PHYSICAL REVIEW D **86**, 024025 (2012)

Exact and asymptotic black branes with spherical compactification

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(Received 14 February 2012; published 12 July 2012)

In the six-dimensional Kaluza-Klein model with the multidimensional cosmological constant Λ_6, we obtain the black brane with spherical compactification of the internal space. The matter source for this exact solution consists of two parts. First, it is a fine-tuned homogeneous perfect fluid which provides spherical compactification of the internal space. Second, it is a gravitating massive body with the dustlike equation of state in the external space and tension $\hat{p}_1 = -(1/2)\hat{\varepsilon}$ in the internal space. This solution exists both in the presence and absence of Λ_6. In the weak-field approximation, we also get solutions of the linearized Einstein equations for the model with spherical compactification. Here, the gravitating matter source has the dustlike equation of state in the external space and an arbitrary equation of state $\hat{p}_1 = \Omega\hat{\varepsilon}$ in the internal space. In the case $\Lambda_6 > 0$ and $\Omega \neq -1/2$, these approximate solutions tend asymptotically to the weak-field limit of the exact black brane solution. Both the exact and asymptotic black branes satisfy the gravitational experiments at the same level of accuracy as general relativity.

DOI: 10.1103/PhysRevD.86.024025 PACS numbers: 04.50.Kd, 04.25.Nx, 04.50.Cd, 04.80.Cc

I. INTRODUCTION

In our recent paper [1] (see also [2]), we investigated classical gravitational tests (the deflection of light and the time delay of radar echoes) in the six-dimensional Kaluza-Klein (KK) model with spherical compactification of the two-dimensional internal space. These studies were motivated by our previous papers [3–5] devoted to KK models with toroidal compactification, where we have shown that these models failed with the gravitational experiments in the case of a pointlike[1] mass with the dustlike equation of state in all spatial dimensions. It was surprising to us because this approach works well in general relativity [7]. In the models with toroidal compactification, latent solitons (in particular, black strings and black branes) are the only astrophysical objects which satisfy the gravitational experiments at the same level of accuracy as general relativity [4,5]. They are the exact solutions of the Einstein equations in vacuum, i.e. outside of the gravitating source. To get these solutions, the matter source[2] must have tension in the internal space instead of the dustlike equation of state. In other words, the energy-momentum tensor for these solutions has negative components $T_{\mu\mu}$ for $\mu = 4, 5, \ldots$, i.e. for the extra dimensions. Taking into account that, up to the terms of the order $1/c^2$, these components define pressure in the μth space ($T_{\mu\mu} \approx p_\mu$), we get negative pressure/tension in the internal spaces. This is a distinctive feature of these solutions. For black strings and black branes, the notion of tension is defined, e.g., in [8] and it follows from the first law for black hole spacetimes [9–11]. However, the physical meaning of tension for ordinary astrophysical objects (such as our Sun) is still not clear. Black strings/branes have a topology (four-dimensional Schwarzschild spacetime) × (flat internal space).

In the case of models with spherical compactification, the background metrics is not flat. To create such curved background, we should introduce the additional matter in the form of a homogeneous perfect fluid. Then, we perturb this background by a pointlike mass. In the weak-field limit, we have shown that a pointlike mass with the dustlike equation of state can satisfy the gravitational experiments if the model contains a positive cosmological constant [1]. It happens if the Yukawa interaction, generated by the conformal variations of the volume of the internal space [12], becomes negligible and we can drop the admixture of such interaction to the metric perturbations h_{00} and $h_{\alpha\alpha}$, $\alpha = 1, 2, 3$, resulting in equality of h_{00} and $h_{\alpha\alpha}$. A natural question arises whether this approach is the only way to satisfy the gravitational experiments in the case of spherical compactification. Can we find a solution similar to the black strings/branes? In the present paper we give a positive answer. We find the black brane with spherical compactification of the internal space. This is the exact solution of the Einstein equations that is important in itself. We are not aware of such

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[1]In most cases, when we use the word "pointlike," we usually mean a gravitating mass which has a delta-shaped form in the external space and is uniformly smeared over the internal space. In this case, the nonrelativistic gravitational potential exactly coincides with the Newtonian one [6].

[2]This matter source is compact and spherically symmetric in the external/our three-dimensional space and uniformly smeared over the internal space. It follows from the fact that the metric coefficients for these solutions depend only on the absolute value of the three-dimensional radius vector [4,5].

solutions in the literature. Our black brane has the topology (four-dimensional Schwarzschild spacetime) × (two-sphere). This solution exists both in the presence and absence of the multidimensional cosmological constant and has the negative pressure (tension) in the internal space with the equation of state $\hat{p}_1 = -(1/2)\hat{\varepsilon}$ in full analogy with the case of toroidal compactification (where such equation of state takes place in each extra dimension [5]). Additionally, we consider the weak-field limit of the model with spherical compactification in the case of a pointlike (with respect to the external space) mass with an arbitrary equation of state $\hat{p}_1 = \Omega\hat{\varepsilon}$ in the internal space and find the solution of the linearized Einstein equations. If $\Omega = -1/2$, then we reproduce the weak-field limit of the black brane solution. For arbitrary Ω (except for $\Omega = -1/2$), our approximate solution tends asymptotically to the weak-field limit of the black brane in the model with positive cosmological constant. It happens in regions where we can drop the admixture of the Yukawa interaction. This type of approximate solutions we call the asymptotic black branes. Obviously, the exact and asymptotic black branes satisfy the gravitational experiments at the same level of accuracy as general relativity.

The paper is structured as follows: In Sec. II, we obtain the exact black brane solution for the model with spherical compactification of the internal space. In Sec. III, we get solutions of the linearized Einstein equations in the case of a pointlike mass with an arbitrary equation of state in the internal space and single out the asymptotic black brane. The main results are summarized in concluding Sec. IV.

II. EXACT BLACK BRANE

It is well known (see, e.g., [4,5]) that black strings and black branes satisfy the gravitational experiments at the same level of accuracy as general relativity. They have the topology (four-dimensional Schwarzschild spacetime) × (D'-dimensional flat internal space) with $D' \geq 1$, and they are exact solutions of the Einstein equations. In this section we want to get a black brane solution with spherical compactification of the two-dimensional internal space. To obtain such solution, we consider the metrics in the following form:

$$ds^2 = \tilde{A}(\tilde{r}_3)c^2dt^2 + \tilde{B}(\tilde{r}_3)d\tilde{r}_3^2 + \tilde{C}(\tilde{r}_3)(d\theta^2 + \sin^2\theta d\varphi^2) + \tilde{E}(\tilde{r}_3)(d\xi^2 + \sin^2\xi d\eta^2), \tag{1}$$

where tilde denotes the "Schwarzschild-like" parametrization for the metrics and the three-dimensional radial coordinate. Similar to the black strings/branes with the flat internal space, here the metric coefficients depend only on the absolute value of the three-dimensional radius vector. These metric coefficients can be found with the help of the six-dimensional Einstein equation:

$$R_{ik} = \kappa_6\left(T_{ik} - \frac{1}{4}Tg_{ik} - \frac{1}{2}\Lambda_6 g_{ik}\right), \tag{2}$$

where Λ_6 is a bare cosmological constant, $\kappa_6 \equiv 2S_5\tilde{G}_6/c^4$, $S_5 = 8\pi^2/3$ is the total solid angle (the surface area of the four-dimensional sphere of a unit radius) and $\tilde{G}_6$ is the gravitational constant in the six-dimensional spacetime.

In the usual four-dimensional spacetime, the Schwarzschild metrics is created by a compact (e.g., pointlike) spherically symmetric gravitating matter source. However, in the case of the six-dimensional spacetime with spherical compactification of the internal space, we should introduce additional matter[3] which provides such compactification. Let the components of the energy-momentum tensor of this matter read

$$T_{ik} = \begin{cases} \varepsilon(\tilde{r}_3)g_{ik} & \text{for } i,k = 0,\ldots,3; \\ -\omega_1\varepsilon(\tilde{r}_3)g_{ik} & \text{for } i,k = 4,5. \end{cases} \tag{3}$$

Its trace reads $T = 2(2-\omega_1)\varepsilon(\tilde{r}_3)$. In the language of a perfect fluid, we have a vacuumlike equation of state in the external space, but an arbitrary equation of state with the parameter ω_1 in the internal space. Then, taking into account that $R_{33} = R_{22}\sin^2\theta$, $R_{55} = R_{44}\sin^2\xi$ and $T_{33} = T_{22}\sin^2\theta$, $T_{55} = T_{44}\sin^2\xi$, we reduce the Einstein equation [Eq. (2)] to the following system of fundamentally different equations:

$$\frac{R_{00}}{\tilde{A}} = -\frac{1}{4\tilde{A}'\tilde{C}^2\tilde{E}^2}\left(\frac{\tilde{A}'^2\tilde{C}^2\tilde{E}^2}{\tilde{A}\tilde{B}}\right)' = \frac{\kappa_6}{2}(\omega_1\varepsilon - \Lambda_6), \tag{4}$$

$$\frac{R_{11}}{\tilde{B}} = -\frac{1}{4\tilde{A}'}\left(\frac{\tilde{A}'^2}{\tilde{A}\tilde{B}}\right)' - \frac{1}{2\tilde{C}'}\left(\frac{\tilde{C}'^2}{\tilde{B}\tilde{C}}\right)' - \frac{1}{2\tilde{E}'}\left(\frac{\tilde{E}'^2}{\tilde{B}\tilde{E}}\right)' = \frac{\kappa_6}{2}(\omega_1\varepsilon - \Lambda_6), \tag{5}$$

$$\frac{R_{22}}{\tilde{C}} = \frac{1}{\tilde{C}} - \frac{1}{4\tilde{C}'\tilde{A}\tilde{C}\tilde{E}^2}\left(\frac{\tilde{C}'^2\tilde{A}\tilde{E}^2}{\tilde{B}}\right)' = \frac{\kappa_6}{2}(\omega_1\varepsilon - \Lambda_6), \tag{6}$$

$$\frac{R_{44}}{\tilde{E}} = \frac{1}{\tilde{E}} - \frac{1}{4\tilde{E}'\tilde{A}\tilde{E}\tilde{C}^2}\left(\frac{\tilde{E}'^2\tilde{A}\tilde{C}^2}{\tilde{B}}\right)' = -\frac{\kappa_6}{2}[(2+\omega_1)\varepsilon + \Lambda_6], \tag{7}$$

where a prime denotes the derivative with respect to the coordinate $\tilde{r}_3$.

In the case of black strings/branes with toroidal compactification, the internal space is flat. Now, we require that the internal space is exactly the two-sphere, that is $\tilde{E} \equiv -a^2 = \text{const}$. Therefore, Eq. (7) reads

$$-\frac{1}{a^2} = -\frac{\kappa_6}{2}[(2+\omega_1)\varepsilon + \Lambda_6], \tag{8}$$

which is valid for $\varepsilon \equiv \bar{\varepsilon} = \text{const}$. On the other hand, Eqs. (4)–(6) exactly coincide with the vacuum

[3]Obviously, there is no need for such additional matter in the case of KK models with toroidal compactification.

four-dimensional Schwarzschild equations if the following condition holds:

$$\bar{\varepsilon} = \Lambda_6/\omega_1. \tag{9}$$

From this condition and Eq. (8) we obtain the relation

$$\bar{\varepsilon} = \frac{1}{(1+\omega_1)\kappa_6 a^2}. \tag{10}$$

These relations exactly coincide with the relations in Ref. [1]. From these relations we can conclude that $\bar{\varepsilon} > 0 \Rightarrow \omega_1 > -1$ and $\mathrm{sign}\Lambda_6 = \mathrm{sign}\omega_1$. The parameter ω_1 is not fixed and takes part in fine-tuning Eq. (9) between $\bar{\varepsilon}$ and Λ_6. Choosing different values of ω_1 (with the vacuumlike equation of state in the external space), we can simulate different forms of matter. For example, $\omega_1 = 1$ and $\omega_1 = 2$ correspond to the monopole form-fields (the Freund-Rubin scheme of compactification) and the Casimir effect, respectively [1,13–15]. It is worth noting that in the case of the zero cosmological constant $\Lambda_6 = 0$, the parameter ω_1 should also be equal to zero: $\omega_1 = 0$ (Ref. [2]).

As we saw above, the homogeneous matter with the energy-momentum tensor [Eq. (3), where $\varepsilon \equiv \bar{\varepsilon} = \mathrm{const}$ and the conditions in Eqs. (9) and (10) hold] provides spherical compactification of the internal space. However, to get the external spacetime in the form of the Schwarzschild metrics, we have to introduce a compact gravitating object which is spherically symmetric in the external space and uniformly smeared over the internal space [4,6]. Let the energy-momentum tensor of this object read

$$\hat{T}_{00} = \hat{\varepsilon} g_{00}, \qquad \hat{T}_{\alpha\alpha} = 0, \qquad \alpha = 1, 2, 3,$$
$$\hat{T}_{44} = -\hat{p}_1 g_{44}, \qquad \hat{T}_{55} = -\hat{p}_1 g_{55}. \tag{11}$$

Therefore, the total energy-momentum tensor is the sum of Eq. (3) (with $\varepsilon \equiv \bar{\varepsilon}$) and Eq. (11). In the weak-field limit $\hat{\varepsilon} \approx \hat{\rho} c^2$ and for smeared extra dimensions $\hat{\rho} = \hat{\rho}_3/V_2$ where $\hat{\rho}_3$ is the three-dimensional rest mass density and the internal space volume $V_2 = 4\pi a^2$. In the case of a pointlike gravitating mass $\hat{\rho}_3 = m\delta(\tilde{\mathbf{r}}_3)$.

Now, taking into account the gravitating matter source and keeping in mind that we want to get the Schwarzschild solution in the external space, it can be easily realized that the only nonzero components of the Ricci tensor are

$$R_{00} = \frac{1}{2}\kappa_6 \hat{\varepsilon} g_{00} \approx \frac{1}{2}\kappa_N \hat{\rho}_3 c^2 g_{00},$$
$$R_{\alpha\alpha} = -\frac{1}{2}\kappa_6 \hat{\varepsilon} g_{\alpha\alpha} \approx -\frac{1}{2}\kappa_N \hat{\rho}_3 c^2 g_{\alpha\alpha}, \qquad \alpha = 1, 2, 3,$$
$$R_{44} = 1, \qquad R_{55} = \sin^2\xi, \tag{12}$$

where

$$\frac{\kappa_6}{V_2} = \kappa_N \equiv \frac{8\pi G_N}{c^4} \tag{13}$$

and G_N is Newton's gravitational constant. Substitution of these components of the Ricci tensor as well as the components of the total energy-momentum tensor [where we should take into account the relations in Eqs. (9) and (10)] in the Einstein equation [Eq. (2)] shows that these equations are compatible only if the following equation of state holds:

$$\hat{p}_1 = -\frac{1}{2}\hat{\varepsilon}. \tag{14}$$

For example, the 00-component of the Einstein equation is

$$R_{00} = \frac{1}{2}\kappa_6 \hat{\varepsilon} g_{00} = \kappa_6\left[\hat{\varepsilon} - \frac{1}{4}(\hat{\varepsilon} - 2\hat{p}_1)\right] g_{00},$$

where we take into account Eq. (9). This equation results in Eq. (14). Similarly, all other nontrivial components also give Eq. (14). That is, the gravitating matter source should have tension in the internal space as it takes place for the black strings/branes with toroidal compactification. Therefore, the exact solution—the black brane with spherical compactification—reads

$$ds^2 = \left(1 - \frac{r_g}{\tilde{r}_3}\right)c^2 dt^2 - \left(1 - \frac{r_g}{\tilde{r}_3}\right)^{-1} d\tilde{r}_3^2 - \tilde{r}_3^2 d\Omega_2^2$$
$$- a^2(d\xi^2 + \sin^2\xi d\eta^2), \tag{15}$$

where $r_g = 2G_N m/c^2$. The matter source of this black brane consists of two parts. First, it is the homogeneous component of the form of Eq. (3) with fine-tuning conditions Eqs. (9) and (10). This component provides spherical compactification of the internal space. Second, it is the gravitating object of the form of Eq. (11) which is spherically symmetric and compact in the external space and uniformly smeared over the internal space. It has negative pressure [Eq. (14)] in the extra dimensions. This component provides the Schwarzschild-like metrics in the external spacetime.

To calculate formulas for the famous gravitational experiments (the perihelion shift, the light deflection and the time delay of radar echoes) or expressions for parameterized post-Newtonian (PPN) parameters, it is convenient to rewrite the metrics in Eq. (15) in isotropic (with respect to our three-dimensional space) coordinates. The Schwarzschild-like radial coordinate $\tilde{r}_3$ and the isotropic radial coordinate r_3 are connected by the relation (see, e.g., [7]):

$$\tilde{r}_3 = r_3\left(1 + \frac{r_g}{4r_3}\right)^2. \tag{16}$$

For example, in isotropic coordinates

$$ds^2 \approx \left(1 + \frac{2\varphi_N}{c^2}\right)c^2 dt^2 - \left(1 - \frac{2\varphi_N}{c^2}\right)(dx^2 + dy^2 + dz^2)$$
$$- a^2(d\xi^2 + \sin^2\xi d\eta^2), \tag{17}$$

where $r_3 = \sqrt{x^2 + y^2 + z^2}$, $\varphi_N = -G_N m/r_3 = -r_g c^2/(2r_3)$ and we expand the metric coefficients up to the terms $1/c^2$

(the weak-field limit). The metrics [Eq. (17)] shows that the PPN parameter $\gamma = 1$. It is not difficult to demonstrate also that the PPN parameter $\beta = 1$ similar to general relativity. Therefore, our black brane satisfies the gravitational experiments at the same level of accuracy as general relativity.

III. ASYMPTOTIC BLACK BRANE

The matter of the form described by Eq. (3) [with conditions of Eqs. (9) and (10)] provides spherical compactification of the internal space. The corresponding manifold has the topology $\mathbb{R}^4 \times S_2$. To get the Schwarzschild metrics in the external spacetime, we introduced on this background a gravitating mass with negative pressure/tension [Eq. (14)] in the extra dimensions.

In general relativity, the weak-field limit is a good approximation to calculate the mentioned above gravitational experiments. In this limit, a gravitating massive body (e.g., a pointlike mass) has dustlike equations of state [7]. It is natural to generalize such approach to our multidimensional model assuming the dustlike equations of state in all spatial dimensions and to perturb the background $\mathbb{R}^4 \times S_2$ by such massive source. This problem was considered in papers [1,2]. It was shown that the Einstein equations are compatible only if the background matter also undergoes perturbations, i.e. the energy-momentum tensor of the perturbed background is

$$\tilde{T}_{ik} \approx \begin{cases} (\bar{\varepsilon} + \varepsilon^1) g_{ik}, & i,k = 0,\ldots,3; \\ -\omega_1(\bar{\varepsilon} + \varepsilon^1) g_{ik}, & i,k = 4,5, \end{cases} \tag{18}$$

where $\bar{\varepsilon}$ still satisfies the conditions of Eqs. (9) and (10) and the correction ε^1 is of the same order of magnitude as the energy density of perturbation $\hat{\rho} c^2$. We found that in the case $\omega_1 > 0$ this model can satisfy the gravitational experiments [1].

Let us investigate now the more general case, where, instead of the dustlike equations of state in all spatial dimensions, we suppose the following energy-momentum tensor of the perturbation:

$$\begin{aligned} &\hat{T}_{00} \approx \hat{\rho} c^2, \qquad \hat{T}_{\alpha\alpha} = 0, \qquad \alpha = 1,2,3, \\ &\hat{T}_{44} \approx \Omega \hat{\rho} c^2 a^2, \qquad \hat{T}_{55} \approx \Omega \hat{\rho} c^2 a^2 \sin^2\xi. \end{aligned} \tag{19}$$

Therefore, the total energy-momentum tensor is the sum of energy-momentum tensors of the perturbed background [Eq. (18)] and the perturbation [Eq. (19)]: $T_{ik} = \tilde{T}_{ik} + \hat{T}_{ik}$.

As we pointed out in [1], in the case of uniformly smeared (over the internal space) perturbation, the perturbed metrics preserves its diagonal form and in isotropic coordinates reads

$$ds^2 = Ac^2dt^2 + Bdx^2 + Cdy^2 + Ddz^2 + Ed\xi^2 + Fd\eta^2 \tag{20}$$

with

$$\begin{aligned} &A \approx 1 + A^1(r_3), && B \approx -1 + B^1(r_3), \\ &C \approx -1 + C^1(r_3), && D \approx -1 + D^1(r_3), \\ &E \approx -a^2 + E^1(r_3), && F \approx -a^2\sin^2\xi + F^1(r_3), \end{aligned} \tag{21}$$

where we take into account the spherical symmetry of the perturbation with respect to the external space. All perturbed metric coefficients A^1, B^1, C^1, D^1, E^1, and F^1 are of the order of ε^1. To find these coefficients, we should solve Eq. (2) which is reduced now to the system of equations (see also [1])

$$\triangle_3 A^1 = \kappa_6 \omega_1 \varepsilon^1 + \left(\frac{3}{2} + \Omega\right)\kappa_6 \hat{\rho} c^2, \tag{22}$$

$$\triangle_3 B^1 = \triangle_3 C^1 = \triangle_3 D^1 = -\kappa_6 \omega_1 \varepsilon^1 + \left(\frac{1}{2} - \Omega\right)\kappa_6 \hat{\rho} c^2, \tag{23}$$

$$\triangle_3 E^1 = (2 + \omega_1)\kappa_6 a^2 \varepsilon^1 - \frac{2}{a^2} E^1 + \left(\frac{1}{2} + \Omega\right)\kappa_6 \hat{\rho} c^2 a^2, \tag{24}$$

where $\triangle_3$ is the three-dimensional Laplace operator. Equation (23) shows that $B^1 = C^1 = D^1$. With the help of Eqs. (B8) and (B9) in [1], we also obtain that $F^1 = E^1 \sin^2\xi$ and

$$\begin{aligned} \triangle_3 E^1 &= \frac{a^2}{2}(\triangle_3 A^1 - \triangle_3 B^1) \\ &= \frac{a^2}{2}[2\kappa_6 \omega_1 \varepsilon^1 + (1 + 2\Omega)\kappa_6 \hat{\rho} c^2], \end{aligned} \tag{25}$$

where in the latter equality we use Eqs. (22) and (23). The comparison of Eqs. (24) and (25) yields

$$\kappa_6 \varepsilon^1 = \frac{E^1}{a^4}. \tag{26}$$

The substitution of this relation back into Eq. (25) gives

$$\begin{aligned} \triangle_3 E^1 - \frac{\omega_1}{a^2} E^1 &= \left(\frac{1}{2} + \Omega\right)\kappa_6 \hat{\rho} c^2 a^2 \\ &= \left(\frac{1}{2} + \Omega\right)\frac{8\pi G_N}{c^2} a^2 m \delta(\mathbf{r}_3), \end{aligned} \tag{27}$$

where for the smeared extra dimensions $\hat{\rho} = m\delta(\mathbf{r}_3)/(4\pi a^2)$ and we also use the relation in Eq. (13). In the case $\omega_1 > 0$, the solution of this Helmholtz equation reads[4]

$$E^1 = a^2 \frac{\varphi_N}{c^2}(1 + 2\Omega) e^{-r_3/\lambda}, \qquad \lambda \equiv a/\sqrt{\omega_1}, \tag{28}$$

[4]If $\Omega \neq -1/2$, then the negative value of ω_1 results in the nonphysical oscillating solution. Moreover, in the most interesting examples (e.g., Freund-Rubin form-field compactification, Casimir effect) $\omega_1 > 0$. Stabilization of the internal spaces also requires the positiveness of ω_1 [4,15]. The case $\Omega = -1/2$ is discussed below.

where φ_N is defined in Eq. (17). Taking into account Eqs. (26) and (27), we can rewrite Eqs. (22) and (23) in the form

$$\triangle_3\left(A^1 - \frac{E^1}{a^2}\right) = \kappa_6 \hat{\rho} c^2, \tag{29}$$

$$\triangle_3\left(B^1 + \frac{E^1}{a^2}\right) = \kappa_6 \hat{\rho} c^2, \tag{30}$$

and we obtain

$$A^1 = \frac{2\varphi_N}{c^2} + \frac{E^1}{a^2} = \frac{2\varphi_N}{c^2}\left[1 + \left(\frac{1}{2} + \Omega\right)e^{-r_3/\lambda}\right], \tag{31}$$

$$B^1 = \frac{2\varphi_N}{c^2} - \frac{E^1}{a^2} = \frac{2\varphi_N}{c^2}\left[1 - \left(\frac{1}{2} + \Omega\right)e^{-r_3/\lambda}\right]. \tag{32}$$

To get agreement with gravitational experiments, coefficients A^1 and B^1 should be very close to each other. In general relativity, A^1 is exactly equal to B^1. In our model, we can satisfy this condition in two cases.

First, $A^1 = B^1 = 2\varphi_N/c^2$ and $E^1 = \kappa_6 a^4 \varepsilon^1 = 0$ for $\Omega = -1/2$. Obviously, this is the case of the previous section, and we reproduce this exact solution in the weak-field limit. Here, the parameter ω_1 is not fixed and satisfies the condition $\omega_1 > -1$, including the case $\omega_1 = 0$, when a bare cosmological constant is also zero: $\Lambda_6 = 0$.

Second, for $r_3 \gg \lambda$ (roughly speaking, for $r_3/\lambda \to +\infty$) both A^1 and B^1 asymptotically tend to $2\varphi_N/c^2$ and E^1, ε^1 go to zero. Here, the metrics asymptotically approaches to Eq. (17) for any value of $\Omega \neq -1/2$, including the physically reasonable case of the dustlike equation of state $\Omega = 0$. Therefore, the second case is called the asymptotic black brane. The parameter Ω can be set arbitrarily and does not necessarily equal $-1/2$. The positiveness of ω_1 (as well as Λ_6) is the necessary condition of the considered case. The metric correction term $A^1 \sim O(1/c^2)$ describes the nonrelativistic gravitational potential: $A^1 = 2\varphi/c^2$. Therefore, this potential acquires the Yukawa correction term. The Yukawa interaction is characterized by two parameters: the parameter λ, which defines the characteristic range of this interaction, and the parameter α in front of the exponential function. In our case $\alpha = 1/2 + \Omega$. There is a strong restriction on these parameters from the inverse square law experiments. If, for example, $|\Omega| \sim O(1)$ (and is not equal to $-1/2$), the upper limit for λ is $\lambda_{\max} \sim 10^{-3}$ cm (Ref. [16]). In view of the relation $\lambda = a/\sqrt{\omega_1}$, we have also a possibility to estimate the upper limit of the size of the internal space for a fixed value of ω_1 (usually, $\omega_1 \sim O(1)$).

Let us estimate now the Yukawa correction term for the gravitational experiments (the deflection of light and the time delay of radar echoes) in the Solar system. We can take $r_3 \gtrsim r_\odot \sim 7 \times 10^{10}$ cm. Then, for $\lambda \lesssim 10^{-3}$ cm, we get $r_3/\lambda \gtrsim 10^{13}$. Therefore, with very high accuracy we can drop the Yukawa correction term, and we arrive at the case of the asymptotic black brane.

IV. CONCLUSION

In this paper we found a metrics for a black brane with spherical compactification of the internal space. This is the exact solution of the Einstein equations. To get such solution, we should first prepare the corresponding background with the flat external spacetime and the curved internal space (the two-sphere). For this purpose, we should include a matter source in the form of a homogeneous perfect fluid with vacuum equation of state in the external (our) space and an arbitrary equation of state in the internal space. The model can also contain a bare multidimensional cosmological constant Λ_6. To get spherical compactification, parameters of the perfect fluid should be fine-tuned. The presence of such perfect fluid is the main difference from the well-known black branes with toroidal compactification. In the latter case we do not need to introduce an additional perfect fluid, because the background here is flat for both external and internal spaces.

The next step is to construct a Schwarzschild-like metrics in the external spacetime. To perform it, we included a gravitating object which is spherically symmetric and compact in the external space and uniformly smeared over the internal space. We have shown that the Einstein equations are compatible only if this object has negative pressure (i.e. tension) in the internal space with the following equation of state: $\hat{p}_1 = -(1/2)\hat{\varepsilon}$. It should be noted that the gravitating matter source for black branes with toroidal compactification has precisely the same equation of state in the internal space.

Then, we generalized our investigations to the case where the background with spherical compactification is perturbed by a matter source which has the dustlike equation of state in the external space and an arbitrary equation of state $\hat{p}_1 = \Omega\hat{\varepsilon}$ in the internal space. In the weak-field limit, we found solutions of the linearized Einstein equations. The case $\Omega = -1/2$ reproduces the weak-field limit of the exact solution. In the case $\Omega \neq -1/2$ and $\Lambda_6 > 0$, the metric coefficients acquire the Yukawa correction terms which are negligibly small at three-dimensional distances much greater than the characteristic range of the Yukawa interaction. At these distances, the metrics asymptotically tends to the weak-field limit of the exact black brane solution. We named the second case the asymptotic black brane. Obviously, in the case of spherical compactification, the exact black branes and asymptotic black branes satisfy the gravitational experiments at the same level of accuracy as general relativity.

ACKNOWLEDGMENTS

This work was supported in part by the Cosmomicrophysics 2 program of the Physics and Astronomy Division of the National Academy of Sciences of Ukraine.

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