

Reducing the parameter space for unparticle-inspired models using white dwarf masses

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Based on astrophysical constraints derived from Chandrasekhar's mass limit for white dwarfs, we study the effects of the model on the parameters of unparticle-inspired gravity, on scales $\Lambda_U > 1$ TeV and $d_U \approx 1$.

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I. INTRODUCTION

Many proposals for explaining the apparent shortcomings of the standard model have been advanced. The unparticle model proposed by Georgi [1] aimed to include in the standard model massive but scale-invariant particles, sharing the same physics of the scale-dependent counterparts. These objects, called “unparticles,” could play an important role in low-energy physics [2], since the model implies that unparticles can be exchanged between massive particles, leading to a new force called “ungravity.” This “fifth” force would add a perturbation term to the Newtonian gravitational potential, although the exact potential cannot be obtained because the distance at which the perturbed potential matches the Newtonian expression needs to be known. In order to bypass this limitation, the perturbed potential has been assumed to be of the form [2]

$$V(r) = -\frac{GM}{2r} \left[1 + \left(\frac{R_G}{r} \right)^{2d_U-2} \right], \quad (1)$$

where d_U (the scaling dimension of the unparticles operator O_U) is ≈ 1 , as a reasonable approximation, and R_G is the characteristic length scale of ungravity, given by

$$R_G = \frac{1}{\pi \Lambda_U} \left(\frac{M_{Pl}}{M_*} \right)^{1/(d_U-1)} \times \left[\frac{2(2-\alpha) \Gamma(d_U + 1/2) \Gamma(d_U - 1/2)}{\pi \Gamma(2d_U)} \right]^{1/(d_U-1)}, \quad (2)$$

where Λ_U is the energy scale of the unparticle interactions, $M_{Pl} = 2.4 \times 10^{18}$ GeV is the Planck mass, and α is a constant dependent on the type of propagator considered.

The problem addressed in this work is to determine the bounds of the mass of the interaction (un)particle M_* with $d_U \approx 1$. For this purpose, a suitable quasi-Newtonian gravitational system needs to be studied and compared with the pure Newtonian results. A first study of this regime by Bertolami, Páramos, and Santos [3] has addressed the stellar equilibrium problem, deriving a perturbed Lane-Emden equation further applied to the Sun. They explored the well-known similarity of the full stellar

structure to an $n = 3$ polytropic model, and derived limits from the maximum allowed uncertainty in the central temperature $\Delta T_c/T_c = 0.06$.

In spite of the successful derivation of meaningful limits to the unparticle parameters, it is known that the detailed structure of the Sun is actually quite complicated, and many physical factors have to be considered beyond the simplest Chandrasekhar's polytropic model [4]. Therefore, it is worth considering another very well-known system to which the Chandrasekhar theory gives an even better representation: the white dwarf sequence. We shall show below that an important feature of these sequences (the maximum mass) is sensitive to the unparticle quantities and allows us to impose strong limits on them.

II. STELLAR EQUILIBRIUM AND WHITE DWARFS

Since Chandrasekhar's polytropic model is widely known, we briefly recall how the unparticle theory modifies it, as first shown by Bertolami, Páramos, and Santos [3]. The equations of stellar hydrostatic equilibrium and mass conservation can be reduced to a second order differential equation, if a polytropic equation of state of the form $P = K\rho^{1+1/n}$ is assumed to hold. If the density is written as $\rho = \rho_c \theta^n$, and the radius as $r = \beta \xi$, one can easily obtain the original Lane-Emden equation

$$\frac{1}{\xi^2} \frac{d}{d\xi} \left(\xi^2 \frac{d\theta}{d\xi} \right) = -\theta^n, \quad (3)$$

where n is the polytropic index, ξ is the dimensionless radius, and β is given by

$$\beta = \left[\frac{(n+1)K}{4\pi G} \rho_c^{(1/n)-1} \right]^{1/2}, \quad (4)$$

K being the polytropic constant dependent on the specific value of n . This equation is subject to the usual boundary conditions: $\rho(r=0) = \rho_c$ and $dP/dr = 0$ for $r=0$, which translates to $\theta(\xi=0) = 1$ and $\theta'(\xi=0) \equiv d\theta/d\xi = 0$. The detailed derivation of the Lane-Emden equation can be consulted in the classical reference [4]. These results can be used to derive the mass-radius relation, given by Eq. (5):

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$$m(r) = 4\pi \left(\frac{r}{\xi_*}\right)^{(3-n)/(1-n)} \left(\frac{(n+1)K}{4\pi G}\right)^{n/(n-1)} \xi_*^2 |\theta'(\xi_*)|. \quad (5)$$

Considering that white dwarfs are small stars composed by electron-degenerate matter, in which the core material no longer undergoes fusion reactions, the Lane-Emden equation describes very well their behavior as a result of the proximity of the electronic component to a polytropic form. As is well known, in the nonrelativistic limit the white dwarf matter can be represented by a $n = 3/2$ polytrope and in the relativistic case the $n = 3$ is quite accurate.

The same technique employed by Chandrasekhar can be used to obtain the perturbed Lane-Emden equation. From the perturbed potential given by Eq. (1), the perturbed gravitational acceleration can be obtained via $\vec{F} = -\vec{\nabla}V$ [5], used to determine the hydrostatic equilibrium equation, as seen in Eq. (6):

$$\frac{dP}{dr} = -\frac{GM\rho}{2r^2} \left[1 + (2d_U - 1) \left(\frac{R_G}{r}\right)^{2d_U-2} \right]. \quad (6)$$

Then, from this hydrostatic equilibrium equation, Bertolami, Páramos, and Santos derived a perturbed Lane-Emden equation

$$\frac{1}{\xi^2} \frac{d}{d\xi} \left(\xi^2 \frac{d\theta}{d\xi} \right) = \frac{-\theta^n}{2} \left[1 + (2d_U - 1) \left(\frac{\xi_G}{\xi}\right)^{2d_U-2} \right] - (2d_U - 1)(d_U - 1) \frac{1}{\xi} \frac{d\theta}{d\xi} \left(\frac{\xi_G}{\xi}\right)^{2d_U-2} \quad (7)$$

in order to examine meaningful bounds on the parameters R_G in Eq. (2), from which one can obtain the bounds of the mass of the interaction (un)particle M_* based on astrophysical constraints (the parameter $\xi_G = R_G/\beta$ has been defined here to simplify the expressions). We have corrected a slight misprint in the paper [3] which would preclude the derivation of Eq. (7), related to the definition of the variable β as given by (4).

Once these matters are settled, the perturbed mass-radius relation can be obtained in the usual manner, being formally identical to the analogous mass-radius relation given by Chandrasekhar [Eq. (5)], but with the perturbative effects present through the solutions of the perturbed Lane-Emden equation and its derivative ($\theta(\xi_1)$ and $\theta'(\xi_1)$), respectively, both evaluated at the first zero as in Ref. [4]. This modification sets the stage for an analysis leading to novel bounds on the unparticle parameters. Since we are interested on the maximum possible mass for a white dwarf, we set the parameters related to the Lane-Emden model to the relativistic limit (polytropic index $n = 3$), and the white dwarf mass-radius relation becomes

$$M = 4\pi \left(\frac{K}{\pi G}\right)^{3/2} \xi_*^2 |\theta'(\xi_*)| \quad (8)$$

losing any dependence on the radius, as expected.

The key feature pointed out above, related to the explicit dependence of the maximum mass through the zero of the Lane-Emden function and its slope, leads to quite a strong dependence of the maximum mass with d_U and R_G , which can be substantially different from the ‘‘canonical’’ $M_{Ch} = 1.457(2/\mu_e)^2 M_\odot$ [4,6].

III. OBSERVATIONS AND ANALYSIS

Following the framework presented by Bertolami, Páramos, and Santos, we obtained the numerical solutions of Eq. (7) varying both parameters ξ_G and d_U , and used these solutions on Eq. (8) to determine the masses associated to the combination of d_U and ξ_G . It is important to note that the combination ξ_G and d_U generated a broad range of masses, spanning from $M \approx 1.291M_\odot$ to $M \approx 1.874M_\odot$. This in turn means that it is possible to constrain the parameters based on the maximum mass for white dwarfs, because this mass must comply with the values obtained by observational data. Following this reasoning, we assumed three reasonable values for the maximum mass

- (i) the maximum observed masses in a large white dwarf sample, from the recent work by Kepler and collaborators [7], is $M = 1.33M_\odot$. Nevertheless, in order to effectively constrain the values of d_U near 1, we had to consider masses of at least $M = 1.36M_\odot$, otherwise d_U would be far from 1 and the gravitational corrections in Eq. (1) would be too large;
- (ii) the ‘‘canonical’’ limit, $M_{Ch} = 1.457M_\odot$ for carbon-type white dwarfs; in spite of the fact that this widely accepted value is still beyond the actual observed maximum, its use seems very reasonable;
- (iii) an even larger value, above the former ‘‘Chandrasekhar’s’’ limit, $M = 1.60M_\odot$, arbitrarily chosen to represent an extreme limit allowed in nature. If true, the number of objects between the actually observed maximum $M = 1.33M_\odot$ and this proposed extreme value of $M = 1.60M_\odot$ must be substantial in a large sample such as the one analyzed by Kepler *et al.* [7], although none has been actually reported.

From the data above mentioned, we constructed the contour plots where the combination of d_U and ξ_G generates the desired maximum mass. This plots are depicted in Fig. 1 for ranges larger than the radius of the star ($\xi_G > 1$, $d_U \lesssim 1$) and in Fig. 2 for ranges smaller the radius of the star ($\xi_G < 1$, $d_U \gtrsim 1$). One should note that for $d_U \gtrsim 1$, only masses of $M = 1.36M_\odot$ and $M = 1.457M_\odot$ are obtained, while a mass of $M = 1.6M_\odot$ can be imposed for $d_U \lesssim 1$ only.

The connection with the (un)particle mass follows by solving equation Eq. (2), for the ratio M_*/M_{Pl} , namely

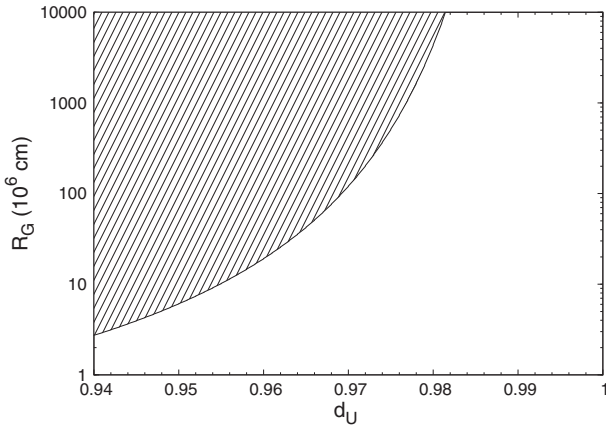


FIG. 1. Contour plot (*locus*) of maximum mass ($M = 1.6M_\odot$) in the R_G - d_U plane, for an interaction range larger than the radius of the star. The dashed area gives masses bigger than $M = 1.6M_\odot$, and should be considered as forbidden.

$$\frac{M_*}{M_{pl}} = [\pi \Lambda_U R_G(d_U)]^{1-d_U} \times \left[\frac{2(2-\alpha) \Gamma(d_U + 1/2) \Gamma(d_U - 1/2)}{\pi \Gamma(2d_U)} \right]^{1/2}, \quad (9)$$

which, by using $R_G(d_U)$ defined in the plots above, it is possible to infer the mass of the interaction (un)particle M_* . We point out that this procedure differs significantly from the method used by Bertolami, Páramos, and Santos [3], where the 6% uncertainty leads to terms $R_-(d_U)$ and $R_+(d_U)$, because we use exact values for the maximum

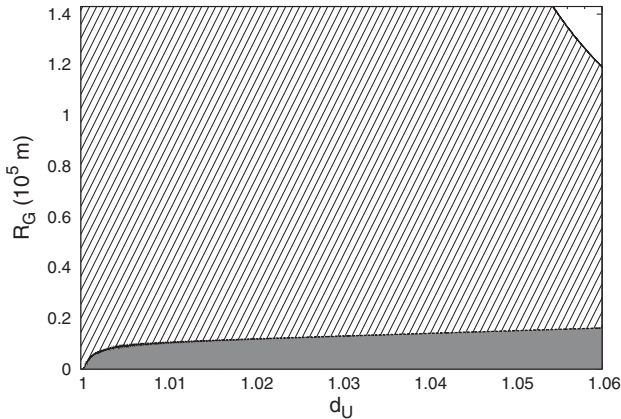


FIG. 2. Contour plot (*locus*) of maximum masses $M = 1.36M_\odot$ and $M = 1.457M_\odot$ in the R_G - d_U plane, for an interaction range smaller than the radius of the star. The curve spanning from $1.055 < d_U < 1.06$ (upper right corner of the graphic) produces a maximum mass of $M = 1.36M_\odot$ and the curve between $1 < d_U < 1.06$ (lower side of the graphic) the mass of $M = 1.457M_\odot$. The upper white area gives maximum masses lower than $M = 1.36M_\odot$, the dashed area yields masses in the range $1.36M_\odot < M < 1.457M_\odot$, and the lower area produces masses bigger than $1.457M_\odot$. If one accepts the latter value as the maximum allowed, the dark area is forbidden.

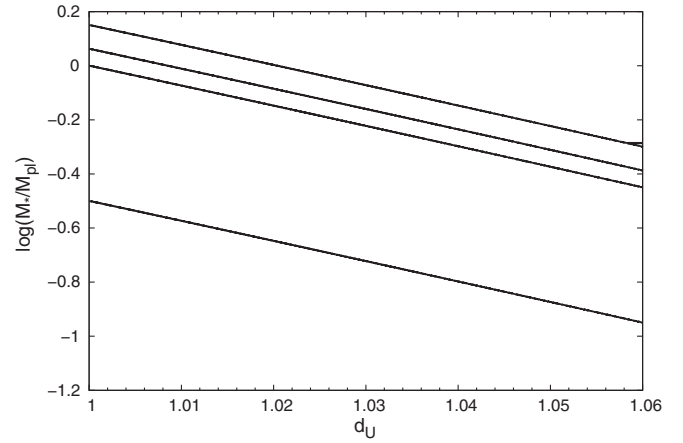


FIG. 3. Lower bounds on $\log(M_*/M_{pl})$, for $\alpha = 0$, $\alpha = 2/3$, $\alpha = 1$, $\alpha = 1.9$, from top to bottom respectively. The fixed values are $\Lambda_U = 1$ TeV and $M = 1.457M_\odot$.

masses. Nevertheless, it is clear that the ratio must be interpreted as a lower bound to the ratio M_*/M_{pl} , considering that lower maximum masses (which would be in conflict with observations, as stated before) would result in even lower ratios. Plotting the ratio as a function of d_U and fixing the parameters α and Λ_U in the same way as Bertolami, Páramos, and Santos, we obtain the ranges for the mass of the interaction (un)particle depicted in Figs. 3 and 4 for the case of the “canonical” mass and two different choices of Λ_U .

Analyzing the data for $M = 1.36M_\odot$, d_U s fall in the range $1.055 < d_U < 1.060$. Therefore, this maximum mass yields a lower bound to M_* in the range $(0.03\text{--}0.50)M_{pl}$. For $M = 1.457M_\odot$, d_U lies in the range $1 < d_U < 1.06$, yielding a lower bound to M_* in the range $(0.1\text{--}1.6)M_{pl}$. In the last case, with an assumed maximum mass $M = 1.6M_\odot$ and the interaction range larger than the star radius, the only allowed values for

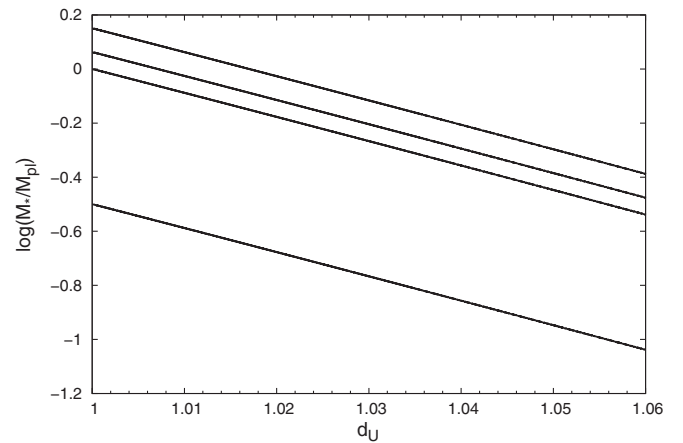


FIG. 4. Lower bounds on $\log(M_*/M_{pl})$, for $\alpha = 0$, $\alpha = 2/3$, $\alpha = 1$, $\alpha = 1.9$ from top to bottom respectively. The fixed values are $\Lambda_U = 10^3$ TeV and $M = 1.457M_\odot$.

d_U 's are in the range $0.94 < d_U < 0.982$, and lower bounds to M_* in the range $(0.5\text{--}6.3)M_{Pl}$.

IV. CONCLUSIONS

We have shown in this work that quite strong limits to the unparticle parameters can be obtained by using a simple form of the polytropic theory of Chandrasekhar adding a perturbation to the Lane-Emden equation, as first obtained by Bertolami, Páramos, and Santos [3], and applying it to the white dwarf sequences.

The key point elaborated here is that a change on the unparticle parameters would affect the maximum mass allowed to white dwarfs, and thereby we explored this characteristic in order to limit the values of such parameters.

The requirement that the maximum mass cannot be too small (because it would conflict with a few massive stars [7] or too big (because it would lead to unobserved super-massive white dwarfs) limit the values of M_* to a confidence range of $0.1M_{Pl} < M_* < 1.6M_{Pl}$ from this analysis alone for the case $d_U \gtrsim 1$. For the case $d_U \lesssim 1$, the mass-radius relation gives only masses bigger than the canonical

value. Considering that until today there is no observation of white dwarfs with such high masses, this analysis may be interpreted to mean that values of $d_U < 1$ are not allowed.

Following a different approach, based on a cosmological scenario, Bertolami and Santos [8] considered the variation of the gravitational coupling at the time of big bang nucleosynthesis, tensor exchange, and the scaling dimension $d_U = 1.1$, and found M_* to be $>0.05M_{Pl}$, which is very close to the bounds found for $1 < d_U < 1.06$. Other works studying complementary bounds [3,9–13] could be combined to address the viability of a general unparticle model, unless one can manage to evade the bounds altogether. Even if so, a general argument to constrain the admissible perturbations to the Newtonian potential can be made *via* the perturbed Lane-Emden equation, resorting to the observed massive white dwarfs.

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