Spontaneous symmetry breaking in inflationary cosmology: On the fate of Goldstone bosons

Daniel Boyanovsky[*]

Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, Pennsylvania 15260, USA
(Received 16 May 2012; published 3 July 2012)

We argue that in an inflationary cosmology a consequence of the lack of time translational invariance is that spontaneous breaking of a continuous symmetry and Goldstone's theorem *do not* imply the existence of *massless* Goldstone modes. We study spontaneous symmetry breaking in an $O(2)$ model, and implications for $O(N)$ in de Sitter space-time. The Goldstone mode acquires a radiatively generated mass as a consequence of infrared divergences, and the continuous symmetry is spontaneously broken for any finite N; however there is a *first order phase transition* as a function of the Hawking temperature $T_H = H/2\pi$. For $O(2)$ the symmetry is spontaneously broken for $T_H < T_c = \lambda^{1/4} v/2.419$ where λ is the quartic coupling and v is the tree-level vacuum expectation value and the Goldstone mode acquires a radiatively generated mass $\mathcal{M}_\pi^2 \propto \lambda^{1/4} H$. The first order nature of the transition is a consequence of the strong infrared behavior of minimally coupled scalar fields in de Sitter space-time; the jump in the order parameter at $T_H = T_c$ is $\sigma_{0c} \simeq 0.61 H/\lambda^{1/4}$. In the strict $N \to \infty$ the symmetry cannot be spontaneously broken. Furthermore, the lack of kinematic thresholds imply that the Goldstone modes *decay* into Goldstone and Higgs modes by emission and absorption of superhorizon quanta.

I. INTRODUCTION

In its simplest realization, inflationary cosmology can be effectively described as a quasi-de Sitter space-time. Early studies [1–6] revealed that de Sitter space-time features infrared instabilities and profuse particle production in interacting field theories. Infrared divergences in loop corrections to correlation functions hinder the reliability of the perturbative expansion [7–9], lead to the suggestion of an infrared instability of the vacuum [10–14], and affect correlation functions during inflation [7,8,15–20] requiring a nonperturbative treatment.

Backreaction from particle production in a de Sitter background has been argued to provide a dynamical "screening" mechanism that leads to relaxation of the cosmological constant [21–23], a suggestion that rekindled the interest on infrared effects in de Sitter space-time. A body of work established that infrared and secular divergences are manifest in super-Hubble fluctuations during de Sitter (or nearly de Sitter) inflation [24–27], thus a consistent program that provides a resummation of the perturbative expansion is required. Nonperturbative methods of resummation of the secular divergences have been implemented in several studies in de Sitter space-time [28] suggesting a dynamical generation of mass [27], a result that was originally anticipated in the seminal work of Ref. [29], and explored and extended in Ref. [30]. More recently a self-consistent mechanism of mass generation for scalar fields through infrared fluctuations has been suggested [24,27,31–37].

The lack of a global timelike killing vector in de Sitter space-time leads to remarkable physical effects, as it implies the lack of particle thresholds (a direct consequence

of energy-momentum conservation) and the decay of fields even in their own quanta [28,38] with the concomitant particle production, a result that was confirmed in Refs. [12,39] and more recently investigated in Refs. [40,41] for the case of heavy fields.

For light scalar fields in de Sitter space-time with mass $M \ll H$, it was shown in Refs. [28] that the infrared enhancement of self-energy corrections is manifest as poles in $\Delta = M^2/3H^2$ in correlation functions and that the most infrared singular contributions to the self-energy can be isolated systematically in an expansion in Δ akin to the ϵ expansion in critical phenomena. A similar expansion was noticed in Refs. [27,31,34,41,42].

Whereas infrared effects in de Sitter (or quasi-de Sitter) cosmology are typically studied via correlation functions, recently the issue of the time evolution of the *quantum states* has begun to be addressed. In Ref. [43] the Wigner-Weisskopf method [44,45] ubiquitous in quantum optics [46] has been adapted and extended as a nonperturbative quantum field theory method in inflationary cosmology to study the time evolution of quantum states. This method reveals how quantum states *decay* in time, it has been shown to be equivalent to the dynamical renormalization group in Minkowski space-time [43,47], and has recently been implemented to study the radiative generation of masses and decay widths of minimally coupled fields during inflation [37].

Early studies [48,49] suggested that infrared divergences during inflation can prevent spontaneous symmetry breaking; however more recently the issue of spontaneous symmetry breaking during inflation has been revisited in view of the generation of masses by radiative corrections [33,34,36]. In Ref. [34] the study of an $O(N)$ model in the large N limit reveals that there is no spontaneous symmetry breaking as a consequence of the infrared

[*]boyan@pitt.edu

023509-1

divergences: if the $O(N)$ symmetry is spontaneously broken there would be *massless* Goldstone bosons that lead to strong infrared divergences, the resolution, as per the results of this reference, is that the symmetry is restored by the strong infrared divergences and no symmetry breaking is possible. This result is in qualitative agreement with those of earlier works, Refs. [48,49]. However, a different study of the same model in Ref. [36] reaches a different conclusion: that indeed the $O(N)$ symmetry is spontaneously broken but Goldstone bosons acquire a radiatively induced mass. In Ref. [33] a scalar model with Z_2 symmetry is studied with the result that radiative corrections tend to restore the symmetry via the nonperturbative generation of mass. Both Refs. [33,36] suggest a discontinuous transition.

A. Motivation, goals, and results

Spontaneous symmetry breaking is an important ingredient in the inflationary paradigm, and as such it merits a deeper understanding of whether radiative corrections modify the familiar picture of slow roll inflation. If, as found in Ref. [34], symmetry breaking is not possible in some models, these would be ruled out at least in the simple small field scenarios of slow roll, as inflation would not be successfully ended by the inflaton reaching the broken symmetry minimum. Furthermore, if the inflaton is part of a Higgs-type mode of multiplet of fields, the question of whether the fields associated with unbroken generators are massless is very important as these could lead to entropy perturbations whose infrared divergences are more severe than those of adiabatic perturbations [9].

In this article we study an $O(2)$ scalar field theory in de Sitter space-time and extract implications for $O(N)$ with the following *goals*: (i) Revisit at a deeper level the content of Goldstone's theorem in an *expanding cosmology* in absence of manifest time translational invariance. In particular, whether spontaneous symmetry breaking of a continuous symmetry does imply the existence of massless Goldstone modes in an inflationary setting. (ii) Study beyond the local mean field approximation of whether a continuous symmetry can be spontaneously broken in de Sitter space-time. (iii) Reveal how the mechanism of self-consistent nonperturbative mass generation can be compatible with symmetry breaking and Goldstone modes.

Recently there has been renewed interest in a deeper understanding of Goldstone's theorem and spontaneous symmetry breaking both in relativistic and nonrelativistic systems [50–52]; thus our study provides a complementary investigation of symmetry breaking in a *cosmological setting* wherein the lack of a global timelike Killing vector leads to unexpected yet very physical consequences.

B. Brief summary of results

(i) We argue that in absence of time translational invariance Goldstone's theorem *does not* imply the existence of massless excitations if a continuous symmetry is spontaneously broken. We revisit the implementation of Goldstone's theorem in a spontaneously broken $O(2)$ symmetry in Minkowski space-time and highlight that the masslessness of Goldstone bosons is a consequence of a cancellation between *space-time local and nonlocal terms* in the loop expansion and discuss the implications for an $O(N)$ theory in the large N limit.

(ii) We then study the same model in de Sitter space-time, and emphasize that whereas in Minkowski space-time the conservation of the Noether current associated with the continuous symmetry directly leads to Goldstone's theorem, in an expanding cosmology this current is *covariantly conserved* and the consequences are, therefore, much less stringent. In conformal coordinates a *conserved* Noether current is manifestly obtained, but the lack of time translational invariance renders the content of Goldstone's theorem much less stringent.

(iii) We implement a self-consistent nonperturbative approach based on the Wigner-Weisskopf method described in Refs. [37,43] that allows one to extract the mass of the single-particle excitations and distinctly shows that the space-time local terms cannot be cancelled by nonlocal self-energy terms in leading order in a Δ expansion. As a result Goldstone modes acquire a radiatively generated mass as a consequence of infrared divergences in agreement with the results in Refs. [34,36]. The lack of a timelike Killing vector entails that there are no kinematic thresholds, and as a consequence Goldstone modes acquire a *width* from processes of absorption and emission of superhorizon quanta of both Goldstone and Higgs-like modes.

(iv) We show that for finite N there is a symmetry breaking first order transition as a function of the Hawking temperature $T_H = H/2\pi$, Goldstone modes acquire a radiatively infrared generated self-consistent mass but also a *decay width*, and that the symmetry cannot be spontaneously broken in the strict $N \rightarrow \infty$ limit. We argue that a first order transition is a distinct and expected consequence of infrared effects, because a continuous transition would entail that at the critical point there should be massless excitations which would lead to infrared divergences. Radiative corrections relieve the infrared singularities by generating a mass but at the expense of turning the symmetry breaking transition into first order.

II. SPONTANEOUS SYMMETRY BREAKING AND GOLDSTONE BOSONS IN MINKOWSKI SPACE-TIME

A. General aspects

We consider the $O(2)$ linear sigma model as a simple example of a scalar theory with spontaneous symmetry

breaking (SSB) and extract consequences for the case of $O(N)$ in the large N limit.

The Lagrangian density for the $O(2)$ sigma model is

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \pi)^2 - V(\sigma^2 + \pi^2), \quad (2.1)$$

which is invariant under the infinitesimal transformations

$$\pi \to \pi + \epsilon\sigma, \qquad \sigma \to \sigma - \epsilon\pi, \quad (2.2)$$

with ϵ a space-time constant infinitesimal angle. The canonical momenta conjugate to the π, σ fields are, respectively,

$$P_\pi(x) = \dot{\pi}(x), \qquad P_\sigma(x) = \dot{\sigma}(x), \quad (2.3)$$

with the equal time canonical commutation relations

$$[P_\pi(\vec{x}, t), \pi(\vec{y}, t)] = -i\delta^3(\vec{x} - \vec{y}),$$
$$[P_\sigma(\vec{x}, t), \sigma(\vec{y}, t)] = -i\delta^3(\vec{x} - \vec{y}). \quad (2.4)$$

The conserved Noether current associated with the global symmetry (2.2) is

$$J^\mu(x) = i(\sigma(x)\partial^\mu\pi(x) - \pi(x)\partial^\mu\sigma(x)),$$
$$\partial_\mu J^\mu(x) = 0, \quad (2.5)$$

with the conserved charge

$$Q = i\int d^3x(\sigma(\vec{x}, t)P_\pi(\vec{x}, t) - \pi(\vec{x}, t)P_\sigma(\vec{x}, t)). \quad (2.6)$$

Consider the following identity resulting from current conservation (2.5):

$$\int d^3x \langle 0|[\vec{\nabla}\cdot\vec{J}(\vec{x}, t), \pi(\vec{y}, t')]|0\rangle$$
$$= \frac{\partial}{\partial t}\int d^3x \langle 0|[J^0(\vec{x}, t), \pi(\vec{y}, t')]|0\rangle. \quad (2.7)$$

Assuming spatial translational invariance we introduce

$$S(\vec{k}; t, t') = \int d^3x e^{-i\vec{k}\cdot(\vec{x}-\vec{y})}\langle 0|[J^0(\vec{x}, t), \pi(\vec{y}, t')]|0\rangle. \quad (2.8)$$

If the surface integral on the left-hand side of Eq. (2.7) vanishes, then it follows that

$$\lim_{k\to 0}\frac{\partial}{\partial t}S(\vec{k}; t, t') = 0. \quad (2.9)$$

In general this result implies that

$$\lim_{k\to 0}S(\vec{k}; t, t') = \langle 0|[Q(t), \pi(\vec{y}, t')]|0\rangle$$
$$= \langle 0|\sigma(\vec{y}, t')|0\rangle = v(t'), \quad (2.10)$$

namely, Q is time independent. In absence of time translational invariance the results (2.9) and (2.10) are the only statements that can be extracted from the conservation of the current. However, if *time tranlational invariance holds* then $S(\vec{k}; t, t') = S(\vec{k}; t - t')$ and introducing the spectral representation

$$S(\vec{k}, t - t') = \int\frac{d\omega}{2\pi}S(\vec{k}, \omega)e^{-i\omega(t-t')}, \quad (2.11)$$

it follows from (2.9) that (i) $v(t') = v$ in (2.10) is time independent and (ii)

$$\lim_{k\to 0}S(\vec{k}; \omega) = 2\pi v\delta(\omega), \qquad v = \langle 0|\sigma(\vec{0}, 0)|0\rangle, \quad (2.12)$$

where we have used Eqs. (2.6) and (2.4).

When space-time translational invariance is available, further information is obtained by writing $S(\vec{k}, \omega)$ in term of a complete set of eigenstates of the momentum and Hamiltonian operators by inserting this complete set of states in the commutators

$$e^{i\vec{P}\cdot\vec{x}}e^{-iHt}|n\rangle = e^{i\vec{p}_n\cdot\vec{x}}e^{-iE_n t}|n\rangle, \quad (2.13)$$

from which we obtain

$$S(\vec{k}, \omega) = 2\pi\sum_n\{\langle 0|J^0(\vec{0}, 0)|n\rangle\langle n|\pi(\vec{0}, 0)|0\rangle$$
$$\times \delta^3(\vec{p}_n - \vec{k})\delta(E_n - \omega) - \langle 0|\pi(\vec{0}, 0)|n\rangle$$
$$\times \langle n|J^0(\vec{0}, 0)|0\rangle\delta^3(\vec{p}_n + \vec{k})\delta(E_n + \omega)\}. \quad (2.14)$$

Then the result (2.12) implies an intermediate state with vanishing energy for vanishing momentum. This is the general form of Goldstone's theorem valid even for nonrelativistic systems [50–53]. The result has a clear interpretation: under the assumption that the current flow out of the integration boundaries vanishes, the total charge is a constant of motion. *If the theory is manifestly time translational invariant,* this automatically implies that $S(\vec{k}, t - t')$ in (2.8) does not depend on $t - t'$ by charge conservation; therefore, it follows directly that in the limit $k \to 0$ the spectral density $S(\vec{k}, \omega)$ can *only* have support at $\omega = 0$.

The standard intuitive explanation for gapless long wavelength excitations relies on the fact that the continuous symmetry entails that the manifold of minima away from the origin form a continuum of degenerate states. A rigid rotation around the minimum of the potential does not cost any *energy* because of the degeneracy; therefore the energy cost of making a long wavelength spatial rotation vanishes in the long wavelength limit precisely because of the degeneracy. Both this argument and the more formal proof (2.12) rely on the existence of a conserved energy and energy eigenstates, which is not available in the cosmological setting.

The main reason for going through this textbook derivation of Goldstone's theorem is to highlight that *time translational invariance* is an essential ingredient in the statement that the Goldstone theorem implies a *gapless excitation* if the symmetry is spontaneously broken.[1]

---

[1]Under the assumption that the current flow out of a boundary vanishes, see discussion in [53].

Precisely this point will be at the heart of the discussion of symmetry breaking in inflationary cosmology.

B. Tree-level, one-loop, and large N

In order to compare the well-known results in Minkowski space-time with the case of inflationary cosmology, we now study how Goldstone's theorem is implemented at tree and one-loop levels in the $O(2)$ case, and in the large N limit in the case of $O(N)$ symmetry, as this study will highlight the main differences between Minkowski and de Sitter space-times.

To be specific, we now consider the $O(2)$ model with potential

$$V(\sigma^2 + \pi^2) = \frac{\lambda}{8}\left(\sigma^2 + \pi^2 - \frac{\mu^2}{\lambda}\right)^2. \qquad (2.15)$$

Shifting the field

$$\sigma = \sigma_0 + \chi \qquad (2.16)$$

the potential (2.15) becomes

$$V(\chi, \pi) = \frac{M_\chi^2}{2}\chi^2 + \frac{M_\pi^2}{2}\pi^2 + \frac{\lambda}{2}\sigma_0 J\chi + \frac{\lambda}{2}\sigma_0\chi^3$$
$$+ \frac{\lambda}{2}\sigma_0\pi^2\chi + \frac{\lambda}{8}\chi^4 + \frac{\lambda}{8}\pi^4 + \frac{\lambda}{4}\chi^2\pi^2, \quad (2.17)$$

where

$$J = \sigma_0^2 - \frac{\mu^2}{\lambda}, \qquad M_\chi^2 = \lambda\left(\sigma_0^2 + \frac{J}{2}\right),$$
$$M_\pi^2 = \frac{\lambda}{2}J \Rightarrow M_\chi^2 - M_\pi^2 = \lambda\sigma_0^2. \qquad (2.18)$$

The value of σ_0 is found by requiring that the expectation value of χ vanishes in the correct vacuum state, thus it departs from the tree-level value μ^2/λ by radiative corrections.

Tree level.—At tree level $\sigma_0^2 = \mu^2/\lambda$, $M_\pi^2 = 0$, $M_\chi^2 = \mu^2$, and the π field obeys the equation of motion

$$\ddot{\pi}(\vec{x}, t) - \nabla^2\pi(\vec{x}, t) = 0. \qquad (2.19)$$

The π field is quantized in a volume V as usual

$$\pi(\vec{x}, t) = \sum_{\vec{k}} \frac{1}{\sqrt{2Vk}}[a_{\vec{k}}e^{-i(kt - \vec{k}\cdot\vec{x})} + a_{\vec{k}}^\dagger e^{i(kt - \vec{k}\cdot\vec{x})}]. \quad (2.20)$$

The conserved current (2.5) becomes

$$J^\mu = i\sigma_0\partial^\mu\pi + i(\chi\partial^\mu\pi - \pi\partial^\mu\chi). \qquad (2.21)$$

At tree level only the first term contributes to the spectral density (2.14), since at this level the π field creates a single-particle state out of the vacuum, which is the *only* state that contributes to (2.14). We refer to the first term as J_{tl}^μ and its conservation is a result of the equation of motion (2.19) and σ_0 being a space-time constant. It is straightforward to find

$$\langle 0|J_{tl}^0(\vec{0}, 0)|1_{\vec{p}}\rangle\langle 1_{\vec{p}}|\pi(\vec{0}, 0)|0\rangle$$
$$= -\langle 0|\pi(\vec{0}, 0)|1_{\vec{p}}\rangle\langle 1_{\vec{p}}|J_{tl}^0(\vec{0}, 0)|0\rangle = \frac{\sigma_0}{2V}, \qquad (2.22)$$

where V is the quantization volume. Therefore

$$S(\vec{k}, \omega) = 2\pi\sigma_0 \int \frac{d^3p}{(2\pi)^3}\frac{1}{2}[\delta(p + \omega)\delta^3(\vec{p} + \vec{k})$$
$$+ \delta(p - \omega)\delta^3(\vec{p} - \vec{k})], \qquad (2.23)$$

and

$$\lim_{k\to 0}S(\vec{k}, \omega) = 2\pi\sigma_0\delta(\omega). \qquad (2.24)$$

One loop.—We now focus on understanding how the $\pi-$ field remains massless with radiative corrections. We carry out the loop integrals in four-dimensional Euclidean space-time; the result is independent of this choice. The interaction vertices are depicted in Fig. 1.

The vacuum expectation value σ_0 is fixed by the requirement that

$$\langle\chi\rangle = 0, \qquad (2.25)$$

to which we refer as the *tadpole condition*; it is depicted in Fig. 2. We find

$$\langle\chi\rangle = 0 \Rightarrow \frac{\lambda\sigma_0}{2M_\chi^2}[J + 3I_\chi + I_\pi] = 0, \qquad (2.26)$$

where

$$I_\chi = \int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2 + M_\chi^2}, \qquad I_\pi = \int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2 + M_\pi^2}. \qquad (2.27)$$

This condition ensures that the matrix element of the interaction Hamiltonian H_I between the vacuum and single-particle states vanishes, namely,
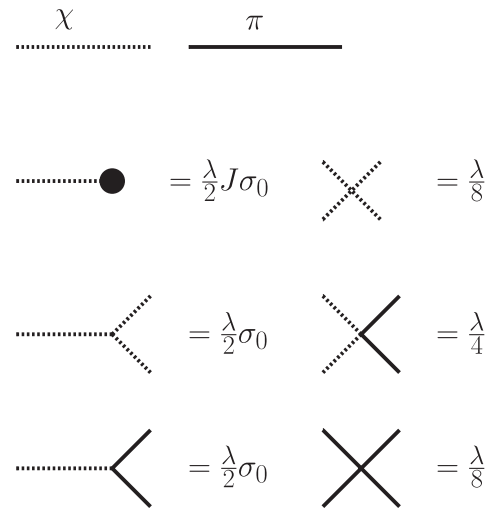

FIG. 1. Vertices in broken symmetry. The broken line ending in the black dot refers to the *linear* term in χ in Eq. (2.17).

FIG. 2.   Tadpole condition (2.25).

$$\langle 1_{\vec{k}}|H_I|0\rangle = 0. \qquad (2.28)$$

There are two solutions of the tadpole equation

$$\sigma_0 = 0, \qquad (2.29)$$

$$J = -3I_\chi - I_\pi \Rightarrow \sigma_0^2 = \frac{\mu^2}{\lambda} - 3I_\chi - I_\pi \neq 0. \qquad (2.30)$$

If available, the second solution (2.30) leads to spontaneous symmetry breaking.

At finite temperature

$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + M_{\chi,\pi}^2} \Rightarrow T\sum_{\omega_n} \frac{d^3k}{(2\pi)^3} \frac{1}{\omega_n^2 + \vec{k}^2 + M_{\chi,\pi}^2},$$

$$\omega_n = 2\pi n T, \qquad (2.31)$$

where ω_n are the Matsubara frequencies. For $T^2 \gg M_{\chi,\pi}^2$ both integrals are proportional to T^2 and the symmetry breaking solution becomes

$$\sigma_0^2 = C(T_c^2 - T^2) \qquad (2.32)$$

with C a positive numerical constant. This well-known observation will become relevant below in the discussion of symmetry breaking in de Sitter space-time because the (physical) event horizon of de Sitter space-time $1/H$ determines the Hawking temperature $T_H = H/2\pi$.

The π propagator becomes

$$G_\pi(k) = \frac{1}{k^2 + M_\pi^2 - \Sigma_\pi(k)}, \qquad (2.33)$$

where the Feynman diagrams for the self-energy are shown in Fig. 3.
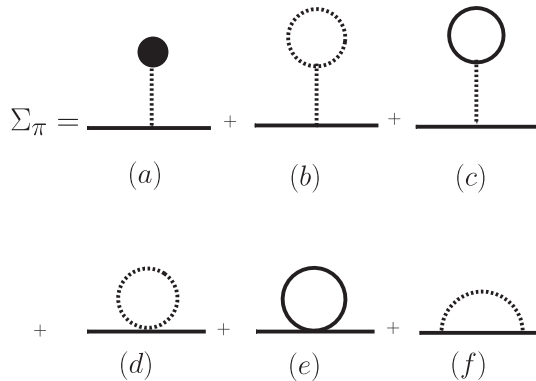
FIG. 3.   One-loop diagrams that contribute to the π field self-energy $\Sigma_\pi(k)$.

The contributions from diagrams (a), (b), (c) yield

$$\Sigma_{\pi,a}(k) + \Sigma_{\pi,b}(k) + \Sigma_{\pi,c}(k) = \frac{\lambda^2 \sigma_0^2}{2M_\chi^2}[J + 3I_\chi + I_\pi] = 0 \qquad (2.34)$$

as a consequence of the tadpole condition (2.26). The remaining diagrams yield

$$\Sigma_{\pi,d}(k) + \Sigma_{\pi,e}(k) + \Sigma_{\pi,f}(k)$$
$$= -\frac{\lambda}{2}\Bigg[I_\chi + 3I_\pi - 2\lambda\sigma_0^2 \int \frac{d^4q}{(2\pi)^4}$$
$$\times \frac{1}{(q^2 + M_\chi^2)((q+k)^2 + M_\pi^2)}\Bigg]. \qquad (2.35)$$

The pole in the π propagator determines the physical mass of the π field; we find

$$k^2 + M_\pi^2 - \Sigma_\pi(k) = k^2 + \frac{\lambda}{2}\Bigg[J + I_\chi + 3I_\pi - 2\lambda\sigma_0^2 \int \frac{d^4q}{(2\pi)^4}$$
$$\times \frac{1}{(q^2 + M_\chi^2)((q+k)^2 + M_\pi^2)}\Bigg], \qquad (2.36)$$

where we have used M_π^2 given by Eq. (2.18).

If there is spontaneous symmetry breaking, $J = -3I_\chi - I_\pi$ leading to

$$M_\pi^2 - \Sigma_\pi(k) = \lambda \int \frac{d^4q}{(2\pi)^4}\Bigg[\frac{1}{q^2 + M_\pi^2} - \frac{1}{q^2 + M_\chi^2}$$
$$- \frac{\lambda\sigma_0^2}{((q+k)^2 + M_\pi^2)(q^2 + M_\chi^2)}\Bigg]. \qquad (2.37)$$

Therefore the inverse propagator is given by

$$k^2 + M_\pi^2 - \Sigma_\pi(k) = k^2 + \lambda\sigma_0^2 \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 + M_\chi^2}$$
$$\times \Bigg[\frac{1}{q^2 + M_\pi^2} - \frac{1}{(q+k)^2 + M_\pi^2}\Bigg], \qquad (2.38)$$

where we used Eq. (2.18). Obviously (2.37) and (2.38) vanish as $k^2 \to 0$ (and are proportional to k^2 in this limit by Lorentz invariance); therefore the propagator for the Goldstone mode π features a pole at $k^2 = 0$. We emphasize that the vanishing of the mass is a consequence of a precise *cancellation* between the local tadpole terms, Figs. 3(d) and 3(e) and the nonlocal (in space-time) contribution Fig. 3(f), in the $k \to 0$ limit.

The propagator for χ—the Higgs-like mode—is obtained in a similar manner; the Feynman diagrams for the self-energy $\Sigma_\chi(k)$ are similar to those for Σ_π with χ external lines and the only difference being the combinatoric factors for diagrams (a)–(e), and two exchange diagrams of the (f)-type with intermediate states of two χ particles and two π particles, respectively. Again diagrams

of the type (a)–(c) are cancelled by the tadpole condition (2.26), and we find

$$k^2 + M_\chi^2 - \Sigma_\chi(k) = k^2 + \frac{\lambda}{2}\bigg[2\sigma_0^2 + J + 3I_\chi + I_\pi$$
$$- \lambda\sigma_0^2 \tilde{I}_\pi(k) - 9\lambda\sigma_0^2 \tilde{I}_\chi(k)\bigg], \quad (2.39)$$

where

$$\tilde{I}_{\chi,\pi}(k) = \int \frac{d^4q}{(2\pi)^4} \frac{1}{((q+k)^2 + M_{\chi,\pi}^2)^2}. \quad (2.40)$$

If the symmetry is spontaneously broken, using the condition (2.30) we find

$$k^2 + M_\chi^2 - \Sigma_\chi(k) = k^2 + \lambda\sigma_0^2\bigg[1 - \frac{\lambda}{2}\tilde{I}_\pi(k) - \frac{9\lambda}{2}\tilde{I}_\chi(k)\bigg]. \quad (2.41)$$

Large N limit.—If rather than an $O(2)$ symmetry we consider the $O(N)$ case, after symmetry breaking along the σ direction the $\vec{\pi}$ fields belong to an $O(N-1)$ multiplet. In the large N limit the leading term in the tadpole condition $\langle\chi\rangle = 0$ (2.25) is given by the last diagram (solid circle) in Fig. 2,

$$\langle\chi\rangle = 0 \Rightarrow \frac{\lambda\sigma_0}{2M_\chi^2}[J + NI_\pi] = 0, \quad (2.42)$$

where we have neglected terms of $\mathcal{O}(1/N)$ in the large N limit. In this limit the leading contribution to the π self-energy is given by Fig. 3(e),

$$\Sigma_\pi = -\frac{\lambda}{2}NI_\pi, \quad (2.43)$$

where again we neglected terms of $\mathcal{O}(1/N)$. Therefore the inverse π propagator in the large N limit is given by

$$k^2 + M_\pi^2 - \Sigma_\pi = k^2 + \mathcal{M}_\pi^2, \quad (2.44)$$

where

$$\mathcal{M}_\pi^2 = \frac{\lambda}{2}[J + NI_\pi], \quad (2.45)$$

thus in the large N limit, the tadpole condition (2.42) can be written as

$$\langle\chi\rangle = 0 \Rightarrow \sigma_0 \mathcal{M}_\pi^2 = 0. \quad (2.46)$$

Therefore, if this condition is fulfilled with $\sigma_0 \neq 0$, namely, with spontaneous symmetry breaking, automatically the π field becomes massless.

C. Counterterm approach

An alternative approach that is particularly suited to the study of radiative corrections to masses in the cosmological setting is the familiar method of introducing a mass counterterm in the Lagrangian by writing the mass term in the Lagrangian density as

$$M_\pi^2 \pi^2 = \mathcal{M}_\pi^2 \pi^2 + \delta M_\pi^2 \pi^2, \qquad \delta M_\pi^2 = M_\pi^2 - \mathcal{M}_\pi^2, \quad (2.47)$$

and requesting that the counterterm δM^2 subtracts the π self-energy at zero four momentum

$$- \delta M_\pi^2 + \Sigma_\pi(0) = 0 \Rightarrow \mathcal{M}_\pi^2 = M_\pi^2 - \Sigma_\pi(0), \quad (2.48)$$

and the inverse propagator becomes

$$G_\pi^{-1}(k) = k^2 + \mathcal{M}_\pi^2 - [\Sigma_\pi(k) - \Sigma_\pi(0)] \quad (2.49)$$

in the broken symmetry phase $\mathcal{M}_\pi^2 = 0$ from Eqs. (2.37) and (2.38) and the propagator features a pole at zero four momentum.

The main reason to go through this exercise is to highlight the following important points:

  (i) The tadpole type diagrams (a), (b), (c) are cancelled by the tadpole condition (2.26) which is tantamount to the requirement that the interaction Hamiltonian has a vanishing matrix element between the vacuum and a single χ particle state.

  (ii) At one-loop level the vanishing of the π mass in the case of spontaneous symmetry breaking is a consequence of the cancellation between the *local* tadpole diagrams (d), (e) and the *nonlocal* one-loop diagram (f) in the $k \to 0$ limit (the nonlocality is in configuration space not in Fourier space). This point will be at the heart of the discussion in inflationary space-time below.

  (iii) In the large N limit, only the local tadpole Fig. 3(e) contributes to the π self-energy and the tadpole condition (2.26), for which a symmetry breaking solution immediately yields a vanishing π mass. The tadpole and nonlocal diagrams Figs. 3(d) and 3(f) are suppressed by a power of $1/N$ in this limit compared to the diagram [3(e)].

  (iv) The general, nonperturbative proof of the existence of gapless long wavelength excitations as a consequence of the results (2.12) and (2.14) manifestly relies on time translational invariance and energy eigenstates. In its most general form, without invoking time translational invariance, the result (2.10) is much less stringent on the long wavelength spectrum of excitations without an (obvious) statement on the mass spectrum of the theory. Such a situation, the lack of time translational invariance (global timelike Killing vector), is a hallmark of inflationary cosmology and it is expected that—unlike in Minkowski space-time—Goldstone modes *may acquire a mass radiatively.*

These points are relevant in the discussion of the fate of Goldstone bosons in de Sitter space-time discussed below.

III. GOLDSTONE BOSONS IN DE SITTER SPACE-TIME

We consider the $O(2)$ linear sigma model minimally coupled in a spatially flat de Sitter space-time with metric given by

$$ds^2 = dt^2 - a^2(t)d\vec{x}^2, \qquad a(t) = e^{Ht}, \qquad (3.1)$$

defined by the action (the different notation for the fields as compared to the previous section will be explained below)

$$L = \int d^4x \sqrt{|g|} \left\{ \frac{1}{2} g^{\mu\nu} \partial_\mu \vec{\Phi} \cdot \partial_\nu \vec{\Phi} - V(\vec{\Phi} \cdot \vec{\Phi}) \right\},$$

$$\vec{\Phi} = (\phi_1, \phi_2), \qquad (3.2)$$

where

$$V(\vec{\Phi} \cdot \vec{\Phi}) = \frac{\lambda}{8} \left(\phi_1^2 + \phi_2^2 - \frac{\mu^2}{\lambda} \right)^2. \qquad (3.3)$$

We follow the method of Ref. [54] to obtain the conservation law associated with the global $O(2)$ symmetry: consider a space-time dependent infinitesimal transformation that vanishes at the boundary of space-time

$$\phi_1(\vec{x}, t) \rightarrow \phi_1(\vec{x}, t) - \epsilon(\vec{x}, t)\phi_2(\vec{x}, t),$$

$$\phi_2(\vec{x}, t) \rightarrow \phi_2(\vec{x}, t) + \epsilon(\vec{x}, t)\phi_1(\vec{x}, t), \qquad (3.4)$$

under which the change in the action is given by

$$\delta L = \int d^4x \sqrt{|g|} \partial_\mu \epsilon(\vec{x}, t) J^\mu(\vec{x}, t), \qquad (3.5)$$

where

$$J^\mu(\vec{x}, t) = ig^{\mu\nu}[\phi_1 \partial_\nu \phi_2 - \phi_2 \partial_\nu \phi_1] \qquad (3.6)$$

upon integration by parts assuming a vanishing boundary term,

$$\delta L = -\int d^4x \sqrt{|g|} \epsilon(\vec{x}, t) J^\mu_{;\mu}(\vec{x}, t), \qquad (3.7)$$

from which upon using the variational principle [54] we recognize that the current (3.6) is *covariantly conserved*

$$J^\mu_{;\mu}(\vec{x}, t) = \frac{1}{\sqrt{|g|}} \partial_\mu(\sqrt{|g|} J^\mu)$$

$$= \dot{J}^0 + 3HJ^0 - \frac{1}{a^2(t)} \nabla \cdot (\phi_1 \nabla \phi_2 - \phi_2 \nabla \phi_1)$$

$$= 0, \qquad (3.8)$$

where the dot stands for d/dt. This covariant conservation law can be seen to follow from the Heisenberg equations of motion for the fields,

$$\ddot{\phi}_a + 3H\dot{\phi}_a - \frac{\nabla^2}{a^2(t)}\phi_a + 2\left(\frac{dV(\rho^2)}{d\rho^2}\right)\phi_a = 0,$$

$$a = 1, 2, \qquad \rho^2 = \phi_1^2 + \phi_2^2. \qquad (3.9)$$

It is the second term in (3.8) that prevents a straightforward generalization of the steps leading to Goldstone's theorem as described in the previous section. Fundamentally it is this difference that is at the heart of the major discrepancies in the corollary of Goldstone's theorem in the expanding cosmology as compared to Minkowski space-time.

It is convenient to pass to conformal time

$$\eta = -\frac{e^{-Ht}}{H}, \qquad a(\eta) = -\frac{1}{H\eta}, \qquad (3.10)$$

and to rescale the fields

$$\phi_1(\vec{x}, t) = \frac{\sigma(\vec{x}, \eta)}{a(\eta)}, \qquad \phi_2(\vec{x}, t) = \frac{\pi(\vec{x}, \eta)}{a(\eta)} \qquad (3.11)$$

in terms of which the covariant conservation law (3.8) becomes

$$\frac{\partial}{\partial\eta} \mathcal{J}^0(\vec{x}, \eta) + \vec{\nabla} \cdot \vec{\mathcal{J}}(\vec{x}, \eta) = 0, \qquad (3.12)$$

where

$$\mathcal{J}^0(\vec{x}, \eta) = i[\sigma\pi' - \pi\sigma'], \qquad (3.13)$$

$$\vec{\mathcal{J}}(\vec{x}, \eta) = -i[\sigma\vec{\nabla}\pi - \pi\vec{\nabla}\sigma], \qquad (3.14)$$

where $\prime \equiv d/d\eta$.

In terms of the rescaled fields the action becomes (after dropping a total surface term)

$$L = \int d^3x \, d\eta \left\{ \frac{1}{2} \left[\sigma'^2 - (\nabla\sigma)^2 + \pi'^2 - (\nabla\pi)^2 \right. \right.$$

$$\left. \left. + \frac{a''}{a}(\sigma^2 + \pi^2) \right] - \mathcal{V}(\sigma^2 + \pi^2; \eta) \right\}, \qquad (3.15)$$

where

$$\mathcal{V}(\sigma^2 + \pi^2; \eta) = \frac{\lambda}{8}\left(\sigma^2 + \pi^2 - a^2(\eta)\frac{\mu^2}{\lambda}\right)^2. \quad (3.16)$$

Therefore, although the Noether current (3.13) and (3.14) is conserved and looks similar to that in Minkowski space-time, the Hamiltonian is manifestly time dependent; there is no time translational invariance and no energy conservation and no spectral representation available—all of these are necessary ingredients for Goldstone's theorem to guarantee massless excitations.

The Heisenberg equations of motion are

$$\sigma'' - \nabla^2\sigma + \left[2\frac{d\mathcal{V}(r^2)}{dr^2} - \frac{a''}{a} \right]\sigma = 0, \qquad (3.17)$$

$$\pi'' - \nabla^2\pi + \left[2\frac{d\mathcal{V}(r^2)}{dr^2} - \frac{a''}{a} \right]\pi = 0, \qquad (3.18)$$

where $r^2 = \pi^2 + \sigma^2$. Using these Heisenberg equations of motion it is straightforward to confirm the conservation law (3.12) with (3.13) and (3.14).

Now making an η-dependent shift of the field σ

$$\sigma(\vec{x}, \eta) = \sigma_0 a(\eta) + \chi(\vec{x}, \eta), \qquad (3.19)$$

the action (3.15) becomes

$$
L = \int d^3x \, d\eta \left\{ \frac{1}{2} \left[\chi'^2 - (\nabla\chi)^2 + \pi'^2 - (\nabla\pi)^2 \right. \right.
$$
$$
\left. - \frac{1}{\eta^2}\left(\frac{M_\chi^2}{H^2} - \frac{1}{2}\right)\chi^2 - \frac{1}{\eta^2}\left(\frac{M_\pi^2}{H^2} - \frac{1}{2}\right)\pi^2 \right]
$$
$$
+ \frac{\lambda}{2\eta^3} \frac{\sigma_0 J}{H^3} \chi + \frac{\lambda}{2\eta} \frac{\sigma_0}{H} \chi^3 + \frac{\lambda}{2\eta} \frac{\sigma_0}{H} \pi^2 \chi
$$
$$
\left. - \frac{\lambda}{8}\chi^4 - \frac{\lambda}{8}\pi^4 - \frac{\lambda}{4}\chi^2\pi^2 \right\}, \qquad (3.20)
$$

where $M_{\chi,\pi}$, J are the same as in the Minkowski space-time case given by Eq. (2.18). The Heisenberg equations of motion for the spatial Fourier modes of wave vector k of the fields in the noninteracting ($\lambda = 0$) theory are given by

$$
\chi_{\vec{k}}''(\eta) + \left[k^2 - \frac{1}{\eta^2}\left(\nu_\chi^2 - \frac{1}{4}\right) \right]\chi_{\vec{k}}(\eta) = 0, \qquad (3.21)
$$

$$
\pi_{\vec{k}}''(\eta) + \left[k^2 - \frac{1}{\eta^2}\left(\nu_\pi^2 - \frac{1}{4}\right) \right]\pi_{\vec{k}}(\eta) = 0, \qquad (3.22)
$$

where

$$
\nu_{\chi,\pi}^2 = \frac{9}{4} - \frac{M_{\chi,\pi}^2}{H^2}. \qquad (3.23)
$$

We will focus on the case of "light" fields, namely, $M_{\chi,\pi}^2 \ll H^2$ and choose Bunch-Davies vacuum conditions for which the two linearly independent solutions are given by

$$
g_{\chi,\pi}(k;\eta) = \frac{1}{2} i^{\nu_{\chi,\pi}+(1/2)}\sqrt{-\pi\eta}\, H_{\nu_{\chi,\pi}}^{(1)}(-k\eta), \qquad (3.24)
$$

$$
f_{\chi,\pi}(k;\eta) = \frac{1}{2} i^{-\nu_{\chi,\pi}-(1/2)}\sqrt{-\pi\eta}\, H_{\nu_{\chi,\pi}}^{(2)}(-k\eta)
$$
$$
= g_{\chi,\pi}^*(k;\eta), \qquad (3.25)
$$

where $H_\nu^{(1,2)}(z)$ are Hankel functions. Expanding the field operator in this basis in a comoving volume V

$$
\chi(\vec{x}, \eta) = \frac{1}{\sqrt{V}} \sum_{\vec{k}} [a_{\vec{k}} g_\chi(k;\eta) e^{i\vec{k}\cdot\vec{x}} + a_{\vec{k}}^\dagger g_\chi^*(k;\eta) e^{-i\vec{k}\cdot\vec{x}}],
$$
$$
(3.26)
$$

$$
\pi(\vec{x}, \eta) = \frac{1}{\sqrt{V}} \sum_{\vec{k}} [b_{\vec{k}} g_\pi(k;\eta) e^{i\vec{k}\cdot\vec{x}} + b_{\vec{k}}^\dagger g_\pi^*(k;\eta) e^{-i\vec{k}\cdot\vec{x}}].
$$
$$
(3.27)
$$

The Bunch-Davies vacuum is defined so that

$$
a_{\vec{k}}|0\rangle = 0, \qquad b_k|0\rangle = 0, \qquad (3.28)
$$

and the Fock states are obtained by applying creation operators $a_{\vec{k}}^\dagger$, $b_{\vec{k}}^\dagger$ onto the vacuum.

After the shift (3.19), the current (3.13) and (3.14) becomes

$$
\mathcal{J}^0(\vec{x}, \eta) = \mathcal{J}_{tl}^0(\vec{x}, \eta) + i[\chi\pi' - \pi\chi'],
$$
$$
\mathcal{J}_{tl}^0(\vec{x}, \eta) = i[\sigma_0 a \pi' - \pi \sigma_0 a'], \qquad (3.29)
$$

$$
\vec{\mathcal{J}}(\vec{x}, \eta) = \vec{\mathcal{J}}_{tl}(\vec{x}, \eta) - i[\chi\vec{\nabla}\pi - \pi\vec{\nabla}\chi],
$$
$$
\vec{\mathcal{J}}_{tl}(\vec{x}, \eta) = -i\sigma_0 a \vec{\nabla}\pi. \qquad (3.30)
$$

The terms $\mathcal{J}_{tl}^0(\vec{x}, \eta)$, $\vec{\mathcal{J}}_{tl}(\vec{x}, \eta)$ on the right-hand sides of (3.29) and (3.30) are the *tree-level* contributions to the conserved current as these terms create single-particle π states out of the vacuum.

The interaction vertices are the same as those for the Minkowski space-time case depicted in Fig. 1 but with the replacements

$$
\sigma_0 \rightarrow -\frac{\sigma_0}{H\eta}, \qquad J \rightarrow -\frac{J}{H\eta}. \qquad (3.31)
$$

In Refs. [28,31,37] it is found that the tadpole contributions in Figs. 2, 3(d), and 3(e) are given by

$$
\langle 0|\chi^2(\vec{x}, \eta)|0\rangle_{\text{ren}} = \frac{1}{8\pi^2\eta^2}\frac{1}{\Delta_\chi}[1 + \cdots], \qquad (3.32)
$$

$$
\langle 0|\pi^2(\vec{x}, \eta)|0\rangle_{\text{ren}} = \frac{1}{8\pi^2\eta^2}\frac{1}{\Delta_\pi}[1 + \cdots], \qquad (3.33)
$$

where the renormalization regularizes ultraviolet divergences, and

$$
\Delta_\chi = \frac{M_\chi^2}{3H^2}, \qquad \Delta_\pi = \frac{M_\pi^2}{3H^2}. \qquad (3.34)
$$

The dots in Eqs. (3.32) and (3.33) stand for terms subleading in powers of $\Delta_{\chi,\pi} \ll 1$. In order to maintain a notation consistent with the previous section we introduce

$$
I_{\chi,\pi} \equiv \frac{1}{8\pi^2\Delta_{\chi,\pi}}. \qquad (3.35)
$$

The tadpole condition now becomes

$$
\langle\chi\rangle = 0 \Rightarrow \frac{\lambda a \sigma_0}{2\eta^2}\left[\frac{J}{H^2} + 3I_\chi + I_\pi\right] = 0. \qquad (3.36)
$$

A symmetry breaking solution corresponds to $\sigma_0 \neq 0$, $J/H^2 = -3I_\chi - I_\pi$. At tree level

$$
\sigma_0^2 = \frac{\mu^2}{\lambda} \Rightarrow J = 0 \Rightarrow M_\pi^2 = 0, \qquad (3.37)
$$

and using that $a''/a = 2/\eta^2$ the tree-level conservation law becomes

$$\frac{\partial}{\partial \eta} \mathcal{J}_{tl}^0 + \vec{\nabla} \cdot \vec{\mathcal{J}}_{tl} = 0 \Rightarrow \sigma_0 a(\eta) \left[\pi'' - \frac{2}{\eta^2} - \nabla^2 \pi \right]$$
$$= 0, \tag{3.38}$$

which is fulfilled by the Heisenberg equation of motion for the π field (3.22) with $M_\pi = 0$, namely, $\nu_\pi = 3/2$.

It is illuminating to understand how the result (2.10) is fulfilled at tree level. With the expansion of the π field given by (3.27) and $\nu_\pi = 3/2$ introduced in $\mathcal{J}_{tl}^0(\vec{x}, \eta)$, we find

$$S(\vec{k}; \eta, \eta') = -2\sigma_0 a(\eta) \text{Im} \left[g_\pi^*(k; \eta') \left(g_\pi'(k; \eta) \right. \right.$$
$$\left. \left. + \frac{g_\pi(k; \eta)}{\eta} \right) \right], \tag{3.39}$$

and the long wavelength limit is given by

$$\lim_{k \to 0} S(\vec{k}; \eta, \eta') = \sigma_0 a(\eta'). \tag{3.40}$$

Again, we note that it is precisely the lack of time translational invariance that restricts the content of Eq. (3.40); while this equation is satisfied with $M_\pi = 0$ at tree level, there is no constraint on the mass of the single-particle excitations from the general result (2.10). Thus whether the Goldstone fields acquire a mass via radiative corrections now becomes a dynamical question.

There are two roadblocks to understanding radiative corrections to the mass, both stemming from the lack of time translational invariance: (i) in general, there is no simple manner to resum the series of one particle irreducible diagrams into a Dyson propagator, whose poles reveal the physical mass, (ii) there is no Fourier transform in time that when combined with a spatial Fourier transform would allow one to glean a dispersion relation for single-particle excitations. Obviously these two problems are related. In Refs. [33,34,36] only the local tadpoles were considered, this is a local mean field approximation and the space-time local nature of the tadpole allows one to extract a mass. However, while the mean field tadpole is the leading contribution in the large N limit as discussed in the previous section, for finite N the nonlocal diagram equivalent to Fig. 3(f) is of the same order, and in Minkowski space-time it is *this* diagram that cancels the tadpole (mean field) contribution to the π mass. Thus for finite N the question is whether the nonlocal self-energy contribution 3(f) can cancel the tadpole contributions of Fig. 3(d) and 3(e) even when these feature very different time dependence and 3(f) does not have a time Fourier transform that renders it local in frequency space.

It is at this point where the Wigner-Weisskopf method introduced in Refs. [37,43] proves to be particularly useful.

A. Wigner-Weisskopf theory in de Sitter space-time

In order to make the discussion self-contained, we highlight the main aspects of the Wigner-Weisskopf nonpertur-

bative approach to study the time evolution of quantum states pertinent to the self-consistent description of mass generation discussed in the previous sections. For a more thorough discussion and comparison to results in Minkowski space-time, the reader is referred to Refs. [37,43]. Expanding the interaction picture state $|\Psi(\eta)\rangle_I$ in Fock states $|n\rangle$ obtained as usual by applying the creation operators on to the (bare) vacuum state (here taken to be the Bunch-Davies vacuum) as

$$|\Psi(\eta)\rangle_I = \sum_n C_n(\eta)|n\rangle, \tag{3.41}$$

the evolution of the state in the interaction picture is given by [43]

$$i\frac{d}{d\eta}|\Psi(\eta)\rangle_I = H_I(\eta)|\Psi(\eta)\rangle_I, \tag{3.42}$$

where $H_I(\eta)$ is the interaction Hamiltonian in the interaction picture. In terms of the coefficients $C_n(\eta)$ Eq. (3.42) becomes

$$\frac{dC_n(\eta)}{d\eta} = -i\sum_m C_m(\eta)\langle n|H_I(\eta)|m\rangle. \tag{3.43}$$

It is convenient to separate the diagonal matrix elements, that represent *local contributions,* from those that represent transitions and are associated with nonlocal self-energy corrections, writing

$$\frac{dC_n(\eta)}{d\eta} = -iC_n(\eta)\langle n|H_I(\eta)|n\rangle$$
$$- i\sum_{m \neq n} C_m(\eta)\langle n|H_I(\eta)|m\rangle. \tag{3.44}$$

Although this equation is exact, it yields an infinite hierarchy of simultaneous equations when the Hilbert space of states $|n\rangle$ is infinite dimensional. However, progress is made by considering the transition between states connected by the interaction Hamiltonian at a given order in H_I: consider the case when one state, say $|A\rangle$, couples to a set of states $|\kappa\rangle$, which couple back to $|A\rangle$ via H_I, to lowest order in the interaction the system of equations closes in the form

$$\frac{dC_A(\eta)}{d\eta} = -i\langle A|H_I(\eta)|A\rangle C_A(\eta)$$
$$- i\sum_{\kappa \neq A} \langle A|H_I(\eta)|\kappa\rangle C_\kappa(\eta), \tag{3.45}$$

$$\frac{dC_\kappa(\eta)}{d\eta} = -iC_A(\eta)\langle \kappa|H_I(\eta)|A\rangle, \tag{3.46}$$

where the $\sum_{\kappa \neq A}$ is over all the intermediate states coupled to $|A\rangle$ via H_I representing transitions.

Consider the initial value problem in which at time $\eta = \eta_0$ the state of the system is given by $|\Psi(\eta = \eta_0)\rangle = |A\rangle$ so that

$$C_A(\eta_0) = 1, \qquad C_{\kappa \neq A}(\eta = \eta_0) = 0. \qquad (3.47)$$

Solving (3.46) and introducing the solution into (3.45) we find

$$C_\kappa(\eta) = -i \int_{\eta_0}^{\eta} \langle \kappa | H_I(\eta') | A \rangle C_A(\eta') d\eta', \qquad (3.48)$$

$$\frac{dC_A(\eta)}{d\eta} = -i \langle A | H_I(\eta) | A \rangle C_A(\eta)$$
$$- \int_{\eta_0}^{\eta} \Sigma_A(\eta, \eta') C_A(\eta') d\eta', \qquad (3.49)$$

where[2]

$$\Sigma_A(\eta, \eta') = \sum_{\kappa \neq A} \langle A | H_I(\eta) | \kappa \rangle \langle \kappa | H_I(\eta') | A \rangle. \qquad (3.50)$$

In Eq. (3.46) we have not included the diagonal term as in (3.45)[3]; it is clear from (3.48) that with the initial condition (3.47) the amplitude of C_κ is of $\mathcal{O}(H_I)$; therefore, a diagonal term would effectively lead to higher order contributions to (3.49). The integro-differential equation (3.49) with *memory* yields a nonperturbative solution for the time evolution of the amplitudes and probabilities, which simplifies in the case of weak couplings. In perturbation theory the time evolution of $C_A(\eta)$ determined by Eq. (3.49) is *slow* in the sense that the time scale is determined by a weak coupling kernel Σ_A, hence an approximation in terms of an expansion in derivatives of C_A emerges as follows: introduce

$$W(\eta, \eta') = \int_{\eta_0}^{\eta'} \Sigma_A(\eta, \eta'') d\eta'', \qquad (3.51)$$

so that

$$\Sigma_A(\eta, \eta') = \frac{d}{d\eta'} W(\eta, \eta'), \qquad W(\eta, \eta_0) = 0. \quad (3.52)$$

Integrating by parts in Eq. (3.49) we obtain

$$\int_{\eta_0}^{\eta} \Sigma_A(\eta, \eta') C_A(\eta') d\eta'$$
$$= W(\eta, \eta) C_A(\eta) - \int_{\eta_0}^{\eta} W(\eta, \eta') \frac{d}{d\eta'} C_A(\eta') d\eta'. \qquad (3.53)$$

The second term on the right-hand side is formally of *higher order* in H_I. Integrating by parts successively yields a systematic approximation scheme as discussed in Ref. [43].

---

[2] In Ref. [43] it is proven that in Minkowski space-time the retarded self-energy in the single-particle propagator is given by $i\Sigma$.

[3] These diagonal terms represent local self-energy insertions in the propagators of the intermediate states, hence higher orders in the perturbative expansion.

Therefore to leading order in the interaction we find

$$C_A(\eta) = e^{-\int_{\eta_0}^{\eta} \tilde{W}(\eta', \eta') d\eta'},$$
$$\tilde{W}(\eta', \eta') = i \langle A | H_I(\eta') | A \rangle + \int_{\eta_0}^{\eta'} \Sigma_A(\eta', \eta'') d\eta''. \quad (3.54)$$

Following Ref. [37] we introduce the *real quantities* $\mathcal{E}_A(\eta); \Gamma_A(\eta)$ as

$$i \langle A | H_I(\eta') | A \rangle + \int_{\eta_0}^{\eta'} \Sigma_A(\eta', \eta'') d\eta''$$
$$\equiv i\mathcal{E}_A(\eta') + \frac{1}{2} \Gamma_A(\eta') \qquad (3.55)$$

in terms of which

$$C_A(\eta) = e^{-i \int_{\eta_0}^{\eta} \mathcal{E}_A(\eta') d\eta'} e^{-(1/2) \int_{\eta_0}^{\eta} \Gamma_A(\eta') d\eta'}. \qquad (3.56)$$

When the state A is a single-particle state, radiative corrections to the mass are extracted from $\mathcal{E}_A$ and

$$\Gamma_A(\eta) = -\frac{d}{d\eta} \ln \left[|C_A(\eta)|^2 \right] \qquad (3.57)$$

is identified as a (conformal) time-dependent decay rate.

Extracting the mass.—In Minkowski space-time for $|A\rangle = |1_{\vec{k}}\rangle$, a single-particle state of momentum $\vec{k}$, $\mathcal{E}_{1_{\vec{k}}}$ includes the self-energy correction to the mass of the particle [37,43,46]. Consider adding a mass counterterm to the Hamiltonian density, in terms of the spatial Fourier transform of the fields, it is given by

$$H_{ct} = \frac{\delta M^2}{2} \sum_{\vec{k}} \pi_{\vec{k}} \pi_{-\vec{k}} \qquad (3.58)$$

the matrix element

$$\langle 1_{\vec{k}}^\pi | H_{ct} | 1_{\vec{k}}^\pi \rangle = \delta M^2 | g_\pi(\eta) |^2. \qquad (3.59)$$

Hence it is clear that only the imaginary part of $\tilde{W}$ can be interpreted as a mass term, thus only the imaginary part of $\Sigma_{1_{\vec{k}}}$ contributes to the mass. However, the nonlocal nature of $\Sigma_{1_{\vec{k}}}$ also includes transient behavior from the initial state preparation, thus a mass term must be isolated in the asymptotic long time limit when transient phenomena has relaxed. Last but not least, momentum dependence can mask a constant mass term, which can only be identified in the long wavelength limit. In particular in Refs. [37,43] it is shown that in Minkowski space-time (see the Appendix)

$$\text{Im} \int_0^{t \to \infty} \Sigma_{1_{\vec{k}}}(t, t') dt' = \delta E_{1_{\vec{k}}}, \qquad (3.60)$$

where $\delta E_{1_{\vec{k}}}$ is the second order correction to the energy of a single-particle state with momentum $\vec{k}$ obtained in quantum mechanical perturbation theory (see also the Appendix).

The program of renormalized perturbation theory begins by writing the free field part of the Lagrangian in terms of the renormalized mass and introducing a counterterm in the interaction Lagrangian so that it cancels the radiative corrections to the mass from the self-energy. Namely, the counterterm in the interaction Lagrangian is fixed by requiring that $\mathcal{E}_{1\vec{k}}(\eta') = 0$, in the long time limit $\eta' \to 0^-$ and in the long wavelength limit. Therefore, as per the discussion above, we extract the mass term from the condition

$$\mathcal{E}_{1\vec{k}}(\eta') = \langle 1_{\vec{k}} | H_I(\eta') | 1_{\vec{k}} \rangle + \int_{\eta_0}^{\eta'} \text{Im}[\Sigma_1(k; \eta', \eta'')] d\eta''$$
$$= 0 \qquad (3.61)$$

in the long wavelength limit.

In Minkowski space-time, the condition (3.61) is tantamount to requiring that the real part of the pole in the propagator be at the physical mass [43] and is equivalent to the counterterm approach described in Sec. II C. In the Appendix we carry out this program and show explicitly how the Wigner-Weisskopf approach reproduces the results in Minkowski space-time obtained in Sec. II and how the mass is reliably extracted in the long time, long wavelength limit.

We implement the *same strategy* to obtain the self-consistent radiatively generated mass in de Sitter space-time where Eq. (3.61) will determine the *self-consistent condition* for the mass.

In the mass terms in the Lagrangian (3.20) we implement the counterterm method by introducing the renormalized masses $\mathcal{M}_{\chi,\pi}^2$ that include the radiative corrections, and writing

$$-\frac{M_\chi^2}{2H^2\eta^2}\chi^2 - \frac{M_\pi^2}{2H^2\eta^2}\pi^2 \equiv -\frac{\mathcal{M}_\chi^2}{2H^2\eta^2}\chi^2 - \frac{\mathcal{M}_\pi^2}{2H^2\eta^2}\pi^2 - \mathcal{L}_{ct}$$
$$(3.62)$$

leading to the counterterm *Hamiltonian*

$$H_{ct} = \frac{1}{2H^2\eta^2} \int d^3x[(M_\chi^2 - \mathcal{M}_\chi^2)\chi^2 + (M_\pi^2 - \mathcal{M}_\pi^2)\pi^2]$$
$$(3.63)$$

included in the interaction Hamiltonian $H_I(\eta)$, and redefining

$$\Delta_\chi = \frac{\mathcal{M}_\chi^2}{3H^2}, \qquad \Delta_\pi = \frac{\mathcal{M}_\pi^2}{3H^2}. \qquad (3.64)$$

In what follows we assume that $\Delta_{\chi,\pi} \ll 1$; therefore the leading order contributions arise from poles in $\Delta_{\chi,\pi}$ as a result of the strong infrared divergences of minimally coupled light fields.

The contributions from diagrams like those of Figs. 3(a)–3(c) are cancelled by the tadpole condition (3.36). For the $\pi - \chi$ fields, respectively, we find

$$\langle 1_{\vec{k}}^\pi | H_I(\eta) | 1_{\vec{k}}^\pi \rangle = \frac{|g_\pi(k,\eta)|^2}{H^2\eta^2} \left[\frac{\lambda}{2}\left(\frac{J}{H^2} + 3I_\pi + I_\chi \right) - \frac{\mathcal{M}_\pi^2}{H^2} \right],$$
$$(3.65)$$

where $I_{\chi,\pi}$ are given by Eqs. (3.35) with the redefined $\Delta_{\chi,\pi}$ given by (3.64).

The nonlocal contribution is given by (see [37])

$$\Sigma_\pi(k; \eta; \eta') = \frac{\lambda^2\sigma_0^2}{H^2\eta\eta'} g_\pi^*(k; \eta)g_\pi(k; \eta')$$
$$\times \int \frac{d^3q}{(2\pi)^3} g_\chi(q; \eta)g_\chi^*(q; \eta')$$
$$\times g_\pi(|\vec{q} - \vec{k}|; \eta)g_\pi^*(|\vec{q} - \vec{k}|; \eta'). \qquad (3.66)$$

For $\Delta_{\pi,\chi} \sim 0$ the integral features infrared divergences in the regions $q \sim 0$, $|\vec{q} - \vec{k}| \sim 0$, which are manifest as poles in $\Delta_{\pi,\chi}$ [37]. These regions are isolated following the procedure of Ref. [37] and the poles in $\Delta_{\pi,\chi}$ can be extracted unambiguously. To leading order in these poles we find

$$\Sigma_\pi(k; \eta; \eta') = \frac{\lambda^2\sigma_0^2}{8\pi^2 H^2(\eta\eta')^2} g_\pi^*(k; \eta)g_\pi(k; \eta')$$
$$\times \left[\frac{g_\pi(k; \eta)g_\pi^*(k; \eta')}{\Delta_\chi} + \frac{g_\chi(k; \eta)g_\chi^*(k; \eta')}{\Delta_\pi} \right].$$
$$(3.67)$$

As discussed in detail in Ref. [37] the poles originate in the emission and absorption of superhorizon quanta and arise from the integration of a band of superhorizon wave vectors $0 \leq q \leq \mu_{ir} \to 0$ (see Ref. [37] for details).

As per the discussion in Minkowski space-time, a vanishing mass for a Goldstone boson after radiative correction requires that the tadpole terms in (3.65) be *exactly* cancelled by the nonlocal self-energy contribution in the long time, long wavelength limit. In particular the poles in $\Delta_{\chi,\pi}$ in (3.65) must be exactly cancelled by similar poles in Σ_π (3.67). Therefore, to leading order in $\Delta_{\pi,\chi}$, we can set $\Delta_\pi = \Delta_\chi = 0$, namely, $\nu_{\pi,\chi} = 3/2$ in the mode functions $g_{\pi,\chi}$ given by (3.24), whence it follows that to leading order in $\Delta_{\pi,\chi}$

$$\Sigma_\pi(k; \eta; \eta') = \frac{\lambda^2\sigma_0^2}{8\pi^2 H^2} \frac{|g(k; \eta)|^2|g(k; \eta')|^2}{(\eta\eta')^2}\left[\frac{1}{\Delta_\pi} + \frac{1}{\Delta_\chi} \right]$$
$$\times [1 + \mathcal{O}(\Delta_\pi, \Delta_\chi) + \cdots], \qquad (3.68)$$

where

$$g(k; \eta) = -\frac{1}{2}\sqrt{-\pi\eta}H_{3/2}^{(1)}(-k\eta). \qquad (3.69)$$

Therefore, to leading order in poles in $\Delta_{\chi,\pi}$, $\Sigma_\pi(k; \eta; \eta')$ is *real* and *does not contribute to the radiatively generated π mass*.

Therefore, to leading order in the poles in $\Delta_{\pi,\chi}$, the self-consistent condition that determines the mass, Eq. (3.61) becomes

$$\langle 1_{\vec{k}}^{\pi}|H_I(\eta)|1_{\vec{k}}^{\pi}\rangle = 0. \tag{3.70}$$

This observation is important: unlike Minkowski space-time where the diagram [3(f)] *cancels* the local tadpole contributions, in de Sitter space-time the similar diagram *cannot* cancel the local contributions because the leading infrared divergences yield a real contribution whereas the tadpoles yield a purely imaginary contribution as befits a mass insertion. Therefore, the self-consistent mass is obtained solely from the local tadpole terms that determine the mean field contribution. This validates the results of [34,36] that rely solely on the mean field approximation (which is exact only in the strict $N \to \infty$ limit).

Assuming spontaneous symmetry breaking so that Eq. (3.36) is fulfilled with $\sigma_0 \neq 0$, namely,

$$\frac{J}{H^2} = -3\mathcal{I}_\chi - \mathcal{I}_\pi, \tag{3.71}$$

it follows that

$$\frac{\mathcal{M}_\pi^2}{H^2} = \frac{\lambda}{8\pi^2}\left[\frac{1}{\Delta_\pi} - \frac{1}{\Delta_\chi}\right]. \tag{3.72}$$

For the χ field we find the following contributions:

$$\langle 1_{\vec{k}}^{\chi}|H_I(\eta)|1_{\vec{k}}^{\chi}\rangle = \frac{|g_\chi(k,\eta)|^2}{H^2\eta^2}\left[\frac{\lambda}{2}\left(\frac{J}{H^2} + 2\frac{\sigma_0^2}{H^2}\right.\right.$$
$$\left.\left. + 3\mathcal{I}_\chi + \mathcal{I}_\pi\right) - \frac{\mathcal{M}_\chi^2}{H^2}\right], \tag{3.73}$$

where $\mathcal{I}_{\chi,\pi}$ are given by Eq. (3.35), and for $\Sigma_\chi(k;\eta,\eta')$ we find

$$\Sigma_\chi(k;\eta;\eta') = \frac{\lambda^2\sigma_0^2}{2H^2\eta\eta'}g_\chi^*(k;\eta)g_\chi(k;\eta')$$
$$\times \int\frac{d^3q}{(2\pi)^3}[9g_\chi(q;\eta)g_\chi^*(q;\eta')g_\chi(|\vec{q}-\vec{k}|;\eta)$$
$$\times g_\chi^*(|\vec{q}-\vec{k}|;\eta') + g_\pi(q;\eta)g_\pi^*(q;\eta')$$
$$\times g_\pi(|\vec{q}-\vec{k}|;\eta)g_\pi^*(|\vec{q}-\vec{k}|;\eta')]. \tag{3.74}$$

Extracting the poles in $\Delta_{\pi,\chi}$ the leading order result is given by

$$\Sigma_\chi(k;\eta;\eta') = \frac{\lambda^2\sigma_0^2}{8\pi^2H^2(\eta\eta')^2}g_\chi^*(k;\eta)g_\chi(k;\eta')$$
$$\times\left[\frac{g_\pi(k;\eta)g_\pi^*(k;\eta')}{\Delta_\pi} + 9\frac{g_\chi(k;\eta)g_\chi^*(k;\eta')}{\Delta_\chi}\right]. \tag{3.75}$$

Again, just as for the π field above, to leading order in the poles in $\Delta_{\pi,\chi}$ we can set $\Delta_\pi = \Delta_\chi = 0$, namely, $\nu_{\pi,\chi} = 3/2$ in the mode functions $g_{\pi,\chi}$, leading to

$$\Sigma_\chi(k;\eta;\eta') = \frac{\lambda^2\sigma_0^2}{8\pi^2H^2}\frac{|g(k;\eta)|^2|g(k;\eta')|^2}{(\eta\eta')^2}\left[\frac{1}{\Delta_\pi} + \frac{9}{\Delta_\chi}\right], \tag{3.76}$$

where $g(k;\eta)$ is given by Eq. (3.69).

The result is that to leading order in the poles, *both* $\Sigma_{\pi,\chi}$ are *real* and *do not contribute to the radiatively generated masses* but will contribute to the *decay* of the single-particle excitations discussed below (see Sec. III D).

Therefore, *assuming* spontaneous symmetry breaking so that the condition (3.71) holds, we find that

$$\frac{\mathcal{M}_\chi^2}{H^2} = \frac{\lambda\sigma_0^2}{H^2}. \tag{3.77}$$

Now identifying self-consistently the masses in the definition (3.64) with $\mathcal{M}_{\pi,\chi}$, and defining

$$\varepsilon = \sqrt{\frac{\lambda}{24\pi^2}}, \qquad \Delta_\pi = \varepsilon\delta_\pi, \qquad \Delta_\chi = \frac{\lambda}{3}\frac{\sigma_0^2}{H^2} \equiv \varepsilon\delta_\chi, \tag{3.78}$$

Eq. (3.72) becomes

$$\delta_\pi = \frac{1}{\delta_\pi} - \frac{1}{\delta_\chi} \tag{3.79}$$

with the (positive) solution

$$\delta_\pi = \frac{1}{2\delta_\chi}[\sqrt{1 + 4\delta_\chi^2} - 1], \tag{3.80}$$

the negative root would lead to an instability and an uncontrollable infrared divergence in the loop integrals, which would not yield a self-consistent solution.

Now we are in position to understand whether spontaneous symmetry breaking does occur. The condition (3.71) is

$$\frac{\sigma_0^2}{H^2} = \frac{\mu^2}{\lambda H^2} - \frac{3}{8\pi^2\Delta_\chi} - \frac{1}{8\pi^2\Delta_\pi} \neq 0, \tag{3.81}$$

which when written in terms of the definitions (3.78) and using (3.80) becomes

$$F[\delta_\chi] \equiv \delta_\chi + \frac{1}{2\delta_\chi}[7 + \sqrt{1 + 4\delta_\chi^2}] = \frac{\mu^2}{3\varepsilon H^2}. \tag{3.82}$$

The function $F[\delta_\chi]$ and its intersection with $\mu^2/3\varepsilon H^2$ is displayed in Fig. 4.

As shown in Fig. 4, $F[\delta_\chi]$ features a minimum at $\delta_{\chi,\min} = 1.906\cdots$ at which $F[\delta_{\chi,\min}] = 4.77614\cdots$; therefore, there are symmetry breaking solutions for

$$\frac{\mu^2}{3\varepsilon H^2} > 4.77614\dots. \tag{3.83}$$

This condition can be written in a more illuminating manner as

$$T_H < T_c, \qquad T_H = \frac{H}{2\pi},$$

$$T_c = \frac{\mu}{2.419 \cdots \lambda^{1/4}} = \frac{\lambda^{1/4} v}{2.419 \cdots}, \qquad (3.84)$$

where T_H is the Hawking temperature of de Sitter space-time[4] and $v = \mu/\sqrt{\lambda}$ is the tree-level vacuum expectation value (minimum of the tree-level potential). From Eq. (3.79) it follows that

$$\frac{\delta_\chi}{\delta_\pi} = \frac{1 + \sqrt{1 + 4\delta_\chi^2}}{2} \qquad (3.85)$$

and $\delta_\chi > 1.906 \cdots$; therefore, in the broken symmetry phase we find that

$$\frac{\delta_\chi}{\delta_\pi} \simeq \delta_\chi + \frac{1}{2} \quad \text{for} \quad T_H < T_c, \qquad (3.86)$$

in the spontaneously broken phase. At weak coupling, for $\mu^2 \gg 3\varepsilon H^2$ (but $\mu^2 \ll H^2$ for consistency) we find that

$$\mathcal{M}_\chi \simeq |\mu| + a\lambda^{1/4}H, \qquad \mathcal{M}_\pi = b\lambda^{1/4}H, \qquad (3.87)$$

where a, b are positive constants.

For $T_H > T_c$ the unbroken symmetry solution $\sigma_0 = 0$ is the only solution of the tadpole condition (3.36). In this case we find

$$\frac{\mathcal{M}_\pi^2}{H^2} = \frac{\lambda}{2}\left(\frac{J}{H^2} + 3I_\pi + I_\chi\right), \qquad (3.88)$$

$$\frac{\mathcal{M}_\chi^2}{H^2} = \frac{\lambda}{2}\left(\frac{J}{H^2} + 3I_\chi + I_\pi\right). \qquad (3.89)$$

Subtracting (3.89) from (3.88) we find

$$\delta_\pi - \delta_\chi = \frac{1}{\delta_\pi} - \frac{1}{\delta_\chi}, \qquad (3.90)$$

if $\delta_\pi > (<)\delta_\chi$ the left-hand side is positive (negative) but the right-hand side is negative (positive); therefore the only solution is

$$\delta_\pi = \delta_\chi = \frac{\mu^2}{12\varepsilon H^2}\left[\sqrt{1 + \left(\frac{12\varepsilon H^2}{\mu^2}\right)^2} - 1\right]. \qquad (3.91)$$

Inserting this result in (3.89) we find for $T_H > T_c$

$$\mathcal{M}_\pi = \mathcal{M}_\chi = \frac{\mu^2}{4}\left[\sqrt{1 + 0.701\left(\frac{T_H^2}{T_c^2}\right)^2} - 1\right], \qquad (3.92)$$

as expected $\mathcal{M}_\chi = \mathcal{M}_\pi$ if the symmetry is unbroken.

FIG. 4. $F[\delta_\chi]$ vs δ_χ and its intersection with $\mu^2/3\varepsilon H^2$. The function features a minimum at $\delta_{\chi,\mathrm{min}} = 1.906\cdots$ with $F[\delta_{\chi,\mathrm{min}}] = 4.77614\cdots$. The value of $\delta_\pi(\delta_{\chi,\mathrm{min}}) = 0.772\cdots$.

B. A first order phase transition

Figure 4 shows that for $T_H < T_c$ there are *two solutions* of the equation that determines symmetry breaking and the question arises: which of the two solutions describes the broken symmetry phase? The answer is gleaned by analyzing the weak coupling limit $\varepsilon \to 0$ ($\lambda \to 0$). In this limit the left most intersection in Fig. 4 corresponds to the solution

$$\delta_\chi^{(-)} \simeq 12\varepsilon\frac{H^2}{\mu^2} \Rightarrow \mathcal{M}_\chi^2 \simeq \frac{\lambda}{2\pi^2}\frac{H^4}{\mu^2} \overset{\lambda\to0}{\to} 0, \qquad (3.93)$$

whereas the right-most intersection corresponds to the solution

$$\delta_\chi^{(+)} \simeq \frac{\mu^2}{3\varepsilon H^2} \Rightarrow \mathcal{M}_\chi^2 \simeq \mu^2, \qquad \mathcal{M}_\pi^2 \simeq \varepsilon H^2 \to 0. \qquad (3.94)$$

Obviously the solution $\delta_\chi^{(+)}$ is the correct one since for $\lambda \to 0$ the expectation value $\lambda\sigma_0 = \mu^2$, the loop corrections vanish and the mass of the χ, π fields should be the tree-level ones, namely, $\mathcal{M}_\chi^2 = \mu^2$, $\mathcal{M}_\pi^2 = 0$ respectively. However, as εH^2 increases beyond the critical value at which $\mu^2/3\varepsilon H^2 = F[\delta_{\chi,\mathrm{min}}]$, there is no available symmetry breaking solution and this occurs for a nonvanishing value of σ_0 signaling a *first order phase transition* at $T_H = T_c$ given by (3.84). The value of the order parameter at $T_H = T_c$ is given by

$$\sigma_{0c} \simeq 0.61\frac{H}{\lambda^{1/4}}. \qquad (3.95)$$

These results are in general agreement with those of Ref. [36]. The first order nature of the phase transition can also be understood within the context of the infrared divergences: if the transition (as a function of coupling or T_H) were of second order, then at the critical point the

---

[4]In comoving time t, the mode functions g_π, g_χ are functions of $\eta = -e^{-Ht}/H$ and therefore *periodic* in imaginary time $\tau = it$ with period $\beta = 2\pi/H = 1/T_H$. See [38].
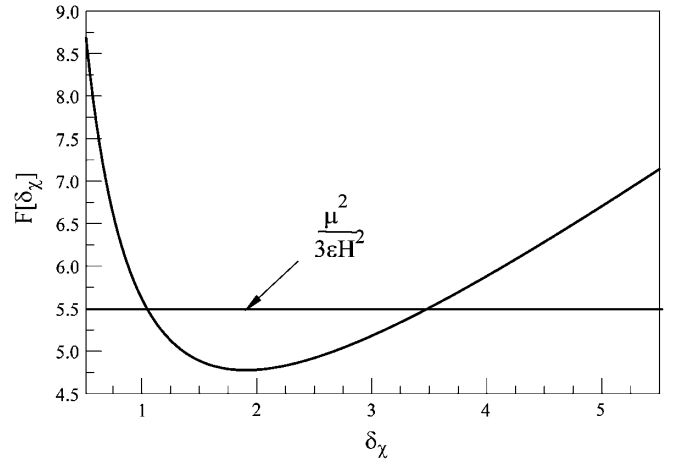
masses of both χ, π fields must necessarily vanish, but the vanishing of the masses would lead to strong infrared divergences. Therefore a first order transition with a finite mass (correlation length) and a jump in the order parameter is a natural consequence of the strong infrared behavior of minimally coupled nearly massless fields in de Sitter space-time. The infrared singularities are self-consistently relieved by the radiative generation of a mass at the expense of turning the phase transition into first order.

C. Large N limit

The above results can be simply generalized to the $O(N)$ case where the π fields form an $O(N-1)$ multiplet. Now the tadpole condition becomes

$$\langle \chi \rangle = 0 \Rightarrow \frac{\lambda a \sigma_0}{2\eta^2} \left[\frac{J}{H^2} + 3I_\chi + (N-1)I_\pi \right] = 0, \quad (3.96)$$

and the χ, π masses become

$$\frac{\mathcal{M}_\pi^2}{H^2} = \frac{\lambda}{2} \left[\frac{J}{H^2} + (N+1)I_\pi + I_\chi \right], \quad (3.97)$$

$$\frac{\mathcal{M}_\chi^2}{H^2} = \frac{\lambda}{2} \left[2\frac{\sigma_0^2}{H^2} + \frac{J}{H^2} + (N-1)I_\pi + 3I_\chi \right]. \quad (3.98)$$

In the strict $N \to \infty$ limit these equations simplify to

$$\sigma_0 \left[\frac{J}{H^2} + NI_\pi \right] = 0, \quad (3.99)$$

$$\frac{\mathcal{M}_\pi^2}{H^2} = \frac{\lambda}{2} \left[\frac{J}{H^2} + NI_\pi \right], \quad (3.100)$$

$$\frac{\mathcal{M}_\chi^2}{H^2} = \frac{\lambda}{2} \left[2\frac{\sigma_0^2}{H^2} + \frac{J}{H^2} + NI_\pi \right], \quad (3.101)$$

with $I_{\pi,\chi}$ given by Eq. (3.34) and self-consistently $\Delta_{\pi,\chi} = \mathcal{M}_{\pi,\chi}^2/3H^2$. Clearly, Eqs. (3.99) and (3.100) lead to conclude that the only symmetry breaking solution corresponds to $\mathcal{M}_\pi^2 = 0$ but this is obviously in contradiction with the self-consistent solution because of the infrared singularity in $I_\pi \propto 1/\mathcal{M}_\pi^2$. Therefore, the only available solution of (3.99) that is also self-consistent and infrared finite must be the unbroken symmetry solution $\sigma_0 = 0$ that results in equal masses for χ, π fields. Thus in the strict $N \to \infty$, neglecting the $1/N$ corrections the $O(N)$ symmetry *cannot* be spontaneously broken because of the strong infrared effects. This is the conclusion of Ref. [34]. However the analysis presented above for finite N, and, in particular, for $N = 2$ suggests that this conclusion holds *only* in the strict $N \to \infty$ limit but for any finite N *there is spontaneous symmetry breaking*, along with infrared radiatively induced masses for the Goldstone fields without contradicting Goldstone's theorem, but the transition is first order as a consequence of infrared divergences.

D. Decay of π, χ particles

As discussed above the nonlocal self-energies $\Sigma_{\pi,\chi}(k; \eta, \eta')$ are real and do not contribute to the mass to leading order in $\Delta_{\pi,\chi}$; however, they determine the *decay* of single-particle states as described in Ref. [37]. We now focus on obtaining the decay amplitudes arising from these contributions. Using the relations given by Eqs. (3.77) and (3.78) to leading order in poles in $\Delta_{\pi,\chi}$, the one-loop results (3.68) and (3.76) can be written as

$$\Sigma_\pi^{(1)}(k; \eta; \eta') = \frac{3\lambda}{8\pi^2} \frac{|g(k;\eta)|^2 |g(k;\eta')|^2}{(\eta\eta')^2} \left[1 + \frac{\delta_\chi}{\delta_\pi} \right], \quad (3.102)$$

$$\Sigma_\chi^{(1)}(k; \eta; \eta') = \frac{27\lambda}{8\pi^2} \frac{|g(k;\eta)|^2 |g(k;\eta')|^2}{(\eta\eta')^2} \left[1 + \frac{\delta_\chi}{9\delta_\pi} \right]. \quad (3.103)$$

Thus formally the real part of the single-particles self-energy are of $\mathcal{O}(\lambda)$.

In Ref. [37] it was found that quartic self-interactions with strength λ yield *two-loop* self-energies that are *also* of $\mathcal{O}(\lambda)$ as a consequence of infrared divergences that are manifest as *second order* poles in Δ. Implementing the "infrared rules" obtained in Ref. [37] in the two-loop diagrams for $\Sigma_{\pi,\chi}$ Figs. 5(a)–5(d), respectively, we find the leading order two-loop contributions

$$\Sigma_\pi^{(2)}(k; \eta; \eta') = \frac{3\lambda}{16\pi^2} \frac{|g(k;\eta)|^2 |g(k;\eta')|^2}{(\eta\eta')^2}$$
$$\times \left[\frac{9}{\delta_\pi^2} + \frac{1}{\delta_\chi^2} + \frac{2}{\delta_\pi \delta_\chi} \right], \quad (3.104)$$

$$\Sigma_\chi^{(2)}(k; \eta; \eta') = \frac{3\lambda}{16\pi^2} \frac{|g(k;\eta)|^2 |g(k;\eta')|^2}{(\eta\eta')^2}$$
$$\times \left[\frac{9}{\delta_\chi^2} + \frac{1}{\delta_\pi^2} + \frac{2}{\delta_\pi \delta_\chi} \right]. \quad (3.105)$$

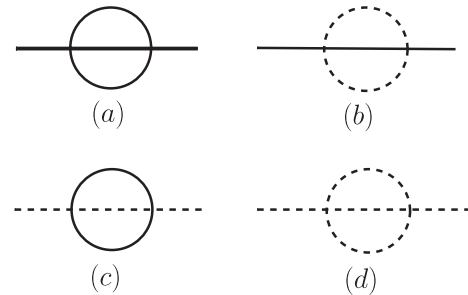From (3.55) we obtain the conformal time-dependent single-particle decay rates (3.57)

FIG. 5. Two-loop contributions to Σ_π (a),(b) and Σ_χ (c),(d). Solid lines = π, dashed lines = χ.

$$\frac{1}{2}\Gamma_{\pi,\chi}(k;\eta) = \int_{\eta_0}^{\eta} \Sigma_\pi(k;\eta;\eta')d\eta'$$

$$= \lambda \mathcal{C}_{\pi,\chi} k \frac{|H_{3/2}^{(1)}(z)|^2}{z} \int_z^{z_0} \frac{dz'}{z'} |H_{3/2}^{(1)}(z')|^2,$$

$$z = -k\eta, \qquad (3.106)$$

with

$$\mathcal{C}_\pi = \frac{3}{256}\left[2\left(1 + \frac{\delta_\chi}{\delta_\pi}\right) + \left(\frac{9}{\delta_\pi^2} + \frac{1}{\delta_\chi^2} + \frac{2}{\delta_\pi \delta_\chi}\right)\right],$$
$$(3.107)$$

$$\mathcal{C}_\chi = \frac{3}{256}\left[18\left(1 + \frac{\delta_\chi}{9\delta_\pi}\right) + \left(\frac{9}{\delta_\pi^2} + \frac{1}{\delta_\chi^2} + \frac{2}{\delta_\pi \delta_\chi}\right)\right].$$
$$(3.108)$$

As discussed in Refs. [37,43] the decay $\pi \rightarrow \pi + \chi$ is a consequence of emission and absorption of superhorizon quanta; in the superhorizon limit $z \ll 1$, $z_0 \sim 1$ the integrals can be done simply [37], leading to the results for the single-particle amplitudes in the superhorizon limit

$$|C_{1_{\vec{k}}}^{\pi,\chi}| \simeq e^{-\gamma_{\chi,\pi}(-k\eta)},$$

$$\gamma_{\chi,\pi}(-k\eta) = \frac{2\lambda}{9\pi^2} \mathcal{C}_{\chi,\pi}\left[\frac{H}{k_{\text{phys}}(\eta)}\right]^6. \qquad (3.109)$$

Possible caveats.—There are other two-loop diagrams that have not been accounted for above. The generic form of these diagrams are displayed in Fig. 6 (we have not displayed specific π, χ lines but just showed the generic form of the diagrams) and can be interpreted as a renormalization of the internal propagator and the vertex. Both of these diagrams are $\propto (\lambda\sigma_0/H)^4 \simeq \lambda^2\Delta_\chi^2$; therefore, *if* the "infrared rules" of Ref. [37] apply to these diagrams, the two loops imply an infrared factor $\propto 1/\Delta_\chi^2$, $1/\Delta_\pi^2$, $1/\Delta_\chi\Delta_\pi$, in which case the overall coupling dependence of these diagrams is $\propto \lambda^2$ and would be subdominant as compared to the two-loop diagrams of Fig. 5. The possible caveat in this argument is that the rules to obtain the leading contributions in poles in Δ given in Ref. [37] do not directly apply to the diagrams above because if the bubble that renormalizes the propagator in the first diagram dresses a line in which the wavevector is within an infrared band $0 < q < \mu_{\text{ir}} \rightarrow 0$, then both lines in this bubble are within this band. This situation is not contemplated in the rules provided in Ref. [37] that apply to the case when in a loop integral only one of the lines carries momenta within an infrared band whereas the other line carries a finite value of the momentum (even if superhorizon) (see the arguments in Ref. [37]). Thus in absence of a sound proof that the diagrams in Fig. 6 are subleading, the result for the damping rate $\Gamma(k;\eta)$ given by Eq. (3.106) should be taken as indicative. Nevertheless the analysis of symmetry breaking and the emerging conclusions on the mass generation
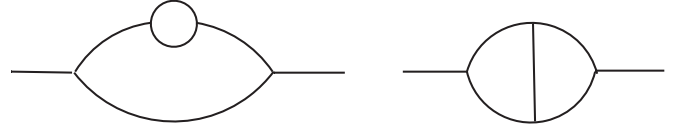

FIG. 6. Other two-loop contributions to $\Sigma_{\pi,\chi}$.

of Goldstone bosons and the order of the transition are not affected by this possible caveat on the damping rate. Further study on the infrared aspects of diagrams in Fig. 6 is certainly worthy but beyond the scope of this article.

IV. CONCLUSIONS

Spontaneous symmetry breaking is an important ingredient in the inflationary paradigm. In this article we have studied SSB of continuous symmetry in an $O(2)$ model of scalar fields minimally coupled to gravity in de Sitter space-time, focusing, in particular, on understanding whether Goldstone's theorem implies massless Goldstone bosons and trying to shed light on conflicting previous results [34,36] that implemented a local mean field approximation. We first revisited the general results of Goldstone's theorem in Minkowski space-time highlighting the fact that it is through *time translational invariance that the conservation of the Noether theorem guarantees massless Goldstone bosons.* We emphasized that in absence of time translational invariance Goldstone's theorem is much less stringent and does not rule out radiatively generated masses for Goldstone modes. We followed with an analysis of the implementation of Goldstone's theorem at one-loop level in Minkowski space-time by studying the self-energies of Goldstone and Higgs-like modes; we showed that at one-loop level the masslessness of the Goldstone boson is a consequence of a precise cancellation between local tadpole and nonlocal (in space-time) contributions; and we analyzed in detail the implementation of Goldstone's theorem in the large N limit of an $O(N)$ scalar theory. These results paved the way towards a deeper understanding of Goldstone's theorem and its consequences in de Sitter cosmology.

Our conclusions are summarized as follows:

(i) In absence of a global timelike Killing vector, Goldstone's theorem *does not* imply massless Goldstone bosons when a continuous symmetry is spontaneously broken.

(ii) We implemented a nonperturbative Wigner-Weisskopf method that allows one to obtain the masses and decay widths of single-particle states in a cosmological setting. Strong infrared behavior associated with light particles minimally coupled to gravity are treated in a self-consistent manner.

(iii) Whereas in Minkowski space-time at one-loop level the masslessness of Goldstone modes in the broken symmetry phase is a consequence of a

precise cancellation between tadpole and nonlocal (absorptive) contributions to the self-energy, we find that in de Sitter space-time no such cancellation is possible. Goldstone modes acquire a self-consistent radiatively generated mass resulting from the buildup of infrared singularities in self-energies. We find that in a weak coupling the mass of the Goldstone modes is $\mathcal{M}_\pi \propto \lambda^{1/4} H$, where λ is the quartic coupling of the $O(2)$ theory.

(iv) We find a *first order phase transition* between the broken and unbroken symmetry phase as a function of $T_H = H/2\pi$ the Hawking temperature of de Sitter space-time. For the $O(2)$ model we find SSB for $T_H < T_c = \lambda^{1/4} v / 2.419\ldots$ where v is the tree-level vacuum expectation value. For $T_H > T_c$ the symmetry is restored. The value of the order parameter at $T_H = T_c$ is $\sigma_{0c} \simeq 0.61 H/\lambda^{1/4}$. The first order nature of the transition and concomitant jump in the order parameter is a consequence of the strong infrared behavior of correlation functions: if the transition were second order both fields would be massless at T_c leading to strong infrared singularities. Thus radiatively induced masses relieve the infrared singularities at the expense of a first order transition and a jump in the order parameter. These results are in qualitative agreement with those of Ref. [36] and also confirm the validity of the local mean field approximation since the nonlocal radiative corrections do not contribute to the masses of either Goldstone or Higgs-like modes but only to their decay widths.

(v) In the strict $N \rightarrow \infty$ limit of an $O(N)$ scalar theory there is no possibility of SSB in agreement with the result of Ref. [34], but SSB is available for any finite N. This result reconciles the conflicting conclusions of Refs. [34,36].

(vi) The lack of a global timelike Killing vector prevents the existence of kinematic thresholds; as a result we find that Goldstone modes *decay* into Goldstone and Higgs modes via the emission and absorption of superhorizon quanta. We have obtained the decay width of Goldstone modes in the superhorizon limit—the amplitude of single-particle Goldstone modes $|C_{1_{\vec{k}}}^\pi| \simeq e^{-\gamma_\pi(-k\eta)}$ where $\gamma_\pi(-k\eta) \propto \lambda (H/k_{\mathrm{phys}}(\eta))^6$.

Further questions.—The discussion in Sec. III on the applicability and corollary of Goldstone's theorem in an expanding cosmology highlights the consequences of a covariant conservation law in a time-dependent background geometry as contrasted with the strict conservation law in Minkowski space-time and is general for any cosmological background. Our study focused on de Sitter space-time wherein infrared divergences associated with minimally coupled massless particles lead to the self-consistent generation of masses for Goldstone bosons as described above. There remains the very important question of whether Goldstone bosons acquire a mass in other cosmologies, for example, during the radiation dominated stage, where the arguments on the time dependence of the background are valid but there may not be infrared divergences that lead to a self-consistent generation of mass as in de Sitter space-time. A deeper understanding of this case certainly merits further study as it may yield to novel and unexpected phenomena in cosmology and is relegated to future work.

APPENDIX A: WIGNER-WEISSKOPF APPROACH TO GOLDSTONE'S THEOREM IN MINKOWSKI SPACE-TIME

In Minkowski space-time and for a single-particle π state of momentum $\vec{k}$ we need [see Eq. (3.54)]

$$\tilde{W}(t) = i\langle 1_{\vec{k}}^\pi | H_I(t) | 1_{\vec{k}}^\pi \rangle + \int_0^t \Sigma_\pi(\vec{k}; t, t') dt', \qquad (\mathrm{A1})$$

from which the total correction to the energy of a single-particle state is obtained from the long time limit

$$\mathcal{E}_{1_{\vec{k}}}^\pi = \langle 1_{\vec{k}}^\pi | H_I(0) | 1_{\vec{k}}^\pi \rangle + \int_0^{t \to \infty} \mathrm{Im}[\Sigma_\pi(k; t, t')] dt', \quad (\mathrm{A2})$$

Including the counterterm Hamiltonian in the interaction as described in Sec. II C leads to the requirement that in the long wavelength limit

$$\mathcal{E}_{1_{\vec{k} \to 0}}^\pi = 0. \qquad (\mathrm{A3})$$

The interaction Hamiltonian is read off from the vertices in Eq. (2.17) including the mass counterterm

$$H_{ct} = \frac{1}{2}(M_\pi^2 - \mathcal{M}_\pi^2), \qquad M_\pi^2 = \frac{\lambda}{2} J. \qquad (\mathrm{A4})$$

The contribution $\langle 1_{\vec{k}}^\pi | H_I(t) | 1_{\vec{k}}^\pi \rangle$ is recognized as the first order shift in the energy.

The tadpole condition eliminates the contributions from the tadpoles in Figs. 3(a)–3(c) because the matrix element of the Hamiltonian between the vacuum and a single-particle state vanish by dint of the tadpole condition. We find

$$\langle 1_{\vec{k}}^\pi | H_I(0) | 1_{\vec{k}}^\pi \rangle = \frac{1}{2\omega^\pi(k)} \left[(M_\pi^2 - \mathcal{M}_\pi^2) + \frac{\lambda}{2}(I_\chi + 3I_\pi) \right],$$
$$(\mathrm{A5})$$

where $I_{\chi, \pi}$ are given by Eq. (2.27). Upon using the tadpole condition assuming spontaneous symmetry breaking it follows that $J = -3I_\chi - I_\pi$ and (A5) becomes

$\langle 1_{\vec{k}}^{\pi}|H_I(0)|1_{\vec{k}}^{\pi}\rangle$

$$= \frac{1}{2\omega^{\pi}(k)}\left[-\mathcal{M}_{\pi}^2 + \lambda \int \frac{d^3q}{(2\pi^3)}\left(\frac{1}{2\omega^{\pi}(q)} - \frac{1}{2\omega^{\chi}(q)}\right)\right].$$
(A6)

The self-energy

$$\Sigma_{\pi}(k; t, t') = \sum_{\kappa \neq 1_{\vec{k}}^{\pi}} \langle 1_{\vec{k}}^{\pi}|H_I(t)|\kappa\rangle\langle\kappa|H_I(t')|1_{\vec{k}}^{\pi}\rangle$$

$$= \sum_{\kappa \neq 1_{\vec{k}}^{\pi}} |\langle 1_{\vec{k}}^{\pi}|H_I(0)|\kappa\rangle|^2 e^{i(\omega^{\pi}(k)-E_{\kappa})(t-t')}, \quad \text{(A7)}$$

where the intermediate states $|\kappa\rangle = |1_{\vec{q}}^{\chi}; 1_{\vec{q}+\vec{k}}^{\pi}\rangle$ [see Fig. 3(f)].

Carrying out the time integral in (A1) in the long time limit we find

$$\int_0^{t\to\infty} \Sigma_{\pi}(k; t, t')dt'$$

$$= i\sum_{\vec{q}} \frac{|\langle 1_{\vec{k}}^{\pi}|H_I(0)|1_{\vec{q}}^{\chi}; 1_{\vec{q}+\vec{k}}^{\pi}\rangle|^2}{\omega^{\pi}(k) - \omega^{\pi}(|\vec{q}+\vec{k}|) - \omega^{\chi}(q) + i\epsilon}$$

$$\equiv i\delta E_{\pi}^{(2)} + \frac{\Gamma_{\pi}}{2}, \quad \text{(A8)}$$

thus the imaginary part of the time integral yields the second order energy shift $\delta E_{\pi}^{(2)}$ and the real part yields half of the decay rate Γ_{π} *a la* Fermi's golden rule. In the case of the π field the imaginary part vanishes by kinematics.

The matrix element is computed straightforwardly and we find

$$\mathcal{E}_{1\vec{k}} = -\frac{1}{2\omega^{\pi}(k)}\left[\mathcal{M}_{\pi}^2 - \frac{\lambda}{2}\int\frac{d^3q}{(2\pi^3)}\left(\frac{1}{\omega^{\pi}(q)} - \frac{1}{\omega^{\chi}(q)}\right)\right.$$

$$\left. - \frac{\lambda\sigma_0^2}{\omega^{\pi}(|\vec{q}+\vec{k}|)\omega^{\chi}(q)(\omega^{\pi}(|\vec{q}+\vec{k}|)+\omega^{\chi}(q)-\omega^{\pi}(k))}\right].$$
(A9)

To leading order in perturbation theory one can set $\omega_{\pi}(k) = k$ in (A9) leading to vanishing of the integral in the long wavelength limit and $\mathcal{M}_{\pi} = 0$ from the condition (A3). However, keeping the π mass self-consistently, in the long wavelength limit (setting $k \to 0$ in the denominator inside the integral) the bracket in (A9) becomes

$$\left[\cdots\right]_{k\to 0} = \mathcal{M}_{\pi}^2 + \frac{\lambda^2\sigma_0^2}{4}|\mathcal{M}_{\pi}|\int\frac{d^3q}{(2\pi^3)}$$

$$\times \frac{1}{\omega^{\pi}(\vec{q})\omega^{\chi}(q)(\omega^{\pi}(\vec{q})+\omega^{\chi}(q)-|\mathcal{M}_{\pi}|)},$$
(A10)

thus the requirement (A3) leads to

$$\mathcal{M}_{\pi} = 0. \quad \text{(A11)}$$

It is straightforward to check that the result (A9) coincides with (2.37) for the (off-shell) value $k = 0$ in (2.37) upon integrating q_0 in the complex plane.

[1] A. M. Polyakov, Sov. Phys. Usp. **25**, 187 (1982); Usp. Fiz. Nauk **136**, 538 (1982).

[2] N. P. Myhrvold, Phys. Lett. **132B**, 308 (1983); Phys. Rev. D **28**, 2439 (1983).

[3] L. H. Ford, Phys. Rev. D **31**, 710 (1985).

[4] B. Allen, Phys. Rev. D **32**, 3136 (1985); B. Allen and A. Folacci, Phys. Rev. D **35**, 3771 (1987).

[5] A. Folacci, Phys. Rev. D **46**, 2553 (1992).

[6] A. D. Dolgov, M. B. Einhorn, and V. I. Zakharov, Phys. Rev. D **52**, 717 (1995); Acta Phys. Pol. B **26**, 65 (1995); A. Dolgov and D. N. Pelliccia, Nucl. Phys. **B734**, 208 (2006).

[7] S. Weinberg, Phys. Rev. D **72**, 043514 (2005); Phys. Rev. D **74**, 023508 (2006).

[8] D. Seery, Classical Quantum Gravity **27**, 124005 (2010); J. Cosmol. Astropart. Phys. 05 (2009) 021; J. Cosmol. Astropart. Phys. 02 (2008) 006; J. Cosmol. Astropart. Phys. 11 (2007) 025.

[9] W. Xue, X. Gao, and R. Brandenberger, arXiv:1201.0768.

[10] A. M. Polyakov, Nucl. Phys. **B797**, 199 (2008); Nucl. Phys. **B834**, 316 (2010).

[11] D. Krotov and A. M. Polyakov, Nucl. Phys. **B849**, 410 (2011).

[12] E. T. Akhmedov, A. Roura, and A. Sadofyev, Phys. Rev. D **82**, 044035 (2010); E. T. Akhmedov and P. V. Buividovich, Phys. Rev. D **78**, 104005 (2008); E. T. Akhmedov, Mod. Phys. Lett. A **25**, 2815 (2010); E. T. Akhmedov, P. V. Buividovich, and D. A. Singleton, Phys. At. Nucl. **75**, 525 (2012); E. T. Akhmedov, J. High Energy Phys. 01 (2012) 066.

[13] A. Higuchi, Classical Quantum Gravity **26**, 072001 (2009).

[14] E. Alvarez and R. Vidal, J. High Energy Phys. 10 (2009) 045; J. Cosmol. Astropart. Phys. 11 (2010) 043; Phys. Rev. D **84**, 105009 (2011); arXiv:1111.2178.

[15] S. B. Giddings and M. S. Sloth, J. Cosmol. Astropart. Phys. 01 (2011) 023.

[16] W. Xue, K. Dasgupta, and R. Brandenberger, Phys. Rev. D **83**, 083520 (2011).

[17] D. Chialva and A. Mazumdar, arXiv:1103.1312.

[18] J. Kumar, L. Leblond, and A. Rajaraman, Phys. Rev. D **82**, 023525 (2010).

[19] S. P. Miao, N. C. Tsamis, R. P. Woodard, J. Math. Phys. (N.Y.) **51**, 072503 (2010); R. P. Woodard, arXiv:astro-ph/0310757; T. M. Janssen, S. P. Miao, T. Prokopec, and R. P. Woodard, Classical Quantum Gravity **25**, 245013 (2008);

N. C. Tsamis and R. P. Woodard, Phys. Lett. B **301**, 351 (1993); N. C. Tsamis and R. P. Woodard, Ann. Phys. (N.Y.) **238**, 1 (1995); N. C. Tsamis and R. P. Woodard, Phys. Rev. D **78**, 028501 (2008).

[20] D. Marolf and I. A. Morrison, Phys. Rev. D **84**, 044040 (2011); S. Hollands, arXiv:1010.5367; A. Higuchi, D. Marolf, and I. A. Morrison, Phys. Rev. D **83**, 084029 (2011).

[21] I. Antoniadis, P. O. Mazur, and E. Mottola, New J. Phys. **9**, 11 (2007); E. Mottola and R. Vaulin, Phys. Rev. D **74**, 064004 (2006); M. Giannotti and E. Mottola, Phys. Rev. D **79**, 045014 (2009); E. Mottola, Acta Phys. Pol. B **41**, 2031 (2010); Int. J. Mod. Phys. A **25**, 2391 (2010).

[22] I. Antoniadis, J. Iliopoulos, and T. N. Tomaras, Phys. Rev. Lett. **56**, 1319 (1986); I. Antoniadis and E. Mottola, J. Math. Phys. (N.Y.) **32**, 1037 (1991).

[23] L. R. W. Abramo, R. H. Brandenberger, and V. F. Mukhanov, Phys. Rev. D **56**, 3248 (1997); V. F. Mukhanov, L. R. W. Abramo, and R. H. Brandenberger, Phys. Rev. Lett. **78**, 1624 (1997); L. R. W. Abramo and R. P. Woodard, Phys. Rev. D **60**, 044010 (1999); L. R. Abramo and R. P. Woodard, Phys. Rev. D **65**, 063515 (2002); G. Geshnizjani and R. Brandenberger, J. Cosmol. Astropart. Phys. 04 (2005) 006; R. H. Brandenberger, arXiv:hep-th/0210165.

[24] G. Petri, arXiv:0810.3330.

[25] K. Enqvist, S. Nurmi, D. Podolsky, and G. I. Rigopoulos, J. Cosmol. Astropart. Phys. 04 (2008) 025.

[26] A. Riotto and M. S. Sloth, J. Cosmol. Astropart. Phys. 04 (2008) 030.

[27] C. P. Burgess, R. Holman, L. Leblond, and S. Shandera, J. Cosmol. Astropart. Phys. 03 (2010) 033; J. Cosmol. Astropart. Phys. 10 (2010) 017.

[28] D. Boyanovsky and H. J. de Vega, Phys. Rev. D **70**, 063508 (2004); D. Boyanovsky, H. J. de Vega, and N. G. Sanchez, Phys. Rev. D **71**, 023509 (2005); Nucl. Phys. **B747**, 25 (2006).

[29] A. A. Starobinski and J. Yokoyama, Phys. Rev. D **50**, 6357 (1994).

[30] R. P. Woodard, J. Phys. Conf. Ser. **68**, 012032 (2007); S.-P. Miao and R. P. Woodard, Phys. Rev. D **74**, 044019 (2006); R. P. Woodard, Nucl. Phys. B, Proc. Suppl. **148**, 108 (2005); T. Brunier, V. K. Onemli, and R. P. Woodard, Classical Quantum Gravity **22**, 59 (2005); T. Prokopec, O. Tornkvist, and R. Woodard, Phys. Rev. Lett. **89**, 101301 (2002).

[31] B. Garbrecht and G. Rigopoulos, Phys. Rev. D **84**, 063516 (2011).

[32] B. Garbrecht and T. Prokopec, Phys. Rev. D **73**, 064036 (2006).

[33] T. Arai, arXiv:1111.6754.

[34] J. Serreau, Phys. Rev. Lett. **107**, 19110 (2011).

[35] A. Rajaraman, Phys. Rev. D **82**, 123522 (2010).

[36] T. Prokopec, arXiv:1110.3187.

[37] D. Boyanovsky, Phys. Rev. D **85**, 123525 (2012).

[38] D. Boyanovsky, R. Holman, and S. Prem Kumar, Phys. Rev. D **56**, 1958 (1997).

[39] J. Bros, H. Epstein, M. Gaudin, U. Moschella, and V. Pasquier, Commun. Math. Phys. **295**, 261 (2009); J. Bros, H. Epstein, and U. Moschella, Annales Henri Poincare **11**, 611 (2010); J. Bros, H. Epstein, and U. Moschella, J. Cosmol. Astropart. Phys. 02 (2008) 003.

[40] D. Marold and I. A. Morrison, Phys. Rev. D **82**, 105032 (2010).

[41] D. P. Jatkar, L. Leblond, and A. Rajaraman, Phys. Rev. D **85**, 024047 (2012).

[42] M. van der Meulen and J. Smit, J. Cosmol. Astropart. Phys. 11 (2007) 023.

[43] D. Boyanovsky and R. Holman, J. High Energy Phys. 5 (2011) 47.

[44] V. Weisskopf and E. Wigner, Z. Phys. **63**, 54 (1930).

[45] D. Boyanovsky, Phys. Rev. D **84**, 065001 (2011).

[46] M. O. Scully and M. S. Zubairy, *Quantum Optics* (Cambridge University Press, Cambridge, England, 1997); M. Sargent III, M. Scully, and W. E. Lamb, *Laser Physics* (Addison-Wesley, Reading, Mass. 1974); W. Louisell, *Quantum Statistical Properties of Radiation* (Wiley, New York, 1974).

[47] D. Boyanovsky and H. J. de Vega, Ann. Phys. (N.Y.) **307**, 335 (2003); D. Boyanovsky, H. J. de Vega, and S.-Y. Wang, Phys. Rev. D **67**, 065022 (2003); D. Boyanovsky, H. J. de Vega, D.-S. Lee, S.-Y. Wang, and H.-L. Yu, Phys. Rev. D **65**, 045014 (2002).

[48] L. H. Ford and A. Vilenkin, Phys. Rev. D **33**, 2833 (1986).

[49] B. Ratra, Phys. Rev. D **31**, 1931 (1985).

[50] T. Brauner, Phys. Rev. D **74**, 085010 (2006); T. Brauner, Symmetry **2**, 609 (2010); H. Watanabe and T. Brauner, Phys. Rev. D **85**, 085010 (2012).

[51] A. Nicolis and F. Piazza, arXiv:1204.1570.

[52] H. Watanabe and H. Murayama, Phys. Rev. Lett. **108**, 251602 (2012).

[53] R. V. Lange, Phys. Rev. Lett. **14**, 3 (1965).

[54] S. Coleman, *Aspects of Symmetry* (Cambridge University Press, New York, 1985) (see pages 45 onward.)