Thermal pions in a magnetic background

Jens O. Andersen

Department of Physics, Norwegian University of Science and Technology, N-7491 Trondheim, Norway and Niels Bohr International Academy, Niels Bohr Institute and Discovery Center, Blegdamsvej 17, DK-2100 Copenhagen, Denmark

(Received 16 February 2012; published 18 July 2012)

We use chiral perturbation theory for $SU(2)$ to compute the leading loop corrections to the thermal mass of the pions and the pion decay constant in the presence of a constant magnetic field B. The magnetic field gives rise to a splitting between M_{π^0} and $M_{\pi^\pm}$ as well as F_{π^0} and $F_{\pi^\pm}$. We also calculate the free energy and the quark condensate to next-to-leading order in chiral perturbation theory. The results suggest that the critical temperature T_c for the chiral transition is larger in the presence of a constant magnetic field, in agreement with most model calculations but in disagreement with recent lattice calculations.

DOI: 10.1103/PhysRevD.86.025020 PACS numbers: 11.10.Wx, 12.38.Mh, 12.39.Fe

I. INTRODUCTION

Chiral perturbation theory (ChPT) provides a systematic framework for calculating properties of QCD at low energies [1–4]. ChPT is not an expansion in powers of some small coupling constant, but it is a systematic expansion in powers of momenta p where a derivative counts as one power and the quark masses count as two powers. Chiral perturbation theory is a nonrenormalizable quantum field theory in the old sense of the word. This means that a calculation at a given order in momentum p requires that one add higher-order operators in order to cancel the divergences that arise in the calculation at that order. This implies that one needs more and more couplings and therefore more experiments to determine them. However, this poses no problem, as long as one is content with finite precision. This is the essence of effective field theory [5]. The chiral Lagrangian that describes the (pseudo) Goldstone bosons is uniquely determined by the global symmetries of QCD and the assumption of symmetry breaking. The Lagrangian $\mathcal{L}_{\rm eff}$ consists of a string of terms that involve an increasing number of derivatives or quark mass factors, each multiplied by a low-energy constant (LEC) l_i. However, QCD is a confining and strongly interacting theory at low energies. Thus the couplings l_i of the chiral Lagrangian cannot be calculated from QCD. Instead, the couplings are fixed by experiments.

The thermodynamics of a pion gas using ChPT was studied in detail in a series of papers 25 years ago [6–8]. The thermal pion mass and the thermal pion decay constant were evaluated at leading order (LO), while the pressure and the temperature dependence of the quark condensate were calculated to next-to-next-to-leading order in ChPT. In the chiral limit, this expansion is controlled by the parameter $T^2/8F_\pi^2$, where F_π is the pion decay constant. In this paper, we present calculations of the pion masses M_{π^0} and $M_{\pi^\pm}$ as well as the decay constants F_{π^0} and $F_{\pi^\pm}$ to leading order, and the free energy and the quark condensate to next-to-leading order (NLO) in ChPT in the presence of a constant magnetic background B. The details of the calculations will be presented elsewhere [9].

QCD in external magnetic fields has received a lot of attention in recent years due to its relevance in several physical situations. For example, large magnetic fields exist inside ordinary neutron stars as well as magnetars [10]. In the latter case, the cores may be color superconducting and so it is important to study the effects of external magnetic fields in this phase [11–18]. Similarly, it has been suggested that strong magnetic fields are created in heavy-ion collisions at the Relativistic Heavy-Ion Collider (RHIC) and the Large Hadron Collider (LHC) and that these play an important role [19]. In this case, the magnetic field strength has been estimated to be up to $B \sim 10^{19}$ gauss, which corresponds to $|qB| \sim 6M_\pi^2$, where $|q|$ is the electric charge of the pion. Even larger fields could be reached due to the effects of event-by-event fluctuations, see for example [20]. This has spurred the interest in studying QCD in external fields. At zero baryon chemical potential this can be done from first principles using lattice simulations and some recent results are found in [21–24].

Chiral perturbation theory has been used to study the quark condensate in strong magnetic fields at zero temperature [25–28] and finite temperature [29]. In Ref. [30], the leading thermal corrections to M_{π^0} and F_{π^0} in a magnetic background were computed. In Ref. [31], the quark-hadron phase transition was studied using ChPT to calculate the free energy at leading order. The effects of external magnetic fields on the chiral transition have been studied in detail using the Nambu—Jona-Lasinio (NJL) model [32–41], the Polyakov-loop extended NJL model [42,43], the quark-meson model [40,41,44–46], the Polyakov-loop quark-meson model [47,48], the linear sigma model [49], and the MIT bag model [50].

II. CHIRAL PERTURBATION THEORY

As explained in the Introduction, chiral perturbation theory is a low-energy effective field theory that can be used to systematically calculate physical quantities as a power series in momentum. The effective Lagrangian is given by an infinite string of operators involving an increasing number of derivatives or quark masses. Schematically, we can write $\mathcal{L}_{\rm eff} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \mathcal{L}^{(6)} + \cdots$ where the superscript indicates the powers of momentum. The leading term is given by

$$\mathcal{L}^{(2)} = \frac{1}{4}F^2 \,{\rm Tr}[(D_\mu U)^\dagger (D_\mu U) - M^2(U + U^\dagger)], \quad (1)$$

which is simply is the Lagrangian of the nonlinear sigma model. Here $U = e^{i\tau_i\pi_i/F}$ is a unitary $SU(2)$ matrix, where π_i are the pion fields and τ_i are the Pauli spin matrices. The low-energy constants M and F are the tree-level values for the pion mass M_π and the pion decay constant F_π, respectively. Moreover D_μ is the covariant derivative. By expanding the Lagrangian $\mathcal{L}^{(2)}$ to fourth order in the pion fields π^i, we obtain

$$\mathcal{L}^{(2)} = -F^2M^2 + \frac{1}{2}(\partial_\mu \pi^0)^2 + \frac{1}{2}M^2(\pi^0)^2 + (\partial_\mu + iqA_\mu)\pi^+(\partial_\mu - iqA_\mu)\pi^- + M^2\pi^+\pi^- - \frac{M^2}{24F^2}[(\pi^0)^2 + 2\pi^+\pi^-]^2 + \frac{1}{6F^2}[-2(\pi^0)^2(\partial_\mu\pi^+)(\partial_\mu\pi^-) - 2\pi^+\pi^-(\partial_\mu\pi^0)^2 + [\partial_\mu(\pi^+\pi^-)]^2 + 2\pi^0[\partial_\mu\pi^0][\partial_\mu(\pi^+\pi^-)] - 4\pi^+\pi^-(\partial_\mu\pi^+)(\partial_\mu\pi^-)], \quad (2)$$

where we have defined the complex pion fields as $\pi^\pm = \frac{1}{\sqrt{2}}(\pi_1 \pm i\pi_2)$ and $A_\mu = B\delta_{\mu 2}x_1$. Similarly, expanding $\mathcal{L}^{(4)}$ to second order in the pion fields yields [26]

$$\mathcal{L}^{(4)} = \frac{1}{4}F_{\mu\nu}^2 + \frac{2l_5}{F^2}(qF_{\mu\nu})^2\pi^+\pi^- + \frac{2il_6}{F^2}qF_{\mu\nu}[(\partial_\mu\pi^-)(\partial_\nu\pi^+) + iqA_\mu\partial_\nu(\pi^+\pi^-)] + (l_3 + l_4)\frac{M^4}{F^2}(\pi^0)^2 + 2(l_3 + l_4)\frac{M^4}{F^2}\pi^+\pi^- + l_4\frac{M^2}{F^2}(\partial_\mu\pi^0)^2 + 2l_4\frac{M^2}{F^2}(\partial_\mu + iqA_\mu)\pi^+(\partial_\mu - iqA_\mu)\pi^-, \quad (3)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength tensor. The Lagrangian $\mathcal{L}^{(6)}$ is very complicated as it contains more than 50 terms for $SU(2)$ [4]. However, only one term is relevant for the present problem [26,28], namely

$$\mathcal{L}^{(6),\rm relevant} = -4c_{34}M^2(qF_{\mu\nu})^2. \quad (4)$$

We have used the parametrization $U = e^{i\pi_i\tau_i/F}$. This parametrization is different from the one used in [26–28] and so the expressions for $\mathcal{L}$ also differ. However, we get identical results for physical quantities independent of parametrization. Moreover, we note that flavor symmetry is broken in an external electromagnetic field due to the different charges of the u and the d quarks. In particular, the $SU(2)_A$ symmetry is broken down to $U(1)_A^3$, which corresponds to the rotation of the u and d quarks by opposite angles. The formation of a quark condensate breaks this Abelian symmetry and gives rise to a Goldstone boson, namely the neutral pion. The charged pions are therefore no longer Goldstone modes. In fact, the presence of the external electromagnetic field allows for an effective mass term even when $M = 0$, cf. the second and third terms in Eq. (3).

The chiral Lagrangian comes with a number of undetermined parameters or low-energy constants l_i. These parameters can be determined by experiments; however, loop corrections involve renormalization of them. The relation between the bare and renormalized parameters can be expressed as

$$l_i = -\frac{\gamma_i}{2(4\pi)^2}\left[\frac{1}{\epsilon} + 1 - \bar{l}_i\right], \quad (5)$$

where γ_i are coefficients and $\bar{l}_i$ are scale-independent parameters [2], i.e. they are the renormalized couplings evaluated at the renormalization scale $\Lambda = M$. In the present calculations, we need $\gamma_3 = -\frac{1}{2}$, $\gamma_4 = 2$, $\gamma_5 = -\frac{1}{6}$, and $\gamma_6 = -\frac{1}{3}$ [2,3].

III. PION MASSES AND PION DECAY CONSTANTS

The pion masses M_{π^0} and $M_{\pi^\pm}$ are defined by the position of the pole of the propagator. At leading order, their expressions are divergent and require renormalization of the parameters l_3, l_5, and l_6. The result is

$$M_{\pi^0}^2 = M_\pi^2\left[1 - \frac{1}{(4\pi)^2F^2}\left(I_B(M) + \frac{1}{2}J_1(\beta M)T^2 - J_1^B(\beta M)|qB|\right)\right], \quad (6)$$

$$M_{\pi^\pm}^2 = M_\pi^2\left[1 + \frac{T^2}{2(4\pi)^2F^2}J_1(\beta M)\right] + \frac{(qB)^2}{3(4\pi)^2F^2}(\bar{l}_6 - \bar{l}_5), \quad (7)$$

where the pion mass M_π^2 in the vacuum is given by

$$M_\pi^2 = M^2\left[1 - \frac{M^2}{2(4\pi)^2F^2}\bar{l}_3\right], \quad (8)$$

the function $I_B(M)$ is defined by

$$I_B(M) = M^2\log\frac{M^2}{2|qB|} - M^2 - 2\zeta^{(1,0)}\left(0, \frac{1}{2} + x\right)|qB|, \quad (9)$$

where $\zeta(q,s)=\sum_{m=0}^{\infty}(q+m)^{-s}$ is the Hurwitz zeta-function and $x=\frac{M^2}{2|qB|}$. The thermal integrals are

$$J_1(\beta M) = 8\beta^2\int_0^\infty \frac{p^2dp}{\sqrt{p^2+M^2}}\frac{1}{e^{\beta\sqrt{p^2+M^2}}-1}, \tag{10}$$

$$J_1^B(\beta M) = 8\sum_{m=0}^{\infty}\int_0^\infty \frac{dp}{\sqrt{p^2+M_B^2}}\frac{1}{e^{\beta\sqrt{p^2+M_B^2}}-1}, \tag{11}$$

where $M_B^2=M^2+(2m+1)|qB|$ and m denotes the mth Landau level.

In order to calculate the pion decay constant, we need to evaluate the matrix elements $\langle 0|\mathcal{A}_\mu^0|\pi^0\rangle$ and $\langle 0|\mathcal{A}_\mu^{\pm}|\pi^{\mp}\rangle$, where $\mathcal{A}_\mu^0$ and $\mathcal{A}_\mu^{\pm}$ are the axial currents for π^0 and $\pi^{\pm}$. At zero magnetic field, these are identical, but there are two pion decay constants at finite temperature; one for the time component and one for the spatial component of $\mathcal{A}_\mu$ since Lorentz invariance is broken. The difference between them is an order-p^4 effect, i.e. appears at the two-loop level [51] and this is beyond the scope of this paper. The matrix elements are proportional to iP_μ and the prefactors are denoted by F_{π^0} and $F_{\pi^\pm}$, respectively. The expressions are divergent and require renormalization of l_4 and the renormalized result is

$$F_{\pi^0} = F_\pi\left[1+\frac{1}{(4\pi)^2F^2}(I_B(M)-J_1^B(\beta M)|qB|)\right], \tag{12}$$

$$F_{\pi^\pm} = F_\pi\left[1+\frac{1}{2(4\pi)^2F^2}(I_B(M)-J_1(\beta M)T^2 - J_1^B(\beta M)|qB|)\right], \tag{13}$$

where the pion decay constant F_π in the vacuum is

$$F_\pi = \left[1+\frac{M^2}{(4\pi)^2F^2}\bar{l}_4\right]. \tag{14}$$

Note that F_{π^0} differs from $F_{\pi^\pm}$ in a magnetic field. The reason is that the loop corrections to the former involve charged pions only, while loop corrections to the latter involve both neutral and charged pions [9].

IV. FREE ENERGY AND QUARK CONDENSATE

We are interested in the contributions to the free energy $\mathcal{F}$ that are due to a nonzero magnetic field and finite temperature. We therefore write the contribution to the free energy at the nth loop order, $\mathcal{F}_n$, as a sum of three terms: $\mathcal{F}_n=\mathcal{F}_n^{\rm vac}+\mathcal{F}_n^B+\mathcal{F}_n^T$, where $\mathcal{F}_n^{\rm vac}$ is the free energy in the vacuum, i.e. $B=T=0$, $\mathcal{F}_n^B$ is the zero-temperature contribution due to a finite magnetic field, and $\mathcal{F}_n^T$ is the finite-temperature contribution. The strategy is to isolate the term $\mathcal{F}_n^{\rm vac}$ and subtract it from $\mathcal{F}_n$. This term contains ultraviolet divergences which are removed by renormalization of the low-energy constants of the chiral Lagrangian and the renormalized $\mathcal{F}_n^{\rm vac}$ represents the vacuum energy of the theory. The term $\mathcal{F}_n^B$ generally contains ultraviolet divergences as well and it is rendered finite by renormalizing the l_is. In the present case, $\bar{l}_5$ and $\bar{l}_6$ in Eq. (3), and c_{34} in Eq. (4) require renormalization. If we express the contributions $\mathcal{F}_1^B$ and $\mathcal{F}_1^T$ in terms of the physical pion masses $M_{\pi^0}(0)$, Eq. (6), and $M_{\pi^\pm}(0)$, Eq. (7), at zero temperature, instead of M, most of the dependence on the constants $\bar{l}_i$s cancels in the expressions for $\mathcal{F}_{1+2}^B$ and $\mathcal{F}_{1+2}^T$. After a lengthy calculation, one finds [9]

$$\begin{aligned}\mathcal{F}_{1+2}^B &= \frac{M_{\pi^\pm}^4(0)}{2(4\pi)^2}\left[1-2\log\frac{M_{\pi^\pm}^2(0)}{2|qB|}\right] \\ &\quad+\frac{4(qB)^2}{(4\pi)^2}\zeta^{(1,0)}\left(-1,\frac{1}{2}+x_{\pi^\pm}\right)+\frac{(qB)^2}{6(4\pi)^2}\log\frac{\Lambda^2}{2qB} \\ &\quad-\frac{(qB)^2}{(4\pi)^4F^2}\bar{d}(M^2)M^2,\end{aligned} \tag{15}$$

$$\begin{aligned}\mathcal{F}_{1+2}^T &= -\frac{1}{2(4\pi)^2}[J_0(\beta M_{\pi^0}(0))T^4 \\ &\quad+2J_0^B(\beta M_{\pi^\pm}(0))|qB|T^2] \\ &\quad+\frac{M^2}{8(4\pi)^4F^2}[-J_1^2(\beta M)T^4 \\ &\quad+4J_1(\beta M)J_1^B(\beta M)T^2|qB|\},\end{aligned} \tag{16}$$

where

$$\bar{d}(M^2) = 8(4\pi)^4c_{34}^r-\frac{1}{3}(\bar{l}_6-\bar{l}_5)\log\frac{M^2}{\Lambda^2}, \tag{17}$$

$x_{\pi^\pm}=\frac{M_{\pi^\pm}^2(0)}{2|qB|}$, and Λ is the renormalization scale. The term $\frac{(qB)^2}{6(4\pi)^2}\log\frac{\Lambda^2}{2qB}$ arises from wave function renormalization of the term $\frac{1}{2}B^2$ in the tree-level expression for the free energy $\mathcal{F}_0=\frac{1}{2}B^2-F^2M^2$. It cancels a logarithmic divergence in $\mathcal{F}_1^B$ proportional to $(qB)^2$. This term is typically ignored in the literature since it is independent of T and the parameters of the chiral Lagrangian.

We note that the NLO correction to the free energy in the chiral limit ($M=0$) does not vanish since $\pi^\pm$ are no longer Goldstone modes and $M_{\pi^\pm}(0)$ is nonzero. This is in contrast to the case of zero magnetic field [6–8].

At finite temperature, the quark condensate is

$$\langle\bar{q}q\rangle = \langle 0|\bar{q}q|0\rangle\left(1-\frac{c}{F^2}\frac{\partial(\mathcal{F}^T+\mathcal{F}^B)}{\partial M_\pi^2}\right), \tag{18}$$

where the constant c is defined by [8]

$$c = -F^2\frac{\partial M_\pi^2}{\partial m_q}\langle 0|\bar{q}q|0\rangle^{-1}. \tag{19}$$

Here m_q is the quark mass. In the chiral limit, we have $c=1$. In that case, the quark condensate reduces to

$$\langle\bar{q}q\rangle=\langle 0|\bar{q}q|0\rangle\Big\{1+\frac{|qB|}{(4\pi)^2F^2}I_B(M_{\pi^\pm}(0))+\frac{(qB)^2}{(4\pi)^4F^4}\bar{d}(|qB|)-\frac{1}{2(4\pi)^2F^2}(J_1(0)T^2+2J_1^B(\beta M_{\pi^\pm}(0))|qB|)+\frac{T^2}{8(4\pi)^4F^4}(J_1^2(0)T^2-4J_1(0)J_1^B(0)|qB|+4\log 2J_1(0)|qB|)\Big\}. \tag{20}$$

This is the main result of the present paper and will be discussed in the next section.

V. RESULTS AND DISCUSSION

We first notice that we in the limit $B\to 0$ recover the temperature dependence of M_π, F_π, $\mathcal{F}$, and $\langle\bar{q}q\rangle$ as in [6–8]. Similarly, we obtain the $T=0$ result for the free energy and the B dependence of the quark condensate as in [26–28]. The results in Eq. (6) for $M^2_{\pi^0}$ and Eq. (12) for F_{π^0} were first obtained in [30]. The neutral pion decay constant depends on the magnetic field, which perhaps is unexpected. However, it is simply due to a cubic term $(\pi^+\pi^-)\partial_\mu\pi^0$ in the expression for the axial current $\mathcal{A}^0_\mu$ and gives rise to a charged pion loop [9,30].

We also notice that the temperature dependence of the charged pion mass is the same as for vanishing magnetic field. The only difference is a temperature-independent constant proportional to $(qB)^2/F^2$ arising from the second and third terms in Eq. (3). Thus the charged pions are massive excitations even in the limit when the quark mass m_q goes to zero. This simply reflects that only the neutral pion is a Goldstone mode in an external electromagnetic field.

The temperature dependence of $M^2_{\pi^\pm}$ may seem surprising at first since there are loop corrections to $M^2_{\pi^\pm}$ involving charged pion loops. However, these loop corrections cancel after having taken appropriately into account wave function renormalization of the charged pion fields [9].

In the remainder we focus on the chiral limit. In this case there are two dimensionless ratios, namely $|qB|/T^2$ and T^2/F^2. The integrals J_n^B are functions only of the dimensionless ratio $|qB|/T^2$. It is straightforward to show that $J_1T^2\geq J_1^B|qB|$ for all values of B and T. This implies that the pion decay constants F_{π^0} and $F_{\pi^\pm}$ are larger than F_π. Moreover, for small values of $|qB|$, i.e. for $|qB|\ll T^2$, we can calculate the first corrections due to nonzero B as a power series in $\sqrt{|qB|}/T$. One finds

$$F_{\pi^0}=F_\pi\left(1+\frac{|qB|\log 2}{(4\pi)^2F^2}-\frac{T^2}{12F^2}+\frac{5\sqrt{|qB|}T}{48\pi F^2}+\cdots\right), \tag{21}$$

$$F_{\pi^\pm}=F_\pi\left(1+\frac{|qB|\log 2}{2(4\pi)^2F^2}-\frac{T^2}{12F^2}+\frac{5\sqrt{|qB|}T}{96\pi F^2}+\cdots\right). \tag{22}$$

Similarly, we can expand the quark condensate around $|qB|=0$ and obtain the first correction proportional to $\sqrt{|qB|}/T$:

$$\langle\bar{q}q\rangle=\langle 0|\bar{q}q|0\rangle\left(1+\frac{|qB|\log 2}{(4\pi)^2F^2}-\frac{T^2}{8F^2}+\frac{5\sqrt{|qB|}T}{48\pi F^2}+\cdots\right). \tag{23}$$

In the limit $|qB|\to\infty$, $J_1^B\to 0$ since the terms in the sum in Eq. (11) are effectively Boltzmann suppressed. Eq. (6) then shows that the dominant contribution to $M^2_{\pi^0}$ goes like $-I_B(0)=-|qB|\log 2$ and so $M^2_{\pi^0}$ eventually turns negative which obviously is unphysical. From Eq. (12), we see that F_{π^0} becomes temperature independent.

In Fig. 1 (left panel), we show the quark condensate Eq. (20) as a function of temperature for $|qB|=5(140\text{ MeV})^2$ at LO and NLO in chiral perturbation theory including the $T=0$ contribution. For comparison, we also show the quark condensate for $|qB|=0$. We are using the experimental value $F_\pi=93$ MeV and $\bar{l}_6-\bar{l}_5=3.0\pm 0.3$ [52,53]. There is a large uncertainty in the constant $\bar{d}(|qB|)$ and its value is consistent with zero and we choose this value for simplicity. In Fig. 1 (right panel), we show the quark condensate Eq. (20) except that we have excluded the zero-temperature contribution. We do this to disentangle the effects of the magnetic field at $T=0$ and the finite-temperature effects. We notice that the LO and NLO results for the condensate in both cases are very close to each other in the entire temperature range. In fact, the LO and NLO curves lie significantly closer than do the corresponding curves for $B=0$. This suggests that chiral perturbation theory converges at least as well in the presence of a magnetic field.

The quark condensate for vanishing B goes to zero faster than it does in the presence of a magnetic field. This effect is caused by two separate mechanisms. First, there is the enhanced quark condensate at $T=0$, which to leading order is determined by the function $I_B(M)$. This is the well-known enhancement of the chiral condensate in the presence of a magnetic field. Second, there are finite-temperature corrections. The basic effect here is that J_1^B is a decreasing function of B and thus $J_1T^2>J_1^B|qB|$ for all $B>0$. Using this inequality, it is straightforward to show that the decrease of the quark condensate [Eq. (20)] due to thermal effects is smaller for nonzero B. The two separate effects are clearly demonstrated if one compares the two panels in Fig. 1.

Comparing the results for the condensate for $B=0$ and $|qB|=5(140\text{ MeV})^2$, it is clear the effects of the magnetic field are quantitatively large. This is due to a very strong magnetic field. For smaller values of $|qB|$, the gaps

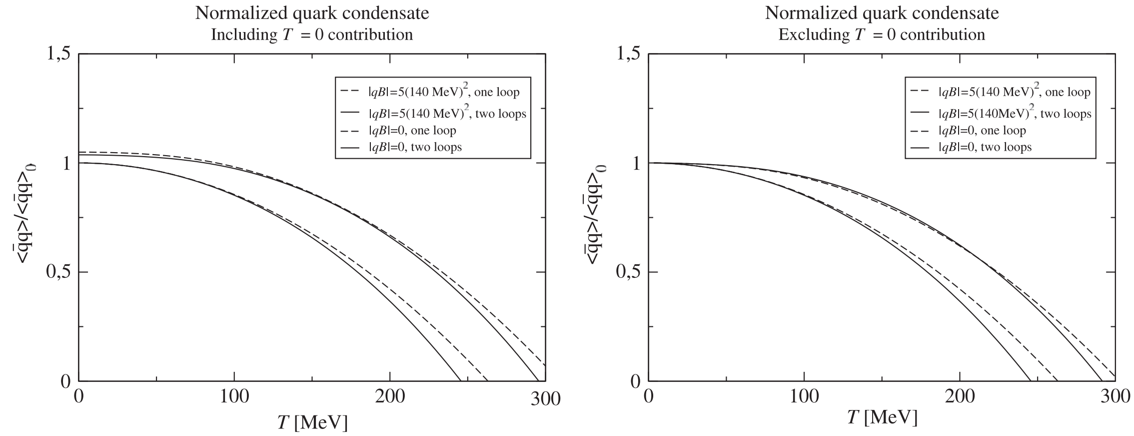

FIG. 1. Temperature dependence of the quark condensate including the $T = 0$ contribution normalized to its vacuum value $qB = 5(140\ \mathrm{MeV})^2$ at LO and NLO in chiral perturbation theory (left panel). For comparison, we show the LO and NLO results for $qB = 0$ as well (right panel).

between the two sets of curves will be smaller too. The calculations indicate that the critical temperature T_c for the chiral transition is higher in a nonzero magnetic field. Of course, this conclusion is cautious since the behavior of the quark condensate in the vicinity of T_c is beyond the reach of chiral perturbation theory. This result is in line with most model calculations, both mean-field type [42,43,47,49] and beyond [46,48]. Model calculations that seem to indicate a decrease of T_c as a function of magnetic field can be found in Refs. [31,50].

In Ref. [31], the authors use ChPT at leading order to investigate the quark-hadron phase transition as a function of the magnetic field at the physical point. They compare the pressure in the hadronic phase with that of the quark-gluon plasma phase for an ideal gas of quarks and gluons, and subtracting the vacuum energy due to a nonzero gluon condensate. For weak magnetic fields, the transition is first order. The line of first-order transitions ends at a critical point. From this temperature onwards, the transition is a crossover. The critical temperature defined this way is a decreasing function of B. Typically, however, the critical temperature is determined by the behavior of the quark condensate. At the physical point, the condensate never vanishes and the transition is a crossover. The crossover temperature is often defined by the inflection point of $\langle \bar{q}q \rangle$ as a function of temperature.

D'Elia *et al.* have carried out lattice simulations in a constant magnetic background at zero chemical potential [21,22]. They explored various constituent quark masses corresponding to a pion mass of 200–480 MeV and different magnetic fields, up to $|qB| \sim 20\, M_\pi^2$ for the lightest quark masses. For these values of the pion mass, they found that there is a slight increase in the critical temperature T_c for the chiral transition. These results have been confirmed by Bali *et al.* [23,24,54]. The same group has also carried out lattice simulations for physical values of the pion mass, i.e. $M_\pi = 140$ MeV. Their results which are extrapolated to the continuum limit show that the critical temperature is a decreasing function of the magnetic field [23,24,54]. Hence the critical temperature for fixed $|qB|$ as a function of the quark mass is nontrivial. This is in stark contrast to most model calculations that imply an increasing critical temperature as a function of B. This is irrespective of whether one goes beyond mean field or not. The discrepancy is perhaps somewhat surprising since at $T = 0$, the lattice results confirm the magnetic catalysis predicted by model calculations.

In conclusion, we have used chiral perturbation theory to calculate the pion masses, the decay constants, the free energy and the quark condensate at finite temperature in a magnetic background. Given the conflicting results for T_c as a function of B of various model calculations and lattice calculations, clearly more work needs to be done.

ACKNOWLEDGMENTS

The author would like to thank G. Bali, G. Endrődi, and F. Bruckmann for useful discussions on their lattice simulations, and N. O. Agasian for discussions on ChPT. He thanks the Niels Bohr International Academy and the Discovery Center for kind hospitality during the course of this work.

[1] S. Weinberg, Physica (Amsterdam) **96A**, 327 (1979).
[2] J. Gasser and H. Leutwyler, Ann. Phys. (N.Y.) **158**, 142 (1984).
[3] J. Gasser and H. Leutwyler, Nucl. Phys. **B250**, 465 (1985).
[4] J. Bijnens, G. Colangelo, and G. Ecker, Ann. Phys. (N.Y.) **280**, 100 (2000).
[5] G. P. Lepage, in *Proceedings of TASI '89: From Actions to Answers, Boulder, CO, 1989,*, edited by T. DeGrand and D. Toussaint (World Scientific, Singapore, 1989).
[6] J. Gasser and H. Leutwyler, Phys. Lett. B **184**, 83 (1987).
[7] J. Gasser and H. Leutwyler, Phys. Lett. B **188**, 477 (1987).
[8] P. Gerber and H. Leutwyler, Nucl. Phys. **B321**, 387 (1989).
[9] J. O. Andersen (to be published).
[10] J. M. Lattimer and M. Prakash, Phys. Rep. **442**, 109 (2007).
[11] M. G. Alford, J. Berges, and K. Rajagopal, Nucl. Phys. **B571**, 269 (2000).
[12] E. V. Gorbar, Phys. Rev. D **62**, 014007 (2000).
[13] E. J. Ferrer, V. de la Incera, and C. Manuel, Phys. Rev. Lett. **95**, 152002 (2005); Nucl. Phys. **B747**, 88 (2006).
[14] E. J. Ferrer and V. de la Incera, Phys. Rev. D **76**, 045011 (2007).
[15] K. Fukushima and H. J. Warringa, Phys. Rev. Lett. **100**, 032007 (2008).
[16] J. L. Noronha and I. A. Shovkovy, Phys. Rev. D **76**, 105030 (2007).
[17] B. Feng, D. Hou, H.-C. Ren, and P.-P. Wu, Phys. Rev. Lett. **105**, 042001 (2010).
[18] S. Fayazbakhsh and N. Sadooghi, Phys. Rev. D **82**, 045010 (2010); **83**, 025026 (2011).
[19] D. E. Kharzeev, L. D. McLerran, and H. J. Warringa, Nucl. Phys. **A803**, 227 (2008).
[20] W.-T. Deng and X-G. Huang, Phys. Rev. C **85**, 044907 (2012).
[21] M. D'Elia, S. Mukherjee, and F. Sanfilippo, Phys. Rev. D **82**, 051501(R) (2010).
[22] M. D'Elia and F. Negro, Phys. Rev. D **83**, 114028 (2011).
[23] G. S. Bali, F. Bruckmann, G. Endrődi, Z. Fodor, S. D. Katz, S. Krieg, A. Schafer, and K. K. Szabo, J. High Energy Phys. 02 (2012) 044.
[24] G. S. Bali, F. Bruckmann, G. Endrődi, Z. Fodor, S. D. Katz, and A. Schafer, arXiv:1206.4205.
[25] I. A. Shuspanov and A. V. Smilga, Phys. Lett. B **402**, 351 (1997).
[26] N. O. Agasian and I. A. Shushpanov, Phys. Lett. B **472**, 143 (2000).
[27] T. D. Cohen, D. A. McGady, and E. S. Werbos, Phys. Rev. C **76**, 055201 (2007).
[28] E. S. Werbos, Phys. Rev. C **77**, 065202 (2008).
[29] N. O. Agasian, Phys. Lett. B **488**, 39 (2000); Phys. At. Nucl. **64**, 554 (2001).
[30] N. O. Agasian and I. A. Shushpanov, J. High Energy Phys. 10 (2001) 006.
[31] N. O. Agasian and S. M. Fedorov, Phys. Lett. B **663**, 445 (2008).
[32] S. P. Klevansky and R. H. Lemmer, Phys. Rev. D **39**, 3478 (1989).
[33] V. P. Gusynin, V. A. Miransky, and I. A. Shovkovy, Phys. Lett. B **349**, 477 (1995); Nucl. Phys. **B462**, 249 (1996).
[34] A. Yu. Babansky, E. V. Gorbar, and G. V. Shchepanyuk, Phys. Lett. B **419**, 272 (1998).
[35] D. Ebert and K. G. Klimenko, Nucl. Phys. **A728**, 203 (2003).
[36] B. Hiller, A. A. Osipov, A. H. Blin, and J. da Providencia, SIGMA **4**, 024 (2008).
[37] J. K. Boomsma and D. Boer, Phys. Rev. D **81**, 074005 (2010).
[38] B. Chatterjee, H. Mishra, and A. Mishra, Phys. Rev. D **84**, 14016 (2011).
[39] S. S. Avancini, D. P. Menezes, and C. Providencia, Phys. Rev. C **83**, 065805 (2011).
[40] M. Frasca and M. Ruggieri, Phys. Rev. D **83**, 094024 (2011).
[41] A. Rabhi and C. Providencia, Phys. Rev. C **83**, 055801 (2011).
[42] R. Gatto and M. Ruggieri, Phys. Rev. D **82**, 054027 (2010); **83**, 034016 (2011).
[43] K. Kashiwa, Phys. Rev. D **83**, 117901 (2011).
[44] E. S. Fraga and A. J. Mizher, Phys. Rev. D **78**, 025016 (2008).
[45] J. O. Andersen and R. Khan, Phys. Rev. D **85**, 065026 (2012).
[46] J. O. Andersen and A. Tranberg, arXiv:1204.3360.
[47] A. J. Mizher, M. N. Chernodub, and E. S. Fraga, Phys. Rev. D **82**, 105016 (2010).
[48] V. Skokov, Phys. Rev. D **85**, 034026 (2012).
[49] D. C. Duarte, R. L. S. Farias, and R. O. Ramos, Phys. Rev. D **84**, 083525 (2011).
[50] E. S. Fraga and L. F. Palhares, arXiv:1201.5881.
[51] R. D. Pisarski and M. Tytgat, Phys. Rev. D **54**, R2989 (1996).
[52] S. Bellucci, J. Gasser, and M. E. Sainio, Nucl. Phys. **B423**, 80 (1994).
[53] J. Bijnens and P. Talavera, Nucl. Phys. **B489**, 387 (1997).
[54] G. S. Bali, G. Endrődi, and F. Bruckmann (private communication).