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Chiral symmetry of heavy-light scalar mesons with $U_A(1)$ symmetry breaking

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In a previous paper, based on a calculation in the nonrelativistic quark model, we advanced the hypothesis that the $D_s(2317)$, $D_0(2308)$ mesons are predominantly four-quark states lowered in mass by the flavor-dependent Kobayashi-Kubo-Maskawa 't Hooft $U_A(1)$ symmetry breaking effective interaction. Here we show similar results and conclusions in a relativistic effective chiral model calculation, based on three-light-quark (i.e., two q plus one $\bar{q}$) local interpolators. To this end we classify the four-quark (three light plus one heavy quark) local interpolators according to their chiral transformation properties and then construct chiral invariant interactions. We evaluate the diagonal matrix elements of the Kobayashi-Kubo-Maskawa 't Hooft interaction between different interpolating fields and show that the lowest-lying one is always the (antisymmetric) $SU(3)_F$ antitriplet belonging to the chiral (3, 3) multiplet. We predict bottom-strange B_{s0} and the bottom-nonstrange B_0 scalar mesons with equal masses at 5720 MeV, the strange meson being some 100 MeV lower than in most of the quark potential models. We also predict the $J^P = 1^+$ bottom-nonstrange B_1 and the bottom-strange B_{s1} meson masses as 5732 MeV and 5765 MeV, respectively, using the Bardeen-Hill-Nowak-Rho-Zahed scalar-vector mass relation.

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I. INTRODUCTION

Nine years have passed since the discovery of the positive-parity charm-strange mesons $D_s^+(2317)$ [1] and $D_s^+(2460)$ [2,3], some 160 MeV below their predicted masses as $c\bar{s}$ states calculated in potential quark models (see Refs. [4–6]). The corresponding bottom-up/down $b\bar{u}/\bar{d}$ and bottom-strange $b\bar{s}$ scalar states still await their discovery, though the odd-parity ground states [the $J^P = 0^-$ mesons $B(5279)$ and $B_s(5366)$, and their $J^P = 1^-$ partners $B^*(5325)$ and $B_s^*(5415)$], and several even-parity states, viz. the $J^P = 1^+$ mesons $B^*(5721)$ and $B_s^*(5830)$ and $J^P = 2^+$ mesons $B^*(5747)$ and $B_s^*(5840)$, have been observed. Early potential quark models (Refs. [4,5]) predicted $J^P = 2^+$ mesons around $B_0(5800)$ and $B_{s0}(5880)$, whereas the post-$D_s^+(2317)$ model (Ref. [7]) moves them to somewhat lower values around 5760 MeV and 5850 MeV, respectively.

Eight years ago we suggested, on the basis of a nonrelativistic quark model calculation involving the 't Hooft interaction, that the $D_s^+(2317)$ is a predominantly tetraquark state (see Refs. [8–11]). Although tetraquark is by no means the only possible explanation of the low mass of these states, to our knowledge it is the only one that predicted the degeneracy of $D_s^+(2317)$ and $D_0^*(2308)$ before the experimental values were published (Ref. [12]).

Bardeen-Hill-Nowak-Rho-Zahed [BHNRZ)] (Refs. [13–15]), on the other hand, used the chiral symmetry in association with the heavy-quark symmetry to predict striking "sum rules" for the $0^\pm$ and $1^\pm$ D and D_s meson masses:

$$D_{0^+}(2308) - D_{1^+}(2420) = D(1869) - D^*(2010) \quad (1)$$

$$D_{s0^+}(2317) - D_{s1^+}(2460) = D_s(1968) - D_s^*(2112). \quad (2)$$

The two relations [Eqs. (1) and (2)] do not predict the (approximate) degeneracy of $D_s^+(2317)$ and $D_0^*(2308)$, as the sum rules for the strange and nonstrange D mesons are independent, but the experimental values allow for, and indeed are consistent with, such a degeneracy. The (numerical) validity of the sum rules was interpreted in Ref. [15] as evidence "that the two orthogonal linear combinations of mesons … have well defined transformation properties under $SU_L(3) \times SU_R(3)$, transforming as (approximately) pure (1, 3) and (3, 1), respectively."

These sum rules can be "read" in two ways: (a) as identities of two hyperfine-interaction (HFI)-induced splittings $0^- - 1^- = 0^+ - 1^+$, which should depend on the light-quark mass/flavor [i.e., on the $SU_F(3)$ symmetry breaking], but in reality do not; and (b) as identities of two parity-doublet splittings $1^+ - 1^- = 0^+ - 0^-$, which need not depend on the light-quark mass/flavor, though in reality they do. This suggests that there are two different broken chiral symmetries at work here: one non-Abelian, $SU_L(3) \times SU_R(3)$, and another Abelian one: $U_L(1) \times U_R(1) \simeq U_V(1) \times U_A(1)$, where $U_V(1) = U_B(1)$ is the (absolutely conserved) baryon number. These perhaps unexpected peculiarities/properties of the D, D_s meson spectrum demand a reexamination of the underlying assumptions of these chiral mass sum rules. That is what we do in this paper, with the following consequences.

Our work is based on three-light-quark (i.e., two q plus one $\bar{q}$ equals one constituent quark) local interpolators. We classify the four-quark (three light plus one heavy quark) local interpolators according to their chiral transformation

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properties: we find five chirally exotic effective constituent quark fields, $Q_1, \ldots, Q_5$. We are interested in the chiral mixtures of these constituent quarks with the ordinary light constituent quark within the scalar mesons: the energy cost of one quantum of orbital angular momentum is roughly the same as the rest mass of a pseudoscalar isosinglet $q\bar{q}$ pair [e.g., the $\eta(550)$ meson mass]. If the interaction between this pair and the rest of the hadron is attractive, as is the case with the Kobayashi-Kubo-Maskawa 't Hooft (KKMT) interaction here, then the total mass may be smaller than that of the orbitally excited meson.

We show that the light constituent antiquark in the scalar (even-parity spinless) heavy-light mesons need not be in an orbitally excited state, but rather it may be in an S wave together with a light pseudoscalar $q\bar{q}$ pair in the flavor $\bar{3}_A$ multiplet, thus forming a chiral (3, 3) "antitriquark" $(\bar{Q}_1 - \bar{Q}_2)$, its mass lowered by the KKMT instanton-induced interaction. The $\bar{3}_A$ flavor $SU_F(3)$ multiplet shows no signs of $SU_F(3)$ symmetry breaking: both the strange and nonstrange states are degenerate even with a strange quark that is heavier than the up/down ones. Thus, the $U_A(1)$ and $SU_L(3) \times SU_R(3)$ symmetry breaking effects "collaborate" to reproduce the observed peculiarities in only one flavor multiplet belonging to only one chiral multiplet.

Thus we have shown, among other things, that the assumption of "(approximately) pure (1, 3) and (3, 1)" chiral multiplets is neither necessary, nor sufficient, at least for the $(0^-, 1^-) - (0^+, 1^+)$ parity doublet(s). Indeed, we have constructed several other effective constituent quark operators that belong to chiral multiplets other than (3, 1) and/or (1, 3), and different Abelian axial charges, but only one of them, the (3, 3), successfully reproduces the observed D meson masses.

We shall not concern ourselves with the technical details of chiral mixing here (that will be done elsewhere), but rather we use these results to make specific predictions for the B mesons: the bottom-strange B_{s0} and the bottom-nonstrange B_0 scalar meson masses ought to have equal mass around 5720 MeV, the strange one being some 100 MeV lower than in most of the quark potential models and the nonstrange one in agreement with most potential models. This is not as marked a decrease as in the case of D_s mesons, where this state was some 160 MeV lower than predicted in potential models.

This paper is organized in four sections. In Sec. II we introduce the framework of our calculation. This is necessary as a prelude to the inclusion of the 't Hooft interaction that follows in Sec. III. Then in Sec. IV we discuss our results, present and future experiments, and compare with a few other theoretical predictions.

II. THEORETICAL FRAMEWORK

We start by defining the theoretical framework that we use. It is based on various interpolating operators for the effective light constituent quark operator. Besides the usual single ("current") quark operator being used in this role, one may define any polynomial function of quark and antiquark operators that preserves the overall quantum numbers (spin, flavor, color, baryon number) as the single quark operator. It need *not* have the chiral properties of the current quark, however, because that symmetry is spontaneously broken. The simplest such operators contain two quarks and one antiquark field; there are five such (linearly independent) objects—we shall call them triquarks. Their chiral transformation properties are generally not identical to those of a current quark, however, and that is precisely what we are looking for: distinct chiral properties, with identical flavor properties that may modify the BHNRZ sum rule.

Chiral properties of interpolating operators for baryons consisting of three quarks have been studied in great detail: see Refs. [16–18]. To construct the triquark fields, we follow the same approach that we used for baryons in Refs. [16–18] and we classify them according to their spin and flavor.

So, in general one must write[1]

$$Q(x) \sim P_{abc}(\mathbf{3_C})\bar{q}_c(x)\Gamma_2(\tilde{q}_a(x)P_{ab}(\bar{\mathbf{3}}_\mathbf{C})\Gamma_1 q_b(x)) + P_{abc}(\mathbf{3_C})\bar{q}_c(x)\Gamma_4(\tilde{q}_a(x)P_{abc}(\mathbf{6_C})\Gamma_3 q_b(x)), \quad (3)$$

where $q(x) = (u(x), d(x), s(x))^T$ is a flavor-triplet quark field at location x, and the indices a, b, and c represent the color. Here the color-space projectors $P_{abc}(\alpha_\mathbf{C})$ ensure that the overall color state of the triquark is the triplet, which implies that any pair of quarks (a "diquark") is either in the color antitriplet $\bar{\mathbf{3}}_\mathbf{C}$ state, or in the color sextet $\mathbf{6_C}$. Here we have introduced the "tilde-transposed" quark field $\tilde{q}$ as follows:

$$\tilde{q} = q^T C\gamma_5, \quad (4)$$

where $C = i\gamma_2\gamma_0$ is the Dirac field charge conjugation operator. The matrices Γ_i ($i = 1, 2, 3, 4$) are tensor products of Dirac and flavor matrices. With a suitable choice of $\Gamma_{1,2}$, the baryon operators are defined so that they form an irreducible representation of the Lorentz and flavor groups, as we shall show below in this section.

A. Local triquark fields

A local interpolating operator $Q(x)$ for a constituent quark consisting of two quarks and one antiquark can be generally written as

$$Q(x) \sim \bar{q}(x)\Gamma_i(\tilde{q}(x)\Gamma_i q(x)) = \bar{q}(x)\Gamma^i D^i(x), \quad (5)$$

where $D^i(x)$ are the local diquark fields at location x (see Sec. II B). The Pauli principle eliminates one half of the local diquark fields in the color $\bar{\mathbf{3}}_\mathbf{C}$-plet and similarly for

[1]Of course one must include the color-dependent and path-dependent "gauge factors." We shall drop them henceforth, to keep the notation simple.

TABLE I. The Abelian $U_A(1)$, the Lorentz group $SO(3,1)$, and the non-Abelian $SU_L(3) \times SU_R(3)$ chiral transformation properties/axial charges and the Lorentz group representations of the diquark fields. In the last column we show the sign under transposition of the two quarks in the color $\bar{3}_C$ state.

	$U_A(1)$	$SO(3,1)$	$SU_L(3) \times SU_R(3)$	$SU_F(3)$	$SU_C(3)$
$D_1 - D_2$	-2	$(0,0)$	$(\bar{\mathbf{3}},1) \oplus (1,\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$
$D_1 + D_2$	$+2$	$(0,0)$	$(\bar{\mathbf{3}},1) \oplus (1,\bar{\mathbf{3}})$	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$
D_3	0	$(\frac{1}{2},\frac{1}{2})$	$(\mathbf{3},\mathbf{3})$	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$
D_4	0	$(\frac{1}{2},\frac{1}{2})$	$(\mathbf{3},\mathbf{3})$	$\mathbf{6}$	$\bar{\mathbf{3}}$
D_5	$+2$	$(1,0) \oplus (0,1)$	$(\mathbf{6},1) \oplus (1,\mathbf{6})$	$\mathbf{6}$	$\bar{\mathbf{3}}$
$D_6 - D_7$	-2	$(0,0)$	$(\mathbf{6},1) \oplus (1,\mathbf{6})$	$\mathbf{6}$	$\mathbf{6}$
$D_6 + D_7$	$+2$	$(0,0)$	$(\mathbf{6},1) \oplus (1,\mathbf{6})$	$\mathbf{6}$	$\mathbf{6}$
D_8	0	$(\frac{1}{2},\frac{1}{2})$	$(\mathbf{3},\mathbf{3})$	$\mathbf{6}$	$\mathbf{6}$
D_9	0	$(\frac{1}{2},\frac{1}{2})$	$(\mathbf{3},\mathbf{3})$	$\bar{\mathbf{3}}$	$\mathbf{6}$
D_{10}	$+2$	$(1,0) \oplus (0,1)$	$(\bar{\mathbf{3}},1) \oplus (1,\bar{\mathbf{3}})$	$\bar{\mathbf{3}}$	$\mathbf{6}$

the local diquarks in the color $\mathbf{6}_C$-plet. Other than that, the Pauli principle does not play a role here. The Fierz transformations that one usually associates with implementing the Pauli principle lead here to identities relating the triquark operators in the diquark basis/channel to triquark operators in the meson basis/channel.

The first step is to apply the Pauli principle to the first and second quarks, i.e., to the diquarks, as discussed in Sec. II B. Secondly, an additional constraint comes from the permutation of the second quark and the third antiquark, which corresponds to the usual Fierz transformation. In this case the Fierz identities do not impose additional Pauli-principle-induced relations (identities) among the fields; rather, they present explicit connection between the fields in the diquark and meson bases.

Note that the Fierz transformation connects only operators belonging to the same Lorentz and flavor group multiplets. We may, therefore, classify the triquark operators according to their Lorentz and flavor representations. It has been known that such operators may couple either to the even-or odd-parity states. In the following discussion, all the triquark operators will be defined as having even parity. We note, however, that two different flavor operators belonging to the same chiral multiplet may have opposite parities.

B. Color $\bar{3}_C$ and 6_C diquarks

We begin with bilinears of two quarks in Eq. (3). There are 10 nonvanishing possibilities for Γ_1 with bilocal fields (besides the familiar five that survive in the local approximation limit):

$$D_1(x) = P_{ab}(\bar{\mathbf{3}}_C)P_{ij}(\bar{\mathbf{3}}_F)\tilde{q}_{a,i}(x)q_{b,j}(x), \qquad (6)$$

$$D_2(x) = P_{ab}(\bar{\mathbf{3}}_C)P_{ij}(\bar{\mathbf{3}}_F)\tilde{q}_{a,i}(x)\gamma^5 q_{b,j}(x), \qquad (7)$$

$$D_3^\mu(x) = P_{ab}(\bar{\mathbf{3}}_C)P_{ij}(\bar{\mathbf{3}}_F)\tilde{q}_{a,i}(x)\gamma^\mu q_{b,j}(x), \qquad (8)$$

$$D_4^\mu(x) = P_{ab}(\bar{\mathbf{3}}_C)P_{ij}(\mathbf{6}_F)\tilde{q}_{a,i}(x)\gamma^\mu\gamma^5 q_{b,j}(x), \qquad (9)$$

$$D_5^{\mu\nu}(x) = P_{ab}(\bar{\mathbf{3}}_C)P_{ij}(\mathbf{6}_F)\tilde{q}_{a,i}(x)\sigma^{\mu\nu} q_{b,j}(x), \qquad (10)$$

where a, b are the color $SU(3)_C$ indices, and i, j are the flavor $SU(3)_F$ indices. There are also five color $\mathbf{6}_C$ ones:

$$D_6(x) = P_{ab}(\mathbf{6}_C)P_{ij}(\mathbf{6}_F)\tilde{q}_{a,i}(x)q_{b,j}(x), \qquad (11)$$

$$D_7(x) = P_{ab}(\mathbf{6}_C)P_{ij}(\mathbf{6}_F)\tilde{q}_{a,i}(x)\gamma^5 q_{b,j}(x), \qquad (12)$$

$$D_8^\mu(x) = P_{ab}(\mathbf{6}_C)P_{ij}(\mathbf{6}_F)\tilde{q}_{a,i}(x)\gamma^\mu q_{b,j}(x), \qquad (13)$$

$$D_9^\mu(x) = P_{ab}(\mathbf{6}_C)P_{ij}(\bar{\mathbf{3}}_F)\tilde{q}_{a,i}(x)\gamma^\mu\gamma^5 q_{b,j}(x), \qquad (14)$$

$$D_{10}^{\mu\nu}(x) = P_{ab}(\mathbf{6}_C)P_{ij}(\bar{\mathbf{3}}_F)\tilde{q}_{a,i}(x)\sigma^{\mu\nu} q_{b,j}(x). \qquad (15)$$

These quark bilinear fields, D_1, D_6, D_2, D_7, D_3^μ, D_8^μ, D_4^μ, D_9^μ and $D_5^{\mu\nu}$, $D_{10}^{\mu\nu}$, shall be referred to as the flavor antitriplet or sextet, depending on their flavor dependence; and scalar, pseudoscalar, vector, axial-vector and tensor diquarks, respectively, according to their Lorentz transformation properties[2].

All spin 0 and 1 diquark operators were classified according to their Lorentz and isospin group representations in Table I. From Table I we see that only the Lorentz rep. $(\frac{1}{2},\frac{1}{2})$ diquark fields have the same chiral properties with both signs under the Pauli principle/two-particle exchange. This means that only the triquark fields made from these diquarks are subject to "Pauli mixing." These are the $J = \frac{1}{2}$ and $J = \frac{3}{2}$ fields.

[2]Throughout the present paper, Latin indices i, j, etc. run over the flavor space $1, 2, \ldots 8$, and Greek indices μ, ν, etc. run over the Lorentz space 0, 1, 2, 3.

C. The constituent quark ($J = \frac{1}{2}, \bar{\mathbf{3}}_F$) fields

Firstly, we consider the simplest case $D(\frac{1}{2}, 0)_{F=\bar{\mathbf{3}}}$, where $D(\frac{1}{2}, 0)$ denotes the representation of the Lorentz group and $F = \bar{\mathbf{3}}_F$ denotes the flavor. There are 10 local quark operators with $J = \frac{1}{2}$ and $\bar{\mathbf{3}}_F$ (we omit the color and flavor projectors here, for brevity's sake) that can be further separated into the color antitriplet diquark ones,

$$
\begin{aligned}
Q_1(x) &= \bar{q}(x)(\tilde{q}(x)q(x)),\\
Q_2(x) &= \bar{q}(x)\gamma_5(\tilde{q}(x)\gamma_5 q(x)),\\
Q_3(x) &= \bar{q}(x)\gamma^{\mu}(\tilde{q}(x)\gamma_{\mu} q(y)),\\
Q_4(x) &= \bar{q}(x)\gamma^{\mu}\gamma_5\tau^i(\tilde{q}(x)\gamma_{\mu}\gamma_5\tau^i q(y)),\\
Q_5(x) &= \bar{q}(x)\sigma^{\mu\nu}\tau^i(\tilde{q}(x)\sigma_{\mu\nu}\tau^i q(y)),
\end{aligned}
$$

and the color sextet diquark ones,

$$
\begin{aligned}
Q_6(x) &= \bar{q}(x)\tau^i(\tilde{q}(x)\tau^i q(x)),\\
Q_7(x) &= \bar{q}(x)\gamma_5\tau^i q(x)(\tilde{q}(x)\gamma_5\tau^i q(x)),\\
Q_8(x) &= \bar{q}(x)\gamma^{\mu}\tau^i q(x)(\tilde{q}(x)\gamma_{\mu}\tau^i q(x)),\\
Q_9(x) &= \bar{q}(x)\gamma^{\mu}\gamma_5(\tilde{q}(x)\gamma_{\mu}\gamma_5 q(x)),\\
Q_{10}(x) &= \bar{q}(x)\sigma^{\mu\nu}q(x)(\tilde{q}(x)\sigma_{\mu\nu}q(y)).
\end{aligned}
$$

Here we use the $SU_F(2)$ isospin τ^i matrices as a shorthand to indicate that the diquark involved is a flavor-sextet one and thus differentiate it from the corresponding flavor antitriplet diquark; otherwise, the flavor and color projector operator conventions for a constituent quark field hold, as specified by Eq. (5). As the color antitriplet diquark tetraquarks are lighter than the color sextet diquark ones, we shall consider only the former. Their chiral properties are summarized in Table II. Note that these constituent quarks may have either parity: the above formulas imply even-parity quarks, but changing it to odd-parity requires a mere multiplication by a γ_5 matrix. Moreover, note that the chiral properties are all ‘‘nonstandard’’ for a single quark: for example $Q_1 + Q_2$ and Q_5 belong to the chiral $(\mathbf{3}, 1) \oplus (1, \mathbf{3})$ multiplet, just like the ‘‘ordinary’’ quark field, but they carry triple of the usual axial baryon charge $g_A^{(0)} = +3$. Similarly, the only other standard chiral $(\mathbf{3}, 1) \oplus (1, \mathbf{3})$ multiplet $Q_3 - Q_4$ is an Abelian chiral mirror field $g_A^{(0)} = -1$. The remaining two effective constituent quark fields, $Q_1 - Q_2$ and $Q_3 + \frac{1}{3}Q_4$, belong to manifestly exotic chiral multiplets $(\bar{\mathbf{3}}, \bar{\mathbf{3}})$ and $(\mathbf{3}, \mathbf{8}) \oplus (\mathbf{8}, \mathbf{3})$. Some of these chirally exotic constituent quarks are subject to additional selection rules that reduce or eliminate entirely their contributions to the scalar mesons. In the next section we shall show that the KKMT interaction may then lower the mass of such chirally exotic scalar tetraquarks below that of the orbitally excited $Q\bar{q}$ meson.

TABLE II. The Abelian $U_A(1)$, the non-Abelian $SU_L(3) \times SU_R(3)$ chiral transformation properties/axial charges, and the Lorentz group representation $(\frac{1}{2}, 0)$.

	$U_A(1)$	$SU_L(3) \times SU_R(3)$
$Q_1 - Q_2$	-1	$(\bar{\mathbf{3}}, \bar{\mathbf{3}})$
$Q_1 + Q_2$	$+3$	$(\mathbf{3}, 1) \oplus (1, \mathbf{3})$
$Q_3 - Q_4$	-1	$(\mathbf{3}, 1) \oplus (1, \mathbf{3})$
$Q_3 + \frac{1}{3}Q_4$	-1	$(\mathbf{3}, \mathbf{8}) \oplus (\mathbf{8}, \mathbf{3})$
Q_5	$+3$	$(\mathbf{3}, 1) \oplus (1, \mathbf{3})$

III. INTERACTIONS

A. The $U_A(1)$-breaking interaction

The KKMT interaction, as induced by instantons in QCD [19], reads

$$\mathcal{L}_{\text{tH}}^{(6)} = -K[\det_f(\bar{\psi}(1+\gamma_5)\psi) + \det_f(\bar{\psi}(1-\gamma_5)\psi)], \tag{16}$$

where $\det_f(\bar{\psi}(1+\gamma_5)\psi)$ is a determinant in the flavor space only. Its primary purpose was to correct the η, η' masses, but in the meantime it has been shown that this interaction also affects the scalar meson [20–22] and baryon spectra [23]. The effective coupling constant K has been determined in Ref. [20] as

$$12K = -\langle\bar{q}q\rangle_0^{-3}[f_{\eta'}^2 m_{\eta'}^2 + f_{\eta}^2 m_{\eta}^2 - 2f_K^2 m_K^2]. \tag{17}$$

Inserting the experimental values of the pseudoscalar meson masses and decay constants into Eq. (17), as well as the quark condensate taken here as $\langle\bar{q}q\rangle = -(225\ \text{MeV})^3$, the KKMT coupling constant comes out at $K = 390\ \text{GeV}^{-5}$. Note, however, that the large uncertainty and the high exponent of the quark condensate $\langle\bar{q}q\rangle_0 = [-(225 \pm 25)\ \text{MeV}]^3$ lead to an even larger uncertainty in the value of K. This uncertainty translates into a wide margin for tetraquark mass predictions.

The KKMT interaction [Eq. (16)] leads to the following three-quark potential:

$$V_{123} = 12KP_{123}^1\left(1 + \sum_{i<j}^{3}\gamma^5{}_i\gamma^5{}_j\right)\delta(\mathbf{r}_1 - \mathbf{r}_2)\delta(\mathbf{r}_3 - \mathbf{r}_2)$$

$$12P_{123}^1 = \left[\frac{4}{9} - \frac{1}{3}\sum_{i<j}^{3}\boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j + d^{abc}\boldsymbol{\lambda}_1^a\boldsymbol{\lambda}_2^b\boldsymbol{\lambda}_3^c\right]. \tag{18}$$

As can be seen from Eq. (18), the flavor-dependent part of the KKMT three-quark potential is just the three-quark-flavor SU(3) singlet projection operator P_{123}^1 [20]. Fierz rearrangement identities lead to additional terms with $(\frac{1}{N_c}\sum_{i<j}^{3}\sigma_{(\mathrm{i})}^{\mu\nu}\sigma_{\mu\nu}^{(\mathrm{j})})$ and explicit color SU(3) dependence.

Such terms have the same chiral symmetries as the original KKMT term, but are generally $\mathcal{O}(1/N_c)$ suppressed with respect to the latter [21]. As such they are not expected to play a significant phenomenological role. These terms are nevertheless interesting, as they introduce $U_A(1)$ symmetry-breaking effects, however small, into vector-and axial-vector mesons and affect a different chiral representation.

1. C conjugation and KKMT interaction for systems with antiquarks

Conversion of the above interactions to systems with one or two antiquarks can be accomplished according to the rules and formulas spelled out in Refs. [24,25]. One must be careful about the definition of the SU(3) factors in the three-body potential involving antiquarks as they are sensitive to the C-conjugation properties of the relativistic interaction from which the potential was derived. More specifically, one finds a difference between the Lorentz scalar and zeroth component of Lorentz vector models.

The quark SU(3) group generator matrices F^a are related to the SU(3) *charge* (i.e., Lorentz vector) operators of an antiquark by

$$\bar{F}^a = -\frac{1}{2}\boldsymbol{\lambda}^{aT} = -\frac{1}{2}\boldsymbol{\lambda}^{a*}. \tag{19}$$

The minus sign in this relation stems from the C-conjugation properties of the *vector* (and/or axial-vector) current and not from the SU(3) algebra itself. So for Lorentz scalar, pseudoscalar, and antisymmetric (Pauli) tensor interactions, such as the KKMT one, this sign changes into a plus one $\boldsymbol{\lambda} \to -\bar{\boldsymbol{\lambda}}_i$ and leads to

$$CP^1_{123}C^{-1} = \frac{1}{27} - \frac{1}{9}\sum_{i<j}^{3}(-)^j F_i \cdot \bar{F}_j - \frac{2}{3}d^{abc}F^a_1F^b_2\bar{F}^c_3. \tag{20}$$

We can express the two SU(3) invariant flavor factors in terms of SU(3) Casimir operators. The first one remains unchanged,

$$\sum_{i<j}^{3} F_i \cdot F_j = F_1 \cdot F_2 + F_1 \cdot \bar{F}_3 + F_2 \cdot \bar{F}_3 = \frac{1}{2}C^{(1)}_{i+j+k} - 2, \tag{21}$$

whereas the second one becomes

$$d^{abc}F^a_1F^b_2\bar{F}^c_3 = \frac{1}{6}\left[C^{(2)}_{i+j+k} - \frac{5}{2}C^{(1)}_{i+j} + \frac{50}{9}\right]. \tag{22}$$

Note that in the second factor [Eq. (22)], the first (quadratic) Casimir is evaluated between the two-quark (sub) state $i + j$, which leads to a distinction between the two overall flavor triplets (which are symmetric and antisymmetric in the quark indices). This leads to the results shown in Table III.

2. Matrix elements

We need to evaluate the KKMT interaction's effects in first-order perturbation theory. There are (only) two kinds of chiral multiplets that are affected by this interaction: the (3, 3) and the $(\bar{6}, 3)$. The former involves only scalar and pseudoscalar fields, the latter only Pauli-tensor (spin-1) interpolators. The (3, 3) contains a flavor-antitriplet and a sextet: $\bar{3}_A \oplus 6 \subset (3, 3)$; whereas the $(\bar{6}, 3)$ contains a flavor-antitriplet and an anti-15-plet: $\bar{3}_S \oplus \overline{15} \subset (\bar{6}, 3)$. The latter does not show up on our list of spin $-\frac{1}{2}$ local effective constituent quarks, however (i.e., such an operator has spin $\frac{3}{2}$ and need not be considered with regard to scalar heavy-light mesons); it does appear in axial-vector heavy-light mesons, however.

TABLE III. Diagonal matrix elements of the three-body SU(3) operators for various SU(3) $q^2\bar{q}$ states.

	$\mathbf{3}_A$	$\mathbf{3}_S$	$\bar{\mathbf{6}}$	$\mathbf{15}$
$\langle\sum_{i<j}^3 F_i \cdot F_j\rangle$	$-\frac{4}{3}$	$-\frac{4}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$
$\langle\sum_{i<j}^3 (-)^j F_i \cdot \bar{F}_j\rangle$	0	2	-1	0
$\langle d^{abc}F^a_1F^b_2\bar{F}^c_3\rangle$	$\frac{5}{9}$	$-\frac{5}{18}$	$-\frac{5}{18}$	$\frac{1}{18}$

Using Table III, we find the following SU(3) projection operators:

$$P(\mathbf{3}_A) = \frac{1}{9} - \frac{1}{12}\sum_{i<j}^{3}(-)^j F_i \cdot \bar{F}_j - \frac{1}{4}\sum_{i<j}^{3} F_i \cdot \bar{F}_j + d^{abc}F^a_1F^b_2\bar{F}^c_3 \tag{23}$$

$$P(\bar{\mathbf{6}}) = \frac{2}{9} - \frac{5}{12}\sum_{i<j}^{3}(-)^j F_i \cdot \bar{F}_j - \frac{1}{4}\sum_{i<j}^{3} F_i \cdot \bar{F}_j - d^{abc}F^a_1F^b_2\bar{F}^c_3 \tag{24}$$

$$P(\mathbf{3}_S) = \frac{1}{9} + \frac{7}{12}\sum_{i<j}^{3}(-)^j F_i \cdot \bar{F}_j - \frac{1}{8}\sum_{i<j}^{3} F_i \cdot \bar{F}_j - \frac{1}{2}d^{abc}F^a_1F^b_2\bar{F}^c_3 \tag{25}$$

$$P(\mathbf{15}) = \frac{5}{9} + \frac{5}{24}\sum_{i<j}^{3}(-)^j F_i \cdot \bar{F}_j + \frac{5}{8}\sum_{i<j}^{3} F_i \cdot \bar{F}_j + \frac{1}{2}d^{abc}F^a_1F^b_2\bar{F}^c_3. \tag{26}$$

We find the following 't Hooft interaction flavor-spin-spatial matrix elements in Eq. (27):

$$CP^1_{123}C^{-1} = -\frac{5}{9}\left[P(\mathbf{3}_A) + \frac{2}{5}P(\mathbf{3}_S) + \frac{1}{15}P(\bar{\mathbf{6}})\right]. \tag{27}$$

The level splitting is shown in Fig. 1.

B. The $U_A(1)$-conserving interactions

The $U_A(1)$-conserving interactions include all gluon exchanges and can be effectively described by chirally invariant Lagrangians, such as those in Ref. [26] connecting the (1, 3) constituent antiquark and the (3, 3) constituent antiquark. This induces chiral mixing:

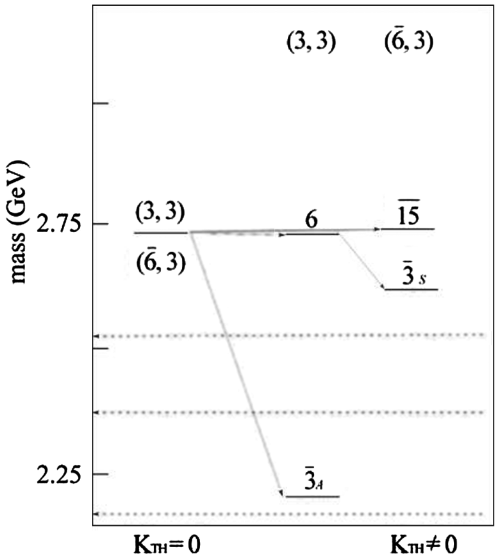

FIG. 1. Masses of the scalar D meson tetraquark states as a function of the KKMT coupling $K_{\rm tH}$: the splitting of the two chiral multiplets $(\bar{\mathbf{6}}, \mathbf{3})$ and $(\mathbf{3}, \mathbf{3})$, and further splitting into flavor SU(3) multiplets $\bar{\mathbf{3}}_{\rm A}$, $\mathbf{6}$, and $\bar{\mathbf{3}}_{\rm A}$, $\overline{\mathbf{15}}$ occurs also due to the $U_A(1)$ symmetry-conserving gluon exchange interactions. Horizontal (dashed) lines denote the corresponding thresholds.

$$|D_{0^+}(2308)\rangle = \cos\phi |D_{0^+}(1, \bar{3})\rangle + \sin\phi |D_{0^+}(3, 3)\rangle \quad (28)$$

$$|D_{s0^+}(2317)\rangle = \cos\phi_s |D_{s0^+}(1, \bar{3})\rangle + \sin\phi_s |D_{s0^+}(3, 3)\rangle. \quad (29)$$

Chiral mixing determines the flavor content of the physical states, which in turn determines their decay channels.

One may try and use chiral Lagrangians such as those in Ref. [26] to evaluate the masses of various flavor SU(3) multiplets within the chiral multiplet: it appears that the ratio of masses of the flavor antitriplet and the sextet in $\bar{3}_{\rm A} \oplus 6 \subset (3, 3)$ is not determined by such symmetry considerations; the only thing that one can show in general is that the $\bar{3}_{\rm A} \subset (3, 3)$ must be lighter than $6 \subset (3, 3)$. The absolute values of these contributions to the total masses remain undetermined by symmetry arguments anyway, and require a dynamical model with some phenomenological free parameter input. Therefore, the ratio of the $U_A(1)$-conserving and the $U_A(1)$-breaking contributions also remains unknown. This fact leads to a potentially variable ordering of higher-lying states in the spectrum; but the lowest-lying state always remains the $\bar{3}_{\rm A} \subset (3, 3)$, in agreement with the nonrelativistic calculation (Ref. [8]).

IV. RESULTS AND EXPERIMENTAL PREDICTIONS

Our model is incomplete: two energy scales, the KKMT and the confining scales, are undetermined. In order to set these scales and thus make absolute predictions in the beauty-meson sector, we must resort either to our previous nonrelativistic calculations or to experimental values (which are, in this case, the same). We adopt the same value(s) of the overall energy shift as in Ref. [8], so as to reproduce the observed D, D_s meson masses and then predict the missing B, B_s meson masses.

Chiral multiplets may involve (only) states with identical values of total angular momentum, and possibly different parities. Therefore, ‘‘truly chiral’’ sum rules may involve only masses of equal-spin states: the BHNRZ sum rule is an exception in this regard, due to the additional heavy-quark Isgur-Wise symmetry.

A. D, D_s scalar mesons

Therefore, we write the 0^-, 0^+ states' masses in a separate spinless meson relation/‘‘sum rule’’:

$$\Delta D_0 = D_{0^+}(2308) - D(1869) \quad (30)$$

$$\Delta D_{s0} = D_{s0^+}(2317) - D_s(1968) \quad (31)$$

$$D_{s0^+} = D_{0^+}, \quad (32)$$

and similarly for the 1^-, 1^+ mesons

$$\Delta D_1 = D_{1^+}(2420) - D^*(2010) \quad (33)$$

$$\Delta D_{s1} = D_{s1^+}(2460) - D_s^*(2112) \quad (34)$$

$$D_{s1^+} \simeq D_{1^+}. \quad (35)$$

We note here that the KKMT interaction does not seem to predict the identity of the two axial-vector masses $D_{s1^+} \simeq D_{1^+}$ [Eq. (35)] the way it predicts the identity of the two spin-0 meson masses $D_{s0^+} = D_{0^+}$ [Eq. (32)]: that is, rather, an experimental fact—i.e., a (possibly fortuitous) coincidence of two scales that are different, in principle. The BHNRZ relations [Eqs. (1) and (2)] imply the equality of the spin-0 and spin-1 ΔD's (i.e., $\Delta D_0 = \Delta D_1$ and $\Delta D_{s1} = \Delta D_{s0}$) that is above and beyond the above relations [Eqs. (32) and (35)].

B. B, B_s mesons

The heavy quark serves (merely) as a (quasistatic) color source for the light constituent (anti)quark(s). Therefore, its mass and/or flavor should not influence the light quark dynamics, at least to lowest approximation, i.e.,

$$\Delta D_0 = \Delta B_0$$
$$\Delta D_{s0} = \Delta B_{s0}$$
$$\Delta D_1 = \Delta B_1$$
$$\Delta D_{s1} = \Delta B_{s1}.$$

We may then use the separate D, D_s ‘‘scalar-pseudoscalar’’ and ‘‘vector-axial vector’’ meson mass relations (sum rules) [Eqs. (32) and (35)], as well as their B, B_s analogons for the 0^-, 0^+ states' masses

TABLE IV. The values of the B meson masses obtained from this model by a "translation" of the D meson masses. The values marked by an asterisk (*) are fit experimental values. We have assigned a "theoretical uncertainty" of ± 25 MeV to all of our predictions that reflects the lack of precise knowledge of the confinement scale and of higher-order correction in this system.

	0^-	1^-	0^+	1^+	1^+	2^+
B	5279*	5322*	5718	5732	5723*	5743*
B_s	5370*	5416*	5719	5765	5829*	5840*

$$\Delta B_0 = B_{0^+}(X) - B(5279) \tag{36}$$

$$\Delta B_{s0} = B_{s0^+}(X_s) - B_s(5370) \tag{37}$$

$$B_{s0^+} = B_{0^+}, \tag{38}$$

and similarly for the 1^-, 1^+ mesons

$$\Delta B_1 = B_{1^+}(Y) - B^*(5322) \tag{39}$$

$$\Delta B_{s1} = B_{s1^+}(Y_s) - B_s^*(5416) \tag{40}$$

$$B_{s1^+} \simeq B_{1^+}, \tag{41}$$

respectively, to predict the (as yet) experimentally missing B, B_s meson masses from

$$D_{0^+}(2308) - D(1869) = B_{0^+}(X) - B(5279) \tag{42}$$

$$D_{s0^+}(2317) - D_s(1968) = B_{s0^+}(X_s) - B_s(5370) \tag{43}$$

$$D_{1^+}(2420) - D^*(2010) = B_{1^+}(Y) - B^*(5322) \tag{44}$$

$$D_{s1^+}(2460) - D_s^*(2112) = B_{s1^+}(Y_s) - B_s^*(5416). \tag{45}$$

There are four (linear) equations with four unknowns (X, Y, X_s, Y_s) that are readily solved with the results shown in Table IV. We see that the nonstrange scalar B meson mass of 5718 MeV is "in the same ballpark" as the predictions of potential models, whereas the strange scalar B_s-meson mass of 5719 MeV is roughly 100 MeV below the predictions of potential models. This shift of the scalar B_s meson mass is smaller than in the D_s meson sector (160 MeV), but should still be clearly visible in experiment. We note here that Bardeen *et al.* (Refs. [15,27]) predicted essentially the same mass for the B_s meson (5718 ± 35 MeV), but a substantially lower one for the nonstrange scalar B meson (5627 ± 35 MeV).

We also note that Matsuki *et al.* [28,29] predicted a scalar B_s meson mass as light as 5617 MeV, with the corresponding scalar B meson at 5592 MeV, some 25 MeV lower than the strange one.

If we accept Eq. (35) as a phenomenological fact, and assume validity of its analogon, Eq. (41), we also predict the $J^P = 1^+$ bottom-nonstrange B_1 and the bottom-strange B_{s1} meson masses at 5732 MeV and 5765 MeV, respectively. We note that Bardeen *et al.* (Refs. [15,27]) predicted essentially the same mass for the B_{s1} meson (5765 ± 35 MeV), but a lower one for the nonstrange axial-vector scalar B_1 meson (5674 ± 35 MeV).

All of these states ought to be observable at the LHCb.

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[1] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **90**, 242001 (2003).
[2] D. Besson *et al.* (CLEO Collaboration), Phys. Rev. D **68**, 032002 (2003); **75**, 119908(E) (2007).
[3] K. Abe *et al.* (Belle Collaboration), Phys. Rev. Lett. **92**, 012002 (2004).
[4] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985).
[5] S. Godfrey and R. Kokoski, Phys. Rev. D **43**, 1679 (1991).
[6] R. N. Cahn and J. D. Jackson, Phys. Rev. D **68**, 037502 (2003).
[7] O. Lakhina and E. S. Swanson, Phys. Lett. B **650**, 159 (2007).
[8] V. Dmitrašinović, Phys. Rev. D **70**, 096011 (2004).
[9] V. Dmitrašinović, Phys. Rev. Lett. **94**, 162002 (2005).
[10] V. Dmitrašinović, Int. J. Mod. Phys. A **21**, 5625 (2006).
[11] V. Dmitrašinović, Mod. Phys. Lett. A **21**, 533 (2006).
[12] K. Abe *et al.* (Belle Collaboration), Phys. Rev. D **69**, 112002 (2004).
[13] M. A. Nowak, M. Rho, and I. Zahed, Phys. Rev. D **48**, 4370 (1993).
[14] W. A. Bardeen and C. T. Hill, Phys. Rev. D **49**, 409 (1994).
[15] W. A. Bardeen, E. J. Eichten, and C. T. Hill, Phys. Rev. D **68**, 054024 (2003).

[16] K. Nagata, A. Hosaka, and V. Dmitrašinović, Eur. Phys. J. C **57**, 557 (2008).

[17] H.-X. Chen, V. Dmitrašinović, A. Hosaka, K. Nagata, and S.-L. Zhu, Phys. Rev. D **78**, 054021 (2008).

[18] H.-X. Chen, V. Dmitrašinović, and A. Hosaka, Phys. Rev. D **81**, 054002 (2010).

[19] G.'t Hooft, Phys. Rev. D **14**, 3432 (1976); **18**, 2199(E) (1978).

[20] V. Dmitrašinović, Phys. Rev. C **53**, 1383 (1996).

[21] V. Dmitrašinović, Phys. Rev. D **56**, 247 (1997).

[22] V. Dmitrašinović, in *Hadrons and Nuclei: First International Symposium*, edited by I.-T. Cheon, T. Choi, S.-W. Hong, and S. H. Lee, AIP Conf. Proc. No. 594 (AIP, New York, 2001).

[23] For baryons, see V. Dmitrašinović, in *Bled Workshops in Physics, Ljubljana, Slovenia, 2002*, edited by B. Golli, M. Rosina, and S. Širca (DMFA, Založništvo, 2002); V. Dmitrašinović and H. Toki, Ann. Phys. (N.Y.) **321**, 355 (2006).

[24] V. Dmitrašinović, Phys. Lett. B **499**, 135 (2001).

[25] V. Dmitrašinović, Phys. Rev. D **67**, 114007 (2003).

[26] H.-X. Chen, V. Dmitrašinović, and A. Hosaka, Phys. Rev. D **83**, 014015 (2011).

[27] W. A. Bardeen, Mod. Phys. Lett. A **23**, 2209 (2008).

[28] T. Matsuki, T. Morii, and K. Sudoh, Prog. Theor. Phys. **117**, 1077 (2007).

[29] T. Matsuki, T. Morii, and K. Sudoh, Phys. Lett. B **659**, 593 (2008).