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Quark mass hierarchy in 3-3-1 models

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We study the mass spectrum of the quark sector in a special type-I-like model with gauge symmetry $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$. By considering couplings with scalar triplets at large ($\sim$ TeV) and small ($\sim$ GeV) scales, we obtain specific zero-texture mass matrices for the quarks which predict three massless quarks (u, d, s) and three massive quarks (c, b, t) at the electroweak scale ($\sim$ GeV). Taking into account mixing couplings with three heavy quarks at large scales predicted by the model, the three massless quarks obtain masses at small order that depend on the inverse of the large scale. Thus, masses of the form $m_u \lesssim m_d < m_s \sim$ MeV and $m_{c,b,t} \sim$ GeV can be obtained naturally from the gauge structure of the model.

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I. INTRODUCTION

Although the Standard Model (SM) [1] is the simplest model that successfully explains most of the phenomena and experimental observations in particle physics, it contains unanswered fundamental questions which many theorists associate with an underlying theory beyond the SM. In particular, the observed fermion mass hierarchies, their mixing, and the three-family structure are not explained in the SM. From the phenomenological point of view, it is possible to describe some features of the mass hierarchy by assuming zero-texture Yukawa matrices [2]. Models with spontaneously broken flavor symmetries may also produce hierarchical mass structures. These horizontal symmetries can be continuous and Abelian, as the original Froggatt-Nielsen model [3], or non-Abelian as, for example, SU(3) and SO(3) family models [4]. Models with discrete symmetries may also predict mass hierarchies for leptons [5] and quarks [6]. Other models with horizontal symmetries have been proposed in the literature [7].

On the other hand, the origin of the family structure of the fermions can be addressed in family-dependent models where a symmetry distinguishes fermions of different families. An interesting alternative that may provide a clue to this puzzle are the models with gauge symmetry $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$, also called 3-3-1 models, which introduce a family nonuniversal $U(1)_X$ symmetry [8–11]. These models have a number of phenomenological advantages. First of all, from the cancellation of chiral anomalies [12] and asymptotic freedom in QCD, the 3-3-1 models can explain why there are three fermion families. Secondly, since the third family is treated under a different representation, the large mass difference between the heaviest quark family and the two lighter ones may be understood [13]. Also, these models contain a natural Peccei-Quinn symmetry, necessary to solve the strong-*CP* problem [14].

In particular, the 3-3-1 models introduce three $SU(3)_L$ scalar triplets: one heavy triplet field with a vacuum expectation value (VEV) at high energy scale $\langle\chi\rangle = v_\chi$, which produces the breaking of the symmetry $SU(3)_L \otimes U(1)_X$ into the SM electroweak group $SU(2)_L \otimes U(1)_Y$, and two lighter triplets with VEVs at the electroweak scale $\langle\rho\rangle = v_\rho$ and $\langle\eta\rangle = v_\eta$, which induce the electroweak breakdown. Thus, the model may provide masses to all fermions and gauge bosons at tree level. On the other hand, the 3-3-1 model possess a specialized two Higgs doublet model type III (2HDM-III) in the low energy limit, where both electroweak triplets ρ and η are decomposed into two hypercharge-one $SU(2)_L$ doublets plus charged and neutral singlets. Thus, like the 2HDM-III, the 3-3-1 model can predict huge flavor changing neutral currents (FCNC) and *CP*-violating effects, which are severely suppressed by experimental data at electroweak scales. One way to remove these effects is by imposing discrete symmetries, obtaining two types of 3-3-1 models (type-I and -II models), which exhibits the same Yukawa interactions as the 2HDM type I and II at low energy. In the first case, one Higgs electroweak triplet (for example, ρ) provides masses to the phenomenological up- and down-type quarks simultaneously. In the type-II model, one Higgs triplet (ρ) gives masses to the up-type quarks and the other triplet (η) to the down-type quarks. In this paper we obtain in the framework of the *I*-type model specific mass matrix structures from the gauge symmetry, where only one of the down-type quarks acquires mass (which could be associated with the phenomenological bottom quark), and two are massless (d and s quarks), while two of the up-type quarks acquire masses (c and t quarks) and one is massless (u quark). We also show by the method of recursive expansion [15] that if mixing couplings with the heavy quark sector of the 3-3-1 model is considered, then the massless quarks indeed may obtain masses at small order that depends on the inverse of the heavy scale (represented by three heavy quarks) without introducing either effective operators or one-loop corrections [16,17]. Thus, at first glance we can obtain

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masses with the structures $m_u \lesssim m_d < m_s \sim \text{MeV}$ and $m_b \sim m_c \sim m_t \sim \text{GeV}$.

This paper is organized as follows. In Sec. II we briefly describe some theoretical aspects of the 3-3-1 model and its particle content, in particular, in the fermionic and scalar sector in order to obtain the mass spectrum. Section III is devoted to obtain the mass matrices in the low energy limit. In Sec. IV we consider the method of recursive expansion to diagonalize the mass matrices taking into account mixing couplings between light and heavy fermions. Finally in Sec. V, we summarize and discuss our results.

II. YUKAWA COUPLINGS OF THE 3-3-1 MODEL

We consider a 3-3-1 model where the electric charge is defined by

$$Q = T_3 - \frac{1}{\sqrt{3}} T_8 + X, \tag{1}$$

with $T_3 = \frac{1}{2}\,\mathrm{Diag}(1, -1, 0)$ and $T_8 = (\frac{1}{2\sqrt{3}})\,\mathrm{Diag}(1, 1, -2)$. In order to avoid chiral anomalies, the model introduces in the fermionic sector the following $(SU(3)_c, SU(3)_L, U(1)_X)$ left-handed representations: one $(3, 3, 1/3)$ quark triplet, two $(3, 3^*, 0)$ quark triplets, and three $(1, 3, -1/3)$ lepton triplets. For the right-handed sector, we introduce the following singlets in order to obtain Dirac-type charged fermions: three $(3^*, 1, -1/3)$ down-type quarks, three $(3^*, 1, 2/3)$ up-type quarks, three $(1, 1, -1)$ electron-type leptons. In addition we introduce three $(3^*, 1, Q_{J_{2,3},T_1})$ and three $(1, 1, 0)$ right-handed singlets associated with the new non-SM quarks and neutral Majorana leptons, respectively. In summary, we have the following representations free from chiral anomalies:

$$\begin{aligned}
Q_L^1 &= \begin{pmatrix} U^1 \\ D^1 \\ T^1 \end{pmatrix}_L : (3, 3, 1/3), \quad \begin{cases} U_R^1\text{: } (3^*, 1, 2/3) \\ D_R^1\text{: } (3^*, 1, -1/3) \\ T_R^1\text{: } (3^*, 1, 2/3), \end{cases} \\
Q_L^{2,3} &= \begin{pmatrix} D^{2,3} \\ U^{2,3} \\ J^{2,3} \end{pmatrix}_L : (3, 3^*, 0), \quad \begin{cases} D_R^{2,3}\text{: } (3^*, 1, -1/3) \\ U_R^{2,3}\text{: } (3^*, 1, 2/3) \\ J_R^{2,3}\text{: } (3^*, 1, -1/3), \end{cases} \\
L_L^{1,2,3} &= \begin{pmatrix} \nu^{1,2,3} \\ e^{1,2,3} \\ (\nu^{1,2,3})^c \end{pmatrix}_L : (1, 3, -1/3), \quad \begin{cases} e_R^{1,2,3}\text{: } (1, 1, -1) \\ N_R^{1,2,3}\text{: } (1, 1, 0), \end{cases}
\end{aligned} \tag{2}$$

where U_L^i and D_L^i for $i = 1, 2, 3$ are three up- and down-type quark components in the flavor basis, while ν_L^i and e_L^i are the neutral and charged lepton families. The right-handed sector transforms as singlets under $SU(3)_L$ with $U(1)_X$ quantum numbers equal to the electric charges. In addition, we see that the model introduces heavy fermions with the following properties: a single flavor quark T^1 with electric charge $2/3$, two flavor quarks $J^{2,3}$ with charge $-1/3$, three neutral Majorana leptons $(\nu^{1,2,3})_L^c$, and three right-handed Majorana leptons $N_R^{1,2,3}$ (recently, a discussion about neutrino masses via double and inverse seesaw mechanisms was performed in Ref. [18]). On the other hand, the scalar sector introduces one triplet field with VEV $\langle \chi \rangle_0 = \upsilon_\chi$, which provides the masses to the new heavy fermions, and two triplets with VEVs $\langle \rho \rangle_0 = \upsilon_\rho$ and $\langle \eta \rangle_0 = \upsilon_\eta$, which give masses to the SM fermions at the electroweak scale. The $(SU(3)_L, U(1)_X)$ group structure of the scalar fields are

$$\begin{aligned}
\chi &= \begin{pmatrix} \chi_1^0 \\ \chi_2^- \\ \frac{1}{\sqrt{2}}(\upsilon_\chi + \xi_\chi \pm i\zeta_\chi) \end{pmatrix} : (3, -1/3), \\
\rho &= \begin{pmatrix} \rho_1^+ \\ \frac{1}{\sqrt{2}}(\upsilon_\rho + \xi_\rho \pm i\zeta_\rho) \\ \rho_3^+ \end{pmatrix} : (3, 2/3), \\
\eta &= \begin{pmatrix} \frac{1}{\sqrt{2}}(\upsilon_\eta + \xi_\eta \pm i\zeta_\eta) \\ \eta_2^- \\ \eta_3^0 \end{pmatrix} : (3, -1/3).
\end{aligned} \tag{3}$$

With the above spectrum, we obtain the following $SU(3)_L \otimes U(1)_X$ renormalizable Yukawa Lagrangian for the quark sector:

$$\begin{aligned}
-\mathcal{L}_Y = {}& \bar{Q}_L^1(\eta h_{\eta 1j}^U + \chi h_{\chi 1j}^U)U_R^j + \bar{Q}_L^1 \rho h_{\rho 1j}^D D_R^j \\
&+ \bar{Q}_L^1 \rho h_{\rho 1m}^J J_R^m + \bar{Q}_L^1(\eta h_{\eta 11}^T + \chi h_{\chi 11}^T)T_R^1 \\
&+ \bar{Q}_L^n \rho^* h_{\rho nj}^U U_R^j + \bar{Q}_L^n(\eta^* h_{\eta nj}^D + \chi^* h_{\chi nj}^D)D_R^j \\
&+ \bar{Q}_L^n(\eta^* h_{\eta nm}^J + \chi^* h_{\chi nm}^J)J_R^m \\
&+ \bar{Q}_L^n \rho^* h_{\rho n1}^T T_R^1 + \text{H.c,}
\end{aligned} \tag{4}$$

where $n = 2, 3$ is the index that labels the second and third quark triplet shown in Eq. (2), and $h_{\phi ij}^f$ are the i, j components of nondiagonal matrices in the flavor space associated with each scalar triplet ϕ: η, ρ, χ. In order to avoid FCNC terms at tree level, we demand the following discrete symmetry:

$$\begin{aligned}
&\eta \rightarrow -\eta, \quad \rho \rightarrow \rho, \qquad D_R \rightarrow D_R, \quad U_R \rightarrow U_R, \\
&\qquad T_R \rightarrow T_R, \quad J_R \rightarrow J_R.
\end{aligned} \tag{5}$$

Thus, the couplings of the quarks with the triplet η are removed from the Lagrangian in (4), which at low energy is equivalent to 2HDM type I. After the symmetry breaking of the 3-3-1 gauge group, and using Eqs. (3)–(5), we obtain the following mass terms:

$$-\langle \mathcal{L}_Y \rangle = (\bar{U}_L, \bar{T}_L) M_{UT} \begin{pmatrix} U_R \\ T_R \end{pmatrix} + (\bar{D}_L, \bar{J}_L) M_{DJ} \begin{pmatrix} D_R \\ J_R \end{pmatrix} + \text{H.c,} \tag{6}$$

where $U_{L,R} = (U^1, U^2, U^3)_{L,R}$ are the left- and right-handed up-type quark flavor vectors, $D_{L,R} = (D^1, D^2, D^3)_{L,R}$ the corresponding down-type quark vectors, $J_{L,R} = (J^2, J^3)_{L,R}$

are two-dimensional vectors associated with the heavy quarks with electric charge $-1/3$ in (2), and $T_{L,R}$ is the single component of the heavy quark with charge $2/3$. The matrices M_{UT} and M_{DJ} have the following structures in the basis (U, T) and (D, J), respectively:

$$M_{UT} = \begin{pmatrix} M_U & k \\ K & M_T \end{pmatrix}, \quad M_{DJ} = \begin{pmatrix} M_D & s \\ S & M_J \end{pmatrix}, \tag{7}$$

where M_U, k, K, and M_T are 3×3, 3×1, 1×3, and 1×1 matrices, respectively, while M_D, s, S, and M_J are 3×3, 3×2, 2×3, and 2×2 matrices, respectively. The Yukawa Lagrangian in (4) for the model type I provides the following relations between the above mass matrices and the Yukawa couplings through the VEVs:

$$\begin{aligned} M_U &= \frac{1}{\sqrt{2}} h_\rho^U v_\rho, & M_T &= \frac{1}{\sqrt{2}} h_\chi^T v_\chi, \\ k &= \frac{1}{\sqrt{2}} h_\rho^T v_\rho, & K &= \frac{1}{\sqrt{2}} h_\chi^U v_\chi \end{aligned} \tag{8}$$

for M_{UT}, and

$$\begin{aligned} M_D &= \frac{1}{\sqrt{2}} h_\rho^D v_\rho, & M_J &= \frac{1}{\sqrt{2}} h_\chi^J v_\chi, \\ s &= \frac{1}{\sqrt{2}} h_\rho^J v_\rho, & S &= \frac{1}{\sqrt{2}} h_\chi^D v_\chi \end{aligned} \tag{9}$$

for M_{DJ}. We can see in the Yukawa Lagrangian in Eq. (4) that due to the nonuniversal form of the $U(1)_X$ values exhibited by the scalar and quark triplets in (2) and (3), not all couplings between quarks and scalars are allowed by the gauge symmetry, which leads us to the following zero-texture Yukawa coupling constants:

$$h_\rho^U = \begin{pmatrix} 0 & 0 & 0 \\ a & b & c \\ d & e & f \end{pmatrix}, \qquad h_\chi^U = (a'', b'', c''), \tag{10}$$

for the couplings with up-type quarks,

$$h_\rho^D = \begin{pmatrix} A & B & C \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad h_\chi^D = \begin{pmatrix} A'' & B'' & C'' \\ D'' & E'' & F'' \end{pmatrix}, \tag{11}$$

for the couplings with down-type quarks,

$$h_\rho^J = \begin{pmatrix} w & x \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \qquad h_\chi^J = \begin{pmatrix} w'' & x'' \\ y'' & z'' \end{pmatrix}, \tag{12}$$

for the couplings with the doublets J, and

$$h_\rho^T = \begin{pmatrix} 0 \\ W \\ X \end{pmatrix}, \qquad h_\chi^T = W'', \tag{13}$$

for the couplings with the single T quark.

III. MASS MATRICES IN THE LOW ENERGY LIMIT

In the low energy limit ($v_\chi \gg v_{\eta,\rho}$), the quark mass eigenstates for the small and large scales can be obtained separately by unitary transformations of the left- and right-handed weak eigenstates: $U'_{L,R} = V_{L,R}^{U\dagger} U_{L,R}$, $D'_{L,R} = V_{L,R}^{D\dagger} D_{L,R}$, and $J'_{L,R} = V_{L,R}^{J\dagger} J_{L,R}$, while the singlet T quark decouples from other components, obtaining $T'_{L,R} = T_{L,R}$. Thus, the matrices for U,T,D, and J quarks in Eqs. (8) and (9) are diagonalized by

$$\begin{aligned} m_U &= V_L^{U\dagger} M_U V_R^U = \frac{v_\rho}{\sqrt{2}} V_L^{U\dagger} h_\rho^U V_R^U, \\ m_D &= V_L^{D\dagger} M_D V_R^D = \frac{v_\rho}{\sqrt{2}} V_L^{D\dagger} h_\rho^D V_R^D, \\ m_J &= V_L^{J\dagger} M_J V_R^J = \frac{v_\chi}{\sqrt{2}} V_L^{J\dagger} h_\chi^J V_R^J, \\ m_T &= \frac{v_\chi}{\sqrt{2}} h_\chi^T. \end{aligned} \tag{14}$$

Thus, the mass matrices for U- and D-type quarks depend only on the h_ρ Yukawa matrices, which from (8)–(11) become

$$M_U = \frac{v_\rho}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ a & b & c \\ d & e & f \end{pmatrix}, \quad M_D = \frac{v_\rho}{\sqrt{2}} \begin{pmatrix} A & B & C \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{15}$$

which diagonalize through the biunitary transformations $V_{L,R}^{U,D}$. Let us evaluate the eigenvalues of the square mass matrices.

A. Up sector

From (15), we obtain the following structure:

$$M_U M_U^\dagger = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \beta^* & \gamma \end{pmatrix}, \tag{16}$$

where α, β, and γ are of the order $v_\rho^2 \sim (246\ \mathrm{GeV})^2$. The above matrix exhibits one zero eigenvalue and diagonalizes with only V_L^U:

$$V_L^{U\dagger} M_U M_U^\dagger V_L^U = m_U^2 = \mathrm{diag}(0, m_2^2, m_3^2). \tag{17}$$

Thus, if we identify the zero-mass component with the phenomenological u quark and the other two with the c and t quarks, we obtain

$$m_u^2 = 0, \qquad m_{c,t}^2 = m_{2,3}^2 \sim \mathrm{GeV}^2. \tag{18}$$

B. Down sector

From (15), we obtain the matrix

$$M_D M_D^\dagger = \begin{pmatrix} \Sigma & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{19}$$

that exhibits two massless quarks which can be associated with the d and s quarks, while Σ is associated with the b quark:

$$m_{d,s}^2 = 0, \qquad m_b^2 = \Sigma \sim \text{GeV}^2. \tag{20}$$

Thus, from the gauge symmetry, we may obtain zero-texture mass matrices for the quark sector in the low energy limit, obtaining three massless quarks (light quarks) and three massive quarks at the electroweak scale ($\sim$ GeV). The massless quarks are indeed massive particles if we consider small couplings with the extra heavy 3-3-1 quarks, as we show below.

IV. MASS MATRICES WITH MIXING COUPLINGS

In this case we consider the complete mass matrices from Eq. (7), which have the following general structure:

$$M = \begin{pmatrix} M_{\text{light}} & f_{\text{light}} \\ G_{\text{heavy}} & \Lambda_{\text{heavy}} \end{pmatrix}, \tag{21}$$

where $M_{\text{light}} \sim f_{\text{light}} \sim v_\rho \sim 246$ GeV, while $G_{\text{heavy}} \sim \Lambda_{\text{heavy}} \sim v_\chi \gg 246$ GeV. The above mass matrix can be diagonalized by a biunitary transformation: $\tilde{m} = \mathcal{O}_L^\dagger M \mathcal{O}_R$. This transformation can be separated into two rotations: first, we can rotate through the biunitary transformations $V_{L,R} = V_{L,R}^{U,D}$ and $P_{L,R} = V_{L,R}^J$ defined by Eq. (14) in the low energy limit. Second, since the first rotation does not lead to a completely diagonal matrix due to the mixing terms f and G, we must perform another rotation through unitary matrices $B_{L,R}$. Thus, we separate the original rotation into

$$\mathcal{O}_{L,R} = \mathcal{U}_{L,R} W_{L,R} = \begin{pmatrix} V_{L,R} & 0 \\ 0 & P_{L,R} \end{pmatrix} \begin{pmatrix} 1 & B_{L,R} \\ -B_{L,R}^\dagger & 1 \end{pmatrix}, \tag{22}$$

where we require that

$$\mathcal{O}_L^\dagger M \mathcal{O}_R = W_L^\dagger \mathcal{U}_L^\dagger M \mathcal{U}_R W_R = \tilde{m} = \begin{pmatrix} \tilde{m}_{\text{light}} & 0 \\ 0 & \tilde{m}_{\text{heavy}} \end{pmatrix}. \tag{23}$$

After the first rotation (through $\mathcal{U}_{L,R}$) of (21), we obtain mixing matrices of the form:

$$\mathcal{U}_L^\dagger M \mathcal{U}_R = m = \begin{pmatrix} m_l & \tilde{f} \\ \tilde{G} & M_H \end{pmatrix}, \tag{24}$$

where $m_l = V_L^\dagger M_{\text{light}} V_R$ and $M_H = P_L^\dagger \Lambda_{\text{heavy}} P_R$ are diagonal blocks, while $\tilde{f} = V_L^\dagger f_{\text{light}} P_R$ and $\tilde{G} = P_L^\dagger G_{\text{heavy}} V_R$ are nondiagonal mixing blocks. Then, we must find the matrices $B_{L,R}$ in order to obtain the complete diagonalization of the mixing matrices in (24). We can achieve this by constructing the following squared mass matrices from (24):

$$\begin{aligned} m_1^2 = m m^\dagger &= \begin{pmatrix} m_l m_l^\dagger + \tilde{f}\tilde{f}^\dagger & m_l \tilde{G}^\dagger + \tilde{f} M_H^\dagger \\ \tilde{G} m_l^\dagger + M_H \tilde{f}^\dagger & M_H M_H^\dagger + \tilde{G}\tilde{G}^\dagger \end{pmatrix} \\ &= \begin{pmatrix} a_l & x_m \\ x_m^\dagger & y_H \end{pmatrix}, \\ m_2^2 = m^\dagger m &= \begin{pmatrix} m_l^\dagger m_l + \tilde{G}^\dagger \tilde{G} & m_l^\dagger \tilde{f} + \tilde{G}^\dagger M_H \\ \tilde{f}^\dagger m_l + M_H^\dagger \tilde{G} & M_H^\dagger M_H + \tilde{f}^\dagger \tilde{f} \end{pmatrix} \\ &= \begin{pmatrix} A_H & X_H \\ X_H^\dagger & Y_H \end{pmatrix}, \end{aligned} \tag{25}$$

where $a_l \sim v_\rho^2$ (electroweak scale), $x_m \sim v_\rho v_\chi$ (intermediate scale), and $y_H \sim A_H \sim X_H \sim Y_H \sim v_\chi^2$ (heavy scale). The above squared matrices are diagonalized through $W_{L,R}$ defined in (22):

$$W_{L,R}^\dagger m_{1,2}^2 W_{L,R} = \begin{pmatrix} \tilde{m}_l^2 & 0 \\ 0 & \tilde{M}_H^2 \end{pmatrix}. \tag{26}$$

From the condition of the vanishing of the off-diagonal submatrices in Eq. (26), we obtain

$$B_L (x_m^\dagger) B_L - a_l B_L + B_L y_H - x_m = 0, \tag{27}$$

$$B_R (X_H^\dagger) B_R - A_H B_R + B_R Y_H - X_H = 0. \tag{28}$$

Since $a_l \ll x_m \ll y_H$, Eq. (27) may be solved assuming that B_L expands in powers of $1/y_H$ [15]:

$$B_L = B_{L1} + B_{L2} + B_{L3} + \cdots, \tag{29}$$

where at order B_{L1} Eq. (27) approximates to $B_L y_H - x_m = 0$, obtaining

$$B_L \approx x_m y_H^{-1} = (m_l \tilde{G}^\dagger + \tilde{f} M_H^\dagger)(M_H M_H^\dagger + \tilde{G}\tilde{G}^\dagger)^{-1}. \tag{30}$$

Solving Eq. (28) is less evident, since all coefficients are at the heavy scales. However, we may consider a scenario where the mixing terms in (21) are small with respect to the diagonal components, which implies for the matrix m_2^2 in (25) the hierarchy $Y_H \gg X_H \gg A_H$. Thus, Eq. (28) may also be solved assuming that B_R expands in powers of $1/Y_H$, where at first order Eq. (28) reads $B_R Y_H - X_H = 0$, obtaining

$$B_R \approx X_H Y_H^{-1} = (\tilde{G}^\dagger M_H)(M_H^\dagger M_H)^{-1}. \tag{31}$$

Putting all together into the total rotation in Eq. (23), we finally find the light and heavy diagonal masses:

$$\begin{aligned} \tilde{m}_{\text{light}} &= m_l - \tilde{f} B_R^\dagger - B_L \tilde{G} + B_L M_H B_R^\dagger, \\ \tilde{m}_{\text{heavy}} &= M_H + \tilde{G} B_R + B_L^\dagger \tilde{f} + B_L^\dagger m_l B_R. \end{aligned} \tag{32}$$

In particular, for the light sector we see that

$$\begin{aligned} m_l &= V_L^\dagger M_{\text{light}} V_R, \qquad M_H = P_L^\dagger \Lambda_{\text{heavy}} P_R, \\ B_L &\approx \tilde{f}/M_H, \qquad B_R^\dagger \approx \tilde{G}/M_H, \end{aligned} \tag{33}$$

obtaining for the light fermions in (32):

$$\tilde{m}_{\text{light}} \approx V_L^{\dagger} M_{\text{light}} V_R - \frac{\tilde{f}\tilde{G}}{M_H}. \tag{34}$$

If we apply the above solution to the mass matrices in (7), we will obtain for the light sector

$$\tilde{m}_U \approx V_L^{U\dagger} M_U V_R^U - \frac{\tilde{k}\tilde{K}}{M_T}, \tag{35}$$

$$\tilde{m}_D \approx V_L^{D\dagger} M_D V_R^D - \frac{\tilde{s}\tilde{S}}{M_J}, \tag{36}$$

where the first terms correspond to the diagonal masses in the low energy limit given by (18) and (20), plus small corrections that arise from the mixing terms k, s, K, S and the inverse of the heavy masses of the T and J quarks. Thus, if we make the same identification as in (18) and (20), we obtain for the up sector

$$\begin{aligned} |\tilde{m}_u| &\approx \frac{|\tilde{k}\tilde{K}|_{11}}{M_T} \sim \text{MeV}, \\ |\tilde{m}_{c,t}| &\approx |V_L^{U\dagger} M_U V_R^U|_{22,33} \sim \text{GeV}, \end{aligned} \tag{37}$$

while for the down sector we obtain

$$\begin{aligned} |\tilde{m}_{d,s}| &\approx \frac{|\tilde{s}\tilde{S}|_{22,33}}{M_{J_{2,3}}} \sim \text{MeV}, \\ |\tilde{m}_b| &\approx |V_L^{D\dagger} M_D V_R^D|_{11} \sim \text{GeV}. \end{aligned} \tag{38}$$

Indeed, we see in (38) that $\tilde{m}_{d,s}$ depends on the inverse of the two masses of the J quarks, while in (37), $\tilde{m}_u$ is inverse in M_T. Thus, if we require that the heavy quarks obey $M_T \gtrsim M_{J_2} > M_{J_3}$, we obtain the following forms:

$$\tilde{m}_u \lesssim \tilde{m}_d < \tilde{m}_s \sim \text{MeV}, \qquad \tilde{m}_{b,c,t} \sim \text{GeV}. \tag{39}$$

V. CONCLUSIONS

The 3-3-1 model exhibits an Abelian nonuniversal $U(1)_X$ symmetry in the quark sector, from which not all Yukawa couplings are allowed by the symmetry. Indeed, the family-dependence feature shown by the quark multiplets in (2) arises from the condition of cancellation of the chiral anomalies in order to obtain a realistic renormalizable spectrum beyond the tree level. Thus, from the gauge structure of the model, we obtain zero-texture Yukawa coupling constants $h_{\rho,\chi}$ as shown by Eqs. (10)–(13). These structures may generate quark mass hierarchies if we consider a special basis through appropriate discrete symmetries that suppress the FCNC couplings, analogous to the 2HDM type I. In this case, one Higgs triplet (ρ) provides masses to the up- and down-type quarks simultaneously, obtaining zero-texture mass matrices through the VEV v_ρ, as shown by (15). The above matrices exhibit one massless up-type quark (u quark) and two massless down-type quarks (d and s quarks), while three quarks (c, b, and t quarks) have masses at the scale $v_\rho \sim$ GeV.

On the other hand, we may generate small ($\sim$ MeV) mass components to the above massless quarks without introducing either effective operators or one-loop corrections. If we consider the complete allowed Yukawa couplings, including small mixing couplings with the heavy T, J_2, and J_3 quarks [which according to (14) have masses at large scale $\nu_\chi \sim$ TeV], the mixing mass matrices in (7) can be diagonalized into light and heavy masses. In particular, by the method of recursive expansion, it is possible to decouple both scales at first order, obtaining seesaw-type masses, where the massless quarks acquire masses at scales inverse in the heavy mass quarks: $|\tilde{m}_u| \sim |\tilde{k}\tilde{K}|/M_T$, $|\tilde{m}_{d,s}| \sim |\tilde{s}\tilde{S}|/M_{J_{2,3}}$. If we consider a heavy nondegenerated spectrum, in particular, that $M_T \gtrsim M_{J_2} > M_{J_3}$, we may understand the observed hierarchy $m_u \lesssim m_d < m_s$ exhibited by the phenomenological light quark sector.

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