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Mesons and nucleons in the soft-wall AdS/QCD model with constrained infrared background

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The purpose of this paper is to further study the soft-wall AdS/QCD model with constrained IR background proposed in [1]. By including a quartic bulk scalar potential we study various meson and nucleon spectra. This model naturally realizes the asymptotical linearity of these mass spectra simultaneously, together with correct patterns of explicit and dynamical chiral symmetry breaking. The agreement between the theoretical calculations and the experimental data is good.

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I. INTRODUCTION

In the 1970s 't Hooft argued [2] that the large N_c limit of QCD, with fixed g^2N_c, should be described by its holographic dual string theory. This idea has been explicitly realized by the AdS/CFT correspondence [3–5]. In the infrared region QCD becomes strongly coupled and the effective dynamical degrees of freedom (instead of quarks and gluons) are hadrons in the particle zoo, like π, ρ, N, etc. Therefore, QCD cannot help us very much in our understanding the properties of low-energy strong interactions. However, the idea of large N_c expansions and holography supply a totally new point of view for these difficult yet important problems. According to its general rule, when the 't Hooft coupling g^2N_c is large, we can use the effective theory in the bulk to study the strongly coupled dynamics of QCD.

Actually this has been an active region of research in recent years. There are two complementary methods of study. One is the top-down method (see e.g. [6,7]), which starts from some brane configurations in string theory. This method has the advantage of theoretical completeness, however, the resulting model only has partial resemblances with the real QCD. The other method is bottom-up, usually called AdS/QCD (see e.g. [8–10]). This method assumes the bulk theory living in the AdS_5 spacetime or its some IR deformation. The model contains several bulk fields each of which corresponds to a QCD operator, people use observed experimental data and/or some properties of QCD, e.g. chiral symmetry breaking, linear confinement, etc., to constrain the possible forms of the model. It supplies necessary conditions that a would-be holographic theory of QCD should have. In this paper we will follow this bottom-up approach.

The so-called hard-wall model, defined on a slice of AdS_5 with a sharp IR cutoff, is developed first. These types of models can correctly realize the pattern of chiral symmetry breaking and low-lying hadron states. For instance, scalar and pseudoscalar mesons were studied in [11], tensor mesons in [12], and b_1/h_1 mesons in [13]. Even hybrid exotic mesons were realized in [14]. In addition to the meson sector, baryons can also be realized in the hard-wall model, see [15–18]. However, the main difficulty of the hard-wall approach is the absence of linear confinement. To remedy this drawback, a soft-wall model is construct in [19], which includes a background dilaton field with quadratic growth at the deep IR region. By WKB-type arguments, it can be shown that the excited meson spectrum exhibits the Regge behavior $m_n^2 \propto n + J$. In [20], by introducing a quartic potential term for the bulk scalar, explicit and spontaneous chiral symmetry breaking are also correctly incorporated in soft-wall AdS/QCD models. The relation with light-front dynamics is also discussed, see e.g. [21,22]. A huge amount of works have been done, a partial list includes [23–33].

In addition to the meson sector, various baryons also exhibit the approximate Regge behavior. One possible explanation [34] of this fact is that the baryon is composed of a quark and a diquark connected by a flux-tube string. So its structure is actually similar with meson. In the literature of AdS/QCD, there are relatively few works considering the baryon linear spectrum, see e.g. [35–38]. In [39] and subsequently [40], we develop a soft-wall AdS/QCD model which realizes asymptotically linear spectra for both mesons and nucleons. We achieve this by a cubic potential term for the bulk scalar and a new parametrization of its vacuum expectation value (VEV). We also calculate the coupling between pion and nucleons. The main drawback of this model is that the slopes of meson mass-squares are different, which is inconsistent with the real data. Unfortunately it is not easy to improve this. However, we will argue in this paper that it is impossible to get parallel meson slopes, while keeping the linear nucleon spectrum, by only varying the scalar VEV or choosing different forms of the potential.

In [1] we argue that, by requiring to have correct Regge-type spectrum in both meson and nucleon sectors, the IR asymptotic behavior of various background fields in the model can be fully determined. The way around the above no-go theorem is to allow the mass of bulk fields being z-dependent.[1] This is actually very natural when considering possible anomalous dimension of the QCD operators. For operators which are not conserved currents, like the quark condensates and baryon operators, the full conformal dimension is not the classical value. The anomalous part is in general scale-dependent due to the running coupling constant, which translates to the z-dependence of the mass term for the corresponding bulk fields according to the well-known mass-dimension relation. Therefore, we assume the z-dependence of the bulk mass for the bulk scalar field and the bulk Dirac field. We only require these masses approaching the value dual to the classical dimension at the UV boundary since the high energy fixed-point of QCD is a free theory.

In the present paper we will further develop the model proposed in [1]. To be more realistic we include a quartic potential for the bulk scalar as in [20]. The trajectories of various meson sectors are parallel with each other, improving the main drawback of our previous model in [39,40]. The remaining parts of this paper are organized as follows. In Sec. II we discuss the meson sector of our model. It includes scalar, vector, axial-vector, and pseudoscalar mesons. We also compare the predicted masses and the corresponding data. In Sec. III we discuss the spin-1/2 nucleons. We show that they also have asymptotically linear spectrum. We summarize this paper in Sec. IV.

II. MESON SECTOR

In our soft-wall AdS/QCD models, all fields are defined in a five-dimensional (5D) Anti-de Sitter (AdS) space with the metric

$$ds^2 = G_{MN}dx^M dx^N = a^2(z)(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2), \quad 0 < z < \infty. \tag{2.1}$$

The bulk action for the meson sector is

$$S_M = \int d^4x dz \sqrt{G} e^{-\Phi} \left\{ -\frac{1}{4g_5^2}(\|F_L\|^2 + \|F_R\|^2) + \|DX\|^2 - m_X^2\|X\|^2 - \lambda\|X\|^4 \right\}. \tag{2.2}$$

Here $g_5^2 = 12\pi^2/N_c = 4\pi^2$ as usual. F_L and F_R are the field strengths of the gauge potentials L and R, respectively. The covariant derivative is defined to be $D_M X = \partial_M X - iL_M X + iXR_M$, with X in the bifundamental representation of $SU(2)_L \times SU(2)_R$. $\|X\|^2$ is the norm of the matrix X, i.e. $\|X\|^2 = \mathrm{Tr}(X^\dagger X)$.

[1]This has been suggested previously in e.g. [41,42] for different purposes.

A. Background fields

First we introduce the bulk scalar X, it is assumed to have a z-dependent VEV as follows:

$$\langle X\rangle = \frac{1}{2}v(z)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{2.3}$$

Then from the bulk action (2.2) we get the equation that combines function $v(z)$ and the background dilaton $\Phi(z)$,

$$\partial_z(a^3 e^{-\Phi}\partial_z v) - a^5 e^{-\Phi}\left(m_X^2 v + \frac{\lambda}{2}v^3\right) = 0. \tag{2.4}$$

We can deduce that the mass-square m_X^2 may be z-dependent due to possible unusual dimension of $\bar{q}_L q_R$. Then according to (2.4), m_X^2 can be expressed as

$$m_X^2 = \frac{v'' + (-\Phi' + 3a'/a)v'}{a^2 v} - \frac{\lambda}{2}v^2. \tag{2.5}$$

The UV limit is still simple to argue, for the warp factor we have

$$a(z) \sim \frac{L}{z}, \qquad z \to 0. \tag{2.6}$$

And for the scalar VEV we have

$$v(z) \sim Az + Bz^3, \qquad z \to 0. \tag{2.7}$$

By the mass-dimension relation $m_X^2 = \Delta(\Delta - 4)$, we have

$$m_X^2(z) \sim -3, \qquad z \to 0. \tag{2.8}$$

These are the behaviors at UV boundary, now we continue to study the IR situation. For the dilaton it must be [19]

$$\Phi(z) \sim O(z^2), \qquad z \to \infty, \tag{2.9}$$

which guarantees the mesons have linear spectra. In order to obtain the spectral linearity of nucleons, the IR limit of the warp factor is [1]

$$a(z) \sim O(z), \qquad z \to \infty. \tag{2.10}$$

To have parallel mass-square lines between vector and axial-vector mesons we get the IR behavior of the scalar VEV

$$v(z) \sim O(z^{-1}), \qquad z \to \infty. \tag{2.11}$$

Now we use simple parametrization to smoothly connect these asymptotes from UV to IR as follows:

$$\Phi(z) = \kappa^2 z^2, \tag{2.12}$$

$$a(z) = \frac{1 + \mu z^2}{z}, \tag{2.13}$$

$$v(z) = \frac{Az + Bz^3}{1 + Cz^4}. \tag{2.14}$$

The parameters are determined by fitting the experimental data of the pseudoscalar, scalar, vector and axial-vector meson masses. We take their values as

$$A = 3.2\ \text{MeV}, \qquad B = (394.5\ \text{MeV})^3,$$
$$C = (786.5\ \text{MeV})^4; \qquad \mu = 1153.6\ \text{MeV}, \tag{2.15}$$
$$\kappa = 413.1\ \text{MeV}, \qquad \lambda = 5.99.$$

In the following sections we will use them to calculate mass spectra and to compare them with the experimental data.

B. Quadratic order action

To get the fluctuation filed we define

$$X = \left(\frac{v}{2} + S\right)e^{2iP}, \tag{2.16}$$

here S is a real scalar, P is a real pseudoscalar, and X, S, P are all 2×2 matrices. Next we define

$$V_M = \frac{1}{2}(L_M + R_M), \qquad A_M = \frac{1}{2}(L_M - R_M). \tag{2.17}$$

We expand the action (2.2) to the quadratic order of these new fields

$$S_M^{(2)} = \int d^4xdz(\mathcal{L}_{P,A_5}^{(2)} + \mathcal{L}_S^{(2)} + \mathcal{L}_V^{(2)} + \mathcal{L}_A^{(2)}). \tag{2.18}$$

Each of them is as follows:

$$\mathcal{L}_{P,A_5}^{(2)} = -\frac{1}{2}a^3v^2e^{-\Phi}P^a\partial^2P^a - \frac{1}{2}a^3v^2e^{-\Phi}(\partial_5P^a - A_5^a)^2 - \frac{1}{2g_5^2}ae^{-\Phi}A_5^a\partial^2A_5^a, \tag{2.19}$$

$$\mathcal{L}_S^{(2)} = -\frac{1}{2}a^3e^{-\Phi}S^a\left\{\partial^2 - \frac{1}{a^3e^{-\Phi}}\partial_5(a^3e^{-\Phi}\partial_5) + m_X^2a^2 - \frac{3}{4}\lambda a^2v\right\}S^a, \tag{2.20}$$

$$\mathcal{L}_V^{(2)} = -\frac{1}{2g_5^2}ae^{-\Phi}V_\mu^a\left\{-\eta^{\mu\nu}\partial^2 + \partial^\mu\partial^\nu + \frac{1}{ae^{-\Phi}}\partial_5(ae^{-\Phi}\partial_5)\eta^{\mu\nu}\right\}V_\nu^a, \tag{2.21}$$

$$\mathcal{L}_A^{(2)} = -\frac{1}{2g_5^2}ae^{-\Phi}A_\mu^a\left\{-\eta^{\mu\nu}\partial^2 + \partial^\mu\partial^\nu + \frac{1}{ae^{-\Phi}}\partial_5(ae^{-\Phi}\partial_5)\eta^{\mu\nu} - g_5^2a^2v^2\eta^{\mu\nu}\right\}A_\nu^a. \tag{2.22}$$

Some cross terms have been canceled by gauge fixing terms,

$$\mathcal{L}_{\text{G.F.}} = -\frac{ae^{-\Phi}}{2g_5^2\xi_V}\left\{\partial^\mu V_\mu^a - \frac{\xi_V}{ae^{-\Phi}}\partial_5(ae^{-\Phi}V_5^a)\right\}^2 - \frac{ae^{-\Phi}}{2g_5^2\xi_A}\left\{\partial^\mu A_\mu^a - \frac{\xi_A}{ae^{-\Phi}}\partial_5(ae^{-\Phi}A_5^a) + g_5^2\xi_Aa^2v^2P^a\right\}^2. \tag{2.23}$$

By using the unitary gauge $\xi \to \infty$ as in [10], we have

$$\partial_5(ae^{-\Phi}V_5^a) = 0, \tag{2.24}$$

$$\partial_5(ae^{-\Phi}A_5^a) = g_5^2a^3v^2e^{-\Phi}P^a. \tag{2.25}$$

We can write P^a in terms of A_5^a. Then Eq. (2.19) becomes

$$\mathcal{L}_{A_5}^{(2)} = -\frac{1}{2g_5^2}ae^{-\Phi}A_5^a\partial^2D^2A_5^a - \frac{1}{2}a^3v^2e^{-\Phi}(D^2A_5^a)(D^2A_5^a), \tag{2.26}$$

and the quadratic order differential operator D^2 is defined by

$$D^2f = -\partial_5\left(\frac{\partial_5(ae^{-\Phi}f)}{g_5^2a^3v^2e^{-\Phi}}\right) + f. \tag{2.27}$$

C. Scalar mesons

Next we should use Kaluza-Klein (KK) expansion to get the four-dimensional (4D) effective action

$$S(x,z) = \sum_{n=0}^{\infty}\phi^{(n)}(x)f_S^{(n)}(z), \tag{2.28}$$

where $f_S^{(n)}$'s are eigenfunctions of the following problem:

$$-\frac{1}{a^3e^{-\Phi}}\partial_5(a^3e^{-\Phi}\partial_5f_S^{(n)}) + \left(m_X^2a^2 - \frac{3}{4}\lambda a^2v\right)f_S^{(n)} = M_S^{(n)2}f_S^{(n)}, \tag{2.29}$$

with the boundary conditions

$$f_S^{(n)}|_{z\to0} = 0, \qquad f_S^{(n)}|_{z\to\infty} = 0.$$

And the orthonormality condition is

$$\int_0^\infty a^3e^{-\Phi}f_S^{(n)}f_S^{(n')}dz = \delta_{nn'}. \tag{2.30}$$

Then we insert (2.28) into (2.20) and do the integration over the z-coordinate andget exactly an effective 4D action for a cluster of scalar fields $\phi^{(n)}$. We can also transform the Sturm-Liouville Eq. (2.29) into a Schrödinger form as $-\psi_S^{(n)\prime\prime} + V_S\psi_S^{(n)} = M_S^{(n)2}\psi_S^{(n)}$ in which we introduce the V_S below. By setting $f_S^{(n)} = e^{\omega_S/2}\psi_S^{(n)}$ with $\omega_S = \Phi - 3\log a$, the effective potential V_S for scalar mesons is

$$V_S = \frac{1}{4}\omega_S'^2 - \frac{1}{2}\omega_S'' + m_X^2a^2 + \frac{3}{4}\lambda a^2v^2. \tag{2.31}$$

TABLE I. The experimental and theoretical values of scalar meson masses. The average error is 2.03%.

n	0	1	2	3	4	5	6	7
$m_{\rm exp}$	550	980	1350	1505	1724	1992	2103	2189
$m_{\rm th}$	550	999	1301	1544	1753	1939	2108	2265
error	0.0%	2.0%	3.6%	2.6%	1.7%	2.7%	0.3%	3.5%

Here we have

$$V_S \sim O(z^2), \qquad z \to \infty \tag{2.32}$$

due to the background dilaton. The eigenvalue problem (2.29) cannot be solved analytically. We have to rely on numerical calculations. We use the former parameters listed in (2.15) to calculate the scalar meson masses. The result and comparison with experimental data are shown in Table I. The agreement between the theoretical and experimental values is good.

D. Pseudoscalar mesons

Similarly, we expand the field A_5 in terms of its KK modes

$$A_5(x, z) = \sum_{n=0}^{\infty} \pi^{(n)}(x) f_P^{(n)}(z), \tag{2.33}$$

with $f_P^{(n)}$ being the eigenfunction of the differential operator D^2,

$$-\partial_5\left(\frac{\partial_5(ae^{-\Phi}f_P^{(n)})}{g_5^2 a^3 v^2 e^{-\Phi}}\right) + f_P^{(n)} = \frac{M_P^{(n)2}}{g_5^2 a^2 v^2} f_P^{(n)} \tag{2.34}$$

and with the boundary condition [11]

$$\partial_5(ae^{-\Phi}f_P^{(n)})|_{z\to 0} = 0, \qquad f_P^{(n)}|_{z\to\infty} = 0. \tag{2.35}$$

According to general theories of the Sturm-Liouville problem, we can normalize $f_P^{(n)}$ by the following orthonormality relation

$$\int_0^\infty \frac{e^{-\Phi}}{av^2} f_P^{(n)} f_P^{(n')} dz = \frac{g_5^4}{M_P^{(n)2}} \delta_{nn'}. \tag{2.36}$$

We can rewrite this eigenvalue problem in a Schrödinger form as in the scalar meson field. Define

$$p = \frac{1}{g_5^2 a^3 v^2 e^{-\Phi}}, \qquad q = \frac{1}{ae^{-\Phi}}, \tag{2.37}$$

and $\psi_P^{(n)} = ae^{-\Phi}p^{1/2} f_P^{(n)}$, which satisfies the Schrödinger equation $-\psi_P^{(n)\prime\prime} + V_P \psi_P^{(n)} = M_P^{(n)2}\psi_P^{(n)}$ with the effective potential

$$V_P = \frac{2pp'' - p'^2 + 4pq}{4p^2}. \tag{2.38}$$

Here we also have

$$V_P \sim O(z^2), \qquad z \to \infty.$$

TABLE II. The experimental and theoretical values of pseudoscalar meson masses. The average error is 7.67%.

n	0	1	2	3	4
$m_{\rm exp}$	139	1300	1816	2070	2360
$m_{\rm th}$	139	1662	1860	2040	2204
error	0.0%	27.9%	2.4%	1.5%	6.6%

Then we find the asymptotical spectrum is linear with respect to the radial quantum number n. The resulting mass spectra are listed in Table II.

E. Vector mesons

With the same procedure as the scalar and pseudoscalar mesons, the field V_μ is expanded as

$$V_\mu(x, z) = \sum_{n=0}^{\infty} \rho_\mu^{(n)}(x) f_V^{(n)}(z), \tag{2.39}$$

with $f_V^{(n)}$ being eigenfunctions of the following problem:

$$-\frac{1}{ae^{-\Phi}}\partial_5(ae^{-\Phi}\partial_5 f_V^{(n)}) = M_V^{(n)2} f_V^{(n)}, \qquad f_V^{(n)}|_{z\to 0} = 0,$$
$$f_V^{(n)}|_{z\to\infty} = 0. \tag{2.40}$$

We normalize $f_V^{(n)}$ by the following orthonormality condition

$$\int_0^\infty ae^{-\Phi} f_V^{(n)} f_V^{(n')} dz = \delta_{nn'}. \tag{2.41}$$

Then we can get the effective 4D action for a tower of massive vector fields $\rho_\mu^{(n)}$, which can be identified as the fields of ρ mesons by inserting (2.39) into (2.21) and integrating over the z-coordinate. Then we also transform (2.40) into a Schrödinger form by setting $f_V^{(n)} = e^{\omega/2}\psi_V^{(n)}$ with $\omega = \Phi - \log a$. The effective potential V_V for vector mesons is

$$V_V = \frac{1}{4}\omega'^2 - \frac{1}{2}\omega''. \tag{2.42}$$

It is also of order $O(z^2)$ in the deep IR region, i.e. $(z \to \infty)$ and gives us asymptotically linear spectra for vector mesons. The resulting mass spectra are listed in Table III.

TABLE III. The experimental and theoretical values of vector meson masses. The average error is 6.61%.

n	0	1	2	3	4	5	6
$m_{\rm exp}$	775.5	1465	1570	1720	1909	2149	2265
$m_{\rm th}$	982.9	1288	1533	1743	1930	2100	2257
error	26.8%	12.1%	2.4%	1.3%	1.1%	2.3%	0.4%

F. Axial-vector mesons

We expand the field A_μ in terms of its KK modes

$$A_\mu(x,z) = \sum_{n=0}^{\infty} a_\mu^{(n)}(x) f_A^{(n)}(z), \tag{2.43}$$

with $f_A^{(n)}$ being eigenfunctions of the following problem:

$$-\frac{1}{ae^{-\Phi}}\partial_5(ae^{-\Phi}\partial_5 f_A^{(n)}) + g_5^2 a^2 v^2 f_A^{(n)} = M_A^{(n)2} f_A^{(n)},$$
$$f_A^{(n)}|_{z\to 0} = 0, \qquad f_A^{(n)}|_{z\to\infty} = 0. \tag{2.44}$$

The orthonormality condition for $f_A^{(n)}$ is

$$\int_0^\infty ae^{-\Phi} f_A^{(n)} f_A^{(n')} dz = \delta_{nn'}, \tag{2.45}$$

the same as vector mesons. Again, inserting (2.43) into (2.22) and integrating over the z-coordinate, we get exactly an effective 4D action for a tower of massive axial-vector fields $a_\mu^{(n)}$. With the former steps, we rewrite (2.44) in a Schrödinger form by setting $f_A^{(n)} = e^{\omega/2}\psi_A^{(n)}$ with $\omega = \Phi - \log a$. The effective potential V_A for axial-vector mesons is

$$V_A = \frac{1}{4}\omega'^2 - \frac{1}{2}\omega'' + g_5^2 a^2 v^2. \tag{2.46}$$

It also has the quadratic behavior at $z \to \infty$ and asymptotically linear spectra follows. Note also that the last term is only $O(1)$ so the first term dominates, which results in the same spectral slope with that vector meson. Having different slopes is a main drawback of our previous model [39,40]. Now it has been removed by the proper choice of background fields. The theoretical and the experimental values of axial-vector mesons are listed in Table IV.

III. NUCLEON SECTOR

To realize the spin-1/2 nucleon in the AdS/QCD, we can introduce two 5D Dirac spinors $\Psi_{1,2}$ in the bulk as suggested in [15]. Each of them is also a isospin doublet. They are charged under the gauge fields L_M and R_M, respectively. The action of nucleon sector is

TABLE IV. The experimental and theoretical values of axial-vector meson masses. The average error is 4.75%.

n	0	1	2	3	4	5
$m_{\rm exp}$	1230	1647	1930	2096	2270	2340
$m_{\rm th}$	1438	1647	1844	2023	2188	2340
error	16.9%	0.0%	4.4%	3.5%	3.6%	0.0%

$$S_N = \int d^5x\sqrt{G}(\mathcal{L}_K + \mathcal{L}_I),$$
$$\mathcal{L}_K = i\bar\Psi_1\Gamma^M\nabla_M\Psi_1 + i\bar\Psi_2\Gamma^M\nabla_M\Psi_2 - m_\Psi\bar\Psi_1\Psi_1 + m_\Psi\bar\Psi_2\Psi_2,$$
$$\mathcal{L}_I = -g\bar\Psi_1 X\Psi_2 - g\bar\Psi_2 X^\dagger\Psi_1. \tag{3.1}$$

Here we have $\Gamma^M = e_A^M\Gamma^A = z\delta_A^M\Gamma^A$ with $\{\Gamma^A, \Gamma^B\} = 2\eta^{AB}$. We choose $\Gamma^A = (\gamma^a, -i\gamma^5)$ with $\gamma^5 = \mathrm{diag}(I, -I)$. The covariant derivatives for spinors are

$$\nabla_M\Psi_1 = \partial_M\Psi_1 + \frac{1}{2}\omega_M^{AB}\Sigma_{AB}\Psi_1 - iL_M\Psi_1, \tag{3.2}$$

$$\nabla_M\Psi_2 = \partial_M\Psi_2 + \frac{1}{2}\omega_M^{AB}\Sigma_{AB}\Psi_2 - iR_M\Psi_2. \tag{3.3}$$

Here $\Sigma_{AB} = \frac{1}{4}[\Gamma_A, \Gamma_B]$ and the nonzero components of the spin connection ω_M^{AB} are $\omega_\mu^{a5} = -\omega_\mu^{5a} = \frac{1}{z}\delta_\mu^a$.

A. Nucleon spectrum

The second order action is

$$S_N^{(2)} = \int d^5x\sqrt{G}(\mathcal{L}_K^{(2)} + \mathcal{L}_I^{(2)}),$$
$$\mathcal{L}_K^{(2)} = \frac{1}{a}\sum_{i=1,2}\bar\Psi_i\left(i\gamma^\mu\partial_\mu + \gamma^5\partial_5 + \frac{2a'}{a}\gamma^5 - m_\Psi a\right)\Psi_i,$$
$$\mathcal{L}_I^{(2)} = -\frac{1}{2}gv(\bar\Psi_1\Psi_2 + \bar\Psi_2\Psi_1) \tag{3.4}$$

and we expand $\Psi_{1,2}$ in terms of their KK modes,

$$\Psi_1(x,z) = \begin{pmatrix} \sum_n N_L^{(n)}(x) f_{1L}^{(n)}(z) \\ \sum_n N_R^{(n)}(x) f_{1R}^{(n)}(z) \end{pmatrix}$$
$$\Psi_2(x,z) = \begin{pmatrix} \sum_n N_L^{(n)}(x) f_{2L}^{(n)}(z) \\ \sum_n N_R^{(n)}(x) f_{2R}^{(n)}(z) \end{pmatrix}. \tag{3.5}$$

Here the $N_{L,R}^{(n)}$ are two-component objects, which will be interpreted as the left-handed and right-handed parts of a tower of 4D nucleon fields, respectively, which means

$$N^{(n)}(x) = (N_L^{(n)}, N_R^{(n)})^{\mathrm{T}} \tag{3.6}$$

when reducing to a 4D effective action. We also have the following equations which the four internal functions $f^{(n)}$ satisfy:

$$\begin{pmatrix} \partial_z - m_\Psi a + 2a'/a & -u(z) \\ -u(z) & \partial_z + m_\Psi a + 2a'/a \end{pmatrix}\begin{pmatrix} f_{1L}^{(n)} \\ f_{2L}^{(n)} \end{pmatrix} = -M_N^{(n)}\begin{pmatrix} f_{1R}^{(n)} \\ f_{2R}^{(n)} \end{pmatrix}, \tag{3.7}$$

$$\begin{pmatrix} \partial_z + m_\Psi a + 2a'/a & u(z) \\ u(z) & \partial_z - m_\Psi a + 2a'/a \end{pmatrix}\begin{pmatrix} f_{1R}^{(n)} \\ f_{2R}^{(n)} \end{pmatrix} = +M_N^{(n)}\begin{pmatrix} f_{1L}^{(n)} \\ f_{2L}^{(n)} \end{pmatrix}, \tag{3.8}$$

with $u(z) = \frac{1}{2} g_Y a(z) v(z)$. Note that these equations are general for any form of various background fields, so it is the generalization of the corresponding equations in [15]. The UV boundary conditions are [15]

$$f_{1L}^{(n)}(z \to 0) = 0, \qquad f_{2R}^{(n)}(z \to 0) = 0. \tag{3.9}$$

The IR condition is as in [39], which is proper for soft-wall models

$$f_{1R}^{(n)}(z \to \infty) = 0, \qquad f_{2L}^{(n)}(z \to \infty) = 0. \tag{3.10}$$

To reduce the 5D bulk action to 4D, we also need the following orthonormality condition

$$\int_0^\infty a^4 f_{aL}^{(n)} f_{aL}^{(n')} dz = \int_0^\infty a^4 f_{aR}^{(n)} f_{aR}^{(n')} dz = \delta_{nn'}. \tag{3.11}$$

From (3.7) and (3.8) it can be seen that only two of f's are linear independent,

$$f_{2L}^{(n)} = -\epsilon f_{1R}^{(n)}, \qquad f_{2R}^{(n)} = \epsilon f_{1L}^{(n)}, \tag{3.12}$$

where $\epsilon = \pm 1$ is the 4D parity. We can transform (3.7) and (3.8) into a two-component vector-valued Sturm-Liouville problem for $f_L^{(n)} = (f_{1L}^{(n)}, f_{2L}^{(n)})^{\mathrm{T}}$ or $f_R^{(n)} = (f_{1R}^{(n)}, f_{2R}^{(n)})^{\mathrm{T}}$. We can further rewrite the vector-valued Sturm-Liouville problem, e.g. $f_L^{(n)}$ into a Schrödinger form $-\chi_L^{(n)\prime\prime} + V_N \chi_L^{(n)} = M_N^{(n)2} \chi_L^{(n)}$ by setting $f_L^{(n)} = a^{-2} \chi_L^{(n)}$. The potential matrix V_N is

$$V_N = \begin{pmatrix} m_\Psi^2 a^2 + (m_\Psi a)' + u^2 & u' \\ u' & m_\Psi^2 a^2 - (m_\Psi a)' + u^2 \end{pmatrix}. \tag{3.13}$$

We also have similar equations for the right-handed fields.

Based on the similar arguments about the anomalous dimensions, we parametrize the bulk spinor mass also as a function of z as

$$m_\Psi = \frac{\frac{5}{2} + \mu_1 z}{1 + \mu_2 z}. \tag{3.14}$$

For we have the mass-dimensional relation for spinors

$$m_\Psi = \Delta - 2. \tag{3.15}$$

TABLE V. The experimental and theoretical values of the spin-1/2 nucleon masses. The average error is 3.06%.

n	0	1	2	3	4	5	6
m_{exp}	939	1440	1535	1650	1710	2090	2100
m_{th}	941	1402	1536	1767	1819	2026	2057
error	0.2%	2.6%	0.1%	7.1%	6.4%	3.1%	2.0%

So this parametrization gives the correct UV limit 5/2, corresponding to the classical dimension 9/2 of the baryon operator by the equation above. And at IR m_Φ will tend to a constant μ_1/μ_2 and this is also reasonable. By fitting the spin-1/2 nucleon mass we choose

$$\mu_1 = 1.16 \text{ GeV}, \qquad \mu_2 = 7.8 \text{ GeV}, \qquad g_Y = 8.74. \tag{3.16}$$

The resulting mass spectra and the corresponding data are listed in Table V.

IV. SUMMARY

In this paper we further develop the model proposed in [1]. The main motivation of this model is to correctly reproduce the observed spectral pattern of both mesons and nucleons. In the original soft-wall model the quadratic dilaton is introduced for the linear spectra of mesons. To further constrain the IR behavior of other background fields, $a(z)$ and $v(z)$, we need to consider more spectral details. Two key facts which help us to fix this are (1) nucleons also have linear spectra, and (2) various meson sectors have the same spectral slopes. Combining these two requires $a(z) \sim O(z)$ and $v(z) \sim O(z^{-1})$ as $z \to \infty$. In the present work we include a quartic potential for the bulk scalar to improve our model, and carefully study the spectra of various mesons and nucleons. The agreement between the theoretical calculation and the experimental data is rather good. Actually it can be easily generalized to include more baryon sectors, e.g. the Δ. These discussions show a consistent way to consider mesons and baryons simultaneously in one AdS/QCD model. The problem that they need different IR cutoffs in the hard-wall model disappears here just by definition. It is a proper setup to further study meson-baryon interactions in future works.

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