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# Two Higgs bidoublet model with spontaneous *P* and *CP* violation and decoupling limit to the two Higgs doublet model

Jin-Yan Liu, Li-Ming Wang, Yue-Liang Wu, and Yu-Feng Zhou

*State Key Laboratory of Theoretical Physics, Kavli Institute for Theoretical Physics China, and Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, People's Republic of China*

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The two Higgs bidoublet left-right symmetric model (2HBDM) as a simple extension of the minimal left-right symmetric model with a single Higgs bidoublet is motivated to realize both spontaneous parity (*P*) violation and charge conjugation and parity (*CP*) violation while consistent with the low-energy phenomenology without significant fine-tuning. By carefully investigating the Higgs potential of the model, we find that sizable *CP*-violating phases are allowed after the spontaneous symmetry breaking. The mass spectra of the extra scalars in the 2HBDM are significantly different from the ones in the minimal left-right symmetric model. In particular, we demonstrate in the decoupling limit when the right-handed gauge-symmetry-breaking scale is much higher than the electroweak scale, the 2HBDM decouples into the general two Higgs doublet model with spontaneous *CP* violation and has rich induced sources of *CP* violation. We show that in the decoupling limit, it contains extra light Higgs bosons with masses around electroweak scale, which can be directly searched at the ongoing LHC and future International Linear Collider experiments.

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## I. INTRODUCTION

The left-right symmetric models [1–3] based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ are extensions of the standard model (SM) motivated by explaining the origin of parity (*P*) violation and the smallness of neutrino masses. In general, it is expected that charge conjugation and parity (*CP*) violation can also be realized as a consequence of spontaneous symmetry breaking [4] in this type of model [5–9]. One of the extensively studied left-right symmetric models is the minimal left-right symmetric model which contains two $SU(2)$ triplets and one bidoublet in the Higgs sector. Despite its simplicity and success in generating the tiny neutrino masses, it suffers from a series of constraints in the Higgs and fermion sector from low-energy phenomenology. It has been shown that in this model the lightest extra Higgs boson has to be heavier than ~10 TeV in order to suppress the tree-level flavor-changing neutral current (FCNC) in neutral kaon meson mixing [9–11]. The conditions for minimizing the Higgs potential lead to the observation that without significant fine-tuning in the potential parameters, the *CP* phases in the vacuum expectation values (VEVs) of the Higgs fields are nearly vanishing [12–14]. In the minimal left-right symmetric model, the Yukawa couplings for both neutral and charged Higgs bosons are fixed by the quark masses and Cabibbo-Kobayashi-Maskawa (CKM) matrix, so that all the *CP*-violating phases are calculable quantities in terms of quark masses and the ratios of the vacuum expectation values (VEVs) of the bidoublet. It has been shown that in the decoupling limit in which the vacuum expectation of the right-handed triplet approaches infinity, the model fails to reproduce the precisely measured weak phase angle $\sin 2\beta$ from *B* factories [9]. Furthermore, from the VEV seesaw mechanism, the $\beta$ parameters in the Higgs potential have to be fine-tuned to be six to seven orders of magnitude smaller than other model parameters in order to meet the experimental bound on both light and heavy neutrino masses [14], if the right-handed scale remains in the TeV range, which is accessible by the current LHC. Given the above-mentioned difficulties in the minimal left-right symmetric model, one may simply give up the spontaneous *CP* violation in the minimal left-right symmetric model by considering explicit *CP* violation in the Higgs potential and/or the Yukawa sector [15–19]. However, a detailed analysis showed that little improvement can be achieved in phenomenology. An alternative treatment for spontaneous *P* and *CP* violation was to introduce mirror particles in a model based on $[SU(2) \times U(1)]^2$ gauge symmetry [20,21].

Motivated by the success of generating spontaneous *CP* violation from the general two Higgs doublet model (2HDM) [22–25], an extension of the minimal left-right symmetric model with two Higgs bidoublets (2HBDM), which can break the *CP* symmetry spontaneously, has been proposed [26,27]. In this paper we show how the 2HBDM can relax the stringent constraints mentioned above for the minimal left-right symmetric model, and in which case it can decouple to the 2HDM. It has been shown in [26,27] that such a simply extended model can be consistent with the low-energy phenomenology in flavor physics. In this work, we shall concentrate on the details of the generalized Higgs potential and the vacuum minimal conditions, and demonstrate how such a model can avoid the fine-tuning problem in generating sizable *CP*-violating phases, so that the left-right symmetric 2HBDM with spontaneous *P* and *CP* violation could become more realistic at the TeV scale. We focus on the mass spectrum of Higgs bosons in the

 

2HBDM. Different from the minimal model with only one light neutral Higgs boson similar to the standard model, we shall show that there exist in general three light neutral Higgs bosons and one pair of light charged Higgs bosons in the decoupling limit of 2HBDM, which means that the 2HBDM decouples to 2HDM when $v_R \to \infty$. Such a feature differs completely from the minimal left-right symmetric model.

The paper is organized as follows: In Sec. II, we give an overview of the problems appearing in the minimal left-right symmetric model. In Sec. III, we present the most general Higgs potential with two Higgs bidoublets and demonstrate in an explicit way why such a generalization can save the left-right symmetric model from the above-mentioned problems arising in the minimal left-right symmetric model, and the possible new physics at TeV scale. In Sec. IV, we show that the 2HBDM can decouple to 2HDM in the decoupling limit and then extend the result to general cases. The conclusions and remarks are given in the last section.

## II. OVERVIEW OF THE MINIMAL LEFT-RIGHT SYMMETRIC MODEL

The Higgs sector in the minimal model consists of one Higgs bidoublet and two Higgs triplets:

$$\phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix} \sim (2, 2, 0), \tag{1}$$

$$\Delta_L = \begin{pmatrix} \delta_L^+/\sqrt{2} & \delta_L^{++} \\ \delta_L^0 & -\delta_L^+/\sqrt{2} \end{pmatrix} \sim (3, 1, 2), \tag{2}$$

$$\Delta_R = \begin{pmatrix} \delta_R^+/\sqrt{2} & \delta_R^{++} \\ \delta_R^0 & -\delta_R^+/\sqrt{2} \end{pmatrix} \sim (1, 3, 2), \tag{3}$$

where the numbers in the brackets denote the quantum number of Higgs multiplets under the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. The neutral parts of Higgs fields obtain VEV in such pattern:

$$\langle\phi\rangle = \begin{pmatrix} k_1/\sqrt{2} & 0 \\ 0 & k_2 e^{i\theta_2}/\sqrt{2} \end{pmatrix}$$
$$\langle\Delta_{L,R}\rangle = \begin{pmatrix} 0 & 0 \\ v_{L,R} e^{i\theta_{L,R}}/\sqrt{2} & 0 \end{pmatrix}. \tag{4}$$

And the most general Higgs potential is given by [14]:

$$\begin{aligned}
V_{\phi\Delta} = & -\mu_1^2 \mathrm{Tr}(\phi^+\phi) - \mu_2^2[\mathrm{Tr}(\tilde{\phi}\phi^+ + \tilde{\phi}^+\phi)] - \mu_3^2[\mathrm{Tr}(\Delta_L\Delta_L^+) + \mathrm{Tr}(\Delta_R\Delta_R^+)] \\
& + \lambda_1 \mathrm{Tr}^2(\phi\phi^+) + \lambda_2[\mathrm{Tr}^2(\tilde{\phi}\phi^+) + \mathrm{Tr}^2(\tilde{\phi}^+\phi)] \\
& + \lambda_3 \mathrm{Tr}(\tilde{\phi}\phi^+)\mathrm{Tr}(\tilde{\phi}^+\phi) + \lambda_4 \mathrm{Tr}(\phi\phi^+)\mathrm{Tr}(\tilde{\phi}\phi^+ + \tilde{\phi}^+\phi) \\
& + \rho_1[\mathrm{Tr}^2(\Delta_L\Delta_L^+) + \mathrm{Tr}^2(\Delta_R\Delta_R^+)] \\
& + \rho_2[\mathrm{Tr}(\Delta_L\Delta_L)\mathrm{Tr}(\Delta_L^+\Delta_L^+) + \mathrm{Tr}(\Delta_R\Delta_R)\mathrm{Tr}(\Delta_R^+\Delta_R^+)] \\
& + \rho_3 \mathrm{Tr}(\Delta_L\Delta_L^+)\mathrm{Tr}(\Delta_R\Delta_R^+) + \rho_4[\mathrm{Tr}(\Delta_L\Delta_L)\mathrm{Tr}(\Delta_R^+\Delta_R^+) + \mathrm{Tr}(\Delta_L^+\Delta_L^+)\mathrm{Tr}(\Delta_R\Delta_R)] \\
& + \alpha_1 \mathrm{Tr}(\phi\phi^+)[\mathrm{Tr}(\Delta_L\Delta_L^+ + \Delta_R\Delta_R^+)] + \alpha_2 \mathrm{Tr}(\tilde{\phi}\phi^+ + \tilde{\phi}^+\phi)\mathrm{Tr}(\Delta_R\Delta_R^+ + \Delta_L\Delta_L^+) \\
& + \alpha_3 \mathrm{Tr}(\phi\phi^+\Delta_L\Delta_L^+ + \phi^+\phi\Delta_R\Delta_R^+) \\
& + \beta_1 \mathrm{Tr}(\phi\Delta_R\phi^+\Delta_L^+ + \phi^+\Delta_L\phi\Delta_R^+) + \beta_2 \mathrm{Tr}(\tilde{\phi}\Delta_R\phi^+\Delta_L^+ + \tilde{\phi}^+\Delta_L\phi\Delta_R^+) \\
& + \beta_3 \mathrm{Tr}(\phi\Delta_R\tilde{\phi}^+\Delta_L^+ + \phi^+\Delta_L\tilde{\phi}\Delta_R^+).
\end{aligned} \tag{5}$$

There are three independent vacuum minimal conditions, after eliminating $\mu_{1,2,3}$ parameters:

$$(2\rho_1 - \rho_3)v_L v_R = \beta_1 k_1 k_2 \cos(\theta_L - \theta_2) + \beta_2 k_1^2 \cos\theta_L + \beta_3 k_2^2 \cos(\theta_L - 2\theta_2), \tag{6a}$$

$$0 = \beta_1 k_1 k_2 \sin(\theta_L - \theta_2) + \beta_2 k_1^2 \sin\theta_L + \beta_3 k_2^2 \sin(\theta_L - 2\theta_2), \tag{6b}$$

$$0 = k_1 k_2[\alpha_3(v_R^2 + v_L^2) + (4\lambda_3 - 8\lambda_2)(k_1^2 - k_2^2)]\sin\theta_2 + v_R v_L \cdot \beta k^2 \text{ terms}, \tag{6c}$$

where $k^2 = k_1^2 + k_2^2$ represents electroweak (EW) scale. From Eq. (6a) one can obtain the so-called VEV seesaw relation,

$$\gamma \equiv \frac{\beta}{\rho} = \frac{v_L v_R}{k^2}, \tag{7}$$

which indicates a big gap between $v_L$ and $v_R$ to produce a correct small neutrino mass. If $\rho$ and $\beta$ parameters are within their normal range, i.e., there is no fine-tuning in Higgs sector, $v_R$ has to go up to $10^7$ GeV, as shown in the literature [14]. On the contrary, if $v_R$ is set to several TeV to obtain TeV phenomenology, $\beta$ parameters have to be fine-tuned to $10^{-7}$. The third equation would lead to a severe fine-tuning problem and contradict to phenomenological bounds on neutral Higgs mass. By diagonalizing the Higgs mass matrix, one finds the FCNC-violating Higgs mass is

$$M^2_{\rm FCNC} \sim \frac{1}{2}\alpha_3 v_R^2 \frac{1}{\sqrt{1-\frac{2k_1k_2}{k^2}}}. \tag{8}$$

The lower bound of $M_{\rm FCNC}$ constrained by low-energy phenomenology is 10 TeV. Thus it is obvious that the third equation is hardly satisfied unless vacuum phases $\theta_2$ and $\theta_L$ are fine-tuned to very small values and the model fails to produce a right normal-sized vacuum *CP* phase. Combining the constraints from the neutral Higgs mass and the FCNC Higgs mass, one finds immediately that the fine-tuning problem is inevitable in the minimal model: one has to fine-tune $\alpha_3$ when $v_R$ goes up to $10^7$ GeV while keeping $M^2_{\rm FCNC}$ around 10 TeV; or else one has to fine-tune $\beta$ when $\alpha_3$ remains at a normal size.

From the above analysis, one sees clearly that in the minimal model there is severe inconsistence in the Higgs potential for yielding correct phenomenology. The vacuum minimal condition, neutrino mass, and FCNC bounds contradict each other, so that one has to make a big concession on the naturalness of the parameters in the Higgs sector, including the fine-tuned nearly zero vacuum *CP* phase, losing elegance and failing in spontaneous *CP* violation. The fundamental reason of this self-inconsistence results from the fact that the fermion-Higgs couplings are too strongly constrained by the left-right symmetry. This is exactly why we want to add an extra Higgs bidoublet to relax the Yukawa sector.

## III. THE TWO HIGGS BI-DOUBLET LEFT-RIGHT SYMMETRIC MODEL

We simply add an extra Higgs bidoublet $\chi$ into the Higgs sector:

$$\chi = \begin{pmatrix} \chi_1^0 & \chi_2^+ \\ \chi_1^- & \chi_2^0 \end{pmatrix} \sim (2, 2, 0), \tag{9}$$

which has the same gauge property as $\phi$ in the minimal model. The overall neutral parts of Higgs fields obtain VEV in such pattern:

$$\langle\phi\rangle = \begin{pmatrix} \kappa_1 e^{i\theta_1^p}/\sqrt{2} & 0 \\ 0 & \kappa_2 e^{i\theta_2^p}/\sqrt{2} \end{pmatrix}, \quad \langle\chi\rangle = \begin{pmatrix} \omega_1 e^{i\theta_1^c}/\sqrt{2} & 0 \\ 0 & \omega_2 e^{i\theta_2^c}/\sqrt{2} \end{pmatrix}, \tag{10a}$$

$$\langle\Delta_L\rangle = \begin{pmatrix} 0 & 0 \\ v_L e^{i\theta_L}/\sqrt{2} & 0 \end{pmatrix}, \quad \langle\Delta_R\rangle = \begin{pmatrix} 0 & 0 \\ v_R e^{i\theta_R}/\sqrt{2} & 0 \end{pmatrix}. \tag{10b}$$

Note now there are in total six *CP* phases in the vacuum parameters, two of which can be rotated away by gauge group action. We define here the other four gauge-invariant *CP* phases,

$$\theta^p = \theta_1^p + \theta_2^p, \quad \theta^c = \theta_1^c + \theta_2^c,$$
$$\theta^{pc} = (\theta_1^c - \theta_2^c) - (\theta_1^p - \theta_2^p), \quad \theta^{LR} = \theta_L - \theta_R \tag{11}$$

In our following discussion, we take $\theta_1^p = \theta_R = 0$ unless otherwise noted. Next we will comment shortly on the new features of model structures.

### A. New features of 2HBDM

With the extra bidoublet $\chi$, the resulting Lagrangian of 2HBDM has new features in its structure at tree level, which leads to remarkable differences in phenomenological descriptions. To facilitate further discussion, we assume that the VEVs satisfy the hierarchy structure $v_L \ll \kappa_{1,2}, \omega_{1,2} \ll v_R$. Also, the *P* and *CP* symmetry are assumed.

*Gauge sector.*— There is no change on the Fermion-gauge part. The Higgs-gauge sector is altered with more complicated Higgs-gauge interactions. As a result, the gauge boson mass after spontaneous symmetry breaking (SSB) is slightly changed. The mass matrices for charged gauge bosons under basis $\{W_L^+, W_R^+\}$ and for neutral ones under basis $\{W_L^3, W_R^3, B\}$ are

$$\tilde{M}^W = \frac{g^2}{4}\begin{pmatrix} v^2 + 2v_L^2 & -2(\kappa_1^*\kappa_2 + \omega_1^*\omega_2) \\ -2(\kappa_1\kappa_2^* + \omega_1\omega_2^*) & v^2 + 2v_R^2 \end{pmatrix}, \tag{12a}$$

$$\tilde{M}^0 = \frac{1}{2}\begin{pmatrix} \frac{g^2}{2}(v^2 + 4v_L^2) & -\frac{g^2}{2}v^2 & -2gg'v_L^2 \\ -\frac{g^2}{2}v^2 & \frac{g^2}{2}(v^2 + 4v_R^2) & -2gg'v_R^2 \\ -2gg'v_L^2 & -2gg'v_R^2 & 2g'^2(v_L^2 + v_R^2) \end{pmatrix}, \tag{12b}$$

with

$$g \equiv g_L = g_R, \quad v^2 \equiv \kappa_1^2 + \kappa_2^2 + \omega_1^2 + \omega_2^2, \tag{13a}$$

$$\kappa_1^*\kappa_2 = \kappa_1\kappa_2 e^{i(\theta_2^p - \theta_1^p)}, \quad \omega_1^*\omega_2 = \omega_1\omega_2 e^{i(\theta_2^c - \theta_1^c)}. \tag{13b}$$

where $v \simeq 246$ GeV is the electroweak scale. Following the same procedure in [28,29], one can obtain the physical gauge boson mass and the mixing angles, where for $Z_{1,2}$ gauge bosons nothing changed except for the definition of $v$.

$$\begin{pmatrix} Z_L \\ Z_R \\ A \end{pmatrix} = \begin{pmatrix} c_W & -s_W t_W & -t_W\sqrt{c_{2W}} \\ 0 & \sqrt{c_{2W}}/c & -t_W \\ s_W & s_W & \sqrt{c_{2W}} \end{pmatrix}\begin{pmatrix} W_{3L} \\ W_{3R} \\ B \end{pmatrix}, \tag{14}$$

where $A$ is the photon, and $Z_{L,R}$ are neutral gauge bosons with mixing

$$\tilde{M}^Z = \frac{g^2}{4}\begin{pmatrix} (v^2 + 4v_L^2)c_W^2 & -(v^2(1 - t_W^2) - 4v_L^2 t_W^2)/\sqrt{c_{2W}} \\ -(v^2(1 - t_W^2) - 4v_L^2 t_W^2)/\sqrt{c_{2W}} & 4c_W^2(v_R^2 + v^2/s_{2W}^2 + t_W^4 v_L^2)/c_{2W} \end{pmatrix}. \tag{15}$$

The physical $Z_{1,2}$ gauge bosons are defined by

$$\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} \cos\xi & -\sin\xi \\ \sin\xi & \cos\xi \end{pmatrix} \begin{pmatrix} Z_L \\ Z_R \end{pmatrix}, \tag{16}$$

and the physical masses are found to be

$$M^2_{Z_{1,2}} = \frac{1}{4}\{[g^2v^2 + 2(g^2 + g'^2)(v_L^2 + v_R^2)] \mp \sqrt{[g^2v^2 + 2(g^2 + g'^2)(v_L^2 + v_R^2)]^2 - 4g^2(g^2 + 2g'^2)(v_L^2 + v_R^2)v^2}\}, \tag{17}$$

with the mixing angle $\zeta$ given by

$$\sin 2\xi = -\frac{g^2v^2\sqrt{\cos 2\theta_W}}{2\cos^2\theta_W(M^2_{Z_2} - M^2_{Z_1})}. \tag{18}$$

For the physical gauge bosons $W_{1,2}$, they are defined as

$$\begin{pmatrix} W_1^+ \\ W_2^+ \end{pmatrix} = \begin{pmatrix} \cos\zeta & -\sin\zeta e^{-i\eta} \\ \sin\zeta e^{i\eta} & \cos\zeta \end{pmatrix} \begin{pmatrix} W_L^+ \\ W_R^+ \end{pmatrix}, \tag{19}$$

with masses

$$M^2_{W_{1,2}} = \frac{g^2}{4}\left(v^2 + v_L^2 + v_R^2 \mp \frac{v_R^2 - v_L^2}{\cos 2\zeta}\right). \tag{20}$$

It is noted that the difference here is the mixing angle between $W_1$ and $W_2$, which is replaced as

$$\zeta \sim \frac{\tan 2\zeta}{2} = -\frac{|\kappa_1\kappa_2 e^{i(\theta_2^p - \theta_1^p)} + \omega_1\omega_2 e^{i(\theta_2^c - \theta_1^c)}|}{v_R^2 - v_L^2} \simeq -r \times \frac{M^2_{W_1}}{M^2_{W_2}}, \tag{21}$$

where $r = 2|\kappa_1\kappa_2 + \omega_1\omega_2 e^{i\theta^{pc}}|/v^2$.

One may see from Eqs. (12a) and (13b) that the imaginary part of $W_{1,2}$ mixing is

$$-\mathrm{Im}(\tilde{M}^W_{12}) = 2w_1w_2\cos(\theta_2^p - \theta_1^p)\sin\theta^{pc} + 2(\kappa_1\kappa_2 + \omega_1\omega_2\cos\theta^{pc})\sin(\theta_2^p - \theta_1^p) \tag{22}$$

and the complex phase $\eta$ is

$$\sin\eta = \frac{\mathrm{Im}(\tilde{M}^W_{12})}{\tan 2\zeta(v_R^2 - v_L^2)}. \tag{23}$$

The second term in Eq. (22) vanishes when $\theta_2^p - \theta_1^p$ is rotated away by gauge symmetry, whereas the first term always remains nonzero since $\theta^{pc}$ is gauge invariant. This distinguishes 2HBDM from the minimal left-right symmetric model in which the phase of $W_{1,2}$ mixing can be rotated away entirely.

The physical Higgs-gauge interaction depends on the mixings of the Higgs sector. The SM-like Higgs coupling to gauge bosons in the minimal left-right symmetric model resembles those in the SM in the limit $\kappa_2 \ll \kappa_1$ and $\kappa_1 \ll v_R$, whereas in 2HBDM the couplings might differ from the SM ones, which is due to its 2HDM nature in the decoupling limit. We will further discuss it in following sections.

*Yukawa sector.*—The general form of quark Yukawa couplings is

$$\mathcal{L}_Y = -\sum_{i,j}\bar{Q}_{iL}((y_q)_{ij}\phi + (\tilde{y}_q)_{ij}\tilde{\phi} + (h_q)_{ij}\chi + (\tilde{h}_q)_{ij}\tilde{\chi})Q_{jR}, \tag{24}$$

which induces the quark mass term after SSB,

$$M_u = \frac{1}{\sqrt{2}}(y_q\kappa_1 + \tilde{y}_q\kappa_2 e^{i\theta_2^p} + h_q\omega_1 e^{i\theta_1^c} + \tilde{h}_q\omega_2 e^{i\theta_2^c}),$$
$$M_d = \frac{1}{\sqrt{2}}(y_q\kappa_2 e^{-i\theta_2^p} + \tilde{y}_q\kappa_1 + h_q\omega_2 e^{-i\theta_2^c} + \tilde{h}_q\omega_1 e^{-i\theta_1^c}). \tag{25}$$

$P$ symmetry requires

$$y_q = y_q^\dagger, \quad \tilde{y}_q = \tilde{y}_q^\dagger, \quad h_q = h_q^\dagger, \quad \tilde{h}_q = \tilde{h}_q^\dagger. \tag{26}$$

When both $P$ and $CP$ are required to be broken down spontaneously, all the Yukawa coupling matrices are real and symmetric. As there are in total $6 \times 4$ free parameters in $y_q$, $\tilde{y}_q$ and $h_q$, $\tilde{h}_q$, two significant consequences follow:

(1) The very stringent bound on the minimal model largely results from the fact that the CKM phases are all calculable quantities given quark masses, mixing angles and ratio of VEVs; while in 2HBDM, although the relation $V_L^{\mathrm{CKM}} = (V_R^{\mathrm{CKM}})^*$ still holds (pseudomanifest), there are more freedoms in the Yukawa couplings and no direct connection exists between CKM phases and other input parameters.

(2) The general form of Eq. (25) generates large FCNC at tree level. The situation gets worse when the mass of FCNC Higgs is brought down to the EW scale. As shown in our previous works [26,27], the tree level FCNC could be suppressed following the similar treatment in the general 2HDM by considering the mechanism of approximate global $U(1)$ family symmetry [23–25],

$$(u_i, d_i) \to e^{-i\theta_i}(u_i, d_i), \tag{27}$$

which is motivated by the approximate unity of the CKM matrix. As a consequence, $y$, $\tilde{y}$, $h$, and $\tilde{h}$ are nearly diagonal matrices.

*Higgs sector.*—Based on the general form of the Higgs potential in the minimal left-right symmetric model, we carefully write down the most general form of Higgs potential for the 2HBDM, which is listed in Eq. (A1). From that potential, we can obtain the vacuum minimal conditions and find that the tension between spontaneous $CP$ violation and scale hierarchy is largely relaxed; hence, the new model can generate sizable vacuum phases as the source of $CP$ violation, whereas the VEV seesaw problem still leads to the fine-tuning on the $\beta$ parameters. We shall postpone the details to the next section.

The extended model contains in total 28 freedoms in the Higgs sector, including $4+4+2+2$ neutral ones, $2+2+1+1$ pairs of charged ones, and two pair of doubly charged ones. After spontaneous symmetry breaking, two neutral and two pairs of charged freedoms would become Goldstone bosons absorbed into the longitudinal part of gauge vectors, leaving 10 neutral, four pairs of charged, and two pairs of doubly charged physical Higgs bosons. After carefully studying the Higgs mass spectrum, we find that in the 2HBDM there exist more than one light Higgs boson at the electroweak scale.

From the above analysis, it is seen that the 2HBDM improves the minimal one by introducing more flexible Yukawa couplings, hence allowing for free CKM phases, as well as by enlarging the Higgs sector to avoid the inconsistence in the vacuum minimal conditions. As a consequence, the 2HBDM can be the realistic model with spontaneous $CP$ violation.

### B. Generalized vacuum minimal conditions and spontaneous $CP$ violation

In the 2HBDM, there are 10 independent vacuum parameters which correspond to 10 independent vacuum minimal conditions, i.e.,

$$0=\frac{\partial V}{\partial \kappa_1}=\frac{\partial V}{\partial \kappa_2}=\frac{\partial V}{\partial \omega_1}=\frac{\partial V}{\partial \omega_2}=\frac{\partial V}{\partial v_L}=\frac{\partial V}{\partial v_R}=\frac{\partial V}{\partial \theta_2^p}=\frac{\partial V}{\partial \theta_1^c}=\frac{\partial V}{\partial \theta_2^c}=\frac{\partial V}{\partial \theta_L}. \tag{28}$$

Based on the most general form of Higgs potential given in Eq. (A1), we can write down the general form of all 10 vacuum minimal conditions following the same procedure in the minimal model. By eliminating seven mass dimensional parameters $\mu^2$s, we obtain three independent equations:

$$\begin{aligned}(2\rho_1-\rho_3)v_Lv_R &= \beta_1^p k_1k_2\cos(\theta_L-\theta_2^p)+\beta_2^p k_1^2\cos\theta_L+\beta_3^p k_2^2\cos(\theta_L-2\theta_2^p)\\ &\quad+\beta_1^c w_1w_2\cos(\theta_L+\theta_1^c-\theta_2^c)+\beta_2^c w_1^2\cos(\theta_L+2\theta_1^c)+\beta_3^c w_2^2\cos(\theta_L-2\theta_2^c)\\ &\quad+\beta_1^{pc}k_2w_1\cos(\theta_L+\theta_1^c-\theta_2^p)+\beta_2^{pc}k_1w_2\cos(\theta_L-\theta_2^c)+\beta_3^{pc}k_1w_1\cos(\theta_L+\theta_1^c)\\ &\quad+\beta_4^{pc}k_2w_2\cos(\theta_L-\theta_2^p-\theta_2^c),\end{aligned} \tag{29a}$$

$$\begin{aligned}0 &= \beta_1^p k_1k_2\sin(\theta_L-\theta_2^p)+\beta_2^p k_1^2\sin\theta_L+\beta_3^p k_2^2\sin(\theta_L-2\theta_2^p)\\ &\quad+\beta_1^c w_1w_2\sin(\theta_L+\theta_1^c-\theta_2^c)+\beta_2^c w_1^2\sin(\theta_L+2\theta_1^c)+\beta_3^c w_2^2\sin(\theta_L-2\theta_2^c)\\ &\quad+\beta_1^{pc}k_2w_1\sin(\theta_L+\theta_1^c-\theta_2^p)+\beta_2^{pc}k_1w_2\sin(\theta_L-\theta_2^c)+\beta_3^{pc}k_1w_1\sin(\theta_L+\theta_1^c)\\ &\quad+\beta_4^{pc}k_2w_2\sin(\theta_L-\theta_2^p-\theta_2^c),\end{aligned} \tag{29b}$$

$$\begin{aligned}\frac{v_L}{v_R}\beta O(v^2) &= \left(1+\frac{v_L^2}{v_R^2}\right)[\kappa_1\kappa_2\sin\theta_2^p\alpha_3^p+\omega_1\omega_2\sin(\theta_1^c+\theta_2^c)\alpha_3^c\\ &\quad+(\kappa_2\omega_1\sin(\theta_2^p+\theta_1^c)+\kappa_1\omega_2\sin\theta_2^c)\alpha_3^{pc}\\ &\quad+(\kappa_1\omega_1\sin\theta_1^c-\kappa_2\omega_2\sin(\theta_2^p+\theta_2^c))\alpha_4^{pc}]+\lambda O\left(\frac{(v^2)^2}{v_R^2}\right),\end{aligned} \tag{29c}$$

where $\lambda$ in the third equation stands for a group of $\lambda$ parameters. From Eqs. (29a) and (29b), it is noticed that the $\beta$ parameters still need to be fine-tuned to satisfy the neutrino mass bound for fulfilling a phenomenological model with $v_R$ at TeV scale. The possible explanation for the smallness of $\beta$ parameters could be a softly breaking $Z_2$ or the approximate $U(1)_{P-Q}$ symmetry imposed on a Higgs field; however, these arguments inevitably lead to difficulty in generating correct quark mass hierarchy and quark mixings [14] or may violate the $M_{W_2}-M_N$ relation obtained from experimental constraints such as $0\nu\beta\beta$ [30] or from the K/B meson mixings [9]. In this note we ignore the fine-tuned $\beta$'s and focus on the spontaneous $CP$ violation. Eq. (29c) has the following hierarchy structure:

$$\alpha \gg \lambda O\left(\frac{v^2}{v_R^2}\right) \gg \beta\frac{v_L}{v_R}\ \text{(scale hierarchy)}. \tag{30}$$

In the minimal model, the terms proportional to $\alpha_3^c$, $\alpha_3^{pc}$, and $\alpha_4^{pc}$ are vanishing. Only one term $\kappa_1\kappa_2\sin\theta_2^p\alpha_3^p$ exists, which leads to an extremely small $CP$ phase angle $\theta_2^p$ in order to satisfy Eq. (29c). However, in the 2HBDM, as there are many more free parameters, the $\alpha$ terms can cancel among themselves such that the sum of them is of the order $\lambda$ terms ($O(\frac{v^2}{v_R^2})$),

$$\begin{aligned}\lambda O\left(\frac{(v^2)^2}{v_R^2}\right) &\simeq \kappa_1\kappa_2\sin\theta_2^p\alpha_3^p+\omega_1\omega_2\sin(\theta_1^c+\theta_2^c)\alpha_3^c+[\kappa_2\omega_1\sin(\theta_2^p+\theta_1^c)\\ &\quad+\kappa_1\omega_2\sin\theta_2^c]\alpha_3^{pc}+[\kappa_1\omega_1\sin\theta_1^c-\kappa_2\omega_2\sin(\theta_2^p+\theta_2^c)]\alpha_4^{pc},\end{aligned} \tag{31}$$

the sum of all the $\alpha$ terms may cancel with the $\lambda$ terms, and the final result is of the order of $\beta$ terms ($O(\frac{v_L}{v_R})$). Thus in the 2HBDM, Eqs. (29c) and (30) can both be satisfied with sizable $CP$-violating phases. The condition can naturally be satisfied provided $\alpha_3^p \sim \alpha_3^c \sim \alpha_3^{pc} \sim \alpha_4^{pc}$ and $\kappa_1 \sim \kappa_2 \sim \omega_1 \sim \omega_2$. Hence, it is seen that the 2HBDM potential allows sizable vacuum phases $\theta_2^p$, $\theta_1^c$, $\theta_2^c$, which generate spontaneous $CP$ violation after spontaneous symmetry breaking in the gauge, Higgs and Yukawa sectors through gauge boson mixings, Higgs mixings, and quark mixings.

## IV. DECOUPLING LIMIT TO 2HDM

In this section we give the explicit form of the Higgs mass matrix and separate contributions from different symmetry-breaking scales, i.e., $v_{L,R}$ and electroweak scale $k$. We first study the Higgs sector in the so-called special decoupling limit to 2HDM and then extend it to a general decoupling limit to 2HDM.

*The special decoupling limit to 2HDM.—* Let us first consider a special case of Eq. (31) with the following limit:

$$v_L \ll \kappa_2, \omega_2 \ll \kappa_1, \omega_1 \ll v_R, \tag{32}$$

The reasons to apply Eq. (32) include (1) the electroweak precision test and neutrino mass require $v_L \ll v$; (2) $v/v_R \sim 100$ GeV/1 TeV $\ll 1$ for TeV new physics; (3) $\kappa_2, \omega_2 \ll \kappa_1, \omega_1$ ensures $W_{1,2}$ mixing around $10^{-3}$ [29,30] or smaller. Combining Eq. (31), an immediate consequence of Eq. (32) is

$$|\alpha_4^{pc}| \ll 1. \tag{33}$$

Note that in the above limit the gauge-invariant phases defined in Eq. (11) reduce to (with current choice $\theta_1^p = \theta_R = 0$)

$$\theta^{pc} \equiv \theta_1^c, \qquad \theta^{LR} \equiv \theta_L, \tag{34}$$

and the other two phases $\theta^p$ and $\theta^c$ become physically negligible, as $\theta_2^p$ and $\theta_2^c$ compared to $\theta^{pc}$ hardly affect physical processes.

From the Higgs potential Eq. (A1c), it is not difficult to check that there is mass splitting of bidoublets. The symmetry in the Higgs potential is firstly broken down to $SU(2)_L$ due to large $v_R$. The Higgs bidoublets acquire masses around $v_R$ scale through the $\alpha$-type couplings. The $\alpha_{1,2}^{p,c,pc}$ terms do not break the global $SU(2)_R$ symmetry for bidoublets in the Higgs potential; thus, they do not contribute the masses at $v_R$ scale to bidoublets. The remaining $\alpha_3^{p,c,pc}$ terms ($\alpha_4^{pc}$ is omitted) contribute to bidoublet mass in the following way:

$$\langle \mathrm{Tr}[(X + X^\dagger)\Delta_R \Delta_R^\dagger]\rangle = \mathrm{Tr}[(X + X^\dagger)\cdot \mathcal{P}_{\mathcal{R}}]v_R^2, \tag{35}$$

with

$$\mathcal{P}_{\mathcal{R}} = \frac{1}{2}\begin{pmatrix} 0 & \\ & 1 \end{pmatrix}. \tag{36}$$

Here the left-right asymmetric operator $\mathcal{P}_{\mathcal{R}}$ brings the residual effect of $SU(2)_R$ symmetry breaking into the electroweak sector, resulting in mass splitting among components of bidoublets and the scale hierarchy in the vacuum minimal conditions. Thus, the inconsistence inside the minimal model is clearly seen as the left-right asymmetric $\alpha_3$ term is simultaneously linked with both spontaneous $CP$ violation and FCNC; i.e., the spontaneous $CP$ violation requires a fine-tuned $\alpha_3$ of order $k^2/v_R^2$ to generate sizable $CP$ asymmetry, while the FCNC bound requires a large mass splitting of order 10 TeV. In the 2HBDM, such a tension is moderated through more flexible vacuum structure and Yukawa couplings. To be more precise, let us define the following structure

$$\phi \equiv (\phi_1, \phi_2), \qquad \chi \equiv (\chi_1, \chi_2), \tag{37}$$

where $\phi_1$, $\phi_2$, $\chi_1$, and $\chi_2$ are four doublets. When omitting the mixing between electroweak scale and $\nu_R$ scale, we obtain the following Higgs potential after $SU(2)_R$ symmetry breaking:

$$\begin{aligned} V_{\phi,\chi,\langle\Delta_R\rangle} = &-(\mu_1^p)^2\,\mathrm{Tr}[\phi^\dagger\phi] - (\mu_1^c)^2\,\mathrm{Tr}[\chi^\dagger\chi] - (\mu_1^{pc})^2\,\mathrm{Tr}[\phi^\dagger\chi + \mathrm{H.c.}] \\ &+ \frac{\alpha_1^p v_R^2}{2}\,\mathrm{Tr}[\phi^\dagger\phi] + \frac{\alpha_1^c v_R^2}{2}\,\mathrm{Tr}[\chi^\dagger\chi] + \frac{\alpha_1^{pc} v_R^2}{2}\,\mathrm{Tr}[\phi^\dagger\chi + \mathrm{H.c.}] \\ &- (\mu_2^p)^2\,\mathrm{Tr}[\tilde{\phi}^\dagger\phi] - (\mu_2^c)^2\,\mathrm{Tr}[\tilde{\chi}^\dagger\chi] - (\mu_2^{pc})^2\,\mathrm{Tr}[\tilde{\phi}^\dagger\chi + \mathrm{H.c.}] \\ &+ \frac{\alpha_2^p v_R^2}{2}\,\mathrm{Tr}[\tilde{\phi}^\dagger\phi] + \frac{\alpha_2^c v_R^2}{2}\,\mathrm{Tr}[\tilde{\chi}^\dagger\chi] + \frac{\alpha_2^{pc} v_R^2}{2}\,\mathrm{Tr}[\tilde{\phi}^\dagger\chi + \mathrm{H.c.}] \\ &+ \frac{\alpha_3^p v_R^2}{2}\,\mathrm{Tr}[\phi^\dagger\phi\mathcal{P}_{\mathcal{R}}] + \frac{\alpha_3^c v_R^2}{2}\,\mathrm{Tr}[\chi^\dagger\chi\mathcal{P}_{\mathcal{R}}] + \frac{\alpha_3^{pc} v_R^2}{2}\,\mathrm{Tr}[(\phi^\dagger\chi + \mathrm{H.c.})\mathcal{P}_{\mathcal{R}}] \\ &+ (\lambda - \text{terms}) + (\beta - \text{terms}). \end{aligned} \tag{38}$$

In the limit of Eqs. (32) and (33), all seven $\mu^2$s parameters can be solved from the vacuum minimal conditions in the form

$$\frac{(\mu_1^p)^2}{v_R^2}\simeq\frac{\alpha_1^p}{2},\quad \frac{(\mu_1^c)^2}{v_R^2}\simeq\frac{\alpha_1^c}{2},\quad \frac{(\mu_1^{pc})^2}{v_R^2}\simeq\frac{\alpha_1^{pc}}{2},\quad \frac{(\mu_3)^2}{v_R^2}\simeq\frac{\rho_1}{2},$$
$$\frac{(\mu_2^p)^2}{v_R^2}\simeq\frac{\alpha_2^p}{2},\quad \frac{(\mu_2^c)^2}{v_R^2}\simeq\frac{\alpha_2^c}{2},\quad \frac{(\mu_2^{pc})^2}{v_R^2}\simeq\frac{\alpha_2^{pc}}{2}, \tag{39}$$

where approximation is made by omitting all electroweak scale contributions from $\lambda$ terms shown in Eq. (A1d).

With the definition of Eq. (37), we arrive at the corresponding quadratic terms for the four doublets $(\phi_1, \chi_1, \phi_2, \chi_2)$,

$$\begin{aligned} V^{(2)}_{\phi,\chi,\langle\Delta_R\rangle} = &-(\tilde{\mu}_1^p)^2\phi_1^\dagger\phi_1 - (\tilde{\mu}_1^c)^2\chi_1^\dagger\chi_1 - (\tilde{\mu}_1^{pc})^2(\phi_1^\dagger\chi_1 + \chi_1^\dagger\phi_1) \\ &- [2(\tilde{\mu}_2^p)^2\phi_1^T\varepsilon\phi_2 + 2(\tilde{\mu}_2^c)^2\chi_1^T\varepsilon\chi_2 + (\tilde{\mu}_2^{pc})^2(\phi_1^T\varepsilon\chi_2 - \phi_2^T\varepsilon\chi_1)] + \text{H.c.} \\ &+ (\alpha_3^p v_R^2/2 - (\tilde{\mu}_1^p)^2)\phi_2^\dagger\phi_2 + (\alpha_3^c v_R^2/2 - (\tilde{\mu}_1^c)^2)\chi_2^\dagger\chi_2 \\ &+ (\alpha_3^{pc} v_R^2/2 - (\tilde{\mu}_1^{pc})^2)(\phi_2^\dagger\chi_2 + \text{H.c.}) \\ &+ (\lambda - \text{terms}) + (\beta - \text{terms}), \end{aligned} \tag{40}$$

with

$$\varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \tag{41}$$

where we have redefined the electroweak scale parameters

$$\tilde{\mu}_i^2 = \mu_i^2 - \alpha_i v_R^2/2, \tag{42}$$

which are reasonably small when applying the vacuum minimal conditions Eq. (39) resulted from the limit case in Eqs. (32) and (33). It is manifest that $\phi_1$ and $\chi_1$ will acquire small masses at the electroweak scale after $SU(2)_L$ symmetry breaking from $\lambda$ terms in Eq. (A1d), while $\phi_2$ and $\chi_2$ have masses at the $v_R$ scale from $\alpha_3^{p,c,pc}$ terms. Note that approximate mass degeneration of Higgs fields $\phi_2^0$, $\phi_2^\pm$ in doublet $\phi_2$ and $\chi_2^0$, $\chi_2^\pm$ in doublet $\chi_2$ reveals the fact that they are not involved in $SU(2)_L$ symmetry breaking.

When omitting the terms concerning the heavy Higgs fields $\phi_2$, $\chi_2$ and $\Delta_{L,R}$, we yield the Higgs potential for the electroweak symmetry,

$$\begin{aligned} V_{\phi_1,\chi_1,\langle\Delta_R\rangle} = &-(\tilde{\mu}_1^p)^2\phi_1^\dagger\phi_1 - (\tilde{\mu}_1^c)^2\chi_1^\dagger\chi_1 - (\tilde{\mu}_1^{pc})^2(\phi_1^\dagger\chi_1 + \chi_1^\dagger\phi_1) \\ &+ \lambda_1^p(\phi_1^\dagger\phi_1)^2 + \lambda_1^c(\chi_1^\dagger\chi_1)^2 + \lambda_1^{pc}[(\phi_1^\dagger\chi_1)^2 + (\chi_1^\dagger\phi_1)^2] + \lambda_2^{pc}(\phi_1^\dagger\chi_1)(\chi_1^\dagger\phi_1) \\ &+ \lambda_3^{pc}(\phi_1^\dagger\phi_1)(\phi_1^\dagger\chi_1 + \chi_1^\dagger\phi_1) + \lambda_4^{pc}(\chi_1^\dagger\chi_1)(\chi_1^\dagger\phi_1 + \phi_1^\dagger\chi_1) + \lambda_7^{pc}(\phi_1^\dagger\phi_1)(\chi_1^\dagger\chi_1), \end{aligned} \tag{43}$$

which is exactly in the same form (with 10 independent terms—three $\mu$ terms and seven $\lambda$ terms) as the potential in the general 2HDM model with spontaneous $CP$ violation [22,23].

It is easy to check that in this limit the electroweak part of the gauge and Yukawa sectors is 2HDM-like. For the $SU(2)_L$ gauge-Higgs interactions, it reads

$$\begin{aligned} \text{Tr}[(D_L\phi)^\dagger D_L\phi] &= (D_L\phi_1)^\dagger(D_L\phi_1) + (D_L\phi_2)^\dagger(D_L\phi_2), \\ \text{Tr}[(D_L\chi)^\dagger D_L\chi] &= (D_L\chi_1)^\dagger(D_L\chi_1) + (D_L\chi_2)^\dagger(D_L\chi_2), \end{aligned} \tag{44}$$

and for the Yukawa interactions, the quark-Higgs couplings can be written as

$$\begin{aligned} \mathcal{L}^Y &= \bar{Q}_L(y_q\phi + \tilde{y}_q\tilde{\phi} + h_q\chi + \tilde{h}_q\tilde{\chi})Q_R \\ &= \bar{Q}_L(y_q\phi_1 + h_q\chi_1)Q_R^u + \bar{Q}_L(\tilde{y}_q\tilde{\phi}_1 + \tilde{h}_q\tilde{\chi}_1)Q_R^d \\ &\quad + \bar{Q}_L(y_q\phi_2 + h_q\chi_2)Q_R^u + \bar{Q}_L(\tilde{y}_q\tilde{\phi}_2 + \tilde{h}_q\tilde{\chi}_2)Q_R^d. \end{aligned} \tag{45}$$

This is why the limit in Eqs. (32) and (33) is called the 2HDM limit. The decoupling rule of 2HBDM to 2HDM is the basic reason why the 2HBDM can be a realistic model with spontaneous $P$ and $CP$ violation.

Let us now check the Higgs mass matrix. In the limit of Eqs. (32) and (33), the mixings between Higgs bidoublets and triplets vanish, and also the mixings between left-hand triplet $\Delta_L$ and right-hand $\Delta_R$ become negligibly small. As a consequence, the $12\times 12$ mass matrix of the neutral Higgs bosons splits into $(8\times 8)_{pc}\oplus(2\times 2)_L\oplus(2\times 2)_R$ on the $\{\phi_{1,2}^0, \chi_{1,2}^0, \delta_L^0, \delta_R^0\}$ basis, and the $6\times 6$ mass matrix of the charged Higgs bosons splits into $(4\times 4)_{pc}\oplus 1_L\oplus 1_R$ in the $\{\phi_{1,2}^+, \chi_{1,2}^+, \delta_L^+, \delta_R^+\}$ basis. Substituting Eq. (39) into the mass matrix, we find that there is a big mass splitting inside the bidoublets; thus, half of the eight freedoms obtain masses at the $v_R$ scale, while the remaining four freedoms remain at the electroweak scale, one of which becomes the Goldstone boson of $SU(2)_L$ symmetry breaking. The same reasoning applies to the charged Higgs sector. The imaginary part of

$\delta_R^0$ and $\delta_R^+$ becomes the Goldstone bosons of $SU(2)_R$ symmetry breaking and the real parts of $\delta_R^0$ and $\delta_L^0$ become physical Higgs bosons at $v_R$ scale. To conclude, there are three neutral and one pair of charged Higgs bosons at the electroweak scale in the limit of Eqs. (32) and (33). At the $v_R$ scale, there are seven neutral Higgs bosons, and three pairs of charged Higgs bosons, among which two come from Higgs bidoublets and one from $\delta_L^\pm$, as well as two pairs of doubly charged Higgs bosons $\delta_{L,R}^{\pm\pm}$. The specific form of Higgs mass spectrum is listed in Appendix B.

*General decoupling limit to 2HDM.*— In the general case, the form of Higgs mass matrix is rather complicated. All six VEVs and four phases enter the expression. However, the above analysis on bidoublet mass splitting still holds, which means that the heavy freedoms at the $v_R$ scale in two Higgs bidoublets are all dominated by the explicit $SU(2)_R$ symmetry-breaking terms $\alpha_3^{p,c,pc}$.

It is rather tedious to write down the general form for the mass matrix, but we have carefully checked and confirmed that in the general case without imposing the special limit given in Eqs. (32) and (33), there are still three neutral and one pair of charged Higgs bosons at the electroweak scale as long as the $SU(2)_R$ symmetry-breaking scale $v_R$ is taken to be much higher than the electroweak scale, i.e.,

$$v_R \gg \kappa_1, \kappa_2, w_1, w_2, v_L, \tag{46}$$

which may be regarded as the general decoupling limit for 2HBDM approaching a 2HDM-like state. It is also found that the mixings of the Higgs sector have the same pattern as that described in Appendix B.

From the above analysis, we arrive at the conclusion that the 2HBDM will degenerate to the 2HDM in a general decoupling limit [Eq. (46)]. The main difference is that in the general decoupling limit the electroweak sector is separated from the right-hand sector associated with $v_R$ scale in a much more complicated way.

The explicit structures of the mass matrices for physical Higgs bosons are given as follows with different scales:

$$M^0 = \begin{pmatrix} M_h^0 & v^2 & vv_R & vv_L \\ v^2 & M_H^0 & vv_R & vv_L \\ v_Rv & v_Rv & M_R^0 & v_Rv_L \\ v_Lv & v_Lv & v_Lv_R & M_L^0 \end{pmatrix}, \qquad M^\pm = \begin{pmatrix} M_h^\pm & v^2 & vv_L \\ v^2 & M_H^\pm & vv_L \\ v_Lv & v_Lv & M_L^\pm \end{pmatrix}, \tag{47}$$

with $M_h^0$ a $3\times 3$ mass matrix, $M_H^0$ a $4\times 4$ mass matrix, and $M_H^\pm$ a $2\times 2$ mass matrix, and

$$M_H^0 = \begin{pmatrix} M_H^{0R} & v^2 \\ v^2 & M_H^{0I} \end{pmatrix}, \qquad M_L^0 = \frac{(\rho_3 - 2\rho_1)v_R^2}{2}\begin{pmatrix} 1 & \\ & 1 \end{pmatrix},$$
$$M_R^0 = 2\rho_1 v_R^2, \qquad M_L^\pm = \frac{(\rho_3 - 2\rho_1)v_R^2}{2} + \alpha v^2. \tag{48}$$

Thus the neutral Higgs mass matrix $M^0$ in Eq. (47) is a $10\times 10$ matrix, and $h^0(h^\pm)$ and $H^0(H^\pm)$ stand for the (nearly physical) Higgs bosons from the combination of bidoublets with mass scales $k$ and $v_R$, respectively. $M_h^0$ is a $3\times 3$ matrix with elements of order $k^2$, and $M_h^\pm$ is also of order $k^2$, while $M_H^{0R}$ (real part), $M_H^{0I}$ (imaginary part), and $M_H^\pm$ matrix elements are all of order $v_R^2$.

The Goldstone bosons are defined as

$$\begin{aligned}
\tilde{G}_L^0 &= \mathrm{Im}(\kappa_1\phi_1^{0*} + \kappa_2^*\phi_2^0 + \omega_1\chi_1^{0*} + \omega_2^*\chi_2^0 + 2\mathbf{v}_L^*\delta_L^0)/\sqrt{v^2 + 4v_L^2},\\
\tilde{G}_R^0 &= \mathrm{Im}(\kappa_1^*\phi_1^0 + \kappa_2\phi_2^{0*} + \omega_1^*\chi_1^0 + \omega_2\chi_2^{0*} + 2\mathbf{v}_R^*\delta_R^0)/\sqrt{v^2 + 4v_R^2},\\
\tilde{G}_L^+ &= (-\kappa_1\phi_1^+ + \kappa_2^*\phi_2^+ - \omega_1\chi_1^+ + \omega_2^*\chi_2^+ + \sqrt{2}\mathbf{v}_L^*\delta_L^+)/\sqrt{v^2 + 2v_L^2},\\
\tilde{G}_R^+ &= (-\kappa_1^*\phi_2^+ + \kappa_2\phi_1^+ - \omega_1^*\chi_2^+ + \omega_2\chi_1^+ + \sqrt{2}\mathbf{v}_R^*\delta_R^+)/\sqrt{v^2 + 2v_R^2},
\end{aligned} \tag{49}$$

which have been extracted from the mass matrix. Note that the real directions of the neutral and charged Goldstone bosons in the general case correspond to the combination of $\tilde{G}_{L,R}^0$ and $\tilde{G}_{L,R}^\pm$, respectively.

*Low-energy phenomenology.*— In our previous works [26,27], we have shown the low-energy phenomenological constraints and demonstrated that the mentioned stringent phenomenological constraints on the minimal model from neutral meson mixings can be significantly relaxed. In particular, it has been shown that the right-handed gauge boson mass can be as low as 600 GeV, with the charged Higgs mass around 200 GeV. The FCNC will not impose severe constraints on the neural Higgs mass, provided small off-diagonal Yukawa couplings via the mechanism of approximate global $U(1)$ family symmetry [22,23,25]. We have also analyzed the mass difference $\Delta m_K$ and indirect $CP$ violation $\epsilon_K$ in the neutral $K$ system and observed that the right-handed gauge boson contributions to the mass difference $\Delta m_K$ can be the opposite of that from the charged Higgs boson, and a cancelation between the two contributions is possible in a large parameter space. The suppression of right-handed gauge boson contributions to the indirect $CP$ violation $\epsilon_K$ has been found to occur naturally. As a consequence, a light right-handed gauge boson around the current experimental low bound is allowed. For the neutral $B$ meson system, the mass difference $\Delta m_B$ and the time-dependent $CP$ asymmetry in $B^0 \to J/\Psi K_S$ decay have been found to be consistently characterized in the 2HBDM with spontaneous $P$ and $CP$ violation, which is unlike the minimal model with only one Higgs bidoublet.

## V. CONCLUSION

We have discussed the 2HBDM with spontaneous $P$ and $CP$ violation as a simple extension of the minimal left-right symmetric model by adding an extra bidoublet. It has been shown that such an extended 2HBDM can solve the inconsistency between the vacuum minimal conditions on spontaneous $CP$ violation and the low-energy phenomenological bounds on the FCNC Higgs mass. It has been found that the 2HBDM can relax the quark Yukawa sector, which is strictly constrained by left-right symmetry in the minimal one.

In particular, we have demonstrated the existence of a general decoupling limit in the 2HBDM, which states that as long as the $SU(2)_R$ symmetry-breaking scale caused by the $SU(2)_R$ triplet Higgs is much higher than the electroweak symmetry-breaking scale. The 2HBDM will degenerate to the 2HDM-like state, which apparently differs from the minimal model with or without spontaneous $CP$ violation. As a consequence, the Higgs mass spectra in the 2HBDM have been obtained with reasonable approximation, where the three neutral and one pair of charged Higgs become naturally light Higgs bosons with masses at the electroweak scale and may be explored at LHC and International Linear Collider (ILC) [31,32], and the sources of $CP$ violation in the 2HBDM also get much richer and may show up in low-energy processes such as rare $B$ decays [33–36].

As the 2HBDM decouples to the 2HDM in the decoupling limit, it can evade the stringent constraints from the neutral meson mixing and make the allowed mass of right-handed gauge boson to be closing to the current direct experimental search bound. It is expected that the new physics particles in the 2HBDM can directly be searched in upcoming LHC and future ILC experiments.

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## APPENDIX A: THE MOST GENERAL HIGGS POTENTIAL IN 2HBDM

By simply adding a Higgs bidoublet $\chi$, the Higgs potential becomes much more complicated than the minimal model with a single Higgs bidoublet. Except including an identical copy of $\chi$ coupling to triplets $\Delta_{L,R}$, there are also the mixing terms between two Higgs bidoublets $\phi$ and $\chi$. The general form of Higgs potential may be written as follows:

$$V_{\phi,\chi,\Delta_L,\Delta_R} = V_\mu + V_\alpha + V_\rho + V_\lambda + V_\beta, \tag{A1a}$$

with

$$\begin{aligned} V_\mu = &-(\mu_1^p)^2 \operatorname{Tr}[\phi^\dagger \phi] - (\mu_1^c)^2 \operatorname{Tr}[\chi^\dagger \chi] \\ &- (\mu_1^{pc})^2 \operatorname{Tr}[\phi^+ \chi + \chi^+ \phi] \\ &- (\mu_2^p)^2 \operatorname{Tr}[\tilde{\phi} \phi^\dagger + \tilde{\phi}^\dagger \phi] - (\mu_2^c)^2 \operatorname{Tr}[\tilde{\chi} \chi^\dagger + \tilde{\chi}^\dagger \chi] \\ &- (\mu_2^{pc})^2 \operatorname{Tr}[\tilde{\phi} \chi^+ + \tilde{\phi}^+ \chi] \\ &- \mu_3^2 \operatorname{Tr}[\Delta_L \Delta_L^\dagger + \Delta_R \Delta_R^\dagger], \end{aligned} \tag{A1b}$$

$$\begin{aligned} V_\alpha = &\alpha_1^p \operatorname{Tr}[\phi\phi^\dagger] \operatorname{Tr}[\Delta_L \Delta_L^\dagger + \Delta_R \Delta_R^\dagger] + \alpha_1^c \operatorname{Tr}[\chi\chi^\dagger] \operatorname{Tr}[\Delta_L \Delta_L^\dagger + \Delta_R \Delta_R^\dagger] \\ &+ \alpha_1^{pc} \operatorname{Tr}[\phi^\dagger \chi + \chi^\dagger \phi] \operatorname{Tr}[\Delta_L \Delta_L^\dagger + \Delta_R \Delta_R^\dagger] \\ &+ \alpha_2^p \operatorname{Tr}[\phi^\dagger \tilde{\phi} + \phi \tilde{\phi}^\dagger] \operatorname{Tr}[\Delta_L \Delta_L^\dagger + \Delta_R \Delta_R^\dagger] + \alpha_2^c \operatorname{Tr}[\chi^\dagger \tilde{\chi} + \chi \tilde{\chi}^\dagger] \operatorname{Tr}[\Delta_L \Delta_L^\dagger + \Delta_R \Delta_R^\dagger] \\ &+ \alpha_2^{pc} \operatorname{Tr}(\tilde{\phi} \chi^\dagger + \tilde{\phi}^\dagger \chi) \operatorname{Tr}(\Delta_L \Delta_L^\dagger + \Delta_R \Delta_R^\dagger) \\ &+ \alpha_3^p \operatorname{Tr}[\phi\phi^\dagger(\Delta_L \Delta_L^\dagger + \Delta_R \Delta_R^\dagger)] + \alpha_3^c \operatorname{Tr}[\chi\chi^\dagger(\Delta_L \Delta_L^\dagger + \Delta_R \Delta_R^\dagger)] \\ &+ \alpha_3^{pc} \operatorname{Tr}[(\phi\chi^\dagger + \chi\phi^\dagger)\Delta_L \Delta_L^\dagger + (\phi^\dagger \chi + \chi^\dagger \phi)\Delta_R \Delta_R^\dagger] \\ &+ \alpha_4^{pc} \operatorname{Tr}[(\tilde{\phi}\chi^\dagger + \chi\tilde{\phi}^\dagger)\Delta_L \Delta_L^\dagger + (\chi^\dagger \tilde{\phi} + \tilde{\phi}^\dagger \chi)\Delta_R \Delta_R^\dagger], \end{aligned} \tag{A1c}$$

$$
\begin{aligned}
V_\lambda = {} & \lambda_1^p \operatorname{Tr}^2[\phi\phi^\dagger] + \lambda_2^p (\operatorname{Tr}^2[\tilde{\phi}\phi^\dagger] + \operatorname{Tr}^2[\tilde{\phi}^\dagger\phi]) + \lambda_3^p \operatorname{Tr}[\tilde{\phi}\phi^\dagger]\operatorname{Tr}[\tilde{\phi}^\dagger\phi] \\
& + \lambda_4^p \operatorname{Tr}[\phi\phi^\dagger](\operatorname{Tr}[\tilde{\phi}\phi^\dagger] + \operatorname{Tr}[\tilde{\phi}^\dagger\phi]) \\
& + \lambda_1^c \operatorname{Tr}^2[\chi\chi^\dagger] + \lambda_2^c (\operatorname{Tr}^2[\tilde{\chi}\chi^\dagger] + \operatorname{Tr}^2[\tilde{\chi}^\dagger\chi]) + \lambda_3^c \operatorname{Tr}[\tilde{\chi}\chi^\dagger]\operatorname{Tr}[\tilde{\chi}^\dagger\chi] \\
& + \lambda_4^c \operatorname{Tr}[\chi\chi^\dagger](\operatorname{Tr}[\tilde{\chi}\chi^\dagger] + \operatorname{Tr}[\tilde{\chi}^\dagger\chi]) \\
& + \lambda_1^{pc} (\operatorname{Tr}^2[\phi^\dagger\chi] + \operatorname{Tr}^2[\chi^\dagger\phi]) + \lambda_2^{pc} \operatorname{Tr}[\phi^\dagger\chi]\operatorname{Tr}[\chi^\dagger\phi] \\
& + \lambda_3^{pc} \operatorname{Tr}[\phi\phi^\dagger]\operatorname{Tr}[\phi^\dagger\chi + \chi^\dagger\phi] + \lambda_4^{pc} \operatorname{Tr}[\chi\chi^\dagger]\operatorname{Tr}[\phi^\dagger\chi + \chi^\dagger\phi] \\
& + \lambda_5^{pc} (\operatorname{Tr}^2[\tilde{\phi}\chi^\dagger] + \operatorname{Tr}^2[\tilde{\phi}^\dagger\chi]) + \lambda_6^{pc} \operatorname{Tr}[\tilde{\phi}\chi^\dagger]\operatorname{Tr}[\tilde{\phi}^\dagger\chi] \\
& + \lambda_7^{pc} \operatorname{Tr}[\phi\phi^\dagger]\operatorname{Tr}[\chi\chi^\dagger] \\
& + \lambda_8^{pc} \operatorname{Tr}[\phi\phi^\dagger]\operatorname{Tr}[\tilde{\chi}\chi^\dagger + \tilde{\chi}^\dagger\chi] + \lambda_9^{pc} \operatorname{Tr}[\phi\phi^\dagger]\operatorname{Tr}[\tilde{\phi}\chi^\dagger + \tilde{\phi}^\dagger\chi] \\
& + \lambda_{10}^{pc} \operatorname{Tr}[\chi\chi^\dagger]\operatorname{Tr}[\tilde{\phi}\phi^\dagger + \tilde{\phi}^\dagger\phi] + \lambda_{11}^{pc} \operatorname{Tr}[\chi\chi^\dagger]\operatorname{Tr}[\tilde{\chi}\phi^\dagger + \tilde{\chi}^\dagger\phi] \\
& + \lambda_{12,13}^{pc} \operatorname{Tr}[\tilde{\phi}\phi^\dagger \pm \tilde{\phi}^\dagger\phi]\operatorname{Tr}[\tilde{\phi}^\dagger\chi \pm \tilde{\phi}\chi^\dagger] + \lambda_{14,15}^{pc} \operatorname{Tr}[\tilde{\chi}\chi^\dagger \pm \tilde{\chi}^\dagger\chi]\operatorname{Tr}[\tilde{\phi}^\dagger\chi \pm \tilde{\phi}\chi^\dagger] \\
& + \lambda_{16,17}^{pc} \operatorname{Tr}[\phi\chi^\dagger \pm \chi\phi^\dagger]\operatorname{Tr}[\tilde{\phi}^\dagger\phi \pm \tilde{\phi}\phi^\dagger] + \lambda_{18,19}^{pc} \operatorname{Tr}[\phi\chi^\dagger \pm \chi\phi^\dagger]\operatorname{Tr}[\tilde{\chi}^\dagger\chi \pm \tilde{\chi}\chi^\dagger] \\
& + \lambda_{20,21}^{pc} \operatorname{Tr}[\phi\chi^\dagger \pm \chi\phi^\dagger]\operatorname{Tr}[\tilde{\phi}^\dagger\chi \pm \tilde{\phi}\chi^\dagger] \\
& + \lambda_{22,23}^{pc} \operatorname{Tr}[\tilde{\phi}^\dagger\phi \pm \tilde{\phi}\phi^\dagger]\operatorname{Tr}[\tilde{\chi}\chi^\dagger \pm \tilde{\chi}^\dagger\chi],
\end{aligned} \tag{A1d}
$$

$$
\begin{aligned}
V_\beta = {} & \beta_1^p \operatorname{Tr}[\phi\Delta_R\phi^\dagger\Delta_L^\dagger + \phi^\dagger\Delta_L\phi\Delta_R^\dagger] + \beta_1^c \operatorname{Tr}[\chi\Delta_R\chi^\dagger\Delta_L^\dagger + \chi^\dagger\Delta_L\chi\Delta_R^\dagger] \\
& + \beta_1^{pc} \operatorname{Tr}[\phi\Delta_R\chi^\dagger\Delta_L^\dagger + \phi^+\Delta_L\chi\Delta_R^\dagger] \\
& + \beta_2^p \operatorname{Tr}[\tilde{\phi}\Delta_R\phi^\dagger\Delta_L^\dagger + \tilde{\phi}^\dagger\Delta_L\phi\Delta_R^\dagger] + \beta_2^c \operatorname{Tr}[\tilde{\chi}\Delta_R\chi^\dagger\Delta_L^\dagger + \tilde{\chi}^\dagger\Delta_L\chi\Delta_R^\dagger] \\
& + \beta_2^{pc} \operatorname{Tr}[\chi\Delta_R\phi^\dagger\Delta_L^\dagger + \chi^\dagger\Delta_L\phi\Delta_R^\dagger] \\
& + \beta_3^p \operatorname{Tr}[\phi\Delta_R\tilde{\phi}^\dagger\Delta_L^\dagger + \phi^\dagger\Delta_L\tilde{\phi}\Delta_R^\dagger] + \beta_3^c \operatorname{Tr}[\chi\Delta_R\tilde{\chi}^\dagger\Delta_L^\dagger + \chi^\dagger\Delta_L\tilde{\chi}\Delta_R^\dagger] \\
& + \beta_3^{pc} \operatorname{Tr}[\tilde{\phi}\Delta_R\chi^\dagger\Delta_L^\dagger + \tilde{\phi}^\dagger\Delta_L\chi\Delta_R^\dagger] + \beta_4^{pc} \operatorname{Tr}[\phi\Delta_R\tilde{\chi}^\dagger\Delta_L^\dagger + \phi^\dagger\Delta_L\tilde{\chi}\Delta_R^\dagger],
\end{aligned} \tag{A1e}
$$

$$
\begin{aligned}
V_\rho = {} & \rho_1 (\operatorname{Tr}^2[\Delta_L\Delta_L^\dagger] + \operatorname{Tr}^2[\Delta_R\Delta_R^\dagger]) \\
& + \rho_2 (\operatorname{Tr}[\Delta_L\Delta_L]\operatorname{Tr}[\Delta_L^\dagger\Delta_L^\dagger] + \operatorname{Tr}[\Delta_R\Delta_R]\operatorname{Tr}[\Delta_R^\dagger\Delta_R^\dagger]) \\
& + \rho_3 \operatorname{Tr}[\Delta_L\Delta_L^\dagger]\operatorname{Tr}[\Delta_R\Delta_R^\dagger] \\
& + \rho_4 (\operatorname{Tr}[\Delta_L\Delta_L]\operatorname{Tr}[\Delta_R^\dagger\Delta_R^\dagger] + \operatorname{Tr}[\Delta_L^\dagger\Delta_L^\dagger]\operatorname{Tr}[\Delta_R\Delta_R]),
\end{aligned} \tag{A1f}
$$

where upper indices $p$'s denote the terms relevant to $\phi$, $c$'s denote those relevant to $\chi$ and $pc$'s denote those relevant to both $\phi$ and $\chi$.

## APPENDIX B: HIGGS MASS SPECTRA IN THE DECOUPLING LIMIT

Here we explicitly list the Higgs mass spectra in the 2HBDM with approximation by omitting the terms $O(\frac{v_L}{k})$, $O(\frac{k}{v_R})$, $O(\frac{\kappa_2,\omega_2}{\kappa_1,\omega_1})$, and their higher orders.

(i) Light neutral Higgs ($h_1^0, h_2^0, h_3^0$) at the electroweak scale:

$$
\begin{aligned}
(M_{h^0}^{11})^2 &\simeq 2\lambda_1^p\kappa^2 + 2\lambda_1^{pc}\omega^2\cos^2\theta_1^c + 2\lambda_3^{pc}\kappa\omega\cos\theta, \\
(M_{h^0}^{12})^2 &\simeq -2\lambda_1^{pc}\kappa\omega\sin^2\theta_1^c + \lambda_2^{pc}\kappa\omega + \lambda_3^{pc}\kappa\omega\cos + \lambda_4^{pc}\omega^2\cos\theta + \lambda_7^{pc}\kappa\omega, \\
(M_{h^0}^{22})^2 &\simeq 2\lambda_1^c\omega^2 + 2\lambda_1^{pc}\kappa^2\cos^2\theta_1^c + 2\lambda_4^{pc}\kappa\omega\cos\theta, \\
(M_{h^0}^{13})^2 &\simeq -v(2\lambda_1^{pc}\omega\cos\theta + \lambda_3^{pc}\kappa), \\
(M_{h^0}^{23})^2 &\simeq -v(2\lambda_1^{pc}\kappa\cos\theta + \lambda_4^{pc}\omega), \\
(M_{h^0}^{33})^2 &\simeq 2\lambda_1^{pc}v^2\sin^2\theta_1^c,
\end{aligned} \tag{B1}
$$

with

$$\phi_1^0 = h_1^0 - i\sin\beta h_2^0 + i\cos\beta\tilde{G}_1^0,$$
$$\phi_2^0 = e^{i\theta}h_3^0 - e^{-i\theta}\sin\beta h_2^0 - e^{-i\theta}\cos\beta\tilde{G}_1^0, \quad \text{(B2)}$$

where $\theta \equiv \theta_1^c$, $\kappa \equiv \kappa_1$, $\omega \equiv \omega_1$, $v^2 = \kappa^2 + \omega^2$, and $\tan\beta = \omega/\kappa$. $\tilde{G}_1^0$ absorbed by $Z_1$ is the neutral Goldstone of $SU(2)_L$ symmetry breaking.

(ii) Light charged Higgs ($h^\pm$) at the electroweak scale:

$$(M_{h^\pm})^2 = \frac{1}{2}(2\lambda_1^{pc} - \lambda_2^{pc})v^2, \quad \text{(B3)}$$

with

$$\phi_1^+ = -\sin\beta e^{-i\theta}h^+ + \cos\beta G_L^+,$$
$$\chi_1^+ = \cos\beta h^+ + \sin\beta e^{i\theta}G_L^+, \quad \text{(B4)}$$

where $G_L^+$ absorbed by $W_L$ is the charged Goldstone of $SU(2)_L$ symmetry breaking.

(iii) Heavy doublets $(H_1, H_2)$ on the basis of $\{\phi_2, \chi_2\}$ at the $v_R$ scale:

$$(M_{1,2}^H)^2 = \frac{1}{2}(\alpha_3^p + \alpha_3^c \mp \sqrt{(\alpha_3^p - \alpha_3^c)^2 + 4(\alpha_3^{pc})^2})v_R^2, \quad \text{(B5)}$$

with

$$\phi_2 = \cos\xi H_1 - \sin\xi H_2, \quad \chi_2 = \sin\xi H_1 + \cos\xi H_2,$$
$$\tan 2\xi = (\alpha_3^p - \alpha_3^c)/\alpha_3^{pc}. \quad \text{(B6)}$$

Mixings inside doublets $H_1$ or $H_2$ are generally $O(k^2/v_R^2)$.

(iv) Heavy right-handed triplet Higgs (neutral) from $\text{Re}(\delta_R^0)$ at the $v_R$ scale:

$$(M_R^0)^2 \simeq 2\rho_1 v_R^2. \quad \text{(B7)}$$

(v) Heavy doubly charged right-handed triplet Higgs($\delta_R^{\pm\pm}$):

$$(M_R^{\pm\pm})^2 \simeq 2\rho_2 v_R^2. \quad \text{(B8)}$$

(vi) Heavy left-handed triplet $\Delta_L$ (neutral, charged, and doubly charged):

$$(M^{\Delta_L})^2 \simeq \frac{1}{2}(\rho_3 - 2\rho_1)v_R^2. \quad \text{(B9)}$$

(vii) Higgs mixing among different components: The light Higgs $h_{1,2,3}^0$ mixings with heavy Higgs $\text{Re}(\delta_R^0)$ are of order $O(\frac{k}{v_R})$. The light Higgs $h_{1,2,3}^0$ mixings with heavy Higgs $H_{1,2}^0$ are of order $O(\frac{k^2}{v_R^2})$. The light Higgs $h^\pm$ mixings with $H_{1,2}^\pm$ are of order $O(\frac{k^2}{v_R^2})$. The heavy Higgs $H_{1,2}^0$ mixings with $\text{Re}(\delta_R^0)$ are of order $O(\frac{k^2}{v_R^2})$. $\Delta_L$ mixings with others approach to vanishing when $v_L \to 0$.


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