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Correlations of Abelian monopoles in quark-gluon plasma

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In this paper the properties of thermal Abelian monopoles in the deconfinement phase of the $SU(2)$ gluodynamics are considered. In particular, to study the properties of the Abelian monopole component of quark-gluon plasma (QGP), we calculate three-point correlation functions of monopoles for different temperatures from the region $T/T_c \in (1.5, 6.8)$. The results of the calculation show that the three-point correlation functions can be described by independent pair correlations of monopoles. From this, one can conclude that the system of Abelian monopoles in QGP reveals the properties of a dilute gas. In addition, one can assert that the interaction between Abelian monopoles is a pair interaction and there are no three-particle forces acting between monopoles.

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One of the most interesting results obtained at RHIC is a large elliptic flow [1,2]. Interpretation of this result suggests that QGP reveals the properties of a strongly correlated system with very small shear viscosity [3]. An interesting explanation of this peculiarity can be given within the hypothesis that unusual properties of QGP are closely connected with the magnetic degrees of freedom [4–8].

In paper [7] such magnetic degrees of freedom have been related to thermal Abelian monopoles evaporating from the magnetic condensate, which is believed to induce color confinement at low temperatures. Moreover, it has been proposed to detect such thermal monopoles in finite temperature lattice QCD simulations by identifying them with monopole currents having a nontrivial wrapping in the Euclidean temporal direction [7,9,10].

The way one can study the monopoles' properties on the lattice is via an Abelian projection after fixing the maximally Abelian gauge (MAG) [11,12]. This gauge, as well as the properties of monopole clusters, have been investigated in numerous papers both at zero and nonzero temperatures (see for an extensive list of references, e.g. [13]). The evidence was found that the nonperturbative properties of the gluodynamics, such as confinement, deconfining transition, chiral symmetry breaking, etc., are closely related to the Abelian monopoles defined in MAG. This was called a monopole dominance.

Motivated by the hypothesis that thermal Abelian monopoles might be responsible for the unusual properties of QGP, in this paper we continue the study of their properties. In particular, we are going to study monopole correlation functions in order to address the question of collective phenomena of the magnetic component of QGP. Although the study of two-point correlation functions carried out in papers [14–17] revealed rather nontrivial interaction between monopoles, it is rather difficult to draw some conclusions about the properties of monopole medium in QGP. To study the properties of this medium, in this paper we consider three-point correlation functions of monopoles.

The correlation function under consideration can be defined as follows:

$$g^{(3)}(r_{12}, r_{13}, r_{23}) = \frac{\langle \rho(\bar{r}_1)\rho(\bar{r}_2)\rho(\bar{r}_3)\rangle}{\rho^3}, \quad (1)$$

where $\bar{r}_1$, $\bar{r}_2$, $\bar{r}_3$ are the positions of three monopoles, $r_{12} = |\bar{r}_1 - \bar{r}_2|$, $r_{13} = |\bar{r}_1 - \bar{r}_3|$, $r_{23} = |\bar{r}_2 - \bar{r}_3|$ are distances between the monopoles, $\rho(\bar{r})$ is the operator of monopole density at the point $\bar{r}$ and ρ is the averaged density.

To study collective phenomena and medium effects we are going to compare correlation function (1) with the model correlation function

$$G^{(3)}(r_{12}, r_{13}, r_{23}) = g^{(2)}(r_{12})g^{(2)}(r_{13})g^{(2)}(r_{23}), \quad (2)$$

where $g^{(2)}(r)$ is the two-point correlation function

$$g^{(2)}(r_{12}) = \frac{\langle \rho(\bar{r}_1)\rho(\bar{r}_2)\rangle}{\rho^2}, \quad (3)$$

which will be taken from paper [17]. Now two comments are in order:

(1) Model (2) implies that three-particle correlation takes place only through independent correlation of the pairs. Such correlation function is valid for the systems similar to a dilute gas. Evidently in a dilute gas there are no collective phenomena, and

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one can disregard the influence of monopole medium to the system of three monopoles. So, the deviation of correlation function (1) from model function (2) can be considered as a measure of collective phenomena and monopole medium effects.

(2) From correlation function (2) one can conclude that the interaction between monopoles in the monopole medium is a pair interaction described by some universal potential $V(r)$, which can be extracted from the two-point correlation function. The potential $V(r)$ depends on the distance between two monopoles r and the temperature of QGP. We believe that the last property is rather a nontrivial property of nonabelian gluodynamics.

To model the system of Abelian monopoles in QGP, we use $SU(2)$ lattice gauge theory with the standard Wilson action

$$S = \beta \sum_x \sum_{\mu>\nu} \left[1 - \frac{1}{2} \mathrm{Tr}(U_{x\mu} U_{x+\mu;\nu} U^\dagger_{x+\nu;\mu} U^\dagger_{x\nu}) \right],$$

where $\beta = 4/g_0^2$ and g_0 is a bare coupling constant.

Our calculations were performed on the asymmetric lattices with lattice volume $V = L_t L_s^3$, where $L_{t,s}$ is the number of sites in the time (space) direction. The temperature T is given by

$$T = \frac{1}{aL_t}, \tag{4}$$

where a is the lattice spacing.

The MAG is fixed by finding an extremum of the gauge functional

$$F_U(g) = \frac{1}{4V} \sum_{x\mu} \frac{1}{2} \mathrm{Tr}\left(U^g_{x\mu} \sigma_3 U^{g\dagger}_{x\mu} \sigma_3 \right), \tag{5}$$

with respect to gauge transformations g_x. We apply the simulated annealing (SA) algorithm, which proved to be very efficient for this gauge [18] as well as for other gauges such as a center gauge [19] and Landau gauge [20].

In Table I we provide the information about the gauge field ensembles used in our study.

The lattice version of correlation functions (1) can be written as follows:

$$g^{(3)}(r_1, r_2, r_3) = \frac{1}{\rho^3} \frac{dN(r_1, r_2, r_3)}{dV(r_1, r_2, r_3)}, \tag{6}$$

TABLE I. Values of β, lattice sizes, temperatures and number of configurations. To fix the scale we take $\sqrt{\sigma} = 440$ MeV.

β	a [fm]	L_t	L_s	T/T_c	N_{meas}
2.43	0.108	4	32	1.5	1000
2.635	0.054	4	36	3.0	500
2.80	0.034	4	48	4.8	1000
2.93	0.024	4	48	6.8	1000

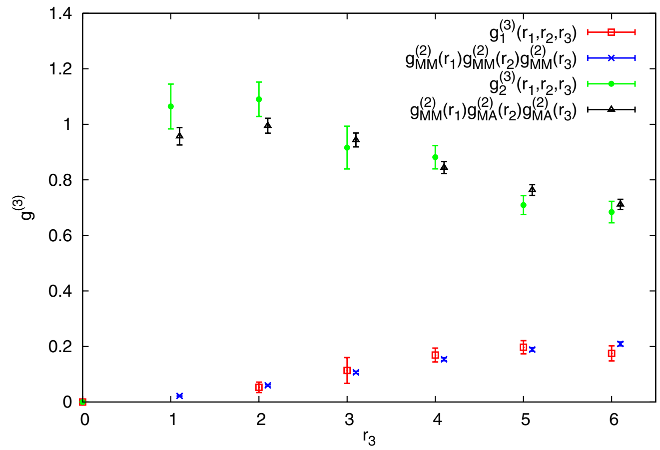

FIG. 1 (color online). $\beta = 2.43$, $T/T_c = 1.5$. The correlation function $g^{(3)}(r_1, r_2, r_3)$ and the model function $G^{(3)}(r_1, r_2, r_3)$. The distances between the first and second, and the first and the third monopoles, are $r_1 = r_2 = 3$ lattice spacings. The distance between the second and the third monopoles r_3 is varied.

where $dN(r_1, r_2, r_3)$ is the total number of triples of monopoles such that the distances between monopoles lie in the domain $r_{12} \in (r_1, r_1 + \Delta r)$, $r_{13} \in (r_2, r_2 + \Delta r)$, $r_{23} \in (r_3, r_3 + \Delta r)$. $dV(r_1, r_2, r_3)$ is the number of lattice cubes located in the same domain. In order to take into account discretization errors, we evaluate the $dV(r_1, r_2, r_3)$ numerically. Δr is the size of one bin. An additional factor $1/\rho^3$ was introduced to normalize the whole expression. At large distances, where there are no correlations at all, $g = 1$.

In our analysis only monopoles with magnetic charge $q = \pm 1$ are taken into account. Our results show that the monopoles with $|q| > 1$ are greatly suppressed. Since one considers only two types of effective particles (monopoles $q = +1$ and antimonopoles $q = -1$), there are four different correlators $g^{(3)}_{MMM}$, $g^{(3)}_{AAA}$, $g^{(3)}_{MMA}$, $g^{(3)}_{AAM}$, where M, A denote monopole and antimonopole. Evidently, monopoles are equivalent to antimonopoles in the sense that one can make magnetic charge conjugation and this does not change the physical properties of QGP. For this reason, instead of four correlators we calculate the following linear combinations:

$$g_1^{(3)} = \frac{1}{2}(g^{(3)}_{MMM} + g^{(3)}_{AAA}), \qquad g_2^{(3)} = \frac{1}{2}(g^{(3)}_{MMA} + g^{(3)}_{AAM}). \tag{7}$$

Now let us proceed to the results of this paper. In Figs. 1–3, we plot the correlation functions $g_1^{(3)}(r_1, r_2, r_3)$, $g_2^{(3)}(r_1, r_2, r_3)$, and model (2) for the configurations with $\beta = 2.43$, $T/T_c = 1.5$. In Fig. 1 we fixed the distances between the first and second, and the first and the third monopoles at the values $r_1 = r_2 = 3$ lattice spacings and varied the distance between the second and the third monopoles r_3. Similarly, in Fig. 2 we take the $r_1 = r_2 = 6$ lattice spacings and varied the r_3. In Fig. 3 the monopoles

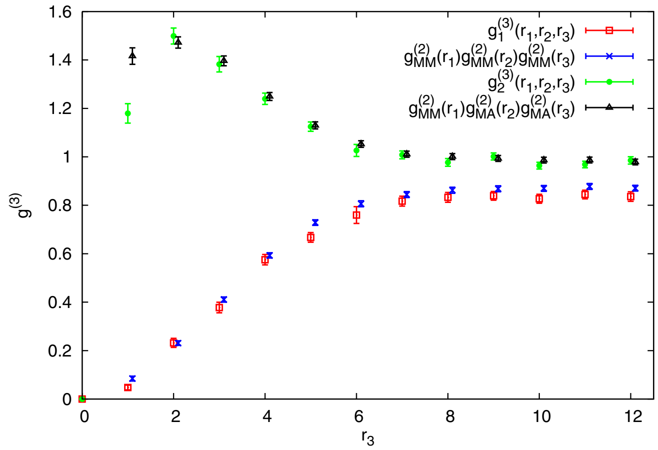

FIG. 2 (color online). $\beta = 2.43$, $T/T_c = 1.5$. The correlation function $g^{(3)}(r_1, r_2, r_3)$ and the model function $G^{(3)}(r_1, r_2, r_3)$. The distances between the first and second, and the first and the third monopoles, are $r_1 = r_2 = 6$ lattice spacings. The distance between the second and the third monopoles r_3 is varied.

are located at the corners of a regular triangle $r_1 = r_2 = r_3 = r$ and the side of this triangle r is varied. From these figures it is seen that up to the statistical uncertainty the correlation functions g_1, g_2 coincide with the corresponding models (2). A similar conclusion can be drawn for the other temperatures $T/T_c = 3.0, 4.8, 6.8$ studied in this paper.

Figures 1–3, can give us only qualitative results. To get the quantitative measurement of the discrepancy between models (1) and (2) we introduce the following quantity:

$$\delta = \frac{1}{N} \sum_{r_1, r_2, r_3} \frac{(g^{(3)}(r_1, r_2, r_3) - G^{(3)}(r_1, r_2, r_3))^2}{\sigma^2(r_1, r_2, r_3)}. \qquad (8)$$

Here, $\sigma(r_1, r_2, r_3)$ is the uncertainty of the calculation of the correlation function $g^{(3)}(r_1, r_2, r_3)$ at the given point

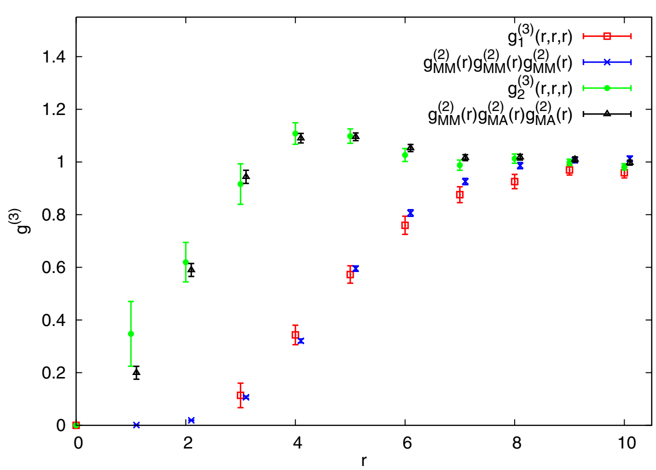

FIG. 3 (color online). $\beta = 2.43$, $T/T_c = 1.5$. The correlation function $g^{(3)}(r_1, r_2, r_3)$ and model function $G^{(3)}(r_1, r_2, r_3)$. The monopoles are located at the corners of the regular triangle $r_1 = r_2 = r_3 = r$ and the side of this triangle r is varied.

TABLE II. The values of β, temperature, the difference δ between correlation functions (1) and the corresponding models (2). The δ_1 corresponds to the correlation function g_1 in (7). The δ_2 corresponds to the correlation function g_2 in (7).

β	T/T_c	δ_1	δ_2
2.43	1.5	0.59	0.67
2.635	3.0	0.67	0.68
2.80	4.8	0.66	0.69
2.93	6.8	0.62	0.68

(r_1, r_2, r_3). Note that we have disregarded the uncertainty in the two-point functions since it is small compared to the $\sigma(r_1, r_2, r_3)$.

There are some restrictions on the values of distances between monopoles. The first one comes from the finite volume effect. Evidently, if $r_1 + r_2 + r_3 > L_s$, then due to periodical boundary conditions new nonphysical triples of monopoles wrapped in a spatial direction appear. We ignored such configurations in the calculation. Another restriction comes from the triangle inequality: $|r_1 - r_2| < r_3 < r_1 + r_2$. We also did not take into account the distances smaller than 3 lattice spacings due to the large statistical uncertainty. In Eq. (8) the sum is taken over all distances with the mentioned restrictions. N is the total number of triples of distances that satisfy these restrictions. Obviously the value of the δ is ≈ 1 if there is no discrepancy between two correlation functions. In Table II we present the values of the δ for the different β. From this table it is seen that up to the uncertainty of the calculation, the three-point correlation function of Abelian monopoles can be described by model (2).

In conclusion, in this paper we studied the properties of thermal Abelian monopoles in the deconfinement phase of the $SU(2)$ gluodynamics. In particular, to study the properties of the Abelian monopole component in QGP we calculated three-point correlation functions of monopoles for different temperatures from the region $T/T_c \in (1.5, 6.8)$. The results of the calculation show that the three-point correlation functions can be described by the independent pair correlation of monopoles. From the last fact one can conclude that *the system of Abelian monopoles in QGP reveals the properties of a dilute gas.* In addition, one can assert that the interaction between Abelian monopoles is a pair interaction and there are no three-particle forces acting between monopoles.

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