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2^{++} glueball

Bing An Li

Department of Physics and Astronomy, University of Kentucky Lexington, Kentucky 40506, USA

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The mixing between the $f_2(1270)$, the $f_2'(1525)$, and the 2^{++} glueball is determined and tested. The mass and the hadronic decay widths of two bodies of the G_2 are predicted. The search for the G_2 state in the radiative decay of the J/ψ is discussed.

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The glueball states are solutions of nonperturbative QCD. The study of 2^{++} glueball has a long history. The MIT bag model predicts $m = 1.29$ GeV for the tensor glueball [1]. In Ref. [2], the mass of the 2^{++} glueball state is predicted in the range of 1.45–1.87 GeV. In Refs. [2,3], it is argued that there are glueball components in the $f_2(1270)$ and the $f_2'(1525)$ mesons. In Ref. [4], the smallness of the ratio of the helicity amplitude $y = \frac{T_2}{T_0}$ of the decay $J/\psi \to \gamma f_2(1270)$ is explained by the fact that the meson $f_2(1270)$ contains a substantial component of the 2^{++} glueball. There are many studies on the 2^{++} glueball [5]. Many 2^{++} isoscalar states have been discovered [6]. It is known that the $f_2(1270)$ and the $f_2'(1525)$ are ground states of the 2^{++} mesons. In this paper, the mixing of these two states and a 2^{++} glueball is revisited.

The $f_2(1270)$, $f_2'(1525)$, and the G_2 glueball are the eigenstates of the mass matrix of the f_8, f_0, and the pure 2^{++} glueball g_2, where f_8, f_0 are the 2^{++} octet and singlet states, respectively. The matrix of the mass square operator $\hat{m}^2$ is expressed as

$$\begin{pmatrix} m_1 \Delta_1 \Delta_2 \\ \Delta_1 m_2 \Delta_3 \\ \Delta_2 \Delta_3 m_3 \end{pmatrix}, \tag{1}$$

where $m_1 = m^2_{f_8}$, $m_2 = m^2_{f_0}$, $m_3 = m^2_{g_2}$, and $\Delta_1 = \langle f_8|\hat{m}^2|f_0|\rangle$, $\Delta_2 = \langle f_8|\hat{m}^2|g_2\rangle$, $\Delta_3 = \langle f_0|\hat{m}^2|g_2\rangle$. The units of the matrix elements (1) are GeV^2. From the quark model and up to the first order in the chiral expansion, we obtain

$$m_1 = \frac{1}{3}(4m^2_{K^*} - m^2_{a_2}), \quad m_2 = \frac{1}{3}(2m^2_{K^*} + m^2_{a_2}). \tag{2}$$

The masses of the $f_2(12770)$ and the $f_2'(1525)$ are taken as inputs. Two more inputs are required to determine all the six elements of Eq. (1). The branching ratios of $f_2'(1525) \to K\bar{K}$, $\pi\pi$ are listed [6] as

$$B(f_2'(1525) \to K\bar{K}) = (88.7 \pm 2.2)\%,$$
$$B(f_2'(1525) \to \pi\pi) = (8.2 \pm 1.5) \times 10^{-3}. \tag{3}$$

The $B(f_2'(1525) \to K\bar{K})$ is larger than the $B(f_2'(1525) \to \pi\pi)$ by two orders of magnitude. On the other hand, both are D-wave decays. The phase space of the $\pi\pi$ channel is much larger than the one of the $K\bar{K}$ channel,

$$\frac{\left(1 - \frac{4m^2_\pi}{m^2_{f_2'}}\right)^{(5/2)}}{\left(1 - \frac{4m^2_K}{m^2_{f_2'}}\right)^{(5/2)}} = 3.61.$$

Therefore, the magnitude of the amplitude of the $f_2'(1525) \to K\bar{K}$ is about 20 times that of the one of $f_2'(1525) \to \pi\pi$. The physical state of the $f_2'(1525)$ contains both the $q\bar{q}$ and the gluon-gluon components. It is reasonable to assume that the $\pi\pi$ is from the gluon-gluon component of the $f_2'(1525)$. Therefore, the $q\bar{q}$ component is dominated by the $s\bar{s}$. The physical state of the $f_2'(1525)$ is expressed as

$$f_2'(1525) = a_2 f_8 + b_2 f_0 + c_2 g_2. \tag{4}$$

The $f_2'(1525)$ is an eigenstate of Eq. (1). The $s\bar{s}$ dominance in the $f_2'(1525)$ leads to

$$a_2 = -\sqrt{2} b_2. \tag{5}$$

The processes $f_2(1270) \to \pi\pi$, $K\bar{K}$ are D-wave decays, and only the quark components contribute to the decays. In the chiral limit, the decay widths are expressed as

$$\Gamma(f_2 \to \pi\pi) = |T|^2 m_{f_2}\left(\frac{a_1}{\sqrt{2}} + b_1\right)^2\left(1 - \frac{4m^2_\pi}{m^2_{f_2}}\right)^{(5/2)}, \tag{6}$$

$$\Gamma(f_2 \to K\bar{K}) = |T|^2 m_{f_2}\frac{1}{3}\left(2b_1 - \frac{a_1}{\sqrt{2}}\right)^2\left(1 - \frac{4m^2_K}{m^2_{f_2}}\right)^{(5/2)}. \tag{7}$$

The $f_2(1270)$ is an eigenstate of Eq. (1) and $f_2(1270) = a_1 f_8 + b_1 f_0 + c_1 g_2$. In the chiral limit, T of Eqs. (6) and (7) are the same. These two decay widths are measured [6]. Inputting the experimental value of the ratio

$$\frac{\Gamma(f_2 \to K\bar{K})}{\Gamma(f_2 \to \pi\pi)} = 0.054[6], \qquad a_1 = 0.504 b_1 \tag{8}$$

is determined. Using these six inputs and solving the three eigen equations, the three off-diagonal matrix elements of Eq. (1) are determined:

$$\Delta_1 = -0.258\ \text{GeV}^2, \qquad \Delta_2 = 0.0186\Delta_3, \qquad \Delta_3 = -0.172\ \text{GeV}^2, \qquad \Delta_2 = 0.0032\ \text{GeV}^2. \tag{9}$$

The masses of the pure glueball g_2 and the new physical state G_2 are predicted:

$$m_{g_2} = 1.352\ \text{GeV}, \qquad m_{G_2} = 1.40\ \text{GeV}. \tag{10}$$

The mixing of the f_2, f_2', G_2 are determined:

$$\begin{aligned} f_2 &= 0.359 f_8 + 0.711 f_0 + 0.605 g_2, \\ f_2' &= 0.801 f_8 - 0.566 f_0 + 0.196 g_2, \\ G_2 &= 0.482 f_8 + 0.402 f_0 - 0.772 g_2. \end{aligned} \tag{11}$$

Using the mixing (11), we can make predictions about the decays, $f_2, f_2', G_2 \to \pi\pi, K\bar{K}, \eta\eta$. The decay width of the $f_2 \to \eta\eta$ is derived as

$$\Gamma(f_2 \to \eta\eta) = |T|^2 m_{f_2} \frac{1}{3}\left(b_1 - \frac{a_1}{\sqrt{2}}\right)^2 \left(1 - \frac{4m_\eta^2}{m_{f_2}^2}\right)^{(5/2)}, \tag{12}$$

$$\frac{\Gamma(f_2 \to \eta\eta)}{\Gamma(f_2 \to K\bar{K})} = 0.0545. \tag{13}$$

In the chiral limit, the amplitude T of Eq. (12) is the same as the one in Eqs. (6) and (7). The experimental data [6] is 0.087 ± 0.025. There is mixing between the η, η' and the 0^{-+} glueball, and the small mixing effect is not taken into account in this study. The theoretical result agrees with the data within the experimental error reasonably well. Replacing a_1, b_1, m_{f_2} by a_2, b_2, $m_{f_2'}$ in Eqs. (7) and (12), the decay widths of $f_2' \to K\bar{K}, \eta\eta$ are obtained and the ratio is predicted:

$$\frac{\Gamma(f_2' \to \eta\eta)}{\Gamma(f_2' \to K\bar{K})} = 0.29. \tag{14}$$

The data of this ratio are following (corresponding to Refs. [7,8,6], respectively):

$$0.069 \pm 0.012 \qquad 0.33 \pm 0.03, \qquad 0.12 \pm 0.03.$$

It is reasonable to assume that in the chiral limit, the $|T|^2$ in Eqs. (6), (7), and (12) are about the same for the two body decays of the f_2, f_2' and the G_2. It is obtained

$$\frac{\Gamma(f_2' \to K\bar{K})}{\Gamma(f_2 \to K\bar{K})} = 6.52. \tag{15}$$

The data [6] is 7.63 ± 1.75. Theory agrees with data within the experimental errors. The decay widths of $\Gamma(G_2 \to \pi\pi, K\bar{K}, \eta\eta)$, whose formulas are obtained by replacing a_1, b_1, m_{f_2} by a_3, b_3, m_{G_2} in Eqs. (6), (7), and (12), are calculated by inputting $|T|^2$ which is determined by $\Gamma(f_2 \to \pi\pi) = 158 \pm 6.32$ MeV [6].

$$\begin{aligned} \Gamma(G_2 \to \pi\pi) &= 102.0 \pm 3.0\ \text{MeV}, \\ \Gamma(G_2 \to K\bar{K}) &= 2.77 \pm 0.15\ \text{MeV}, \\ \Gamma(G_2 \to \eta\eta) &\sim 0.034\ \text{MeV}. \end{aligned} \tag{16}$$

The $\pi\pi$ is the dominant decay channel. Because of small $\frac{1}{3}(2b_3 - \frac{a_3}{\sqrt{2}})^2(1 - \frac{4m_K^2}{m_{G_2}^2})^{(2/5)}$, the $G_2 \to K\bar{K}$ decay is suppressed. Because of very small $\frac{1}{3}(b_3 - \frac{a_3}{\sqrt{2}})^2(1 - \frac{4m_\eta^2}{m_{G_2}^2})^{(2/5)}$, the $G_2 \to \eta\eta$ decay is strongly suppressed. Especially, for the very small $\Gamma(G_2 \to \eta\eta)$, the strong cancellation in $b_3 - \frac{a_3}{\sqrt{2}}$ plays a very important role. Because of this cancellation, the $\Gamma(G_2 \to \eta\eta)$ is very small and very sensitive to the values of a_3 and b_3. The errors of the inputs lead to the errors of the coefficients. Therefore, $\Gamma(G_2 \to \eta\eta)$ cannot be predicted accurately.

Eqs. (9) show that the element Δ_2 is much smaller than the Δ_1, Δ_3. The reason of the smallness of the Δ_2 can be found from the N_C expansion. The N_C expansion of QCD was proposed by 't Hooft [9] and studied by Witten [10]. It is successfully and widely used in hadron physics, including glueballs. The study presented in Ref. [11] is an example of the N_C expansion. In this chiral field theory of mesons, the N_C expansion is naturally embedded. The theory is very successful phenomenologically. In the approach of this paper, there are both chiral and N_C expansions. At the leading order $O(m_q^0)$,

$$m_{a_2}^2 = m_{K^*}^2 = m^2.$$

In the two expansions, the elements m_1, m_2 of Eq. (1) are up to the next leading orders in the chiral expansion and the leading order in the N_C expansions. The m_3 is at the leading orders in both the chiral and the N_C expansions. The orders of the Δ_1, Δ_2, Δ_3 are

$$\Delta_1 \sim m^2 O\left(\frac{m_q}{m}\right), \qquad \Delta_2 \sim m^2 O\left(\frac{m_q}{m}\frac{1}{N_C}\right), \qquad \Delta_3 \sim m^2 O\left(\frac{1}{N_C}\right). \tag{17}$$

The study presented in this paper is up to either next leading order in the chiral expansion $O(\frac{m_q}{m})$ or to the next leading order in the N_C expansion $O(\frac{1}{N_C})$. Obviously, the Δ_2 is at higher order in the two expansions, and for the sake of consistency

$$\Delta_2 = 0 \tag{18}$$

should be taken. Very small Δ_2 determined phenomenologically (9) above is the consequence of the N_C expansion. Now, Eqs. (18), (2), and (5) and the $m_{f_2}^2$, $m_{f_2'}^2$ are taken as inputs. Solving the three eigen equations, it is determined

$$\Delta_1 = -0.275\ \mathrm{GeV}^2, \qquad \Delta_3^2 = 0.0311\ \mathrm{GeV}^4. \tag{19}$$

The masses of the pure glueball g_2 and the new state G_2 are predicted as

$$m_{G_2} = 1.404\ \mathrm{GeV}, \qquad m_{g_2} = 1.356\ \mathrm{GeV}. \tag{20}$$

They are almost the same as the ones of Eq. (10). On the other hand, there are errors in this study. The sources of the errors are the following. The inputs m_1, m_2, $m^2_{f_2}$, $m^2_{f'_2}$ and $a_2 = -\sqrt{2}b_2$ have errors. For example, the input $a_2 = -\sqrt{2}b_2$ causes about 5% error. It is estimated that the errors of this study are about 10%. Therefore, the mass of the G_2 state is expressed as

$$m_{G_2} = 1.40 \pm 0.14\ \mathrm{GeV}. \tag{21}$$

The mass of the new state G_2 is predicted in the range between 1.26 GeV to 1.54 GeV. On the other hand, this study is up to $O(\frac{m_q}{m})$ and $O(\frac{1}{N_C})$. The errors caused by the terms at higher orders in both the chiral and the N_C expansions are unknown. The 10% error is marginal, and it could be greater than 10%.

It is interesting to notice that the values of the Δ_1 and the Δ_3 (19) are much smaller than the values of the m_1 and the m_2. These results (19) show that both the chiral expansion and the $O(N_C)$ expansion work.

Using the values of the Δ_1, Δ_3, $m^2_{G_2}$, the expressions of the three physical 2^{++} states are determined from the three eigen equations of the mass matrix (1),

$$f_2(1270) = 0.357 f_8 + 0.718 f_0 + 0.597 g_2, \tag{22}$$

$$f'_2(1525) = 0.799 f_8 - 0.565 f_0 + 0.204 g_2, \tag{23}$$

$$G_2 = 0.495 f_8 + 0.402 f_0 - 0.770 g_2. \tag{24}$$

The G_2 state contains substantial $q\bar{q}$ components and the glueball component. The glueball component in the f_2 is large and in f'_2 is small. In this study, the g_2 is a pure glueball state, and the G_2 is a new 2^{++} state. Without the pure glueball state g_2, the G_2 doesn't exist.

Using the mixing, the ratios of two-body decays of the f_2, f'_2 are predicted:

$$\frac{\Gamma(f_2 \to K\bar{K})}{\Gamma(f_2 \to \pi\pi)} = 0.0548. \tag{25}$$

The experimental value of this ratio is [6] 0.054 ± 0.007. Theory agrees with the data very well.

$$\frac{\Gamma(f_2 \to \eta\eta)}{\Gamma(f_2 \to K\bar{K})} = 0.055. \tag{26}$$

The experimental data [6] is 0.087 ± 0.025. As mentioned above, there is mixing between the η, η' and the 0^{-+} glueball, and the mixing effect is not taken into account in this study. The theoretical result agrees with the experimental data within the experimental error reasonably well.

$$\frac{\Gamma(f'_2 \to \eta\eta)}{\Gamma(f'_2 \to K\bar{K})} = 0.29 \tag{27}$$

is predicted, which is the same as Eq. (14). The predicted

$$\frac{\Gamma(f'_2 \to K\bar{K})}{\Gamma(f_2 \to K\bar{K})} = 6.34 \tag{28}$$

is in agreement with the data [6] which is 7.63 ± 1.75. Theory agrees with data within the experimental errors.

The decay widths of $\Gamma(G_2 \to \pi\pi, K\bar{K}, \eta\eta)$ are predicted

$$\begin{aligned} &\Gamma(G_2 \to \pi\pi) = 101.2 \pm 3.0\ \mathrm{MeV}, \\ &\Gamma(G_2 \to K\bar{K}) = 2.4 \pm 0.1\ \mathrm{MeV}, \\ &\Gamma(G_2 \to \eta\eta) \sim 0.017\ \mathrm{MeV}. \end{aligned} \tag{29}$$

Comparing the results with $\Delta_2 = 0$ (18) and the results without Eq. (18), it can be found that the most changes are just few percents. Only $\Gamma(G_2 \to \eta\eta)$ has changed by a factor of two. The reason has been mentioned above: this approach predicts a very small $\Gamma(G_2 \to \eta\eta)$ and cannot predict the accurate value of this decay width.

The f_2 state has 4π decays whose branching ratio is about 10%. The 4π decay mode is from the $\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$ component of the f_2 state. The $\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$ component of the G_2 state should lead to the 4π decays, too. The mass of the G_2 is about 10% higher than m_{f_2}. Ignoring the mass difference of the f_2 and G_2, it is estimated

$$\begin{aligned} &\frac{\Gamma(G_2 \to 4\pi)}{\Gamma(f_2 \to 4\pi)} \sim \left(\frac{0.614}{0.793}\right)^2 = 0.6, \\ &\Gamma(G_2 \to 4\pi) \sim 11\ \mathrm{MeV} \\ &B(G_2 \to 4\pi) \sim 10\%. \end{aligned}$$

Detailed study of the 4-body decays of the f_2, f'_2, G_2 are beyond the scope of this paper.

The total width of the G_2 state is

$$\Gamma_{G_2} = 114.6 \pm 11\ \mathrm{MeV}.$$

The estimated 10% error is resulted in the errors of the inputs, the chiral, and the N_C expansion and the estimation of the partial decay width of 4π.

This study predicts a tensor glueball of $1.26\ \mathrm{GeV} < m < 1.54\ \mathrm{GeV}$, $\Gamma = 114 \pm 11$ MeV, and $\pi\pi$ is the dominant decay channel. The $f_2(1430)$ state is listed in Ref. [6]. In Ref. [12], the reaction $pp \to pp\pi^+\pi^-$ is studied. It is mentioned that the "D-wave shows a clear enhancement between 1200 MeV and 1500 MeV which cannot be fitted by $f(1270)$ alone," and a 2^{++} resonance of $m = 1480 \pm 50$ MeV and $\Gamma = 150 \pm 50$ MeV in addition to the $f_2(1270)$ is discovered. No sign of this state is found in the $\eta\eta$ channel. Because of the problem of particle identification, this experiment is unable to investigate whether this state is present in the K^+K^- channel. However, Refs. [6,15] show that the resonance observed in Ref. [12] is not revealed from the analysis of the $K_s^0K_s^0$ system of the reaction $\pi^- p \to K_s^0K_s^0 n$. The study of this paper predicts $B(G_2 \to K_s^0K_s^0) \sim 2\%$. The experimental results [12,13] are in agreement with the predictions of the G_2 state.

On the other hand, very narrow resonances have been observed in the $K\bar{K}$ system. In Ref. [14], in the channel K_sK_s of the reaction $\pi^- p \to K_sK_s n$, a narrow state of $m = 1453 \pm 4$ MeV and $\Gamma = 13 \pm 5$ MeV has been found. However, the quantum numbers of this state are not well-determined: $J^P = 0^{++}$ *or* 2^{++}. In Ref. [15], a state of $m = 1439^{+5}_{-6}$ MeV and $\Gamma = 43^{+17}_{-18}$ MeV has been reported in the reaction $\pi^- p \to K^0\bar{K}^0 n$. Its quantum numbers are unable to be determined. $J^P = 0^+, 2^+, 4^+$ are all possible. In Ref. [16], both structures of $m = 1412 \pm 3$ MeV, $\Gamma = 14 \pm 6$ MeV and $m = 1436^{+26}_{-16}$ MeV, $\Gamma = 81^{+56}_{-29}$ MeV have been observed in $K_s^0K_s^0$ and K^+K^- systems, respectively, in $\pi^- p \to K\bar{K}n$. The quantum numbers of these states are determined to be 2^{++}. According to Ref. [6], the quantum numbers of these states are not well-determined, and it is either 0^{++} or 2^{++}. On the other hand, these narrow resonances of the $K\bar{K}$ are not observed by another experiment of $\pi^- p \to K\bar{K}n$ [6,13]. Comparing with the G_2 state, the decay widths of these resonance states of $K\bar{K}$ systems are too narrow, and it is difficult to associate these very narrow resonances with the G_2 state of $\Gamma = 114 \pm 11$ MeV.

In QCD, the radiative decay of the J/ψ is described as $J/\psi \to \gamma gg$. The contribution of the glueball component of a meson state to the production of this state in the J/ψ radiative decay is at the leading order in the N_C expansion. Therefore, a state with larger glueball component should have larger production rate in J/ψ radiative decay. Eqs. (22)–(24) show that the f_2' contains less glueball component than the f_2 state does. Qualitatively, the theory predicts that the $B(J/\psi \to \gamma f_2)$ is much larger than the $B(J/\psi \to \gamma f_2')$. The experiments [6] support this prediction.

On the other hand, the glueball component of the G_2 state is 0.770 g_2 (24) which is larger than the glueball component, 0.597 g_2 (22), of the f_2 state. Qualitatively, the theory predicts $B(J/\psi \to \gamma G_2)$ is comparable or greater than the $B(J/\psi \to \gamma f_2)$. So far, there is only one measurement of $J/\psi \to \gamma X$, $X \to \pi^+\pi^-$, where m_X is around 1.4 MeV [17]. In 1987, the DM2 collaboration has reported a narrower state [17] in $J/\psi \to \gamma\pi^+\pi^-$:

$$\text{for all } \gamma\pi^+\pi^- \text{events: } m_X = 1421 \pm 7 \text{ MeV}, \quad \Gamma_X = 69 \pm 18 \text{ MeV},$$

$$\text{for events after the } \cos\theta_\pi \text{cut: } m_X = 1421 \pm 5 \text{ MeV}, \quad \Gamma_X = 30 \pm 9 \text{ MeV}.$$

From the first fit, $B(J/\psi \to \gamma X) \times B(X \to \pi^+\pi^-) = (7.9 \pm 2.4 \pm 1.2) \times 10^{-5}$ is determined. If the X state found by this experiment is the G_2 state, the branching ratio of $J/\psi \to \gamma G_2$, $G_2 \to \pi^+\pi^-$ should be much greater than the one reported. However, there are issues about this measurement which should be addressed:

(1) As mentioned in Ref. [17], this small branching ratio is obtained from all events (see Fig. 5a of that paper) and "the cut of the cosθ " enhances the mass region around 1400 MeV (see Fig 5b). The branching ratio from the events after cut of the cosθ is not presented. Figs. 5a, 5b show that the peak around 1400 MeV is really enhanced after the cut, and the branching ratio of the enhanced state should be enhanced, too.

(2) The quantum number of the X state is not well-determined. In Ref. [17], it is only mentioned that "when the $\cos\theta_\pi$ cut is applied, a $J^P = 0^+$ assignment for these events is excluded."

(3) In Ref. [17], the authors tried to explain the excess of events by an interference effect between $f_2(1270)$ and $f_2'(1525)$ amplitudes in the $\pi^+\pi^-$ final state: "the best agreement is obtained, but the f_2 fitted width is still too large (206 ± 7 MeV)."

(4) $B(J/\psi \to \gamma f_2, f_2 \to \pi^+\pi^-) = (7.50 \pm 0.30 \pm \pm 1.12) \times 10^{-4}$ is less than the $B(J/\psi \to \gamma f_2, f_2 \to \pi^+\pi^-) = (9.14 \pm 0.07) \times 10^{-4}$ measured by BES [18] by about 22%.

The present data of DM2 is not enough to make conclusion for the existence of the G_2 state.

Theoretically, this approach predicts that the f_2, G_2 have similar properties. They decay to $\pi\pi$ dominantly, and the $K\bar{K}$, $\eta\eta$ channels are suppressed. Their masses of these two states overlap. This theory predicts coherent productions of the f_2, G_2 states in the $J/\psi \to \gamma X$, $X \to \pi\pi$ where 1.2 GeV $< m_X <$ 1.5 GeV. It is suggested to use these two states together to fit the data of $J/\psi \to \gamma X$, $X \to \pi\pi$ in the mass region of the f_2, G_2, as was done in Ref. [17].

Both the chiral and the N_C expansions are applied to study the mixing of the f_2, f_2' and a tensor glueball. The predictions are in agreement with the data. The mass of the new tensor state G_2 is predicted to be 1.40 ± 0.14 GeV. $\pi\pi$ is the dominant decay mode of the G_2 state, and both the $K\bar{K}$ and $\eta\eta$ decay modes are strongly suppressed. The decay width of the G_2 state is about 110 GeV. So far, the experimental data presented in Refs. [12,13] are in agreement with these predictions. Qualitatively, the study predicts $B(J/\psi \to \gamma f_2') < B(J/\psi \to \gamma f_2)$ which is consistent with the data. Coherent productions of the f_2, G_2 in $J/\psi \to \gamma\pi\pi$ are expected.

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