

Noncommutative magnetic moment, fundamental length, and lepton sizeT. C. Adorno^{*} and D. M. Gitman[†]*Instituto de Física, Universidade de São Paulo, Caixa Postal 66318, CEP 05508-090, São Paulo, São Paulo, Brazil,*A. E. Shabad[‡]*P. N. Lebedev Physics Institute, Leninsky Prospekt 53, Moscow 117924, Russia*

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Upper bounds on fundamental length are discussed that follow from the fact that a magnetic moment is inherent in a charged particle in noncommutative (NC) electrodynamics. The strongest result thus obtained for the fundamental length is still larger than the estimate of electron or muon size achieved following the Brodsky-Drell and Dehlmert approach to lepton compositeness. This means that NC electrodynamics cannot alone explain the whole existing discrepancy between the theoretical and experimental values of the muon magnetic moment. On the contrary, as measurements and calculations are further improved, the fundamental length estimate based on electron data may go down to match its compositeness radius.

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I. INTRODUCTION**A. The problem. Results and conclusions**

The clue issue in checking quantum electrodynamics (QED) is the measurement of the magnetic moment of electron with the subsequent comparison of its measured value with the anomalous electron magnetic moment calculated via the standard model that is mostly QED in this case. Up to now, within every experimental and theoretical accuracy achieved, these two values do coincide. The allowed, within the errors, discrepancy between the experimental and theoretical values of the electron magnetic moment is expected to be diminishing on and on, as the precision grows, and hopefully the coincidence between them will be maintained with better and better accuracy. On the other hand, as far as one is seeking for possible theoretical amendments to the standard model, admissible within the above situation, one should confine their impact on the electron magnetic moment to lie within the present experimental and theoretical indeterminacy. A certain candidate for going beyond the standard QED is proposed by the noncommutative (NC) electrodynamics. It was found recently [1] that in the framework of that theory a static classical charge at rest carries a magnetic moment, called NC magnetic moment, whose smallness is determined by a noncommutativity parameter θ , supplying the theory with the fundamental length¹

$l = \sqrt{\theta}$. By demanding that, for the electron, the NC magnetic moment be less than the existing error in measuring the electron magnetic moment we get an estimate from above on the parameter θ and the associated fundamental length l . Certain restrictions on the fundamental length inherent in the NC theory also follow from the existence of the NC magnetic moment of heavier charged particles. However, the consideration of the noncommutative magnetic moment of the proton and of its contribution to the hyperfine splitting of the energy level $1S_{1/2}$ in a hydrogen atom did not lead [1] to any essentially new estimate for the maximum fundamental length. On the contrary, consideration of leptons did.

Once the NC magnetic moment is found to be inversely proportional to the size of the electric charge, an important role in getting this estimate is played by the size attributed to an electron, the smaller the size, the larger the NC magnetic moment, the smaller the upper estimate on the NC parameter and the fundamental length. We probe different assumptions concerning the “electron size,” the ultimate one being that it is restricted from below only by the fundamental length l itself, since neither object is supposed to be smaller than it. In this way a hitherto lowest upper bound on the fundamental length, as it appears in the noncommutative field theory, was achieved in Ref. [1]. On the other hand, after we update the famous electron size estimate due to Brodsky-Drell-Dehlmert [2], [3] (not based on any noncommutative mechanism, but only on a consideration of a possible compositeness, or divisibility of the electron) by taking into account the most recent measurements of the electron magnetic moment, we find the electron size results to be two orders of magnitude smaller than the boldest estimate of the fundamental length obtained from speculations on noncommutative magnetic moment. As far as in an NC field theory no size of any physical object is admitted to be smaller than the fundamental length, this means that no more than 1/100 part of the

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¹The noncommutativity by no means is the only method of introducing the fundamental, or elementary, length into a theory. Throughout this paper, however, we shall mean namely NC fundamental length when using this notion. On the other hand, we do not know whether the fundamental length as it is proposed by the noncommutativity mechanism is universal for all particles and fields. For this reason we shall discuss its values independently when we deal with different particles.

existing indeterminacy in the knowledge of the electron magnetic moment may be at the best ascribed to the contribution of the noncommutative magnetic moment. Then, two options remain. Either there should be an extra extension beyond the standard QED, other than NC electrodynamics, that may take responsibility for the main part of the admitted, if any, deviation of the magnetic moment from the QED result, or, what is more probable, this admitted deviation will be essentially reduced by further more precise measurements.

The same analysis is repeated in the paper as applied to the μ -meson. The crucial difference with the electron case is that the difference between the theoretical and experimental values of the muon magnetic moment exceeds the limits admitted by the errors. So, no further technical advancement is expected to be able to remove this contradiction, and our results make us more definite in claiming that the noncommutativity cannot provide for the missing part of the muon magnetic moment, a different way for extending the standard model remaining to be needed.

B. Noncommutative magnetic moment

In Ref. [1], classical field equations in $U(1)_*$ -theory (noncommutative electrodynamics) were formulated that—at least within the first order in the noncommutativity parameter θ —restrain the gauge invariance in spite of the presence of external current, known to violate it (at least off-shell). By solving these equations electromagnetic field produced by a finite-size static electric charge was found, and the fact that this charge possesses a magnetic moment depending on its size was established. Let the external current in the field equations of NC electrodynamics be just a static electric charge distributed inside a sphere of a finite radius a with the uniform charge density

$$\rho(\mathbf{r}) = \frac{3}{4\pi} \frac{Ze}{a^3}, \quad r < a, \quad r = |\mathbf{r}|.$$

Outside the sphere there is no charge: $\rho(\mathbf{r}) = 0$, if $r > a$. The above finite-size static total charge Ze , where e is the fundamental charge, produces not only the electrostatic field, but also behaves itself as a magnetic dipole with the magnetic field given in the remote region $r \gg a$ by the following vector-potential

$$\mathbf{A} = \frac{[\mathbf{M} \times \mathbf{r}]}{r^3}, \quad \mathbf{M} = \theta(Ze)^2 \frac{2e}{5a}, \quad (1)$$

where \mathbf{M} was called NC magnetic moment of the charged particle. Here the three spacial components of the vector θ are defined as $\theta^i \equiv (1/2)\epsilon^{ijk}\theta^{jk}$, $i, j, k = 1, 2, 3$ in terms of the antisymmetric noncommutativity tensor $\theta^{\mu\nu}$ that fixes the commutation relations between the operator-valued coordinate components $[X^\mu, X^\nu] = i\theta^{\mu\nu}$, and only the space-space noncommutativity, i.e. the special case of $\theta^{0\nu} = 0$ in a certain Lorentz frame, was considered.

The extension (size) a of the charge in Eq. (1) should be kept nonzero in the spirit of NC theory that does not admit objects with their size smaller than the fundamental length $l = \sqrt{\theta}$, where $\theta = |\theta|$. For a point charge a magnetic solution also exists [4], although in this case it is not a magnetic dipole one. What is more important is that that solution is too singular in the point $r = 0$, where the charge is located, and hence it cannot be given a mathematical sense in terms of the distribution theory in a conventional way.

If we understand the radius a in Eq. (1) as the size of an electrically charged fundamental particle ($Z = 1$), we can speculate on what the contribution of the noncommutativity into its magnetic moment \mathbf{M} may be. Certainly, this is expected to be very small, because of the extreme smallness of the noncommutativity parameter θ . It is primarily supposed [5] that the corresponding length $l = \sqrt{\theta}$ should be of the Plank scale of $l \sim 10^{-33}$ cm (or $\Lambda_{\text{Pl}} \sim 4 \cdot 10^{19}$ GeV in energy units). The reason is that at so small distances unification of gravity with quantum mechanics requires quantization of space-time. Although the Plank scale is far beyond any experimental reach, the everlasting problem is to estimate the upper limits on θ basing on the existing and advancing experimental preciseness. In Ref. [1] it was discussed what new restrictions on the extent of noncommutativity may follow from the newly established fact that a charged fundamental particle is a carrier of the magnetic moment (1) in an NC theory, irrespective of its orbital momentum or spin. In the present article we shall further elaborate this matter addressing the charged leptons e and μ as the “smallest”—and hence providing the maximum contribution of Eq. (1)—particles, to leave alone quarks—also small, but whose magnetic moment is beyond measurements.

II. UPPER BOUNDS FOR FUNDAMENTAL LENGTH FROM NONCOMMUTATIVE MAGNETIC MOMENT

A. Limitations based on high-energy scattering estimates of lepton sizes

In high-energy electron-positron collisions leptons manifest themselves as structureless particles (see e.g. Ref. [2] for an early discussion of this point), described by a fundamental (local), not composite field. No deviation from this rule has been up to now reported. Taking the LEP scale of 200 GeV as an upper limit, to which this statement may be thought of as checked, we must accept that a possible structurelessness of these leptons is below the length (call it divisibility length) $r_0 = 10^{-3}$ Fm. In our further consideration we identify the charge extension a with the divisibility length, because it is hard to imagine a region occupied by a charge that extends above this length, but cannot be divided into parts. (If it could, either the resulting charge would acquire a continuous value, smaller

than e , which contradicts basic assumptions, or the resulting charge would occupy a smaller space and we would be left again with smaller a , down to the divisibility length.)

1. Electron

Bearing in mind that, for electron, the existing local theory perfectly explains the value of its magnetic moment M_e , we expect that the noncommutativity might only contribute into the experimental and theoretical uncertainty δM_e existing in measuring and calculating this quantity. A recent direct measurement of the anomalous magnetic moment of electron, using the magnetic resonance spectroscopy of an individual electron in the Penning trap [3], gives the result [6,7],

$$\left(\frac{M_e}{\mu} - 1\right)\Big|_{\text{MRS}} = 0.00115965218073 \pm 28 \cdot 10^{-14}, \quad (2)$$

where $\mu = e/2m$ is the Bohr magneton. On the other hand, a new report [8] appeared on an *independent* experimental determination of the same magnetic moment with a matching accuracy, obtained with the use of a measurement of the ratio h/m_{Rb} between the Plank constant and the mass of the ^{87}Rb atom. The result is

$$\left(\frac{M_e}{\mu} - 1\right)\Big|_{\text{Rb}} = 0.00115965218113 \pm 84 \cdot 10^{-14}. \quad (3)$$

Authors of Ref. [8] fit the value of the fine structure constant α in such a way as to make Eq. (3) coincide with the theoretical prediction for the electron anomalous magnetic moment, calculated (see Ref. [9] for a review) with the accuracy, including QED calculations up to $(\alpha/\pi)^4$, also electroweak and hadronic contributions (this fit leads to the so far most precise value $\alpha^{-1} = 137.035999037(91)$). For this reason the value (3) is referred to as “theoretical”. (Certainly, the roles of Eqs. (3) and (2) might be reversed.) The theoretical, Eq. (3), and experimental, Eq. (2), values of the electron magnetic moment do not contradict each other, demonstrating the hitherto best confirmation of QED. The discrepancy between them

$$\frac{\delta M_e}{\mu} \sim 10^{-12} \quad (4)$$

lies within the accuracy of measurements and calculations. We demand that a possible contribution of the noncommutative magnetic moment in Eq. (1) should not exceed it:

$$\frac{\delta M_e}{\mu} > \alpha \theta \frac{4m}{5a}, \quad \alpha = e^2. \quad (5)$$

With the high-energy restriction on the size $a < r_0$ accepted above, Eq. (5) implies $\theta < \frac{\delta M_e}{\mu} (5r_0/4m\alpha)$. As $r_0 \sim 10^{-3}$ Fm, we get from here and from Eq. (4) the restriction on the fundamental length $l = \sqrt{\theta} < 7 \cdot 10^{-6}$ Fm = $(28 \text{ TeV})^{-1}$.

2. Muon

The matters stand differently with the μ -meson. In the literature, its anomalous magnetic moment is calculated via the standard model with the inclusion of the QED lowest-order μ - γ vertex, Z-boson, neutrino and hadron lines. The deviation of the measured magnetic moment M_μ from the result of calculations makes the value (see A. Hocker’s and W. J. Marciano’s 2009 update in Ref. [7], also Ref. [9] for a later detailed account)

$$\frac{\delta M_\mu}{\mu} \simeq 25 \cdot 10^{-10}. \quad (6)$$

This exceeds about 3.2 times the estimated 1σ error [7]. It is believed that this discrepancy may be overcome by including supersymmetry for amending the theoretical result. If, on the contrary, we try to explain this discrepancy by the effect of NC magnetic moment of the muon, we get in the way similar to the one described above in this Subsection, using Eq. (6) and the same indivisibility length $r_0 \sim 10^{-3}$ Fm that l is smaller than $2.8 \cdot 10^{-5}$ Fm = $(7 \text{ TeV})^{-1}$ as the high-energy based estimate.

B. Ultimate estimates

Once there is no evidence for any electron extension, it is worth admitting that it may be only restricted by the fundamental length. Then, using $a = l = \sqrt{\theta}$ in Eq. (5) and the indeterminacy (4), we obtain the ultimate bound of $l < 6.6 \cdot 10^{-8}$ Fm = $(3 \cdot 10^3 \text{ TeV})^{-1}$. Dealing with the muon in the same way, but referring to Eq. (6) instead of Eq. (4), we obtain the ultimate estimate of $8 \cdot 10^{-7}$ Fm = $(240 \text{ TeV})^{-1}$.

III. UPPER BOUNDS ON FUNDAMENTAL LENGTH VERSUS COMPOSITENESS SIZES OF LEPTONS

There are [2] much stronger restrictions on the lepton sizes than those following from the high-energy collision experiments. These extend to the energy scale far exceeding the accelerator means. The point is that if one imagines a lepton as a bound state of much heavier particles so that the binding energy compensates the most part of their masses to make the resulting state light, the Bohr radius R of the composite state—to be treated as its size—would be much smaller than the Compton length of the lepton λ_C . According to the Drell-Hearn-Gerasimov sum rule (see Ref. [2] for references) the deviation of the anomalous magnetic moment $(M/\mu - 1)$ from its QED value is proportional to the ratio R/λ_C , which is the measure of compositeness. Based on the experimental data on magnetic moments of the known composite particles—proton and triton—plotted against their measured sizes, a conjecture was formulated by Dehmelt [3] that the proportionality coefficient should be of the order of unity. Then, $R = \lambda_C \delta M/\mu$.

A. Electron

Referring to Eqs. (2) and (3), and using Eq. (4) we may update Dehmelt's 1988 result for the electron of $R < 4 \cdot 10^{-8}$ Fm to $R < 4 \cdot 10^{-10}$ Fm. This is two orders of magnitude smaller than our ultimate estimate of 6.6×10^{-8} Fm for the fundamental length obtained in Subsection II B. (The use of the assertion $R = \lambda_C \delta M / \mu$ together with Eq. (5) would result in the condition $l < \sqrt{5/8\alpha}(\delta M / \mu)\lambda_C = 9.25R$, weaker than the already accepted condition that the fundamental length should be smaller than any size, including the composite electron radius R , that is to $l < R$.) So, Dehmelt's conjecture provides a stronger bound on the fundamental length, than the noncommutative magnetic moment. This means that no more than 10^{-2} part of the measured difference (5) may be at the most attributed to noncommutative contribution.

B. Muon

The muon radius estimated analogously, basing on the compositeness arguments and on the theory-experiment

discrepancy (6), gives the result $R_\mu \simeq 0.5 \cdot 10^{-8}$ Fm. This is smaller than the ultimate estimate of Subsection II B based on muon data. Again, once the muon size cannot be smaller than the fundamental length, this result indicates that the NC magnetic moment alone definitely cannot take on the responsibility for the discrepancy (6) between the theory and experiment, and hence deviations from the standard model other than NC electrodynamics are needed. Unlike the electron case above, one cannot set one's hopes upon the future growth of precession of measurements to abandon this conclusion.

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