

Chiral vortical effect in superfluid

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We consider rotating superfluid pionic liquid, with superfluidity being induced by isospin chemical potential. The rotation is known to result in a chiral current flowing along the axis of the rotation. We argue that in the case of superfluidity the chiral current is realized on fermionic zero modes propagating along vortices. The current evaluated in this way differs by a factor of 2 from the standard one. The reason is that the chiral charge is carried by zero modes which propagate with speed of light, and thus the liquid cannot be described by a single (local) velocity, like it is assumed in standard derivations.

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I. INTRODUCTION

Recently, there were intense studies of hydrodynamics of chiral liquids. A crucial novel point is existence of new transport coefficients, overlooked in the textbook approaches. In particular, it was discovered [1,2] that in the case of liquids with chiral constituents there exists chiral current j_μ^5 proportional to the vorticity ω^μ :

$$\delta j_\mu^5 = \frac{\mu^2}{2\pi^2} \omega_\mu, \quad (1)$$

where $\omega^\mu = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} u_\nu \partial_\alpha u_\beta$, μ is the chemical potential, and u_μ is the 4-velocity of an element of the liquid. The observation (1) can be considered as a kind of generalization of the chiral magnetic effect (CME) known since longer time, for review and further references see, e.g., [3]. In the latter case, the chiral current is directed along the magnetic field in the local rest frame:

$$\delta j_\mu^5 = \frac{\mu}{2\pi^2} q B_\mu, \quad (2)$$

where $B^\mu \equiv \epsilon^{\mu\nu\alpha\beta} u_\nu \partial_\alpha A_\beta$, q is the charge of the constituent fermion.

The currents (1) and (2) represent macroscopic manifestations of the triangle anomaly in the underlying chiral theory. It is a fascinating observation that quantum-loop effects manifest in the hydrodynamic, which is classical approximation. Naturally enough, Eqs. (2) and (1) were considered within various frameworks. Originally, Eq. (1) was derived by considering the entropy current [2]. Later, it was obtained in other ways, in particular, in the language of anomalies in effective theory in the presence of a chemical potential and using other approaches [4–9]. Most recently [10], the assumptions needed to derive (1) and (2) were reduced to general Ward identities of the underlying microscopic theory.

Despite the variety of the assumptions tried, all these approaches treat the liquid as a slowly varying in its properties medium. Since the constituents of the liquid are chiral this assumption might look, though, not well justified. Indeed, the occupation numbers for fermions are

never large and it is not *a priori* clear how to introduce hydrodynamics, or classical approximation for fermions. This problem can be addressed within another approach which starts with a microscopical picture and the central role is played then by low-dimensional defects. This approach goes back to papers [11–13] where it was demonstrated that defects in field theory are closely tied to the realization of anomaly. In particular, anomaly in $2n + 2$ -dimensional theory is connected with $2n$ -dimensional index density and can be understood in terms of fermionic zero modes on strings and domain walls [13]. In all the cases, the chiral current is carried by fermionic zero modes living on the defects.

One can expect, therefore, that microscopically the anomaly is realized on vortexlike strings while the continuum-medium results (1) and (2) arise upon averaging over a large number of defects. In the case of the chiral magnetic effect [see Eq. (2)] such a mechanism was considered, in particular, in Refs. [3,14] and the final result (2) is reproduced on the microscopic level as well.

In Refs. [3,14], the vortices are modeled by regions of space free of the medium substance. In the case of superfluidity, the vortices are much better understood dynamically. The crucial point is that the velocity field of the superfluid is known to be potential, and, naively, the vorticity vanishes everywhere. If this were true, the chiral current (1) would vanish. But it is well known, of course, that the angular momentum is still transferred to the liquid through vortices. The potential is singular on the linear defects, or quantum vortices. The vorticity is not vanishing on these defects and the entire chiral current flows through the vortices. Although the vorticity locally vanishes everywhere but the defects, globally, the whole liquid can be regarded as rotating, e.g., it possesses angular momentum (provided that the angular velocity is large enough) [15]. This is an example of recovering the continuum limit upon averaging over a large number of defects.

In the microscopic picture, the evaluation of chiral effects reduces to counting zero chiral modes. The calculation is in two steps in fact. In the case of the chiral magnetic effect, the technique was elaborated in Ref. [12] where

details can be found. First, one considers plane pierced by magnetic field B . Then, there is an index theorem which relates the number of zero modes to the magnetic flux:

$$N_{0,\perp} \sim q \int d^2x B, \quad (3)$$

where q is the charge of the particle. These zero modes exist for any value of the longitudinal momentum directed along the magnetic field. At the second step, one integrates over the longitudinal momentum up to the chemical potential μ . Combining all the factors one gets for the total number of zero modes relevant to the chiral magnetic effect:

$$N_0 \sim q \cdot B \cdot \mu. \quad (4)$$

Proceed now to the vortical chiral effect. The evaluation of the number of zero modes is the same two-step process as above. However, there is no external electromagnetic field A_μ any longer. Instead, one considers 4-velocity of the liquid u_μ as an external field. One obtains the number of zero modes $N_{0,\perp}$ through the substitution:

$$qA_\mu \rightarrow \mu u_\mu. \quad (5)$$

In this sense, the chemical potential μ plays the role of the strength of interaction (like the charge q). The next step is essentially unchanged. One integrates over the longitudinal momentum up to the chemical potential, $p_{\parallel} \leq \mu$. As a result, the total number of zero modes relevant to the chiral vortical effect (CVE) is proportional to μ^2 . One factor of μ comes from the flux of the vorticity (analogy to the flux of the magnetic field). The origin of the other factor μ is the integration over the longitudinal momentum.

The result for the chiral currents obtained through counting zero modes can be compared to the evaluation of the same currents within effective field theory. In both cases, it is the triangle graphs which control the effect. The chiral magnetic effect is linear both in the interaction qA_μ and μu_μ . The chiral vortical effect is quadratic in μu_μ . In both cases, the triangle graphs reproduce the results (2) and (1).

Our central point is that the chiral current evaluated in terms of the zero modes differs from (1) by a factor of 2. The difference can be traced back to the fact that the fermionic zero modes propagate with speed of light and are not equilibrated to the local 4-velocity u_μ of an element of the liquid. If we consider magnetic field in charged superfluid then we find that the magnetic field plays the role similar to the vorticity and there are strings carrying magnetic flux, and, therefore, zero modes. The chiral magnetic effect in terms of the zero modes turns to be the same Eq. (2), echoing the results of [3,14]. In this sense, the vortical and magnetic chiral effects in superfluid substantially differ from each other. We will further comment on this difference in the conclusions.

The outline of the paper is as follows: We will start by recalling some common points regarding superfluidity, and

the way it arises in quark medium via chiral Lagrangian. We also remind the reader of the derivation of (1) and (2). In the next section, we introduce the interaction between fermions and the Goldstone field, and derive the chiral current via anomaly. We then proceed to the microscopic derivation of the current in terms of zero modes. In the last section, we discuss our results and mention a few open questions.

II. HYDRODYNAMICS, VORTICITY, AND SUPERFLUID

We start our detailed considerations by recalling how superfluidity arises in pion medium at nonzero isospin chemical potential and zero temperature [16]. The pion medium at zero temperature is described by chiral Lagrangian and at nonzero isospin chemical potential μ_I it takes the form

$$L = \frac{1}{4} f_\pi^2 \text{Tr}[D^\mu U (D_\mu U)^\dagger], \quad (6)$$

where $D_0 U = \partial_0 U - \frac{\mu_I}{2} [\tau_3, U]$, $D_i U = \partial_i U$. Here, for simplicity we consider zero quark masses. The chiral symmetry is spontaneously broken to $SU(2)_{L+R}$. Moreover, at nonzero μ_I this symmetry is explicitly broken to $U(1)_{L+R}$. And, finally, the $U(1)_{L+R}$ symmetry is spontaneously broken, triggering superfluidity. The argumentation is as follows: The potential energy in (6) equals to

$$V_{\text{eff}}(U) = \frac{f_\pi^2 \mu_I^2}{8} \text{Tr}[\tau_3 U \tau_3 U^\dagger - 1], \quad (7)$$

minima of that potential can be captured by substitution $U = \cos\alpha + i(\tau_1 \cos\phi + \tau_2 \sin\phi) \sin\alpha$:

$$V_{\text{eff}}(\alpha) = \frac{f_\pi^2 \mu_I^2}{4} (\cos 2\alpha - 1) \quad (8)$$

and for the minimum one readily obtains $\cos\alpha = 0$. Then, depending on sign of μ_I , squared mass of either π^+ or π^- becomes negative, signalling the condensation of the corresponding pion field. This means that the vacuum is described by $U = i(\tau_1 \cos\phi + \tau_2 \sin\phi)$ instead of the ‘‘usual’’ vacuum $U = I$, and the emergence of the new order parameter $\langle \bar{u} \gamma_5 d \rangle + \text{H.c.} = 2 \langle \bar{\psi} \psi \rangle_{\text{vac}} \sin(\alpha) = 2 \langle \bar{\psi} \psi \rangle_{\text{vac}}$. The system is thus a charged superfluid. Degeneracy with respect to the ϕ angle indicates that ϕ can be identified with Goldstone boson, and that the symmetry $U(1)_{L+R}$ is spontaneously broken. More accurately, one needs to make a replacement, $\phi + \mu t \rightarrow \phi$, since ϕ enters stress-energy tensor as a combination $\partial_0 \phi + \mu$; here and thereafter, we consider ϕ as redefined. In addition to the massless Goldstone mode, there are two massive modes. Note that these results can be reproduced holographically [17].

To study vortices we rely on the hydrodynamic approximation. The hydrodynamics of a charged superfluid incorporating the Goldstone field ϕ is worked out in [18] and references therein. The symmetry associated with the

charge is spontaneously broken by a condensate and the Goldstone field ϕ is added to the standard hydrodynamical variables. The ground state is described by the $\partial_0\phi = \mu$ (Josephson equation), which implies nonvanishing charge density. For a general motion of the superfluid the corresponding velocity is given by $u_\mu^s = \partial_\mu\phi/|\partial_\mu\phi|$, $v_i^s = \partial_i\phi/\mu$ (nonrelativistically) and one readily finds that $\text{rot } \mathbf{v}^s = 0$. This seems to forbid rotational motion.

However, it is known that for superfluid (at $T = 0$, so that normal component is absent) put into a rotating bucket the solution with nonzero angular momentum is energetically preferable for angular velocities larger than some critical value $\Omega > \Omega_c$. This implies nonzero velocity field circulation, which means that the velocity potential, given by the Goldstone field, is multivalued. The Goldstone field is then ill-defined on a linear defect called the vortex line, and is given by $\phi = \mu t + n \cdot \varphi$ in the limit of vanishing thickness of the core [19]. Note that this description is valid for distances $r \gg a$ where a is the size of the vortex. The value of critical angular velocity is given by $\Omega_c = \frac{1}{\mu R^2} \times \log \frac{R}{a}$ where R is the radius of the cylindrical bucket [19]. For higher angular velocities the circulation is increased via the generation of additional vortices with $n = 1$, which are energetically favorable to $n > 1$ vortices. This, and the fact that due to mutual repulsion, vortices tend to be distributed uniformly, causes the motion of the liquid induced by the vortices to imitate uniform rotation, $\mathbf{v} = [\mathbf{\Omega} \mathbf{r}]$ for high enough angular velocities $\Omega \gg \Omega_c$. This observation allows to compare the chiral vortical effect evaluated microscopically with (1) obtained macroscopically.

In next section, we consider chiral vortical effect and here it is worth mentioning how it is obtained. We follow [7] and start with action for fermion at nonzero chemical potential (here, for simplicity we consider the case of one flavor):

$$S = \int d^4x (i\bar{\psi}\gamma^\mu(\partial_\mu - iqA_\mu)\psi + \mu u_\mu \bar{\psi}\gamma^\mu\psi), \quad (9)$$

where the substitution $\mu\gamma^0 \rightarrow \mu u_\mu\gamma^\mu$ makes notations relativistically invariant. One can obtain CVE and CME as anomaly-induced currents. At zero axial chemical potential one should expect chiral effects only in the axial current. After calculating of anomaly we get

$$\begin{aligned} \partial_\mu \bar{\psi}\gamma^\mu\gamma_5\psi &= -\frac{1}{4\pi^2}\epsilon_{\mu\nu\alpha\beta}(\partial^\mu(A^\nu + \mu u^\nu) \\ &\times \partial^\alpha(A^\beta + \mu u^\beta)). \end{aligned} \quad (10)$$

Alternatively, one can introduce a modified current:

$$j_\mu^5 = \bar{\psi}\gamma_\mu\gamma_5\psi + \frac{1}{2\pi^2}\mu^2\omega_\mu + \frac{1}{2\pi^2}\mu q B_\mu, \quad (11)$$

which satisfies nonanomalous (in case $\mathbf{E} \cdot \mathbf{B} = 0$) equations and in hydrodynamical limit one replaces $\bar{\psi}\gamma_\mu\gamma_5\psi \rightarrow n_5 u_\mu$. Note the factor of 2 difference between the coefficients in front of the chiral vortical and chiral magnetic effects. This is a consequence of identity of two

vertices in the anomaly triangle diagram in the CVE case. For the CVE case that identity takes place and one should divide the result of diagram on two- $\frac{\mu^2}{4\pi^2}\epsilon_{\mu\nu\alpha\beta}u^\nu\partial^\alpha u^\beta$ in contrast to the case of CME- $\frac{\mu}{2\pi^2}\epsilon_{\mu\nu\alpha\beta}u^\nu\partial^\alpha A^\beta$.

III. CHIRAL CURRENTS VIA ANOMALY

We consider quark matter at finite isospin chemical potential, which forms a superfluid (see [16]) with the corresponding Nambu-Goldstone boson ϕ . It is argued in [18] that $\partial_\mu\phi$ can be identified with non-normalized superfluid velocity. The vortex configuration is in principle determined by the angular velocity. We address a general situation, when the n , the quantum number of circulation, is rather high (but not high enough to ruin the superfluidity). We mentioned above that an energetically preferable configuration is the uniform distribution of vortices with $n = 1$. Nearby any given vortex the Goldstone field is given by [20]

$$\phi = \mu t + \varphi, \quad (12)$$

where φ is the polar angle in the plane orthogonal to the vortex. We will assume that vortices are far from one another $\delta x \gg a$, we will calculate the current for a single vortex and then sum it over all vortices, that is, simply multiply by n .

It is then tempting to introduce the following Lagrangian for the interaction of fermions with the Nambu-Goldstone boson (we will limit ourselves to the case of a single fermion):

$$L = \bar{\psi}i(\partial_\mu + i\partial_\mu\phi)\gamma^\mu\psi. \quad (13)$$

We remind here the following nonrelativistic substitution $\partial_0\phi \rightarrow \mu$, $\partial_i\phi \rightarrow \mu v_i^s$, which relates ϕ to the non-normalized potential for the superfluid velocity. Using standard methods of evaluating the anomalous triangle diagrams one obtains for the axial current (see [4,5,11])

$$j_\mu^5 = \frac{1}{4\pi^2}\epsilon_{\mu\nu\alpha\beta}\partial^\nu\phi\partial^\alpha\partial^\beta\phi. \quad (14)$$

This current seems to vanish identically. However, for the vortex field $\phi = \mu t + \varphi$, and hence:

$$j_3^5 = \frac{\mu}{2\pi}\delta(x, y) \quad (15)$$

since $[\partial_x, \partial_y]\phi = 2\pi\delta(x, y)$. The total current (the sum over the vortices) equals to

$$J_3^5 = \int d^2x j_3^5 = \frac{\mu}{2\pi}n. \quad (16)$$

There is an apparent contradiction between (16) with linear dependence on the chemical potential and (1) where it is quadratic. It is resolved by noting that n , the quantum number for vorticity, depends on μ (see [3], where $\frac{\mu}{2\pi}$ is simply flux measured in units of flux quantum). In order to see this, we have to average over the defects to obtain the

continuum limit, as we mentioned above. Then n has the following continuum limit form:

$$n = \frac{1}{2\pi} \int dx_i \partial_i \phi = \frac{\mu}{2\pi} \int dx_i v_i^s = \frac{\mu}{\pi} \int d^2x \omega^3,$$

since we can replace $\mathbf{rot} \mathbf{v}^s = 2\boldsymbol{\omega}$. Thus, from (16) one can prescribe an average current density exactly matching (1).

This result for CVE conforms with the usual result for CME in the sense that if we considered charged superfluid and turned on magnetic field, a defect would appear, the current would be obtained by substituting the vortex configuration in the usual formula for CME, and would be concentrated on the vortex. However, we show in following section that answers obtained through zero modes calculation are different for CVE and CME.

IV. ZERO MODES

We now proceed to the microscopic picture based on the zero modes. Our considerations in this section are close to those of Ref. [14] but there are several significant differences as well. We can write Lagrangian for our system with coupling to the Goldstone field ϕ in the following form:

$$L = \bar{\psi} i(\partial_\mu + i\partial_\mu \phi) \gamma^\mu \psi. \quad (17)$$

Such coupling with the Goldstone field corresponds in nonsuperfluid limit to naive relativisation $\bar{\psi} \gamma^\mu \psi \partial_\mu \phi \rightarrow \mu u^\mu \bar{\psi} \gamma^\mu \psi$.

Nearby any given vortex the Goldstone field is given by (12). More important, irrelevant of the details of the configuration, the integral $\int dx_i \partial_i \phi = 2\pi n$.

Further calculation is close to the one performed in [14], with a substitution $A_i \rightarrow \partial_i \phi$. The Hamiltonian has the form

$$H = -i(\partial_i - i\partial_i \phi) \gamma^0 \gamma^i \quad (18)$$

then the Dirac equation decomposes:

$$-H_R \psi_L = E \psi_L, \quad H_R \psi_R = E \psi_R. \quad (19)$$

Here, $H_R = (-i\partial_i + \partial_i \phi) \sigma_i$. Hence, any solution ψ_R of $H_R \psi_R = \epsilon \psi_R$ simultaneously generates a solution with $E = \epsilon$,

$$\psi = \begin{pmatrix} 0 \\ \psi_R \end{pmatrix}$$

and a solution with $E = -\epsilon$,

$$\psi = \begin{pmatrix} \psi_R \\ 0 \end{pmatrix}.$$

Because of invariance with the respect to translations in the z direction, we decompose ψ using the momentum eigenstates $-i\partial_3 \psi_R = p_3 \psi_R$ (it is convenient to take the z -direction periodic with length L , and take limit $L \rightarrow \infty$ at the end of the calculation. For each p_3 we can write

$$H_R = p_3 \sigma^3 + H_\perp, \quad H_\perp = (-i\partial_a - \partial_a \phi) \sigma^a, \quad (20)$$

$$a = 1, 2.$$

Notice that $\{\sigma^3, H_\perp\} = 0$. Hence, if $|\lambda\rangle$ is a properly normalized eigenstate of H_\perp with eigenvalue λ , then $\sigma_3 |\lambda\rangle$ is a properly normalized eigenstate with eigenvalue $-\lambda$. This means that all eigenstates of H_\perp with nonzero eigenvalue are of the form $|\lambda\rangle, |-\lambda\rangle = \sigma_3 |\lambda\rangle$, with $\lambda > 0$. Also, σ_3 maps zero eigenstates of H_\perp , so all eigenstates of H_\perp can be classified with respect to σ_3 .

We can now express eigenstates of H_R in terms of eigenstates of H_\perp . Since $[H_R, H_\perp^2] = 0$, H_R will only mix states $|\lambda\rangle, |-\lambda\rangle$. For $\lambda > 0$, one can write

$$\psi_R = c_1 |\lambda\rangle + c_2 \sigma_3 |\lambda\rangle, \quad (21)$$

where c_1, c_2 satisfy

$$\begin{pmatrix} \lambda & p_3 \\ p_3 & -\lambda \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \epsilon \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}. \quad (22)$$

Thus, $\epsilon = \pm \sqrt{\lambda^2 + p_3^2}$ and,

$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}_\pm = (4(\lambda^2 + p_3^2))^{-(1/4)} \times \begin{pmatrix} \pm \text{sgn}(p_3) ((\lambda^2 + p_3^2)^{(1/2)} \pm \lambda)^{(1/2)} \\ ((\lambda^2 + p_3^2)^{(1/2)} \mp \lambda)^{(1/2)} \end{pmatrix}. \quad (23)$$

This means that every eigenstate of H_\perp with eigenvalue $\lambda > 0$ produces two eigenstates of H_R , while the zero modes of H_\perp are eigenstates of H_R with eigenvalue

$$\epsilon = p_3 \sigma^3. \quad (24)$$

Thus, the zero modes of H_\perp are gapless modes of H , capable of traveling up or down the vortex, depending on the sign of σ_3 and chirality. These will be precisely the carriers of the axial current along the vortex. Let N_+ and N_- be the numbers of zero modes with $\sigma_3 = 1$ and $\sigma_3 = -1$, respectively. Consider the zero mode of H_\perp , $|\lambda\rangle = (u, v)$, where u and v are c -functions satisfying

$$\mathcal{D}v = 0, \quad \mathcal{D}^\dagger u = 0, \quad (25)$$

where

$$\mathcal{D} = -i\partial_1 - \partial_2 - (\partial_1 \phi - i\partial_2 \phi). \quad (26)$$

Hence, $N_+ = \dim(\ker(\mathcal{D}^\dagger))$, $N_- = \dim(\ker(\mathcal{D}))$, and

$$\begin{aligned} N &= \text{Index}(H_\perp) = N_+ - N_- \\ &= \dim(\ker(\mathcal{D}^\dagger)) - \dim(\ker(\mathcal{D})). \end{aligned} \quad (27)$$

Note that H_\perp is an elliptic operator. Its index has been computed via various approaches in papers [21,22]. In our case the index is given by

$$N = \frac{1}{2\pi} \int dx_i \partial_i \phi = n. \quad (28)$$

(For $n = 1$ the zero mode is easy to construct, see Appendix.)

One can observe that this result is obtained by direct substitution of $eA_i \rightarrow \partial_i \phi$ in the well-known case of magnetic field parallel to z axis and uniform in that direction, where the index is given by [14,21]

$$N = \frac{e}{2\pi} \int dx_i A_i = \frac{e}{2\pi} \int d^2x B^3. \quad (29)$$

Notice that for the case of superfluid, which we discuss here, the index is essentially an integer.

We now proceed to the computation of the fermion axial current at a finite chemical potential μ . The axial current density in the third direction is given by

$$j_5^3(x) = \bar{\psi}(x) \gamma^3 \gamma^5 \psi(x) = \psi_L^\dagger \sigma^3 \psi_L(x) + \psi_R^\dagger \sigma^3 \psi_R(x). \quad (30)$$

We are interested in the expectation value of the axial current along the vortex, $J_5^3 = \int d^2x \langle j_5^3(x) \rangle$, and at finite chemical potential, we have

$$\begin{aligned} \langle j_5^3(x) \rangle &= \sum_E \theta(\mu - E) \psi_E^\dagger(x) \gamma^0 \gamma^3 \gamma^5 \psi_E(x) \\ &= \sum_\epsilon (\theta(\mu - \epsilon) + \theta(\mu + \epsilon)) \psi_{R\epsilon}^\dagger(x) \sigma^3 \psi_{R\epsilon}(x). \end{aligned} \quad (31)$$

Here, $\theta(\mu - E)$ is the Fermi-Dirac distribution (at zero temperature), ψ_E are eigenstates of H with eigenvalue E , $\psi_{R\epsilon}$ are eigenstates of H_R with eigenvalue ϵ . By substitution of the explicit form of $\psi_{R\epsilon}$ in terms of H_\perp eigenstates, one obtains

$$\begin{aligned} \langle J_5^3 \rangle &= \frac{1}{L} \sum_{p_3} \sum_{\lambda > 0} \sum_{s=\pm} (\theta(\mu - (\lambda^2 + p_3^2)^{1/2}) \\ &\quad + \theta(\mu + (\lambda^2 + p_3^2)^{1/2})) \langle \psi_R^s(\lambda, p_3) | \sigma^3 | \psi_R^s(\lambda, p_3) \rangle \\ &\quad + \frac{1}{L} \sum_{p_3} \sum_{\lambda=0} (\theta(\mu - p_3) + \theta(\mu + p_3)) \langle \lambda | \sigma^3 | \lambda \rangle. \end{aligned} \quad (32)$$

Here, $\lambda > 0$ enumerate eigenstates of H_\perp , which generate eigenstates of H_R , $\psi_R^\pm(\lambda, p_3)$ with momentum p_3 and eigenvalue $\epsilon_\pm = \pm \sqrt{\lambda^2 + p_3^2}$, and $\lambda = 0$ label the zero modes of H_\perp . An explicit calculation gives $\langle \psi_R^s(\lambda, p_3) | \sigma^3 | \psi_R^s(\lambda, p_3) \rangle = s p_3 (\lambda^2 + p_3^2)^{-1/2}$. Consequently, since the result is odd in both s and p_3 , the sum over all nonzero eigenstates vanishes, and only zero modes of H_\perp generate J_5^3 . For the zero modes, $\langle \lambda | \sigma^3 | \lambda \rangle = \sigma^3$, so we obtain

$$\begin{aligned} J_5^3 &= (N_+ - N_-) \frac{1}{L} \sum_{p_3} (\theta(\mu - p_3) + \theta(\mu + p_3)) \\ &= n \int \frac{dp_3}{2\pi} (\theta(\mu - p_3) + \theta(\mu + p_3)) = \frac{\mu}{\pi} n. \end{aligned} \quad (33)$$

This result is similar in structure to (16) but differs from it by a factor of 2. The origin of this difference is discussed in conclusions.

The macroscopic calculation through anomaly and the microscopic one through zero modes seem to be of rather different nature. The triangle diagram for the anomaly requires a particular regularization scheme whereas integrals over momenta of zero modes are cut by Fermi-Dirac distribution. It is worth emphasizing that the two answers coincide with each other for the chiral magnetic effect. It means that the observed discrepancy for vortical current indicates the difference between physical approaches rather than the regularization details. That fact justifies the comparison of macroscopic and microscopic results for CVE.

V. DISCUSSION AND CONCLUSIONS

In this paper we considered the chiral vortical effect in superfluid. Considering superfluidity helps to fix the dynamics to a great extent. The central point is that vorticity is nonvanishing only on linear defects and, therefore, introducing defects is kind of a must. Then, the picture with defects is only one consistent. Moreover, the vortices are quantized and this fixes the index which determines the number of zero modes, which, in turn, are responsible for the chiral current. The quantization of the vortices also accounts for the apparent discrepancy between the usual form of vortical effect (1) and the form (16), since the vorticity quantum number n depends on μ (in the same way as the flux quantum in superconductivity depends on the charge e and magnetic field). On the other hand, it is known that upon averaging over the defects the rotating superfluid looks the same as ordinary liquid and one can compare the result for the current with the continuum limit (1).

A crucial point is that microscopically we get a factor of 2 larger value of the current. To elucidate the origin of this factor it is useful to compare the triangle anomalous graphs for the chiral magnetic and chiral vortical effects, see Fig. 1. The vertices in the graphs are determined by the corresponding terms in the (effective) Lagrangian:

$$L_{\text{int}} = \mu u_\mu \bar{\psi} \gamma^\mu \psi + e A_\mu \bar{\psi} \gamma^\mu \psi, \quad (34)$$

where ψ is fermionic field. In the language of the Feynman graphs, we have in case of the vortical effects two identical

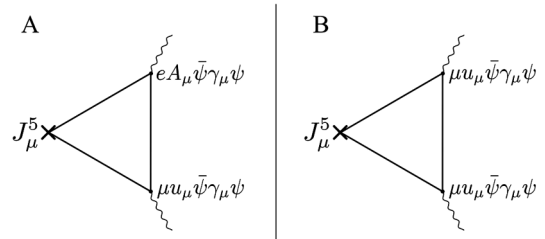


FIG. 1. (A)—contribution to CME, (B)—contribution to CVE.

vertices and this suppresses the result by a factor of 2 compared to the case of the chiral magnetic effect when there are no identical vertices. This is of course a common quantum effect.

This factor of one-half is absent from the counting of the number of zero modes. The result for the vortical effect looks so as if there were no identical vertices. The reason is that the chemical potential μ plays two different roles. First, it determines the strength of the effective interaction, see (34). And second, the chemical potential determines the upper limit on the value of longitudinal component of the momentum of zero modes, see (33). The factor μ^2 which enters the final result (1) is a combined effect of the strength of interaction, see (34), and of the maximal value of the longitudinal momentum, see (33). Thus, the source of the two factors μ is not identical now and, as a result, there is no factor of one-half.

To reiterate: the nowadays common derivation assumes that the liquid is characterized by a single velocity u_μ while the physical picture contains an additional component represented by the fermionic zero modes. Indeed, zero modes always propagate with speed of light and are not equilibrated with the rest of the liquid. This appearance of the additional component does not reduce, to our mind, to the standard introduction of the normal component in the theory of superfluidity. Remarkably, it does not affect the final answer in the case of the chiral magnetic effect because there are no identical vertices in the triangle graph in this case.

As mentioned above, in the case of superfluidity the chiral current is indeed concentrated on the vortices. In this sense, the microscopic picture seems more reliable than the naive continuum limit (1). In fact, as the result of this paper, for the case of superfluidity, the defect picture is the only one possible and the result (33) is the one valid.

In the nonsuperfluid case the defect picture is also more reliable, generally speaking. However, the theory is much less definite because of the absence of vortex quantization.

Thus, the dynamics of the vortices with zero fermionic modes living on them becomes the central issue to evaluate the chiral vortical effect. It is worth emphasizing, therefore, that the model for the vortices considered here can well be oversimplified. Indeed, in the picture considered all zero modes are peaked around the singular core of a vortex. Thus, 4-fermionic interaction could be important. Moreover, in case of quarks the effects of confinement can be crucial.

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APPENDIX

In this section, we explicitly construct the zero mode on the $n = 1$ vortex. As mentioned above, this problem reduces to the analysis of the 2D Dirac operator given by

$$\mathcal{D}^\dagger = -i\partial_1 + \partial_2 - (\partial_1\varphi + i\partial_2\varphi), \quad (\text{A1})$$

here φ stands for the polar angle, since the vortex field is given by $\phi = \mu t + \varphi$. If we substitute $\psi = \frac{f}{r}$ into

$$\mathcal{D}^\dagger\psi = 0, \quad (\text{A2})$$

we obtain for f

$$(\partial_x + i\partial_y)f = 0, \quad (\text{A3})$$

since

$$\left[-i\left(\partial_x + \frac{x}{r^2}\right) + \left(\partial_y + \frac{y}{r^2}\right) \right] \frac{1}{r} = 0. \quad (\text{A4})$$

This implies that f is an entire function of $x + iy$. We now note that if f is not a constant, then the norm of ψ diverges at least powerlike. It forces us to choose $f = 1$. The norm of ψ still diverges, but only logarithmically. We can cut off this divergence on the upper limit by $r = R$, the radius of the bucket, and by $r = a$, the size of the vortex on the lower limit (it is noteworthy that nearby the vortex the equation $\phi = \mu t + \varphi$ is no longer valid anyway, so our consideration actually applies to distances $r \gg a$).

For \mathcal{D} one would analogically obtain the following by substituting $\psi = fr$:

$$(\partial_x - i\partial_y)f = 0, \quad (\text{A5})$$

and f is entire in $x - iy$. But, for any choice of f the norm of ψ diverges. Thus, we have obtained that $N_+ = 1$, since there is one zero mode of \mathcal{D}^\dagger , and $N_- = 0$, and indeed $N = N_+ - N_- = 1$.

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