angles as functions of the Majorana phases α and β of the light neutrino mixing matrix (19).

We plot in Figs. 2 and 3 the dependence of the Majorana phases α and β for both regions I and II as functions of m_1 (in 10^{-3} eV units).

As we see in Fig. 2, in region I the phase α decreases from 90° to 82.1° at $m_1 = 4.1 \times 10^{-3}$ eV and gets back to 90° at $m_1 = r_2$, while β increases from 90° to 180° [notice that we take α in the range $(0^\circ, 90^\circ)$ and β in the range $(90^\circ, 180^\circ)$, but of course (39) holds also with the opposite choice]. As we will see below, the fact that α is rather close to 90° for region I will imply a strong cancellation in the effective mass $\langle m_{ee} \rangle$ of neutrinoless double beta decay.

In region II both Majorana phases can get large values for moderate values of m_1 , where the sides of the triangle (39) are not very different.

Notice now the important remark that for both regions one must have *normal hierarchy*. The reason is the following. From Eq. (39) one gets the relation

$$\tan^2 \theta_s = - \frac{m_1 (e^{-2i\alpha} m_2 + e^{-2i\beta} m_3)}{e^{-2i\alpha} m_2 (m_1 + e^{-2i\beta} m_3)} \quad (43)$$

which, for the inverted hierarchy ($m_3 \ll m_1, m_2$), would be about -1 .

For the normal hierarchy ($m_1, m_2 \ll m_3$), Eq. (43) becomes $\tan^2 \theta_s = - \frac{m_1}{e^{-2i\alpha} m_2}$ and to have $\tan^2 \theta_s \approx 0.4$ [Eq. (2)] one needs

$$\frac{m_1}{m_2} \approx 0.4 \quad \alpha \approx \frac{\pi}{2}. \quad (44)$$

As we have seen above, when A_{33}^L vanishes, one has, from (37),

$$|\langle m_{ee} \rangle| = \frac{m_1 m_2}{m_3} \quad (45)$$

so that, once m_1 is fixed, the three quantities in (38) are also fixed, with m_2 and m_3 given by (1) and (3).

In the present scheme we have therefore for $|\langle m_{ee} \rangle|$ the appealing expression (45), which implies a negative interference between the two terms in (35) for small m_1 , and a positive one when m_1 approaches the cosmological bound.

On the other hand, the mass m_{ν_e} can be obtained from

$$m_{\nu_e} = c_s^2 m_1 + s_s^2 m_2. \quad (46)$$

A further discussion on the constraint

$$M_{R_3} < 3 \times 10^{11} \text{ GeV}$$

Besides the main constraint (39), some words of caution are necessary to prevent the mass of the heavier right-handed neutrino M_{R_3} from being larger than 3×10^{11} GeV. We also have to check that

$$|A_{23}^L| \leq 7.5 \text{ eV}^{-1} \quad (47)$$

because A_{23}^L multiplies the product of the two highest eigenvalues of the Dirac matrix, as we can see in (24), $m_{D_2} m_{D_3} \sim 40 \text{ GeV}^2$. It depends on the θ_{12} mixing angle and is given by

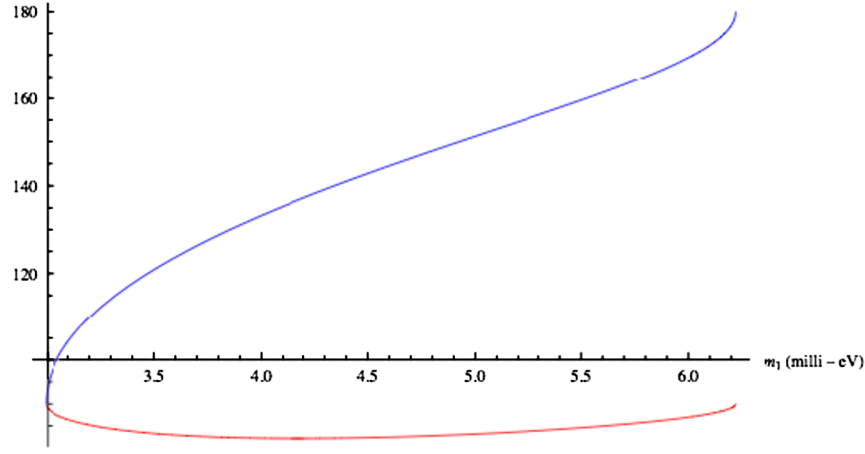


FIG. 2 (color online). The Majorana phases α (red, lower line) and β (blue, upper line) in region I as a function of m_1 in 10^{-3} eV units.

$$\begin{aligned}
 A_{23}^L &= -\frac{1}{2} \left(\frac{s_s^2}{m_1} + \frac{c_s^2}{m_2 e^{-2i\alpha}} - \frac{1}{m_3 e^{-2i\beta}} \right) \cos\theta_{12} \\
 &\quad - \frac{1}{\sqrt{2}} c_s c_s \left(\frac{1}{m_1} - \frac{1}{m_2 e^{-2i\alpha}} \right) \sin\theta_{12} \\
 &= \frac{1}{m_3 e^{-2i\beta}} \cos\theta_{12} - \frac{1}{\sqrt{2}} c_s c_s \left(\frac{1}{m_1} - \frac{1}{m_2 e^{-2i\alpha}} \right) \sin\theta_{12}.
 \end{aligned} \tag{48}$$

At the boundary $m_1 = r_1, r_2, r_3$ of the allowed regions, we can tune the value of θ_{12} in order that $A_{23}^L = 0$ holds, as follows:

$$\tan\theta_{12} = \sqrt{2} \frac{1}{c_s s_s} \frac{m_1 m_2 e^{-2i\alpha}}{m_3 e^{-2i\beta} (m_2 e^{-2i\alpha} - m_1)} \tag{49}$$

implying $\tan\theta_{12} = 0.14, 0.24,$ and $0.6,$ at $m_1 = r_1, r_2,$ and r_3 respectively, where $\sin(2\alpha) = \sin(2\beta) = 0,$ as we have seen above. In the first region, as soon as $\frac{1}{m_3 e^{-2i\beta}}$ forms a large angle with $\frac{1}{m_1} - \frac{1}{m_2 e^{-2i\alpha}}$ in the complex plane, the

cancellation between the two terms in (48) is impossible and, when they are just orthogonal, the coefficient of the term proportional to $m_{D_2} m_{D_3}$ is at least of order $\frac{1}{m_3 e^{-2i\beta}},$ giving rise to two right-handed neutrinos around 8×10^{11} GeV and a lowest state around 0.32×10^6 GeV.

In order to avoid a too small value for the mass of the lightest right-handed neutrino, a necessary condition is that $|A_{22}^L A_{33}^L - (A_{23}^L)^2|$ is smaller than $|(A_{23}^L)^2|.$ This can be obtained by relaxing the condition $A_{33}^L = 0.$ However, one gets a too small mass for the lightest right-handed neutrino anyway because of the range allowed for

$$\begin{aligned}
 A_{22}^L &= \frac{\cos^2\theta_{12}}{2} \left(\frac{s_s^2}{m_1} + \frac{c_s^2}{m_2 e^{-2i\alpha}} + \frac{1}{m_3 e^{-2i\beta}} \right) \\
 &\quad + \sqrt{2} \sin\theta_{12} \cos\theta_{12} c_s s_s \left(\frac{1}{m_1} - \frac{1}{m_2 e^{-2i\alpha}} \right) \\
 &\quad + \sin^2\theta_{12} \left(\frac{c_s^2}{m_1} + \frac{s_s^2}{m_2 e^{-2i\alpha}} \right).
 \end{aligned} \tag{50}$$

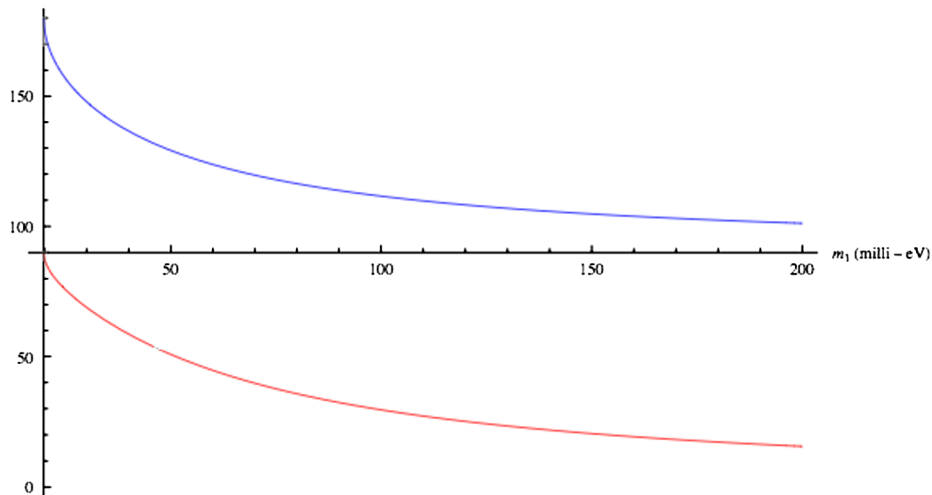


FIG. 3 (color online). The Majorana phases α (red, lower line) and β (blue, upper line) in region II as a function of m_1 in 10^{-3} eV units.

A_{22}^L would be equal to A_{33}^L in the limit of vanishing θ_{12} . So, near the boundaries of region I one gets the choice made recently [12] of a compact neutrino spectrum, as is also the case with a large value of $\tan\theta_{12}$ near r_3 . The other values of m_1 consistent with Eq. (37) imply a value higher than 3×10^{11} GeV for the two heaviest right-handed neutrinos, and a small value for the lightest one.

V. PHENOMENOLOGICAL IMPLICATIONS FOR LOW-ENERGY ν_L PHYSICS

In conclusion, the choice of a compact spectrum seems the most natural, but it is useful to describe the phenomenological consequences of the other scenarios. We shall write the phenomenological consequences for the quantities, for which there are the limits written in (5)–(7) for the two regions (41) and (42) in the triangle

$$\text{Region I} \quad m_{\nu_e} = (4.8 - 7.5) \times 10^{-3} \text{ eV} \quad \sum_i m_i = 0.1 \text{ eV} \quad |\langle m_{ee} \rangle| = (0.6 - 1.3) \times 10^{-3} \text{ eV}, \quad (51)$$

$$\text{Region II} \quad m_{\nu_e} = (2 \times 10^{-2} - 0.2) \text{ eV} \quad \sum_i m_i = (0.1 - 0.6) \text{ eV} \quad |\langle m_{ee} \rangle| = (8.5 \times 10^{-3} - 0.2) \text{ eV}. \quad (52)$$

VI. CONCLUSIONS

With reasonable hypotheses in the framework of $SO(10)$ unified theories, and by imposing the simple assumption of an upper bound on the mass of the heaviest right-handed neutrino $M_{R_3} < 3 \times 10^{11}$ GeV, as suggested by a Pati-Salam intermediate scale of B - L spontaneous symmetry breaking, one gets interesting predictions for the physical quantities related to the effective mass matrix of the light left-handed neutrinos, namely, on the mass of the lightest neutrino and on the Majorana phases.

Using the inverse seesaw formula, we have shown that our hypothesis of an upper bound for the right-handed neutrino masses implies a triangular relation in the complex plane of the light neutrino masses with the Majorana phases. In a straightforward way we thus have predicted, on the one hand, normal hierarchy for the light neutrinos and a lower limit and an exclusion region for the mass of

(39). For the sum of the moduli of the neutrino masses we find in region I values slightly above the lower limit $|\Delta m_d| + |\Delta m_s| \geq 0.06$ eV, while in region II the sum of the neutrino masses is at least 0.96×10^{-1} eV, and it grows almost linearly and saturates the bound at $m_1 = 0.198$ eV.

We always get a small value for $|\langle m_{ee} \rangle|$, in the range $(5.6 \times 10^{-4} - 1.3 \times 10^{-3})$ eV in region I, while in region II the relevant range is $(8.5 \times 10^{-3} - 0.2)$ eV. We have limited the evaluation in region II to $m_1 \leq 0.2$ eV, according to the bound (7).

For m_{ν_e} , the neutrino mass intervening in the tritium decay, it is confined to the ranges $(4.8 - 7.5) \times 10^{-3}$ eV for region I and $(2 \times 10^{-2} - 0.2)$ eV for region II.

To summarize, we obtain the following numerical results:

the lightest left-handed neutrino m_1 , implying an absolute scale for the light neutrino spectrum.

The allowed regions for m_1 are the range $m_1 = (3.0 - 6.2) \times 10^{-3}$ eV and the lower bound $m_1 \geq 2.0 \times 10^{-2}$ eV. For small m_1 , one of the Majorana phases can be close to $\frac{\pi}{2}$, and we get a strong cancellation in the effective mass $|\langle m_{ee} \rangle|$ of neutrinoless double beta decay; for light neutrino masses near the cosmological bound, we obtain a positive interference for this quantity. Within our scheme we also obtain an interesting formula for $|\langle m_{ee} \rangle|$ just in terms of the three light neutrino masses, which is valid in both domains allowed for m_1 .

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