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Heavy and light scalar leptoquarks in proton decay

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We list scalar leptoquarks which mediate proton decay via renormalizable couplings to the standard model fermions. We employ a general basis of baryon number violating operators to parametrize contributions of each leptoquark towards proton decay. This then sets the stage for investigation of bounds on the leptoquark couplings to fermions with respect to the most current Super Kamiokande results on proton stability. We quantify if, and when, it is necessary to have leptoquark masses close to a scale of grand unification in the realistic $SU(5)$ and flipped $SU(5)$ frameworks. The most and the least conservative lower bounds on the leptoquark masses are then presented. We furthermore single out a leptoquark without phenomenologically dangerous tree-level exchanges which might explain discrepancy of the forward-backward asymmetries in $t\bar{t}$ production observed at Tevatron, if relatively light. The same state could also play a significant role in explaining muon anomalous magnetic moment. We identify contributions of this leptoquark to dimension-six operators, mediated through a box diagram, and tree-level dimension-nine operators, which would destabilize the proton if sizable leptoquark and diquark couplings were to be simultaneously present.

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I. PROTON DECAY LEPTOQUARKS

There has been a plethora of low-energy experiments capable of leptoquark discovery thus far. These have generated ever more stringent constraints on available parameter space for their existence. See, for example, Refs. [1–6] for some of the latest results. There also exists a large number of phenomenological studies of leptoquark signatures prompted primarily by various effects they could generate in flavor physics [7–11]. We are interested in a particular subset of scalar leptoquark states which are associated with proton decay. It is well-known that there exists only a small number of these states which can simultaneously violate baryon (B) and lepton (L) numbers [12–14]. The number of scalar leptoquarks which can mediate proton decay at the tree-level is even smaller [15]. Our aim is to present a comprehensive classification of leptoquarks and address a role these have in proton decay processes.

Scalar leptoquarks which mediate proton decay certainly represent qualitatively new physics. Although the relevant operators associated with exchange of these states can be studied from an effective theory point of view, we prefer to trace their origins to a particular unification scenario in order to expose their dependence on underlying couplings. In fact, we will study these states in two different unification frameworks which correspond to the $SU(5)$ [16] and the flipped $SU(5)$ [17–19], i.e., $SU(5) \times U(1)$, embeddings of the matter fields. These two scenarios are general enough to cover other possible embedding schemes.

Let us start by spelling out qualitative differences between the scalar and vector, i.e., gauge boson, leptoquarks which mediate proton decay at the so-called dimension-six ($d = 6$) level. (The latter have been studied much more extensively in the literature. See, for example, Refs. [12–14,17,20–24].) First, vector leptoquarks comprise twenty-four states; whereas, the scalar ones comprise eighteen (fifteen) states in case neutrinos are Dirac (Majorana) particles. Second, while $SU(5)$ contains only a half of all vector leptoquarks, the other half being in flipped $SU(5)$, one can already find all possible proton-decay mediating scalar leptoquarks in either $SU(5)$ or flipped $SU(5)$ framework. Hence, the scalar sector, although smaller, can potentially yield much richer structure with respect to the gauge one. Third, the uncertainty in predictions for partial nucleon decay rates due to the gauge boson exchange resides entirely in a freedom to choose particular unitary rotations which need to be in agreement with observed mixing parameters in the fermionic sector as gauge bosons couple to matter with the gauge coupling strength. Scalar fields, on the other hand, couple to matter through Yukawa couplings. This brings additional uncertainties to potential predictions for relevant decay rates.

The leptoquark states which simultaneously violate B and L quantum numbers tend to mediate proton decay at tree-level and are therefore taken to be very massive. However, we have investigated an $SU(5)$ grand unified

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theory scenario [25] which resulted in a setup with a set of light leptoquarks. Namely, motivated by the need to explain anomalous events in $t\bar{t}$ production at Tevatron [26,27], we have found that a light color triplet weak singlet scalar could contribute to $t\bar{t}$ production and explain the observed increase of the forward-backward asymmetry [28]. We have accordingly demonstrated that the unification of the fundamental interactions is possible if that set of light scalars is a part of the forty-five-dimensional representation [28].

In flavor physics, due to recent accurate measurements at Tevatron and LHCb, the presence of new physics in B systems seems rather unlikely. The muon anomalous magnetic moment, on the other hand, still leaves some room for new physics contributions. The impact of potentially light leptoquark scalars, including the light color triplet weak singlet scalar, on the low energy and hadron collider phenomenology within that context has been investigated in Refs. [28–32].

The color triplet weak singlet scalar state we have singled out does not generate proton decay at the tree-level. However, one can still construct, as we show later, higher-order loop diagrams which yield effective $d = 6$ and tree-level $d = 9$ operators which can destabilize the proton. The natural question then is whether one can simultaneously address the $t\bar{t}$ asymmetry and the muon anomalous magnetic moment by using the very same leptoquark. We investigate this issue in detail in Sec. VI.

This paper is organized as follows. In Secs. II and III we list all proton decay inducing leptoquarks in $SU(5)$ and flipped $SU(5)$ unification frameworks and specify their Yukawa couplings to the SM fermions. In Sec. IV we introduce the effective dimension-six operators for proton decay and calculate associated effective coefficients for each leptoquark state. Section V is devoted to a study of conservative lower bounds on the color triplet leptoquark mass within phenomenologically realistic $SU(5)$ and flipped $SU(5)$ scenarios. In Sec. VI we study leptoquarks which do not contribute to proton decay operators of dimension-six at tree level. We conclude in Sec. VII.

II. LEPTOQUARKS IN $SU(5)$

The scalars which couple to matter at tree-level reside in the five-, ten-, fifteen-, forty-five-, and fifty-dimensional representations of $SU(5)$ because the SM matter fields comprise $\mathbf{10}_i$ and $\bar{\mathbf{5}}_j$, where $i, j = 1, 2, 3$ represent family indices. Namely, $\mathbf{10}_i = (\mathbf{1}, \mathbf{1}, 1)_i \oplus (\bar{\mathbf{3}}, \mathbf{1}, -2/3)_i \oplus (\mathbf{3}, \mathbf{2}, 1/6)_i = (e_i^C, u_i^C, Q_i)$ and $\bar{\mathbf{5}}_j = (\mathbf{1}, \mathbf{2}, -1/2)_j \oplus (\bar{\mathbf{3}}, \mathbf{1}, 1/3)_j = (L_j, d_j^C)$, where

$$Q_i = (u_i \quad d_i)^T$$

and

$$L_j = (\nu_j \quad e_j)^T$$

[16]. Possible contractions of the matter field representations hence read $\mathbf{10} \otimes \mathbf{10} = \bar{\mathbf{5}} \oplus \overline{\mathbf{45}} \oplus \overline{\mathbf{50}}$, $\mathbf{10} \otimes \bar{\mathbf{5}} = \mathbf{5} \oplus \mathbf{45}$, and $\bar{\mathbf{5}} \otimes \bar{\mathbf{5}} = \overline{\mathbf{10}} \oplus \overline{\mathbf{15}}$. Theory also allows for the addition of right-handed neutrinos which can be introduced, for example, in the form of $SU(5)$ fermionic singlets (1) without the need to enlarge the scalar sector. Note that one can also introduce additional nontrivial representations of matter to generate observed fermion mass parameters in the lepton [33] and quark [34] sectors. That, however, would not alter our operator analysis for large enough masses of extra matter fields.

Relevant decomposition of scalar representations to the SM gauge group, i.e., $SU(5) \to SU(3) \times SU(2) \times U(1)$, is given below [35]:

(i) $\mathbf{5} = (\mathbf{1}, \mathbf{2}, 1/2) \oplus (\mathbf{3}, \mathbf{1}, -1/3)$;

(ii) $\mathbf{10} = (\mathbf{1}, \mathbf{1}, 1) \oplus (\bar{\mathbf{3}}, \mathbf{1}, -2/3) \oplus (\mathbf{3}, \mathbf{2}, 1/6)$;

(iii) $\mathbf{15} = (\mathbf{1}, \mathbf{3}, 1) \oplus (\mathbf{3}, \mathbf{2}, 1/6) \oplus (\mathbf{6}, \mathbf{1}, -2/3)$;

(iv) $\mathbf{45} = (\mathbf{8}, \mathbf{2}, 1/2) \oplus (\bar{\mathbf{6}}, \mathbf{1}, -1/3) \oplus (\mathbf{3}, \mathbf{3}, -1/3) \oplus (\bar{\mathbf{3}}, \mathbf{2}, -7/6) \oplus (\mathbf{3}, \mathbf{1}, -1/3) \oplus (\bar{\mathbf{3}}, \mathbf{1}, 4/3) \oplus (\mathbf{1}, \mathbf{2}, 1/2)$;

(v) $\mathbf{50} = (\mathbf{8}, \mathbf{2}, 1/2) \oplus (\mathbf{6}, \mathbf{1}, 4/3) \oplus (\bar{\mathbf{6}}, \mathbf{3}, -1/3) \oplus (\bar{\mathbf{3}}, \mathbf{2}, -7/6) \oplus (\mathbf{3}, \mathbf{1}, -1/3) \oplus (\mathbf{1}, \mathbf{1}, -2)$.

Only **5**, **15** and **45** contain electrically neutral components and are thus capable of developing phenomenologically viable vacuum expectation values (VEVs). Contributions to the up-quark, down-quark, and charged lepton masses can come from both **5** and **45** whereas Majorana (Dirac) masses for neutrinos can be generated by VEV of **15** (**5**).

The scalar leptoquark states which violate both B and L quantum numbers are $(\mathbf{3}, \mathbf{1}, -1/3)$, $(\mathbf{3}, \mathbf{3}, -1/3)$, and $(\bar{\mathbf{3}}, \mathbf{1}, 4/3)$, if one *assumes* neutrinos to be Majorana particles. These states reside in **5**, **45** and **50**. However, if one allows for the possibility that neutrinos are Dirac particles, there is another leptoquark—$(\bar{\mathbf{3}}, \mathbf{1}, -2/3)$—which is found in the **10** of $SU(5)$ which violates both B and L and could thus also destabilize the proton. To that end, we consider both the Majorana and Dirac neutrino cases to keep the analysis as general as possible. Altogether, there are eighteen (fifteen) scalar leptoquarks which could mediate proton decay in case neutrinos are Dirac (Majorana) particles. The leptoquarks in question are all triplets of color as they must contract with lepton and quark states into an $SU(3)$ singlet. Tables I, II, III, and IV, summarize couplings to the matter of relevant states which reside in fifty-, forty-five-, ten-, and five-dimensional representations, respectively.

We observe that in the $SU(5)$ framework the primary obstacle to the proton stability seems to be the need to

TABLE I. Yukawa couplings of the B and L violating scalar in the fifty-dimensional representation of $SU(5)$. $a, b, c = 1, 2, 3$ ($i, j = 1, 2, 3$) are color (flavor) indices. Y_{ij}^{10} are Yukawa matrix elements associated with the relevant contraction in the group space of $SU(5)$.

$SU(5)$	$Y_{ij}^{10}\mathbf{10}_i\mathbf{10}_j\mathbf{50}$
$(\mathbf{3}, \mathbf{1}, -1/3) \equiv \Delta$	$12^{-1/2}\epsilon_{abc}[Y_{ij}^{10} + Y_{ji}^{10}]d_{ai}^T C u_{bj}\Delta_c$
	$3^{-1/2}[Y_{ij}^{10} + Y_{ji}^{10}]e_i^{CT} C u_{aj}^C \Delta_a$

TABLE II. Yukawa couplings of the B and L violating scalars in the forty-five-dimensional representation of $SU(5)$. $a, b, c = 1, 2, 3$ $(i, j = 1, 2, 3)$ are color (flavor) indices. Y^{10}_{ij} and $Y^{\bar{5}}_{ij}$ are Yukawa matrix elements.

$SU(5)$	$Y^{10}_{ij}\mathbf{10}_i\mathbf{10}_j\mathbf{45}$	$Y^{\bar{5}}_{ij}\mathbf{10}_i\bar{\mathbf{5}}_j\mathbf{45}^*$
$(\mathbf{3}, \mathbf{1}, -1/3) \equiv \Delta$	$2^{1/2}[Y^{10}_{ij} - Y^{10}_{ji}]e^{CT}_i C u^C_{aj}\Delta_a$	$2^{-1}Y^{\bar{5}}_{ij}\epsilon_{abc}u^{CT}_{ai}Cd^C_{bj}\Delta^*_c$
		$-2^{-1}Y^{\bar{5}}_{ij}u^T_{ai}Ce_j\Delta^*_a$
		$2^{-1}Y^{\bar{5}}_{ij}d^T_{ai}C\nu_j\Delta^*_a$
$(\mathbf{3}, \mathbf{3}, -1/3) \equiv (\Delta^1, \Delta^2, \Delta^3)$	$2^{1/2}\epsilon_{abc}[Y^{10}_{ij} - Y^{10}_{ji}]d^T_{ai}Cd_{bj}\Delta^1_c$	$Y^{\bar{5}}_{ij}u^T_{ai}C\nu_j\Delta^{1*}_a$
	$-2\epsilon_{abc}[Y^{10}_{ij} - Y^{10}_{ji}]d^T_{ai}Cu_{bj}\Delta^2_c$	$2^{-1/2}Y^{\bar{5}}_{ij}u^T_{ai}Ce_j\Delta^{2*}_a$
		$2^{-1/2}Y^{\bar{5}}_{ij}d^T_{ai}C\nu_j\Delta^{2*}_a$
	$-2^{1/2}\epsilon_{abc}[Y^{10}_{ij} - Y^{10}_{ji}]u^T_{ai}Cu_{bj}\Delta^3_c$	$-Y^{\bar{5}}_{ij}d^T_{ai}Ce_j\Delta^{3*}_a$
$(\bar{\mathbf{3}}, \mathbf{1}, 4/3) \equiv \Delta$	$2^{1/2}[Y^{10}_{ij} - Y^{10}_{ji}]\epsilon_{abc}u^{CT}_{ia}Cu^C_{bj}\Delta_c$	$-Y^{\bar{5}}_{ij}e^{CT}_i Cd^C_{aj}\Delta^*_a$

TABLE III. Yukawa couplings of the B and L violating scalar in the ten-dimensional representation of $SU(5)$. $a, b, c = 1, 2, 3$ $(i, j = 1, 2, 3)$ are color (flavor) indices. Y^1_{ij} and $Y^{\bar{5}}_{ij}$ are Yukawa matrix elements.

$SU(5)$	$Y^1_{ij}\mathbf{10}_i\mathbf{1}_j\mathbf{10}^*$	$Y^{\bar{5}}_{ij}\bar{\mathbf{5}}_i\bar{\mathbf{5}}_j\mathbf{10}$
$(\bar{\mathbf{3}}, \mathbf{1}, -2/3) \equiv \Delta$	$Y^1_{ij}u^{CT}_{ai}C\nu^C_j\Delta^*_a$	$2^{-1/2}\epsilon_{abc}Y^{\bar{5}}_{ij}d^{CT}_{ai}Cd^C_{bj}\Delta_c$
		$Y^{\bar{5}} = -Y^{\bar{5}T}$

generate Yukawa couplings relevant for the charged lepton and down-quark masses. These receive equally important contributions from the $\mathbf{10}_i\bar{\mathbf{5}}_j\mathbf{5}^*$ and $\mathbf{10}_i\bar{\mathbf{5}}_j\mathbf{45}^*$ contractions [36]. It is clear from Tables II and IV that both of these, individually, generate potentially dangerous couplings. The up-quark Yukawa coupling generation, on the other hand, via the $\mathbf{10}_i\mathbf{10}_j\mathbf{45}$ operator, seems not to pose any danger whatsoever as seen from the second column in Table II. However, that operator cannot generate viable masses for all up-quarks due to the antisymmetry of the corresponding mass matrix. The $\mathbf{10}_i\mathbf{10}_j\mathbf{5}$ contraction does provide viable up-quark masses but the price to pay is resurrection of the proton decay issue. To conclude, the only operator which can be considered innocuous in the Majorana neutrino case is the $\mathbf{10}_i\mathbf{10}_j\mathbf{45}$ contraction.

III. LEPTOQUARKS IN FLIPPED $SU(5)$

Another possibility to unify the SM matter into an $SU(5)$-based framework leads to the so-called flipped $SU(5)$ scenario [17–19]. A single family of matter fields in flipped $SU(5)$ can be seen as originating from a sixteen-dimensional representation of $SO(10)$. Actually, flipped $SU(5)$ is not necessarily completely embedded into $SO(10)$. Nevertheless, the generator of electric charge in flipped $SU(5)$ is given as a linear combination of a $U(1)$ generator which resides in $SU(5)$ and an extra $U(1)$ generator as if both of these originate from an $SO(10) \to SU(5) \times U(1)$ decomposition. This guarantees anomaly cancellation at the price of introducing one extra state per family, i.e., the right-handed neutrino ν^C. The transition between the $SU(5)$ and flipped $SU(5)$ embeddings is then provided by $d^C \leftrightarrow u^C$, $e^C \leftrightarrow \nu^C$, $u \leftrightarrow d$ and $\nu \leftrightarrow e$ transformations. Flipped $SU(5)$ thus predicts the existence of three right-handed neutrinos as these transform nontrivially under the underlying gauge symmetry.

The matter fields in flipped $SU(5)$ comprise $\mathbf{10}^{+1}_i$, $\bar{\mathbf{5}}^{-3}_i$ and $\mathbf{1}^{+5}_i$, where the superscripts correspond to the extra $U(1)$ charge assignment. To obtain the SM hypercharge Y one uses the relation $Y = (Y(U(1)) - Y(U(1)_{SU(5)}))/5$, where $Y(U(1))$ and $Y(U(1)_{SU(5)})$ represent the quantum numbers of the extra $U(1)$ and the $U(1)$ in

TABLE IV. Yukawa couplings of the B and L violating scalar in the five-dimensional representation of $SU(5)$. $a, b, c = 1, 2, 3$ $(i, j = 1, 2, 3)$ are color (flavor) indices. Y^{10}_{ij}, $Y^{\bar{5}}_{ij}$ and Y^1_{ij} are Yukawa matrix elements.

$SU(5)$	$Y^{10}_{ij}\mathbf{10}_i\mathbf{10}_j\mathbf{5}$	$Y^{\bar{5}}_{ij}\mathbf{10}_i\bar{\mathbf{5}}_j\mathbf{5}^*$	$Y^1_{ij}\bar{\mathbf{5}}_i\mathbf{1}_j\mathbf{5}$
$(\mathbf{3}, \mathbf{1}, -1/3) \equiv \Delta$	$2\epsilon_{abc}[Y^{10}_{ij} + Y^{10}_{ji}]d^T_{ai}Cu_{bj}\Delta_c$	$2^{-1/2}\epsilon_{abc}Y^{\bar{5}}_{ij}u^{CT}_{ai}Cd^C_{bj}\Delta^*_c$	$Y^1_{ij}d^{CT}_{ai}C\nu^C_j\Delta_a$
	$-2[Y^{10}_{ij} + Y^{10}_{ji}]e^{CT}_i Cu^C_{aj}\Delta_a$	$2^{-1/2}Y^{\bar{5}}_{ij}u^T_{ai}Ce_j\Delta^*_a$	
		$-2^{-1/2}Y^{\bar{5}}_{ij}d^T_{ai}C\nu_j\Delta^*_a$	

TABLE V. Yukawa couplings of the B and L violating scalar in fifty-dimensional representation of flipped $SU(5)$. $a, b, c = 1, 2, 3$ ($i, j = 1, 2, 3$) are color (flavor) indices. Y^{10}_{ij} are Yukawa matrix elements.

$SU(5)\times U(1)$	$Y^{10}_{ij}\mathbf{10}^{+1}_i\mathbf{10}^{+1}_j\mathbf{50}^{-2}$
$(\mathbf{3}, \mathbf{1}, -1/3)^{-2}$ $\equiv$ Δ	$12^{-1/2}\epsilon_{abc}[Y^{10}_{ij}+Y^{10}_{ji}]u^T_{ai}Cd_{bj}\Delta_c$ $3^{-1/2}[Y^{10}_{ij}+Y^{10}_{ji}]\nu^{CT}_i Cd^C_{aj}\Delta_a$

$SU(5)\to SU(3)\times SU(2)\times U(1)$, respectively. One then obtains the electric charge as usual, $Q = Y + T_3$.

The scalar sector which can couple to matter directly is made out of $\mathbf{50}^{-2}$, $\mathbf{45}^{-2}$, $\mathbf{15}^{+6}$, $\mathbf{10}^{+6}$, $\mathbf{5}^{-2}$, and $\mathbf{1}^{-10}$. Representations which can generate contributions to the charged fermion masses and Dirac neutrino masses are $\mathbf{45}^{-2}$ and $\mathbf{5}^{-2}$; whereas, Majorana mass for neutrinos can originate from interactions with $\mathbf{15}^{+6}$. Leptoquarks which violate B and L reside in $\mathbf{50}^{-2}$, $\mathbf{45}^{-2}$, $\mathbf{5}^{-2}$, and $\mathbf{10}^{+6}$ with relevant couplings to matter given in Tables V, VI, VII, and VIII, respectively.

In flipped $SU(5)$ the main obstacle to matter stability is the generation of the up-quark masses. Namely, these can be generated through $\mathbf{10}^{+1}_i\bar{\mathbf{5}}^{-3}_j\mathbf{5}^{*+2}$ and/or $\mathbf{10}^{+1}_i\bar{\mathbf{5}}^{-3}_j\mathbf{45}^{*+2}$ contractions. Both of these are dangerous as far as the proton decay is concerned as can be seen from Tables VI and VII. All other contractions, in the Majorana neutrino case, are actually innocuous.

IV. PROTON DECAY

Let us discuss proton decay operators due to the scalar leptoquark exchange of the lowest possible dimension in detail. These are dimension-six operators made out of three quarks and a lepton which violate B and L by 1 unit. They are summarized below:

$$O_H(d_\alpha, e_\beta) = a(d_\alpha, e_\beta)u^T LC^{-1}d_\alpha u^T LC^{-1}e_\beta, \quad (1)$$

$$O_H(d_\alpha, e^C_\beta) = a(d_\alpha, e^C_\beta)u^T LC^{-1}d_\alpha e^{C\dagger}_\beta LC^{-1}u^{C*}, \quad (2)$$

$$O_H(d^C_\alpha, e_\beta) = a(d^C_\alpha, e_\beta)d^{C\dagger}_\alpha LC^{-1}u^{C*}u^T LC^{-1}e_\beta, \quad (3)$$

$$O_H(d^C_\alpha, e^C_\beta) = a(d^C_\alpha, e^C_\beta)d^{C\dagger}_\alpha LC^{-1}u^{C*}e^{C\dagger}_\beta LC^{-1}u^{C*}, \quad (4)$$

$$O_H(d_\alpha, d_\beta, \nu_i) = a(d_\alpha, d_\beta, \nu_i)u^T LC^{-1}d_\alpha d^T_\beta LC^{-1}\nu_i, \quad (5)$$

$$O_H(d_\alpha, d^C_\beta, \nu_i) = a(d_\alpha, d^C_\beta, \nu_i)d^{C\dagger}_\beta LC^{-1}u^{C*}d^T_\alpha LC^{-1}\nu_i, \quad (6)$$

$$O_H(d_\alpha, d^C_\beta, \nu^C_i) = a(d_\alpha, d^C_\beta, \nu^C_i)u^T LC^{-1}d_\alpha \nu^{C\dagger}_i LC^{-1}d^{C*}_\beta, \quad (7)$$

$$O_H(d^C_\alpha, d^C_\beta, \nu^C_i) = a(d^C_\alpha, d^C_\beta, \nu^C_i)d^{C\dagger}_\beta LC^{-1}u^{C*}\nu^{C\dagger}_i LC^{-1}d^{C*}_\alpha. \quad (8)$$

Here, $i(=1,2,3)$ and α, $\beta(=1,2)$ are generation indices, where all operators which involve a neutrino are bound to have $\alpha+\beta<4$ due to kinematical constraints. $L(=(1-\gamma_5)/2)$ is the left projection operator. The $SU(3)$ color indices are not shown since the antisymmetric contraction $\epsilon_{abc}q_a q_b q_c$ is common to all the above listed operators. The last two operators are relevant only in the case when neutrinos are Dirac particles or when right-handed neutrinos are sufficiently light. This notation has already been introduced in Ref. [24].

These operators allow one to write down explicitly $d=6$ proton decay contributions due to a particular leptoquark exchange [24]. To that end we first specify our convention for the redefinition of the fermion fields which yields the up-quark, down-quark, and charged lepton mass

TABLE VI. Yukawa couplings of the B and L violating scalars in forty-five-dimensional representation of flipped $SU(5)$. $a, b, c = 1, 2, 3$ ($i, j = 1, 2, 3$) are color (flavor) indices. Y^{10}_{ij} and $Y^{\bar{5}}_{ij}$ are Yukawa matrix elements.

$SU(5)\times U(1)$	$Y^{10}_{ij}\mathbf{10}^{+1}_i\mathbf{10}^{+1}_j\mathbf{45}^{-2}$	$Y^{\bar{5}}_{ij}\mathbf{10}_i\bar{\mathbf{5}}^{-3}_j\mathbf{45}^{*+2}$
$(\mathbf{3}, \mathbf{1}, -1/3)^{-2}$ $\equiv$ Δ	$2^{1/2}[Y^{10}_{ij}-Y^{10}_{ji}]\nu^{CT}_i Cd^C_{aj}\Delta_a$	$2^{-1}Y^{\bar{5}}_{ij}\epsilon_{abc}d^{CT}_{ai}Cu^C_{bj}\Delta^*_c$ $-2^{-1}Y^{\bar{5}}_{ij}d^T_{ai}C\nu_j\Delta^*_a$ $2^{-1}Y^{\bar{5}}_{ij}u^T_{ai}Ce_j\Delta^*_a$
$(\mathbf{3}, \mathbf{3}, -1/3)^{-2}$ $\equiv$ $(\Delta^1, \Delta^2, \Delta^3)$	$2^{1/2}\epsilon_{abc}[Y^{10}_{ij}-Y^{10}_{ji}]u^T_{ai}Cu_{bj}\Delta^3_c$	$Y^{\bar{5}}_{ij}d^T_{ai}Ce_j\Delta^{3*}_a$
	$-2\epsilon_{abc}[Y^{10}_{ij}-Y^{10}_{ji}]u^T_{ai}Cd_{bj}\Delta^2_c$	$2^{-1/2}Y^{\bar{5}}_{ij}d^T_{ai}C\nu_j\Delta^{2*}_a$ $2^{-1/2}Y^{\bar{5}}_{ij}u^T_{ai}Ce_j\Delta^{2*}_a$
	$-2^{1/2}\epsilon_{abc}[Y^{10}_{ij}-Y^{10}_{ji}]d^T_{ai}Cd_{bj}\Delta^1_c$	$-Y^{\bar{5}}_{ij}u^T_{ai}C\nu_j\Delta^{1*}_a$
$(\bar{\mathbf{3}}, \mathbf{1}, 4/3)^{-2}$ $\equiv$ Δ	$2^{1/2}[Y^{10}_{ij}-Y^{10}_{ji}]\epsilon_{abc}d^{CT}_{ia}Cd^C_{bj}\Delta_c$	$-Y^{\bar{5}}_{ij}\nu^{CT}_i Cu^C_{aj}\Delta^*_a$

TABLE VII. Yukawa couplings of the B and L violating scalar in a five-dimensional representation of flipped $SU(5)$. a, b, $c = 1, 2, 3$ (i, $j = 1, 2, 3$) are color (flavor) indices. Y_{ij}^{10}, $Y_{ij}^{\bar{5}}$ and Y_{ij}^{1} are Yukawa matrix elements.

$SU(5)\times U(1)$	$Y_{ij}^{10}\mathbf{10}_i^{+1}\mathbf{10}_j^{+1}\mathbf{5}^{-2}$	$Y_{ij}^{\bar{5}}\mathbf{10}_i^{+1}\bar{\mathbf{5}}_j^{-3}\bar{\mathbf{5}}^{*+2}$	$Y_{ij}^{1}\bar{5}_i^{-3}\mathbf{1}_j^{+5}\mathbf{5}^{-2}$
$(\mathbf{3},\mathbf{1},-1/3)^{-2}$ $\equiv$ Δ	$-2\epsilon_{abc}[Y_{ij}^{10}+Y_{ji}^{10}]u_{ai}^{T}Cd_{bj}\Delta_c$ $-2[Y_{ij}^{10}+Y_{ji}^{10}]\nu_i^{CT}Cd_{aj}^{C}\Delta_a$	$2^{-1/2}\epsilon_{abc}Y_{ij}^{\bar{5}}d_{ai}^{CT}Cu_{bj}^{C}\Delta_c^*$ $2^{-1/2}Y_{ij}^{\bar{5}}d_{ai}^{T}C\nu_j\Delta_a^*$ $-2^{-1/2}Y_{ij}^{\bar{5}}u_{ai}^{T}Ce_j\Delta_a^*$	$Y_{ij}^{1}u_{ai}^{CT}Ce_j^{C}\Delta_a$

matrices in physical basis: $M_{U,D,E}\rightarrow M_{U,D,E}^{\rm diag}$. These are $U_C^TM_UU=M_U^{\rm diag}$, $D_C^TM_DD=M_D^{\rm diag}$, and $E_C^TM_EE=M_E^{\rm diag}$. The quark mixing is $U^\dagger D\equiv V_{UD}=K_1V_{\rm CKM}K_2$, where K_1 and K_2 are diagonal matrices containing three and two phases, respectively. In the neutrino sector we have $N_C^TM_NN=M_N^{\rm diag}$ ($N^TM_NN=M_N^{\rm diag}$) in the case of Dirac (Majorana) neutrinos. The leptonic mixing $E^\dagger N\equiv V_{EN}=K_3V_{\rm PMNS}K_4$ in case of Dirac neutrino, or $V_{EN}=K_3V_{\rm PMNS}$ in the Majorana case. K_3 is a diagonal matrix containing three phases; whereas, K_4 contains two phases. $V_{\rm CKM}$ ($V_{\rm PMNS}$) is the Cabibbo-Kobayashi-Maskawa (Pontecorvo-Maki-Nakagawa-Sakata) mixing matrix.

A. Tree-level exchange ($d = 6$) operators in $SU(5)$

The only relevant coefficient for $\Delta\equiv(\mathbf{3},\mathbf{1},-1/3)$ from **50** is

$$a(d_\alpha,e_\beta^C)=\frac{1}{6m_\Delta^2}(U^T(Y^{10}+Y^{10T})D)_{1\alpha}\times(E_C^\dagger(Y^{10}+Y^{10T})^\dagger U_C^*)_{\beta 1}, \tag{9}$$

where m_Δ is a mass of leptoquark in question. (See Table I for details on notation for Yukawa couplings of the fifty-dimensional representation to the matter.)

The relevant coefficients for $\Delta\equiv(\mathbf{3},\mathbf{1},-1/3)$ from **45** are

$$a(d_\alpha^C,e_\beta)=\frac{1}{4m_\Delta^2}(D_C^\dagger Y^{\bar{5}\dagger}U_C^*)_{\alpha 1}(U^TY^{\bar{5}}E)_{1\beta}, \tag{10}$$

$$a(d_\alpha^C,e_\beta^C)=\frac{1}{\sqrt{2}m_\Delta^2}(D_C^\dagger Y^{\bar{5}\dagger}U_C^*)_{\alpha 1}(E_C^\dagger(Y^{10}-Y^{10T})^\dagger U_C^*)_{\beta 1}, \tag{11}$$

TABLE VIII. Yukawa couplings of the B and L violating scalar in ten-dimensional representation of flipped $SU(5)$. a, b, $c=1, 2, 3$ (i, $j=1, 2, 3$) are color (flavor) indices. Y_{ij}^1 and $Y_{ij}^{\bar{5}}$ are Yukawa matrix elements.

$SU(5)\times U(1)$	$Y_{ij}^{1}\mathbf{10}_i^{+1}\mathbf{1}_j^{+5}\mathbf{10}^{*-6}$	$Y_{ij}^{\bar{5}}\bar{\mathbf{5}}_j^{-3}\bar{\mathbf{5}}_j^{-3}\mathbf{10}^{+6}$
$(\bar{\mathbf{3}},\mathbf{1},-2/3)^{+6}\equiv\Delta$	$Y_{ij}^{1}d_{ai}^{CT}Ce_j^{C}\Delta_a^*$	$2^{-1/2}\epsilon_{abc}Y_{ij}^{\bar{5}}u_{ai}^{CT}Cu_{bj}^{C}\Delta_c$ $Y^{\bar{5}}=-Y^{\bar{5}T}$

$$a(d_\alpha,d_\beta^C,\nu_i)=\frac{1}{4m_\Delta^2}(D_C^\dagger Y^{\bar{5}\dagger}U_C^*)_{\beta 1}(D^TY^{\bar{5}}N)_{\alpha i}. \tag{12}$$

The relevant coefficients for $\Delta\equiv(\mathbf{3},\mathbf{1},-1/3)$ from **5** are

$$a(d_\alpha,e_\beta)=-\frac{\sqrt{2}}{m_\Delta^2}(U^T(Y^{10}+Y^{10T})D)_{1\alpha}(U^TY^{\bar{5}}E)_{1\beta}, \tag{13}$$

$$a(d_\alpha,e_\beta^C)=-\frac{4}{m_\Delta^2}(U^T(Y^{10}+Y^{10T})D)_{1\alpha}\times(E_C^\dagger(Y^{10}+Y^{10T})^\dagger U_C^*)_{\beta 1}, \tag{14}$$

$$a(d_\alpha^C,e_\beta)=\frac{1}{2m_\Delta^2}(D_C^\dagger Y^{\bar{5}\dagger}U_C^*)_{\alpha 1}(U^TY^{\bar{5}}E)_{1\beta}, \tag{15}$$

$$a(d_\alpha^C,e_\beta^C)=\frac{\sqrt{2}}{m_\Delta^2}(D_C^\dagger Y^{\bar{5}\dagger}U_C^*)_{\alpha 1}(E_C^\dagger(Y^{10}+Y^{10T})^\dagger U_C^*)_{\beta 1}, \tag{16}$$

$$a(d_\alpha,d_\beta,\nu_i)=\frac{\sqrt{2}}{m_\Delta^2}(U^T(Y^{10}+Y^{10T})D)_{1\alpha}(D^TY^{\bar{5}}N)_{\beta i}, \tag{17}$$

$$a(d_\alpha,d_\beta^C,\nu_i)=-\frac{1}{2m_\Delta^2}(D_C^\dagger Y^{\bar{5}\dagger}U_C^*)_{\beta 1}(D^TY^{\bar{5}}N)_{\alpha i}, \tag{18}$$

$$a(d_\alpha,d_\beta^C,\nu_i^C)=\frac{2}{m_\Delta^2}(U^T(Y^{10}+Y^{10T})D)_{1\alpha}(N_C^\dagger Y^{1\dagger}D_C^*)_{i\beta}, \tag{19}$$

$$a(d_\alpha^C,d_\beta^C,\nu_i^C)=-\frac{1}{\sqrt{2}m_\Delta^2}(D_C^\dagger Y^{\bar{5}\dagger}U_C^*)_{\beta 1}(N_C^\dagger Y^{1\dagger}D_C^*)_{i\alpha}. \tag{20}$$

The relevant coefficients for $\Delta^2\in(\mathbf{3},\mathbf{3},-1/3)$ from **45** are

$$a(d_\alpha,e_\beta)=-\frac{\sqrt{2}}{M_{\Delta^2}^2}(U^T(Y^{10}-Y^{10T})D)_{1\alpha}(U^TY^{\bar{5}}E)_{1\beta}, \tag{21}$$

$$a(d_\alpha,d_\beta,\nu_i)=-\frac{\sqrt{2}}{M_{\Delta^2}^2}(U^T(Y^{10}-Y^{10T})D)_{1\alpha}(D^TY^{\bar{5}}N)_{\beta i}. \tag{22}$$

The relevant coefficient for $\Delta^1 \in (\mathbf{3}, \mathbf{3}, -1/3)$ from **45** is

$$a(d_\alpha, d_\beta, \nu_i) = \frac{2\sqrt{2}}{M^2_{\Delta^1}} (U^T Y^{\bar{5}} N)_{1i} (D^T (Y^{10} - Y^{10T}) D)_{\beta\alpha}, \quad (23)$$

where the extra factor of 2 comes from two terms in Fierz transformation

$$(\overline{s^C} L d)(\overline{\nu^C} L u) = -(\overline{u^C} L s)(\overline{\nu^C} L d) - (\overline{u^C} L d)(\overline{\nu^C} L s). \quad (24)$$

The only relevant coefficient for $\Delta \equiv (\bar{\mathbf{3}}, \mathbf{1}, -2/3)$ from **10** is

$$a(d^C_\alpha, d^C_\beta, \nu^C_i) = -\frac{1}{\sqrt{2} m^2_\Delta} (D^\dagger_C (Y^{\bar{5}} - Y^{\bar{5}T})^\dagger D^*_C)_{\beta\alpha} \times (N^\dagger_C Y^{1\dagger} U^*_C)_{i1}. \quad (25)$$

Finally, $\Delta^3 \in (\mathbf{3}, \mathbf{3}, -1/3)$ and $(\bar{\mathbf{3}}, \mathbf{1}, 4/3)$, both from **45** of $SU(5)$, do not contribute to proton decay at tree level. This is due to antisymmetry, in flavor space, of their couplings to the pair of up-quarks. Nevertheless, both states still induce proton decay through loops at an effective $d = 6$ level. We present a systematic study of these contributions for the $(\bar{\mathbf{3}}, \mathbf{1}, 4/3)$ case in Sec. VI. There, we also spell out contributions of the $(\bar{\mathbf{3}}, \mathbf{1}, 4/3)$ leptoquark to dimension-nine tree-level proton decay amplitudes. Equivalent contributions of $\Delta^3 \in (\mathbf{3}, \mathbf{3}, -1/3)$ are not pursued since the components Δ^1 and Δ^2 from the same state already contribute at leading order. In this manner, higher-order contributions of $\Delta^{1,2,3}$ would only play a role of radiative corrections.

B. Tree-level exchange ($d = 6$) operators in flipped $SU(5)$

The only relevant coefficient for $\Delta \equiv (\mathbf{3}, \mathbf{1}, -1/3)^{-2}$ from $\mathbf{50}^{-2}$ is

$$a(d_\alpha, d^C_\beta, \nu^C_i) = \frac{1}{6 m^2_\Delta} (U^T (Y^{10} + Y^{10T}) D)_{1\alpha} \times (N^\dagger_C (Y^{10} + Y^{10T})^\dagger D^*_C)_{i\beta}. \quad (26)$$

The relevant coefficients for $\Delta \equiv (\mathbf{3}, \mathbf{1}, -1/3)^{-2}$ from $\mathbf{45}^{-2}$ are

$$a(d^C_\alpha, e_\beta) = \frac{1}{4 m^2_\Delta} (D^\dagger_C Y^{\bar{5}*} U^*_C)_{\alpha 1} (U^T Y^{\bar{5}} E)_{1\beta}, \quad (27)$$

$$a(d_\alpha, d^C_\beta, \nu_i) = -\frac{1}{4 m^2_\Delta} (D^\dagger_C Y^{\bar{5}*} U^*_C)_{\beta 1} (D^T Y^{\bar{5}} N)_{\alpha i}, \quad (28)$$

$$a(d^C_\alpha, d^C_\beta, \nu^C_i) = \frac{1}{\sqrt{2} m^2_\Delta} (D^*_C Y^{\bar{5}*} U^*_C)_{\beta 1} \times (N^\dagger_C (Y^{10} - Y^{10T})^\dagger D^*_C)_{i\alpha}. \quad (29)$$

The relevant coefficients for $\Delta \equiv (\mathbf{3}, \mathbf{1}, -1/3)^{-2}$ from $\mathbf{5}^{-2}$ are

$$a(d_\alpha, e_\beta) = \frac{\sqrt{2}}{m^2_\Delta} (U^T (Y^{10} + Y^{10T}) D)_{1\alpha} (U^T Y^{\bar{5}} E)_{1\beta}, \quad (30)$$

$$a(d_\alpha, e^C_\beta) = \frac{2}{m^2_\Delta} (U^T (Y^{10} + Y^{10T}) D)_{1\alpha} (E^\dagger_C Y^{1\dagger} U^*_C)_{\beta 1}, \quad (31)$$

$$a(d^C_\alpha, e_\beta) = \frac{1}{2 m^2_\Delta} (D^\dagger_C Y^{\bar{5}*} U^*_C)_{\alpha 1} (U^T Y^{\bar{5}} E)_{1\beta}, \quad (32)$$

$$a(d^C_\alpha, e^C_\beta) = \frac{1}{\sqrt{2} m^2_\Delta} (D^\dagger_C Y^{\bar{5}*} U^*_C)_{\alpha 1} (E^\dagger_C Y^{1\dagger} U^*_C)_{\beta 1}, \quad (33)$$

$$a(d_\alpha, d_\beta, \nu_i) = \frac{-\sqrt{2}}{m^2_\Delta} (U^T (Y^{10} + Y^{10T}) D)_{1\alpha} (N^T Y^{\bar{5}T} D)_{i\beta}, \quad (34)$$

$$a(d_\alpha, d^C_\beta, \nu_i) = \frac{-1}{2 m^2_\Delta} (D^\dagger_C Y^{\bar{5}*} U^*_C)_{\beta 1} (D^T Y^{\bar{5}} N)_{\alpha i}, \quad (35)$$

$$a(d_\alpha, d^C_\beta, \nu^C_i) = \frac{-4}{m^2_\Delta} (U^T (Y^{10} + Y^{10T}) D)_{1\alpha} \times (N^\dagger_C (Y^{10} + Y^{10T})^\dagger D^*_C)_{i\beta}, \quad (36)$$

$$a(d^C_\alpha, d^C_\beta, \nu^C_i) = -\frac{\sqrt{2}}{m^2_\Delta} (D^\dagger_C Y^{\bar{5}*} U^*_C)_{1\beta} \times (N^\dagger_C (Y^{10} + Y^{10T})^\dagger D^*_C)_{i\alpha}. \quad (37)$$

The relevant coefficients for $\Delta^2 \in (\mathbf{3}, \mathbf{3}, -1/3)^{-2}$ from $\mathbf{45}^{-2}$ are

$$a(d_\alpha, e_\beta) = -\frac{\sqrt{2}}{M^2_{\Delta^2}} (U^T (Y^{10} - Y^{10T}) D)_{1\alpha} (U^T Y^{\bar{5}} E)_{1\beta}, \quad (38)$$

$$a(d_\alpha, d_\beta, \nu_i) = -\frac{\sqrt{2}}{M^2_{\Delta^2}} (U^T (Y^{10} - Y^{10T}) D)_{1\alpha} (D^T Y^{\bar{5}} N)_{\beta i}. \quad (39)$$

The relevant coefficient for $\Delta^1 \in (\mathbf{3}, \mathbf{3}, -1/3)^{-2}$ from $\mathbf{45}^{-2}$ is

$$a(d_\alpha, d_\beta, \nu_i) = \frac{2\sqrt{2}}{M^2_{\Delta^1}} (U^T Y^{\bar{5}} N)_{1i} (D^T (Y^{10} - Y^{10T}) D)_{\beta\alpha}, \quad (40)$$

where the extra factor of 2 comes from Fierz transformation (24).

The only relevant coefficient for $\Delta \equiv (\mathbf{3}, \mathbf{1}, 4/3)^{-2}$ from $\mathbf{45}^{-2}$ is

$$a(d_\alpha^C, d_\beta^C, \nu_i^C) = -\frac{2\sqrt{2}}{M_{\Delta^1}^2}(U_C^\dagger Y^{\bar{5}\dagger} N_C^*)_{i1} \times (D_C^\dagger (Y^{10} - Y^{10T})^\dagger D_C^*)_{\alpha\beta}, \quad (41)$$

where the extra factor of 2 again comes from Fierz transformation.

The relevant coefficients for $(\bar{\mathbf{3}}, \mathbf{1}, -2/3)^{+6} \in 10^{+6}$ and $\Delta^3 \in (\mathbf{3}, \mathbf{3}, -1/3)^{-2} \in \mathbf{45}^{-2}$ are not present at the leading order due to antisymmetry of the couplings to the up-quark pair. The higher-order contributions of the former state are discussed in Sec. VI.

V. LEADING-ORDER CONTRIBUTIONS

Scalar fields couple to matter through Yukawa couplings. This introduces uncertainties to predictions related to any process which involves scalar leptoquark exchange. It is thus natural to ask if, and when, it is necessary to have leptoquark masses close to a scale of grand unification. We will address this issue in the $SU(5)$ and flipped $SU(5)$ frameworks in what follows.

A. Color triplets of $SU(5)$

If the Yukawa sector relevant for proton decay through scalar exchange is not related to the origin of fermion masses and mixing parameters, one cannot make any firm predictions. For example, all operators which correspond to the exchange of the triplet scalar in the five-dimensional representation of $SU(5)$ can be completely suppressed if $(U^T(Y^{10} + Y^{10T})D)_{1\alpha} = 0$ and $(D_C^\dagger Y^{\bar{5}\dagger} U_C^*)_{\alpha 1} = 0$, $\alpha = 1, 2$, in the Majorana neutrino case. (Recall, it was the exchange of this scalar which has led to the so-called doublet-triplet splitting problem within the context of the Georgi-Glashow $SU(5)$ model [16].) The suppression is certainly viable if the entries of U, U_C, D, D_C, Y^{10}, and $Y^{\bar{5}}$ are all free parameters. The first set of conditions can be ensured if, for example, $Y^{10} = -Y^{10T}$. This solution has already been pointed out in Ref. [24]. The second set of conditions can also be easily satisfied although what we find defers from what has been presented in Ref. [24].

Things, however, change in models where the connection between Yukawa sector and fermion masses is strong. Let us thus analyze predictions of the simplest of all possible renormalizable models based on the $SU(5)$ gauge symmetry. We want to find what the current experimental bounds on the partial proton lifetimes for processes presented in Table IX imply for the masses of color triplets if the theory is to be viable with regard to the fermion mass generation. We analyze all these decay modes to make our study as complete as possible.

TABLE IX. Experimental bounds on selected partial proton decay lifetimes at 90% C.L.

Process	τ_p (10^{33} years)
$p \to \pi^0 e^+$	13.0 [37]
$p \to \pi^0 \mu^+$	11.0 [38]
$p \to K^0 e^+$	1.0 [39]
$p \to K^0 \mu^+$	1.3 [39]
$p \to \eta e^+$	4.2 [40]
$p \to \eta \mu^+$	1.3 [40]
$p \to \pi^+ \bar{\nu}$	0.025 [41]
$p \to K^+ \bar{\nu}$	4.0 [37]

We demand in what follows that the theory is renormalizable and thus neglect the possibility that higher-dimensional terms contribute to (super)potential at any level. We furthermore take the simplest possibility for the generation of phenomenologically viable fermion masses and mixing parameters. Namely, we demand that both **5** and **45** of Higgs contribute to the down-quark and charged lepton masses [36]. We further take all mass matrices to be symmetric, i.e., $M_{U,D,E} = M_{U,D,E}^T$. This then allows us to consider two particular scenarios. The first (second) one represents the case when the contributions of the $(\mathbf{3}, \mathbf{1}, -1/3)$ state from the five-dimensional (forty-five-dimensional) representation dominates. Our analysis is self-consistent as the symmetric mass matrix assumption eliminates contributions to proton decay of all other color triplets. Note also that any mixing between the triplets can be accounted for by simple rescaling of relevant operators.

1. The charged antilepton final state

We start our analysis with proton decay due to exchange of the triplet state from the five-dimensional representation. To find widths for the charged antileptons in the final state, one needs to determine $a(d_\alpha, e_\beta)$, $a(d_\alpha, e_\beta^C)$, $a(d_\alpha^C, e_\beta)$, and $a(d_\alpha^C, e_\beta^C)$. If the Yukawa couplings are symmetric, the relevant input for these coefficients reads

$$(U^T(Y^{10} + Y^{10T})D)_{1\alpha} = -\frac{1}{\sqrt{2}v_5}(M_U^{\mathrm{diag}} V_{UD})_{1\alpha}, \quad (42)$$

$$(U^T Y^{\bar{5}} E)_{1\beta} = -\frac{1}{2v_5}(3V_{UD}^* M_D^{\mathrm{diag}} V_{UD}^\dagger U_2^* + U_2 M_E^{\mathrm{diag}})_{1\beta}, \quad (43)$$

$$(D^\dagger Y^{\bar{5}\dagger} U^*)_{\alpha 1} = -\frac{1}{2v_5}(3M_D^{\mathrm{diag}} V_{UD}^T + V_{UD}^\dagger U_2^* M_E^{\mathrm{diag}} U_2^\dagger)_{\alpha 1}, \quad (44)$$

$$(E^\dagger (Y^{10} + Y^{10T})^\dagger U^*)_{\beta 1} = -\frac{1}{\sqrt{2}v_5}(U_2^T M_U^{\mathrm{diag}})_{\beta 1}, \quad (45)$$

where $U_2 = U^T E^*$ and v_5 represents the VEV of the five-dimensional representation. U_2 entries and v_5 are primary sources of uncertainty. Our normalization is such that $|v_5|^2/2 + 12|v_{45}|^2 = v^2$, where $v(= 246\ \text{GeV})$ stands for the electroweak VEV. v_{45} is the VEV in the forty-five-dimensional representation. A connection between Yukawa couplings and charged fermion mass matrices is spelled out elsewhere [31].

We outline details of our calculation using the $p \to e_\delta^+ \pi^0$ channels. Here, $\delta = 1$ ($\delta = 2$) corresponds to e^+ (μ^+) in the final state. The decay widths formulas we use are summarized in Ref. [24]. For these particular channels, we have

$$\Gamma(p \to e_\delta^+ \pi^0) = \frac{(m_p^2 - m_{\pi^0}^2)^2}{64\pi f_\pi^2 m_p^3}(|\alpha a(d_1, e_\delta) + \beta a(d_1^C, e_\delta)|^2 + |\alpha a(d_1, e_\delta^C) + \beta a(d_1^C, e_\delta^C)|^2)(1 + D + F)^2, \quad (46)$$

where α and β are the so-called nucleon matrix elements. (See Sec. VI for more details on α and β.) $F + D$ and $F - D$ combinations are extracted from the nucleon axial charge and form factors in semileptonic hyperon decays, respectively [42,43]. We take $f_\pi = 130$ MeV, $m_p = 938.3$ MeV, $D = 0.80(1)$, $F = 0.47(1)$, and $\alpha = -\beta = -0.0112(25)\ \text{GeV}^3$ [43].

The uncertainty in predicting partial decay rates persists even in the minimal $SU(5)$ scenario with symmetric Yukawa couplings. This is evident from the U_2 dependence of $p \to e_\delta^+ \pi^0$ partial decay widths

$$\Gamma(p \to e_\delta^+ \pi^0) = \frac{(m_p^2 - m_{\pi^0}^2)^2}{64\pi f_\pi^2 m_p^3} \frac{\alpha^2}{v_5^4 m_\Delta^4} \left| (V_{UD})_{11} \times \left[m_u + \frac{3}{4} m_d \right] + \frac{1}{4}(V_{UD}^\dagger U_2^* M_E^{\text{diag}} U_2^\dagger)_{11} \right|^2 \times \left(\left| \frac{3}{2}(V_{UD}^* M_D^{\text{diag}} V_{UD}^\dagger U_2^*)_{1\delta} + \frac{1}{2}(U_2 M_E^{\text{diag}})_{1\delta} \right|^2 + 4|m_u (U_2)_{1\delta}|^2 \right) \times (1 + D + F)^2.$$

One can suppress (enhance) $\Gamma(p \to e_\delta^+ \pi^0)$ with regard to U_2 numerically to generate the least (most) conservative lower bound on the mass of the scalar triplet in the five-dimensional representation of $SU(5)$. This, however, should be done simultaneously with all other partial decay modes to generate a self-consistent solution. We will do that after we address proton decay into neutral antileptons in the final state. The important point is that even in the case of symmetric Yukawa couplings one cannot test the $SU(5)$ theory when the scalar triplet exchange dominates. This is in stark contrast to what one obtains for the gauge $d = 6$ contributions in the $SU(5)$ framework [20].

Nevertheless, we can already outline how one can find a maximum of $\Gamma(p \to e_\delta^+ \pi^0)$ with regard to U_2 to obtain the most conservative bound, from the model building point of view, on m_Δ without resorting to elaborate numerical analysis. The idea is to have Yukawa couplings of the third generation contribute as much as possible towards relevant amplitudes. The only possibility to achieve that is to make the 11 element of $U_2^* M_E^{\text{diag}} U_2^\dagger$ matrix in Eq. (44) as large as possible. This can be done with a simple ansatz

$$U_2 = U^T E^* = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad (47)$$

to obtain the following simplified expressions:

$$\Gamma(p \to e^+ \pi^0) \approx \frac{3^2 m_p \alpha^2}{2^{12} \pi f_\pi^2 v_5^4 m_\Delta^4} |(V_{UD})_{13}|^2 m_b^2 m_\tau^2 \times (1 + D + F)^2, \quad (48)$$

$$\Gamma(p \to \mu^+ \pi^0) \approx \frac{3^2 m_p \alpha^2}{2^{12} \pi f_\pi^2 v_5^4 m_\Delta^4} |(V_{UD})_{12}|^2 m_s^2 m_\tau^2 \times (1 + D + F)^2. \quad (49)$$

Interestingly enough, these expressions when combined with experimental input yield comparable bounds on the mass of the leptoquark in question.

In the previous analysis it was assumed that contributions to proton decay of the triplet in the five-dimensional representation dominate over contributions of triplets in the forty-five-dimensional representation. Let us now see if and when that is truly the case. Our assumption that the mass matrices are symmetric implies that the only other contribution to $p \to e_\delta^+ \pi^0$ ($\delta = 1, 2$) channels originates from exchange of the $(\mathbf{3}, \mathbf{1}, -1/3)$ state in the forty-five-dimensional representation. In fact, the only relevant coefficient is $a(d_\alpha^C, e_\beta)$ with the following entries:

$$(D^\dagger Y^{\bar{5}\dagger} U^*)_{\alpha 1} = \frac{1}{4 v_{45}}(V_{UD}^\dagger U_2^* M_E^{\text{diag}} U_2^\dagger - M_D^{\text{diag}} V_{UD}^T)_{\alpha 1}, \quad (50)$$

$$(U^T Y^{\bar{5}} E)_{1\beta} = \frac{1}{4 v_{45}}(U_2 M_E^{\text{diag}} - V_{UD}^* M_D^{\text{diag}} V_{UD}^\dagger U_2^*)_{1\beta}. \quad (51)$$

The color triplet contribution to $p \to e_\delta^+ \pi^0$ accordingly reads

$$\Gamma(p \to e_\delta^+ \pi^0) = \frac{m_p \alpha^2}{2^{18} \pi f_\pi^2 v_{45}^4 m_\Delta^4} \times \left| (V_{UD}^\dagger U_2^* M_E^{\text{diag}} U_2^\dagger)_{11} - (V_{UD})_{11} m_d \right|^2 \times \left| (U_2 M_E^{\text{diag}} - V_{UD}^* M_D^{\text{diag}} V_{UD}^\dagger U_2^*)_{1\delta} \right|^2 \times (1 + D + F)^2,$$

where v_{45} represents the VEV of the forty-five-dimensional representation. With the ansatz given in Eq. (47) we obtain the following expressions:

$$\Gamma(p \to e^+\pi^0) \approx \frac{m_p \alpha^2}{2^{18}\pi f_\pi^2 v_{45}^4 m_\Delta^4}|(V_{UD})_{13}|^2 m_b^2 m_\tau^2 \times (1 + D + F)^2, \quad (52)$$

$$\Gamma(p \to \mu^+\pi^0) \approx \frac{m_p \alpha^2}{2^{18}\pi f_\pi^2 v_{45}^4 m_\Delta^4}|(V_{UD})_{12}|^2 m_s^2 m_\tau^2 \times (1 + D + F)^2. \quad (53)$$

These should be compared with corresponding results for the exchange of the triplet in the five-dimensional representation given in Eqs. (48) and (49) to obtain

$$\left(\frac{\Gamma(p \to e_\delta^+\pi^0)^5}{\Gamma(p \to e_\delta^+\pi^0)^{45}}\right)_{\max} = 576\left(\frac{v_{45}}{v_5}\right)^4, \qquad \delta = 1, 2. \quad (54)$$

We see that the five-dimensional triplet dominates over the forty-five-dimensional triplet for moderate values of v_{45} if all other relevant parameters are the same.

2. The neutral antilepton final state

In order to incorporate $p \to \pi^+\bar{\nu}$ and $p \to K^+\bar{\nu}$ decay modes in our study we note that one is free to sum over the neutrino flavors in the final state. The relevant coefficients which enter widths for these decays are $a(d_\alpha, d_\beta, \nu_i)$ and $a(d_\alpha, d_\beta^C, \nu_i)$ when the exchanged state is the triplet in the five-dimensional representation. To find them, we need

$$(D^T Y^{\bar{5}} N)_{\beta i} = -\frac{1}{2v_5}(3M_D^{\text{diag}} V_{UD}^\dagger U_2^* V_{EN} + V_{UD}^T U_2 M_E^{\text{diag}} V_{EN})_{\beta i}, \quad (55)$$

where the relevant sums yield

$$\sum_i (D^T Y^{\bar{5}} N)_{\alpha i}(D^T Y^{\bar{5}} N)^*_{\beta i} = \frac{1}{4v_5^2}\sum_j (3M_D^{\text{diag}} V_{UD}^\dagger U_2^* + V_{UD}^T U_2 M_E^{\text{diag}})_{\alpha j} \times (3M_D^{\text{diag}} V_{UD}^T U_2 + V_{UD}^\dagger U_2^* M_E^{\text{diag}})_{\beta j}. \quad (56)$$

The upshot of these results is that widths for decays with neutral antilepton in the final state again depend only on U_2 as far as the mixing parameters are concerned. One can maximize amplitudes for $p \to K^+\bar{\nu}$ and $p \to \pi^+\bar{\nu}$ by taking the contributions proportional to m_τ in Eq. (56) and using the same ansatz for U_2 as before. We find that $\Gamma(p \to K^+\bar{\nu})$ dominates over widths for proton decays into charged antileptons. It reads

$$\Gamma(p \to K^+\bar{\nu}) \approx \frac{(m_p^2 - m_{K^+}^2)^2}{128\pi f_\pi^2 m_p^3}\frac{\alpha^2 m_\tau^4}{v_5^4 m_\Delta^4}|V_{UD})_{12}|^2 \times \left[1 + \frac{m_p}{2m_\Sigma}(D - F) + \frac{m_p}{2m_\Lambda}(D + 3F)\right]^2. \quad (57)$$

This is an important result. It tells us that the most conservative bound on the scalar sector comes from the proton decays into neutral antileptons in the final state.

If one compares the most conservative contributions of the triplets in the five-dimensional and forty-five-dimensional representations towards $p \to K^+\bar{\nu}$ one obtains

$$\left(\frac{\Gamma(p \to K^+\bar{\nu})^5}{\Gamma(p \to K^+\bar{\nu})^{45}}\right)_{\max} = 1024\left(\frac{v_{45}}{v_5}\right)^4. \quad (58)$$

Again, the five-dimensional triplet dominates over the forty-five-dimensional triplet for moderate values of v_{45}.

We can now numerically analyze all the decay modes given in Table IX to find the current bounds on the triplet mass in $SU(5)$ with symmetric Yukawa couplings. We take values of quark and lepton masses at M_Z, as given in Ref. [44], and neglect any running of the relevant coefficients, for simplicity. These effects can be accounted for in a straightforward manner. The CKM angles, when needed, are taken from Ref. [41]. We have randomly generated one million sets of values for nine parameters of U_2 unitary matrix and five phases of V_{UD} to find the bounds presented in Table X. As it turns out, it is $p \to K^+\bar{\nu}$ which dominates in all instances. With that in mind we can write that the most and least conservative bounds read

$$m_\Delta > 1.2 \times 10^{13}\left(\frac{\alpha}{0.0112\,\text{GeV}^3}\right)^{1/2}\left(\frac{100\,\text{GeV}}{v_5}\right)\text{GeV}, \quad (59)$$

$$m_\Delta > 1.5 \times 10^{11}\left(\frac{\alpha}{0.0112\,\text{GeV}^3}\right)^{1/2}\left(\frac{100\,\text{GeV}}{v_5}\right)\text{GeV}. \quad (60)$$

To summarize, if one is to maximize contributions from the triplets in the five-dimensional and forty-five-dimensional representations towards proton decay within renormalizable $SU(5)$ framework with symmetric mass matrices, the current bounds on the triplet mass scale are given in Eqs. (59) and (60) if the color triplet in **5** of Higgs dominates in the most and least conservative scenario, respectively. In other words, any $SU(5)$ scenario where the triplet scalar mass exceeds the most conservative bound of Eq. (59) is certainly safe with regard to the proton decay constraints on the scalar mediated proton decay. If the triplet is to be lighter than that, one needs to explicitly check if particular implementation of Yukawa couplings allows for such scenario. The five-dimensional triplet dominance is determined through relations given in Eqs. (54) and (58). Finally, if the triplet mass is below

TABLE X. The least conservative (second column) and the most conservative (third column) experimental lower bounds on triplet mass in the five-dimensional representation of $SU(5)$ with symmetric Yukawa couplings.

Channel	m_Δ (GeV)	m_Δ (GeV)
$p \to \pi^0 e^+$	1.9×10^{10}	4.9×10^{12}
$p \to \pi^0 \mu^+$	2.8×10^{10}	5.4×10^{12}
$p \to K^0 e^+$	1.7×10^{10}	1.5×10^{12}
$p \to K^0 \mu^+$	2.0×10^{10}	2.1×10^{12}
$p \to \eta e^+$	1.1×10^{10}	4.0×10^{11}
$p \to \eta \mu^+$	7.2×10^{9}	2.7×10^{11}
$p \to \pi^+ \bar{\nu}$	2.2×10^{10}	8.0×10^{12}
$p \to K^+ \bar{\nu}$	1.5×10^{11}	1.2×10^{13}

the least conservative bound of Eq. (60), the $SU(5)$ model is not viable.

B. Color triplet in flipped $SU(5)$

Flipped $SU(5)$ is well-known for the so-called missing partner mechanism which naturally addresses scalar mediated proton decay by making the triplet scalar in the five-dimensional representation heavy enough. Be that as it may, the operators associated with the triplet exchange can be suppressed with ease if $(U^T(Y^{10} + Y^{10T})D)_{1\alpha} = 0$ and $(D_C^\dagger Y^{\bar{5}*} U_C^*)_{\alpha 1} = 0$, $\alpha = 1, 2$, in the Majorana neutrino case. Note that in flipped $SU(5)$ the Dirac mass matrix for neutrinos is proportional to the up-quark mass matrix. This implies that flipped $SU(5)$ predicts Majorana nature of neutrinos.

The operator suppression can be implemented only if the Yukawa couplings are treated as free parameters. That is not the case in the minimal realistic version of flipped $SU(5)$ theory where Yukawa couplings are related to fermion masses and mixing parameters. In the minimal scenario it is sufficient to have only one five-dimensional scalar representation present to generate realistic charged fermion masses. We accordingly analyze predictions of a flipped $SU(5)$ scenario with a single color triplet state. To be able to compare the flipped $SU(5)$ results with the case of ordinary $SU(5)$, we again take $M_{U,D,E} = M_{U,D,E}^T$.

1. The charged antilepton final state

We get the following result for the $p \to e_\delta^+ \pi^0$ ($\delta = 1, 2$) partial decay widths:

$$\Gamma(p \to e_\delta^+ \pi^0) = \frac{m_p}{64\pi f^2} \frac{\alpha^2}{v_5^4 m_\Delta^4} |(V_{UD})_{11}(m_d - m_u)|^2 \times 4(m_u^2 + m_e^2)|(U_2)_{1\delta}|^2 (1 + D + F)^2, \quad (61)$$

where $v_5 (= \sqrt{2} 246$ GeV) represents the VEV of the five-dimensional representation. Interestingly enough, if U_2 takes the form given in Eq. (47), it would yield suppressed partial decay widths for $p \to e_\delta^+ \pi^0$, $\delta = 1, 2$. In other words, the setup which enhances partial proton decay rates with charged antilepton in the final state in $SU(5)$ framework suppresses corresponding rates in flipped $SU(5)$.

If we want to be conservative with regard to the limit on m_Δ it is sufficient to maximize relevant decay widths. This can be done by taking $(U_2)_{1\delta} = 1$ for $p \to e_\delta^+ \pi^0$ ($\delta = 1, 2$) to obtain

$$m_\Delta > 3.6 \times 10^{10} \left(\frac{\alpha}{0.0112 \text{ GeV}^3} \right)^{1/2} \text{ GeV}. \quad (62)$$

This limit comes out to be significantly weaker with respect to the corresponding limit we presented in the $SU(5)$ case.

2. The neutral antilepton final state

To find decay widths for $p \to \pi^+ \bar{\nu}$ and $p \to K^+ \bar{\nu}$ channels, we need to determine the form of $a(d_\alpha, d_\beta, \nu_i)$ and $a(d_\alpha, d_\beta^C, \nu_i)$ coefficients. In the minimal model with symmetric mass matrices the relevant input reads

$$(U^T(Y^{10} + Y^{10T})D)_{1\alpha} = -\frac{1}{\sqrt{2} v_5} (V_{UD}^* M_D^{\text{diag}})_{1\alpha}, \quad (63)$$

$$(D^T Y^{\bar{5}} N)_{\beta i} = -\frac{2}{v_5} (V_{UD}^T M_U^{\text{diag}} U_2^* V_{EN})_{\beta i}, \quad (64)$$

$$(D^\dagger Y^{\bar{5}*} U^*)_{\beta 1} = -\frac{2}{v_5} (V_{UD}^\dagger M_U^{\text{diag}})_{\beta 1}, \quad (65)$$

with the following sum over neutrino flavors:

$$\sum_i (D^T Y^{\bar{5}} N)_{\alpha i} (D^T Y^{\bar{5}} N)_{\beta i}^* = \frac{4}{v_5^2} (V_{UD}^T (M_U^{\text{diag}})^2 V_{UD}^*)_{\alpha\beta}. \quad (66)$$

The sum over neutrino flavors in the final state eliminates dependence on U_2—the matrix which represents mismatch between rotations in the quark and lepton sectors—leaving us with decay widths which depend only on known masses and mixing parameters. This makes minimal flipped $SU(5)$ with symmetric mass matrices rather unique. We find the following limit on the triplet mass which originates from experimental constraints on $p \to K^+ \bar{\nu}$ channel:

$$m_\Delta > 1.0 \times 10^{12} \left(\frac{\alpha}{0.0112 \text{ GeV}^3} \right)^{1/2} \text{ GeV}. \quad (67)$$

The fact that $p \to \pi^+ \bar{\nu}$ is also a clean channel in the sense that it depends only on the CKM angles and known fermion masses means that the minimal flipped $SU(5)$ predicts ratio between $\Gamma(p \to \pi^+ \bar{\nu})$ and $\Gamma(p \to K^+ \bar{\nu})$. We find it to be

$$\frac{\Gamma(p \to \pi^+ \bar{\nu})}{\Gamma(p \to K^+ \bar{\nu})} = 9.0. \quad (68)$$

This result is not sensitive to exact value of the nucleon matrix elements and running of relevant coefficient. It thus represents firm prediction within the framework of the minimal flipped $SU(5)$ with symmetric Yukawa couplings.

VI. HIGHER-ORDER CONTRIBUTIONS

In the $SU(5)$ framework the states $(\bar{\mathbf{3}}, \mathbf{1}, 4/3)$ and $\Delta^3 \in (\mathbf{3}, \mathbf{3}, -1/3)$ violate B and L and do not contribute to dimension-six proton decay operators at tree-level. Antisymmetry of their Yukawa couplings to two up quarks only allows for dimension-six operators involving c or t quarks which produce B number violation in charm or top decays [45], but these operators do not affect the proton stability due to large masses of c and t quarks. However, an additional W boson exchange opens decay channels with final states which are kinematically accessible to proton decay.

A. Box mediated dimension-six operator from $(\bar{\mathbf{3}}, \mathbf{1}, \mathbf{4/3}) \in \mathbf{45}$

One possibility is to make a box diagram with a single W exchange leading to the $d = 6$ operator, as shown in Fig. 1. In the literature, proton decay mediation involving W boson exchanges were considered in Refs. [45–47]. We calculate the box diagram in the approximation where we neglect external momenta; however, we keep both virtual fermions massive since the right-handed Δ interactions force chirality flips on internal fermion lines and thus the diagram would vanish if both fermions were massless. Evaluation of the diagrams with W and would-be Goldstones leads to gauge invariant and finite amplitude. Then we find that $\Delta(\bar{\mathbf{3}}, \mathbf{1}, 4/3)$ generates two effective coefficients:

$$a(d_\alpha, e^C_\beta) = -\frac{G_F}{4\pi^2 m_W^2} \sum_{j,k} [U_C^\dagger (Y^{10*} - Y^{10\dagger}) U_C^*]_{1j} \times [D_C^\dagger Y^{\bar{5}\dagger} E_C^*]_{k\beta} m_{u_j} V_{j\alpha} m_{d_k} V^*_{uk} J(x_\Delta, x_{u_j}, x_{d_k}), \tag{69}$$

$$a(d_\alpha, d^C_\beta, \nu_i) = -\frac{G_F}{4\pi^2 m_W^2} \sum_{j} [U_C^\dagger (Y^{10*} - Y^{10\dagger}) U_C^*]_{1j} \times [D_C^\dagger Y^{\bar{5}\dagger} E_C^*]_{\beta i} m_{u_j} V_{j\alpha} m_{\ell_i} J(x_\Delta, x_{u_j}, x_{\ell_i}). \tag{70}$$

Here, $V \equiv V_{CKM}$, while the leptonic mixing matrix has been set to unity. (For the neutrino final states, one would need to sum over all neutrino flavors.). Mass dependence, apart from helicity flip factors, is encoded in function J (where $x_k \equiv m_k^2/m_W^2$)

$$J(x, y, z) = \frac{(y-4)y \log y}{(y-1)(y-x)(y-z)} + \frac{(z-4)z \log z}{(z-1)(z-y)(z-x)} + \frac{(x-4)x \log x}{(x-1)(x-y)(x-z)}. \tag{71}$$

There are two distinct regimes of dynamics in the box, depending on the presence of t quark in the loop. When $j = 3$ we expand to leading order in $x_{\ell_i}, x_{\ell_i} \ll 1$, and find

$$J(x_\Delta, x_t, x_{\ell_i}) = \frac{1}{x_\Delta - x_t}\left[\frac{x_\Delta - 4}{x_\Delta - 1} \log x_\Delta - \frac{x_t - 4}{x_t - 1} \log x_t\right]. \tag{72}$$

When both fermions are light compared to W, the J function takes the following form:

$$J(x_\Delta, x_{u_j}, x_{\ell_i}) = \frac{1}{x_\Delta}\left[\frac{x_\Delta - 4}{x_\Delta - 1} \log x_\Delta + \frac{4}{x_{u_j} - x_{\ell_i}} \times (x_{\ell_i} \log x_{\ell_i} - x_{u_j} \log x_{u_j})\right]. \tag{73}$$

Contributions of the up-quark Yukawa couplings are weighted approximately by $m_{u_j} V_{jd}$ for $j = 2, 3$ which run in the box. The large mass of the t quark comes with small element V_{td} which makes this product of the same magnitude as $m_c V_{cd}$. Similar cancellation between mass and CKM hierarchies occurs for the down-quarks where the weights obey $m_d V_{ud} \sim m_s V_{us} \sim m_b V_{ub}$.

The $(\bar{\mathbf{3}}, \mathbf{1}, 4/3)$ state has been identified as a suitable candidate to explain the anomalous value of the muon magnetic moment. One of the leptoquark couplings $(D_C^\dagger Y^{\bar{5}\dagger} E_C^*)_{i2}$ between the muon and one of the down quarks d_i must be of the order ~ 2, while other two must be $\lesssim 10^{-3}$ to suppress contributions to other down-quark and charged lepton observables [31]. In addition, the CDF and DØ measurements of forward-backward asymmetry in $t\bar{t}$ production can be explained by a large diquark coupling $(U_C^\dagger (Y^{10*} - Y^{10\dagger}) U_C^*)_{31} \sim 2$ to ut quark pair [28]. Additional couplings between uc and ct quark pairs are constrained by charm and top physics processes and their

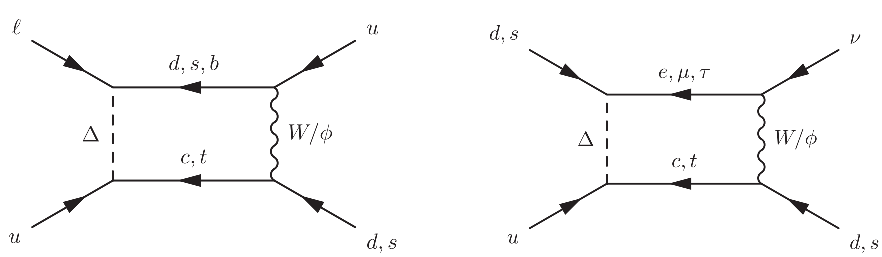

FIG. 1. Box diagrams with $(\bar{\mathbf{3}}, \mathbf{1}, 4/3)$ state which generate $d = 6$ operators of flavor $uud\ell$ and $udd\nu$.

upper bounds are of the order 10^{-1} and 10^{-2}, respectively [32]. Both puzzles can be explained for a mass of the leptoquark of around 400 GeV. However, for a light mass and with the above-mentioned two large couplings the proton would decay much too quickly to a muon final state via dominant contribution of t quark and one of the down quarks in the box [c.f. Eq. (70)]. Therefore, one has to find a second amplitude of equal magnitude and opposite phase in order to achieve cancellation between the first and the second amplitude. As explained in the preceding paragraph, all down quarks in the box which couple to external muon have similar weights which come from loop dynamics and CKM factors. As a result, the hierarchy of d_i contributions to the amplitude follows very closely hierarchy of the leptoquark couplings $(D_C^\dagger Y^{\bar{5}\dagger} E_C^*)_{i2}$ and the required cancellation cannot take place between the different down-quarks in the box. Likewise, cancellation between c and t quarks in the box diagram cannot occur for similar reason.

B. Tree-level dimension-nine operator from $(\bar{\mathbf{3}}, \mathbf{1}, 4/3) \in \mathbf{45}$

The W emission from the up-type quark leads to proton decay amplitudes depicted in Fig. 2. Decays of this type have been already mentioned in Ref. [45]. We focus here on the final state with a single charged lepton whose decay width is most severely bounded experimentally. In this case the following $d = 9$ effective operator is obtained:

$$\mathcal{L}_9 = \sum_{U=c,t} \frac{-8G_F V_{U\alpha} V_{u\gamma}^*}{m_U m_\Delta^2} [U_C(Y^{10} - Y^{10T})^\dagger U_C^*]_{1U} \times [D_C^\dagger Y^{\bar{5}\dagger} E_C^*]_{\beta i} \epsilon_{abc} (\overline{u_a^C} \gamma^\mu L d_{b\alpha})(\overline{d_{c\beta}^C} R \ell_i) \times (\overline{d_{k\gamma}} \gamma_\mu L u_k). \tag{74}$$

Here U labels c or t quark, whereas external leptons $i, j = 1, 2$ and down-type quarks $\alpha, \beta, \gamma = 1, 2$, are all light. a, b, c, k are $SU(3)$ color indices. $R(= (1 + \gamma_5)/2)$ is the right projection operator. We focus immediately on best constrained channels, i.e., $p \to \pi^0 \ell_i^+$, and we set α, β, $\gamma = 1$. The use of Fierz transformations leads to the amplitude with scalar bilinears

$$\mathcal{M}_9^{p\to\pi^0 e_i^+} = \sum_{U=c,t} \frac{8iG_F}{m_\Delta^2 m_U} [U_C(Y^{10} - Y^{10T})^\dagger U_C^*]_{1U} \times [D_C^\dagger Y^{\bar{5}\dagger} E_C^*]_{1i} V_{Ud} V_{ud}^* \epsilon_{abc} \langle \pi^0 \ell_i^+ | (\overline{\ell_i^C} R u_a) \times (\overline{d_k} R d_c)(\overline{u_k^C} L d_b) | p \rangle + \text{tensor terms}. \tag{75}$$

One can estimate the above matrix element by employing the vacuum saturation approximation. We insert the current $\overline{d_k} R d_c$ between the vacuum and π and end up with product of pion creation and proton annihilation amplitudes. The tensor terms which are invoked by the Fierz relations in Eq. (75) cannot contribute in this case. The vacuum-to-pion amplitude is

$$\langle \pi^0 | \overline{d_k} R d_c | 0 \rangle = \frac{-i m_\pi^2 f_\pi}{4\sqrt{2} m_d} \delta_{ck}, \tag{76}$$

whereas the full amplitude is

$$\mathcal{M}_9^{p\to\pi^0 e_i^+} = \sum_{U=c,t} \frac{-\sqrt{2} G_F}{m_\Delta^2 m_U} [U_C(Y^{10} - Y^{10T})^\dagger U_C^*]_{1U} \times [D_C^\dagger Y^{\bar{5}\dagger} E_C^*]_{1i} V_{Ud} V_{ud}^* \frac{m_\pi^2 f_\pi}{m_d} \times \epsilon_{abc} \langle \ell_i^+ | (\overline{u_a^C} L d_b)(\overline{\ell_i^C} R u_c) | p \rangle. \tag{77}$$

The annihilation matrix element of the proton in Eq. (77) has been most precisely evaluated using lattice QCD [48]. These authors have introduced operators $O_{uds}^{\Gamma\Gamma'} = \epsilon_{abc} (\overline{u_a^C} \Gamma d_b) \Gamma' s_c$ and defined constant α as

$$\alpha R u_p = -\langle 0 | O_{udu}^{LR} | p \rangle, \tag{78}$$

where u_p is the Dirac spinor of the proton. The recent value of α obtained from lattice QCD calculation with domain wall fermions [43] is $\alpha = -0.0112(25)\ \text{GeV}^3$. The decay width is then

$$\Gamma(p \to \pi^0 \ell_i^+) = \frac{G_F^2 f_\pi^2 m_\pi^4 \alpha^2}{16\pi m_d^2} \frac{\lambda(m_p^2, m_{\ell_i}^2, m_\pi^2)^{1/2} (m_p^2 + m_{\ell_i}^2 - m_\pi^2)}{m_p^3} \times \left| \sum_{U=c,t} \frac{V_{Ud} [U_C(Y^{10} - Y^{10T})^\dagger U_C^*]_{1U} [D_C^\dagger Y^{\bar{5}\dagger} E_C^*]_{1i}}{m_U m_\Delta^2} \right|^2, \tag{79}$$

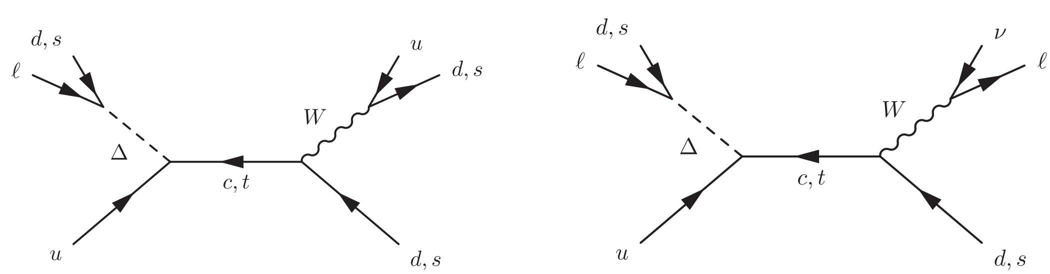

FIG. 2. $d = 6$ proton decay operator induced by tree-level $(\bar{\mathbf{3}}, \mathbf{1}, 4/3) \in 45$ and W exchanges.

where $\lambda(x, y, z) \equiv (x + y + z)^2 - 4(xy + yz + zx)$. From the experimental limits in Table IX one obtains the following bounds:

$$\left| [D_C^\dagger Y^{\bar{5}\dagger} E_C^*]_{11} \sum_{U=c,t} \frac{V_{U1}[U_C(Y^{10} - Y^{10T})^\dagger U_C^*]_{1U}}{m_U} \right| < 2.4 \times 10^{-20} \frac{m_\Delta^2}{(400\ \text{GeV})^2}\ \text{GeV}^{-1}, \quad (80)$$

$$\left| [D_C^\dagger Y^{\bar{5}\dagger} E_C^*]_{12} \sum_{U=c,t} \frac{V_{U1}[U_C(Y^{10} - Y^{10T})^\dagger U_C^*]_{1U}}{m_U} \right| < 2.6 \times 10^{-20} \frac{m_\Delta^2}{(400\ \text{GeV})^2}\ \text{GeV}^{-1}. \quad (81)$$

A comment is in order how phenomenologically preferred values of leptoquark and diquark couplings cope with the above constraints. Couplings to the electrons should be small and are in particular not bounded from below, so the constraint from $\tau(p \to \pi^0 e^+)$ can be avoided by putting $[D_C^\dagger Y^{\bar{5}\dagger} E_C^*]_{11}$ effectively to zero. On the contrary, low-energy leptoquark constraints, especially the $(g-2)_\mu$, indicate that $[D_C^\dagger Y^{\bar{5}\dagger} E_C^*]_{12}$ could be large in some scenarios [31]. In this case we must require cancellation between the c and t quark amplitudes which occurs when

$$\frac{[U_C(Y^{10} - Y^{10T})^\dagger U_C^*]_{12}}{[U_C(Y^{10} - Y^{10T})^\dagger U_C^*]_{13}} \approx -\frac{V_{td}}{V_{cd}} \frac{m_c}{m_t} \approx 2.7 \times 10^{-4} \times e^{-0.37i}. \quad (82)$$

This can be achieved since the $[U_C(Y^{10} - Y^{10T})^\dagger U_C^*]_{12}$ is only bounded from above while at the same time $[U_C(Y^{10} - Y^{10T})^\dagger U_C^*]_{13}$ is bounded from below to satisfy observations in $t\bar{t}$ production. Finally, relative phase between the two couplings can be freely adjusted since it is not probed by any experimental observable to date.

Finally, for the state $(\bar{\mathbf{3}}, \mathbf{1}, -2/3)^{+6}$ present in the flipped $SU(5)$ framework we can easily adapt the results obtained above since the two states are indistinguishable at low energies, provided we make the following substitutions:

$$[U_C^\dagger (Y^{10*} - Y^{10\dagger}) U_C^*] \to \frac{1}{2}[U_C^\dagger Y^{\bar{5}\dagger} U_C^*], \quad [D_C^\dagger Y^{\bar{5}\dagger} E_C^*] \to -[D_C^\dagger Y^{1*} E_C^*]. \quad (83)$$

To conclude, we note that despite the absence of the tree-level contribution to proton decay of the $(\bar{\mathbf{3}}, \mathbf{1}, 4/3)$ state, weak corrections lead to proton destabilizing $d = 6$ and $d = 9$ operators. The effect of the $d = 9$ operators can be rendered adequately small even in the case of simultaneously large leptoquark *and* diquark couplings, a situation which is favored by observables in $t\bar{t}$ production and value of $(g-2)_\mu$. This is achieved by the finely-tuned cancellation of two amplitudes. To the contrary, similar cancellation is impossible in the case of $d = 6$ operator for $p \to \pi^0 \mu^+$ decay and we are required to suppress either all leptoquark couplings involving μ or all diquark couplings. We conclude that the proton decay lifetime constraint allows to fully address either $A_{FB}^{t\bar{t}}$ or $(g-2)_\mu$ observable with the $(\bar{\mathbf{3}}, \mathbf{1}, 4/3)$ state, but not both.

VII. CONCLUSIONS

Lepton and baryon number violating interactions are inherently present within grand unified theories and are most severely constrained by the observed proton stability. Proton decay can be mediated by vector or scalar leptoquarks which violate both baryon and lepton number by one unit. Vector leptoquarks which mediate proton decay have gauge couplings to fermions and are not readily allowed to be far below the unification scale. For the scalar leptoquarks, however, the freedom in Yukawa couplings gives one more maneuverability to realize scenarios with light scalar states. On the other hand, the very same Yukawas which are responsible for proton decay very often need to account for the observed fermion mass spectrum. An example of a setting with light leptoquark states was presented in Refs. [15,31,32] where the low mass of the state $(\bar{\mathbf{3}}, \mathbf{1}, 4/3)$ had an impact on low-energy flavor phenomenology. Most notably, it was found that by tuning independently the two sets of Yukawa couplings, namely the leptoquark and diquark Yukawas, one could reconcile the measured value of forward-backward asymmetry in $t\bar{t}$ production and the value of the magnetic moment of muon.

In this work, we have classified the scalar leptoquarks present in $SU(5)$ and flipped $SU(5)$ grand unification frameworks which mediate proton decay. In both frameworks the considered leptoquark states reside in scalar representations of $SU(5)$ of dimension five, ten, forty-five, or fifty. We integrate out the above states at tree-level and parametrize their contributions in terms of effective coefficients of a complete set of dimension-six effective operators. The mass constraint on the color triplet state contained in the five- and forty-five-dimensional representations is then derived. The precise lower bound depends on the value of the vacuum expectation values of these representations. For the vacuum expectation value of 100 GeV, the least (most) conservative lower bound on the triplet mass which originates from the $p \to K^+ \bar{\nu}$ channel is approximately 10^{11} GeV (10^{13} GeV). The corresponding bound is derived within the flipped $SU(5)$ framework to read 10^{12} GeV and proves to be mixing independent. Moreover, the minimal flipped $SU(5)$ theory with symmetric mass matrices predicts $\Gamma(p \to \pi^+ \bar{\nu})/\Gamma(p \to K^+ \bar{\nu}) = 9$.

The two leptoquark states which do not contribute to proton decay at tree-level are $(\bar{\mathbf{3}}, \mathbf{1}, 4/3)$ and $(\bar{\mathbf{3}}, \mathbf{1}, -2/3)^{+6}$ in the standard and flipped $SU(5)$ frameworks, respectively. We have estimated their contribution to dimension-six operators via box diagram and the tree-level contribution to dimension-nine operators. For the

($\bar{\mathbf{3}}$, $\mathbf{1}$, 4/3) state, it has been found that if it is to explain both the anomalous magnetic moment of the muon and the $t\bar{t}$ forward-backward asymmetry, then the contribution of the dimension-six operator would destabilize the proton in $p \rightarrow \mu^+ \pi^0$ channel. Therefore, only one of the two puzzles can be addressed with this leptoquark state.

Light scalar leptoquarks can be either produced in pairs or in association with SM fermions at the LHC and are a subject of leptoquark and diquark resonance searches [6,49–51]. To conclude, we can expect to find signals of these leptoquark states at the LHC, although it seems very unlikely, in light of the constraints from the proton lifetime measurements, that they would be observed in a baryon number violating processes.

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