Impacts of the Higgs mass on vacuum stability, running fermion masses, and two-body Higgs decays

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The recent results of the ATLAS and CMS experiments indicate 116 GeV $\lesssim M_H \lesssim$ 131 GeV and 115 GeV $\lesssim M_H \lesssim$ 127 GeV, respectively, for the mass of the Higgs boson in the standard model (SM) at the 95% confidence level. In particular, both experiments point to a preferred narrow mass range $M_H \simeq (124 \cdots 126)$ GeV. We examine the impact of this preliminary result of M_H on the SM vacuum stability by using the two-loop renormalization-group equations, and arrive at the cutoff scale $\Lambda_{\rm VS} \sim 4 \times 10^{12}$ GeV (for $M_H = 125$ GeV, $M_t = 172.9$ GeV, and $\alpha_s(M_Z) = 0.1184$), where the absolute stability of the SM vacuum is lost and some kind of new physics might take effect. We update the values of running lepton and quark masses at some typical energy scales, including the ones characterized by M_H, 1 TeV and $\Lambda_{\rm VS}$, with the help of the two-loop renormalization-group equations. The branching ratios of some important two-body Higgs decay modes, such as $H \to b\bar{b}$, $H \to \tau^+\tau^-$, $H \to \gamma\gamma$, $H \to W^+W^-$, and $H \to ZZ$, are also recalculated by inputting the values of relevant particle masses at M_H.

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I. INTRODUCTION

The Higgs mechanism [1] is responsible for the spontaneous gauge symmetry breaking $\rm SU(2)_L \otimes U(1)_Y \to U(1)_{em}$ in the standard model (SM) of electroweak interactions [2], but the Higgs boson itself left no sort of trace in all the previous high-energy collider experiments. The main goal of the Large Hadron Collider (LHC) at CERN is just to discover this elusive particle, which allows the other particles (except the photon and gluons) to gain finite masses. Combined with the indirect bounds obtained from the electroweak precision measurements, the recent data of the ATLAS and CMS experiments lead us to a rather narrow range of the Higgs mass: 114 GeV $\lesssim M_H \lesssim$ 141 GeV [3]. Based on the data collected in 2011 at the center-of-mass energy of 7 TeV, both collaborations have obtained

$$M_H \simeq \begin{cases} (116 \cdots 131)\ \text{GeV} & (\text{ATLAS[4]}) \\ (115 \cdots 127)\ \text{GeV} & (\text{CMS[5]}) \end{cases} \tag{1}$$

at the 95% confidence level. The ATLAS Collaboration has also found a preliminary hint of $M_H \simeq 126$ GeV with the 3.6σ local significance in $H \to \gamma\gamma$ (2.8σ), $H \to ZZ^* \to 4l$ (2.1σ), and $H \to WW^* \to 2l2\nu$ (1.4σ) decay modes [4], and the CMS Collaboration has observed an excess compatible with $M_H \lesssim 124$ GeV with the 2.6σ local significance [5]. These interesting results point to a preferred and narrower range $M_H \simeq (124 \cdots 126)$ GeV for the SM Higgs boson. More recently, the two collaborations have announced their new evidence for the existence of a Higgs-like boson: $M_H = 126.5$ GeV (ATLAS) and $M_H = 125.3 \pm 0.6$ GeV (CMS) at the 5σ significance [6]. Unless something goes wrong, this new particle is expected to be just the long-awaited SM Higgs particle.

Observing the Higgs boson and measuring its mass and other properties may help us solve several fundamental problems in elementary particle physics. Here we mention three of them for examples.

(i) The Higgs mass theoretically suffers significant radiative corrections, and hence new symmetries and (or) new particles should be introduced to stabilize the electroweak scale $\Lambda_{\rm EW} \sim 10^2$ GeV [7]. A solution to this gauge hierarchy problem calls for new physics beyond the SM, such as supersymmetries [8] or extra spatial dimensions [9].

(ii) The Higgs boson is indispensable to the Yukawa interactions of three-family fermions which makes weak *CP* violation possible in the SM or its simple extensions [10]. To some extent, the existence of the scalar fields might also support the Peccei-Quinn mechanism as an appealing solution to the strong *CP* problem [11].

(iii) With the help of the SM Higgs field, one may write out the unique dimension-five operator $\ell\ell HH$ in an effective field theory [12] or implement the seesaw mechanism in a renormalizable quantum field theory [13] to generate finite but tiny neutrino masses.

Therefore, the highest priority of the LHC experiment is to pin down the Higgs boson and its quantum numbers. We are almost making a success in this connection.

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Motivated by the encouraging ATLAS and CMS results, we aim to examine the impact of $M_H \simeq (124 \cdots 126)$ GeV on the vacuum stability of the SM, the running behaviors of fermion masses, and the branching ratios of the Higgs decays. The point is that a relatively small value of M_H is likely to cause the vacuum instability unless new physics takes effect at a proper cutoff scale [14]. Given $M_H \simeq 125$ GeV as indicated by the latest LHC data, it is timely to determine the energy scale at which the effective quartic Higgs coupling $\tilde{\lambda}(\mu)$ runs to zero. Adopting the best-fit values of the relevant input parameters, we find that this cutoff scale is around $\Lambda_{\rm VS} \sim 4 \times 10^{12}$ GeV, which presumably signifies the end of the gauge desert and the beginning of a new physics oasis. Taking account of the allowed range of M_H and the updated values of other SM parameters, we recalculate the running fermion masses at some typical energy scales up to $\Lambda_{\rm VS}$ by means of the renormalization-group equations (RGEs). Such an exercise makes sense because a sufficiently large value of M_H (e.g., $M_H \simeq 140$ GeV) was assumed in the previous works and hence the potential vacuum stability problem did not show up [15]. As a by-product, the branching ratios of some important two-body Higgs decay modes in the SM, such as $H \to b\bar{b}$, $H \to \tau^+\tau^-$, $H \to \gamma\gamma$, $H \to W^+W^-$, and $H \to ZZ$, are also recalculated by using the new values of relevant particle masses obtained at the energy scale $\mu \sim M_H$.

II. THE HIGGS MASS AND VACUUM STABILITY

First of all, let us briefly review the vacuum stability issue in the SM with a relatively light Higgs boson. In order to find out the true vacuum state and analyze its stability, one should calculate the effective scalar potential by taking account of radiative corrections and RGE improvements of the relevant parameters [14,16]. It has been shown that the l-loop scalar potential improved by the $(l+1)$-loop RGEs actually includes all the lth-to-leading logarithm contributions [17]. At the one-loop level, the effective scalar potential in the 't Hooft-Landau gauge and in the $\overline{\rm MS}$ renormalization scheme can be written as [18]

$$\begin{aligned} V_{\rm eff}[\phi(t)] = & -\frac{1}{2}m^2(t)\phi^2(t) + \frac{1}{4}\lambda(t)\phi^4(t) \\ & + \frac{3}{64\pi^2}\left\{2m_W^4[\phi(t)]\left[\ln\left(\frac{m_W^2[\phi(t)]}{\mu^2(t)}\right) - \frac{5}{6}\right]\right. \\ & + m_Z^4[\phi(t)]\left[\ln\left(\frac{m_Z^2[\phi(t)]}{\mu^2(t)}\right) - \frac{5}{6}\right] \\ & \left. - 4m_t^4[\phi(t)]\left[\ln\left(\frac{m_t^2[\phi(t)]}{\mu^2(t)}\right) - \frac{3}{2}\right]\right\}, \end{aligned} \quad (2)$$

in which $t = \ln[\mu(t)/M_Z]$ with $\mu(t) \equiv M_Z e^t$ being the renormalization scale, $m_W^2[\phi(t)] \equiv g^2(t)\phi^2(t)/4$, $m_Z^2[\phi(t)] \equiv [g^2(t) + g'^2(t)]\phi^2(t)/4$, $m_t^2[\phi(t)] \equiv y_t^2(t)\phi^2(t)/2$, and the small contributions from the Goldstone and Higgs bosons have been neglected. The β functions for the dimensionless couplings $g(t)$, $g'(t)$, $\lambda(t)$, and $y_t(t)$ at the two-loop level are already given in Refs. [18,19], so are the γ functions for $m^2(t)$ and $\phi(t)$.

The scalar potential $V_{\rm eff}$ has to develop a realistic minimum at the electroweak scale, corresponding to the SM vacuum. Whether the SM vacuum is stable or not depends on the behavior of $V_{\rm eff}$ in the large-field limit, i.e., $\phi(t) \gg M_Z$. One may find out the extrema $\phi_{\rm ex}(t)$ of the scalar potential via

$$\left.\frac{\partial V_{\rm eff}[\phi(t)]}{\partial \phi(t)}\right|_{\phi(t)=\phi_{\rm ex}(t)} = 0. \quad (3)$$

At the electroweak scale $\mu(0) = M_Z$, one should impose the boundary condition $\phi_{\rm ex}(0) \equiv v \approx 246$ GeV, which is the vacuum expectation value of the Higgs field. At the large-field values, the scalar potential is dominated by the quartic coupling term and the extrema $\phi_{\rm ex}(t)$ can be evaluated at the renormalization scale $\mu(t) = \phi_{\rm ex}(t)$ from Eqs. (2) and (3) as $\phi_{\rm ex}^2 = m^2/\tilde{\lambda}$, where the effective quartic coupling $\tilde{\lambda}$ is defined as

$$\begin{aligned} \tilde{\lambda} = \lambda - \frac{3}{32\pi^2}\left\{\frac{1}{8}(g'^2 + g^2)^2\left[\frac{1}{3} - \ln\left(\frac{g'^2 + g^2}{4}\right)\right]\right. \\ \left. + 2y_t^4\left[\ln\left(\frac{y_t^2}{2}\right) - 1\right] + \frac{1}{4}g^4\left[\frac{1}{3} - \ln\left(\frac{g^2}{4}\right)\right]\right\}. \end{aligned} \quad (4)$$

Now it is clear that $V_{\rm eff} \approx \tilde{\lambda}\phi^4/4$ will develop a minimum much deeper than the realistic minimum if the effective coupling $\tilde{\lambda}$ becomes negative [20–24]. To maintain the absolute stability of the SM vacuum, new physics should come into play below or at the energy scale $\Lambda_{\rm VS}$ where the effective coupling $\tilde{\lambda}$ vanishes, i.e., $\tilde{\lambda}(\Lambda_{\rm VS}) = 0$. One may derive a lower bound of the Higgs mass by requiring that the SM vacuum is absolutely stable up to a possible grand unified theory scale or the Planck scale [14,20–24].

In view of the presently allowed range of the Higgs mass, we shall conversely implement the vacuum stability argument to determine the scale $\Lambda_{\rm VS}$ at which new physics should take effect. Our strategy is as follows. First, we specify the matching conditions which link λ and y_t to the Higgs mass M_H and the top-quark pole mass M_t, respectively. Although the complete effective potential $V_{\rm eff}$ must be scale independent, the approximate one at the one-loop order is not. But one may find out an optimal scale where the effective potential has the least scale dependence. As shown in Ref. [20], $\mu(t^*) = M_t$ is a reasonable choice. So let us just choose the matching conditions for λ and y_t at $\mu(t^*) = M_t$:

$$\begin{aligned} \lambda(M_t) &= \frac{M_H^2}{2v^2}[1 + \delta_H(M_t)], \\ y_t(M_t) &= \frac{\sqrt{2}M_t}{v}[1 + \delta_t(M_t)], \end{aligned} \quad (5)$$

where the correction terms $\delta_H(M_t)$ and $\delta_t(M_t)$ have been given in Refs. [24–26]. The values of the other input parameters are taken from Ref. [27] and will be specified in Sec. 1 when we turn to the running fermion masses. Second, we run $\lambda(\mu)$ to a much higher energy scale by solving the complete two-loop RGEs. Third, the cutoff scale $\Lambda_{\rm VS}$ can be identified as the solution to $\tilde{\lambda}(\Lambda_{\rm VS}) = 0$, where $\tilde{\lambda}$ is related to λ via Eq. (4). Note that the value of $\Lambda_{\rm VS}$ determined by $\tilde{\lambda}(\Lambda_{\rm VS}) = 0$ could be an order of magnitude larger than the one determined by $\lambda(\Lambda_{\rm VS}) = 0$. The latter is less reliable because it does not include the one-loop radiative corrections to the scalar potential [20,22].

Our numerical result for the correlation between the Higgs mass and the energy scale is shown in Fig. 1. Some comments are in order.

(i) If $M_H \gtrsim 129$ GeV holds, the vacuum stability can be guaranteed even up to a possible grand unified theory scale (e.g., 10^{16} GeV) or the Planck scale $\Lambda_{\rm Pl} \sim 10^{19}$ GeV [28]. The cutoff scale $\Lambda_{\rm VS}$ increases as the Higgs mass M_H increases, but this observation is sensitively dependent on the value of the top-quark pole mass M_t.

(ii) Given $M_H \simeq 125$ GeV, some kind of new physics should come out around $\Lambda_{\rm VS} \sim 10^{12}$ GeV to stabilize the SM vacuum if the best-fit values $M_t = 172.9$ GeV and $\alpha_s(M_Z) = 0.1184$ are taken. It is interesting to notice that the canonical seesaw mechanism for the origin of tiny neutrino masses works well around this cutoff scale, so does the leptogenesis mechanism [29] which can account for the observed matter-antimatter asymmetry of the Universe.

(iii) Figure 1 shows that $\Lambda_{\rm VS}$ is sensitive to the Higgs mass in the range [120 GeV, 130 GeV]. Hence, one has to take care of the experimental errors associated with M_t and $\alpha_s(M_Z)$, and the theoretical uncertainties involved in the RGEs and matching conditions [22]. The evaluation of the theoretical uncertainties has recently been improved by taking into account the three-loop RGEs and two-loop matching conditions in Ref. [30], where the absolute stability of the SM vacuum up to the Planck scale is found to be lost for $M_H < 126$ GeV at the 2σ level. For illustration, we have taken the best-fit values of M_t and $\alpha_s(M_Z)$ as our inputs, leading to the output $\Lambda_{\rm VS} \sim 10^{12}$ GeV for $M_H = 125$ GeV. It is worth pointing out that the difference between the quartic Higgs coupling $\lambda(\mu)$ around $\Lambda_{\rm VS}$ via the two-loop RGEs and the one via the three-loop RGEs is smaller than 0.1% [31]. The uncertainty of $\Lambda_{\rm VS}$ is actually dominated by the errors of M_t and $\alpha_s(M_Z)$. Given the same inputs, the result of $\Lambda_{\rm VS}$ based on the two-loop RGEs is therefore of the same order as the one based on the three-loop RGEs.

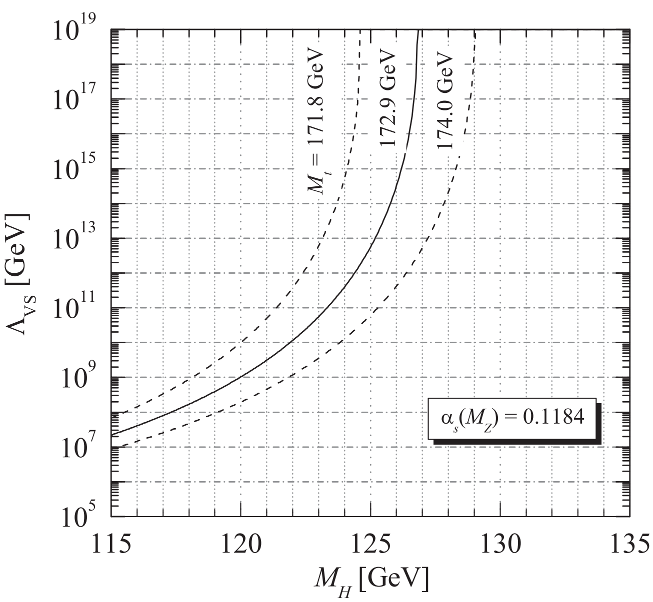

FIG. 1. Correlation between the energy scale $\Lambda_{\rm VS}$ and the Higgs mass M_H as required by the vacuum stability, where the solid curve corresponds to the best-fit value of the top-quark pole mass $M_t = 172.9$ GeV, and the dashed lines stand for the 1σ lower and upper limits.

Even though the existence of a cutoff scale is robust for the SM with a relatively light Higgs boson, it remains unclear what kind of new physics could take effect over there. In any event, if the new physics responsible for the vacuum stability could also offer a solution to the flavor puzzles of leptons and quarks (especially the origin of tiny neutrino masses), the running fermion masses at the cutoff scale $\Lambda_{\rm VS}$ will be very helpful for model building. We shall focus on this issue in the following section.

III. RUNNING LEPTON AND QUARK MASSES

A systematic analysis of the RGE running masses of leptons and quarks has been done in Ref. [15], where $M_H \simeq 140$ GeV was typically taken just for illustration. As discussed above, such a value of the Higgs mass makes the situation simple because it does not give rise to the vacuum instability problem in the SM. Here we want to update the running fermion masses for two good reasons: (a) the latest ATLAS and CMS data point to $M_H \simeq (124 \cdots 126)$ GeV, and hence the issue of vacuum stability should be taken seriously; and (b) the values of some of the input parameters adopted in Ref. [15] have more or less changed in the past few years, and thus an update of the analysis is necessary. Before doing a detailed RGE analysis, let us summarize the input parameters and outline our calculational strategy.

(i) Six quark masses are $m_u(2\,{\rm GeV}) = (1.7 \cdots 3.1)$ MeV, $m_d(2\,{\rm GeV}) = (4.1 \cdots 5.7)$ MeV, $m_s(2\,{\rm GeV}) = (80 \cdots 130)$ MeV, $m_c(m_c) = 1.29^{+0.05}_{-0.11}$ GeV, $m_b(m_b) = 4.19^{+0.08}_{-0.16}$ GeV, and $M_t = 172.9^{+1.1}_{-1.1}$ GeV [27],

TABLE I. Running quark masses at some typical energy scales in the SM, including the Higgs mass $M_H \simeq 125$ GeV and the corresponding cutoff scale $\Lambda_{\rm VS} \simeq 4 \times 10^{12}$ GeV. Note that the values of the pole masses M_q and the running masses $m_q(M_q)$, rather than $m_q(\mu)$ at these mass scales, are given in the last two rows for comparison. However, the pole masses of three light quarks are not listed simply because the perturbative QCD calculation is not reliable in that energy region.

μ	$m_u(\mu)$ (MeV)	$m_d(\mu)$ (MeV)	$m_s(\mu)$ (MeV)	$m_c(\mu)$ (GeV)	$m_b(\mu)$ (GeV)	$m_t(\mu)$ (GeV)
$m_c(m_c)$	$2.79^{+0.83}_{-0.82}$	$5.69^{+0.96}_{-0.95}$	116^{+36}_{-24}	$1.29^{+0.05}_{-0.11}$	$5.95^{+0.37}_{-0.15}$	$385.7^{+8.1}_{-7.8}$
2 GeV	$2.4^{+0.7}_{-0.7}$	4.9 ± 0.8	100^{+30}_{-20}	$1.11^{+0.07}_{-0.14}$	$5.06^{+0.29}_{-0.11}$	$322.2^{+5.0}_{-4.9}$
$m_b(m_b)$	$2.02^{+0.60}_{-0.60}$	$4.12^{+0.69}_{-0.68}$	84^{+26}_{-17}	$0.934^{+0.058}_{-0.120}$	$4.19^{+0.18}_{-0.16}$	$261.8^{+3.0}_{-2.9}$
M_W	$1.39^{+0.42}_{-0.41}$	$2.85^{+0.49}_{-0.48}$	58^{+18}_{-12}	$0.645^{+0.043}_{-0.085}$	$2.90^{+0.16}_{-0.06}$	174.2 ± 1.2
M_Z	$1.38^{+0.42}_{-0.41}$	2.82 ± 0.48	57^{+18}_{-12}	$0.638^{+0.043}_{-0.084}$	$2.86^{+0.16}_{-0.06}$	172.1 ± 1.2
M_H	$1.34^{+0.40}_{-0.40}$	$2.74^{+0.47}_{-0.47}$	56^{+17}_{-12}	$0.621^{+0.041}_{-0.082}$	$2.79^{+0.15}_{-0.06}$	$167.0^{+1.2}_{-1.2}$
$m_t(m_t)$	$1.31^{+0.40}_{-0.39}$	2.68 ± 0.46	55^{+17}_{-11}	$0.608^{+0.041}_{-0.080}$	$2.73^{+0.15}_{-0.06}$	163.3 ± 1.1
1 TeV	1.17 ± 0.35	$2.40^{+0.42}_{-0.41}$	49^{+15}_{-10}	$0.543^{+0.037}_{-0.072}$	$2.41^{+0.14}_{-0.05}$	148.1 ± 1.3
$\Lambda_{\rm VS}$	$0.61^{+0.19}_{-0.18}$	1.27 ± 0.22	26^{+8}_{-5}	$0.281^{+0.02}_{-0.04}$	$1.16^{+0.07}_{-0.02}$	82.6 ± 1.4
M_q	...	...	...	$1.84^{+0.07}_{-0.13}$	$4.92^{+0.21}_{-0.08}$	172.9 ± 1.1
$m_q(M_q)$	...	...	...	$1.14^{+0.06}_{-0.12}$	$4.07^{+0.18}_{-0.06}$	162.5 ± 1.1

where the pole mass M_t is extracted from the direct measurements. The pole masses of three charged leptons are $M_e = (0.510998910 \pm 0.000000013)$ MeV, $M_\mu = (105.658367 \pm 0.000004)$ MeV, and $M_\tau = (1776.82 \pm 0.16)$ MeV [27]. Following the same approach as the one described in Ref. [15], we can calculate the running masses of charged leptons and quarks at some typical energy scales, including $\mu = M_W$, M_Z, M_H, 1 TeV, and $\Lambda_{\rm VS}$.

(ii) The strong and electromagnetic fine-structure constants at M_Z are $\alpha_s(M_Z) = 0.1184 \pm 0.0007$ and $\alpha(M_Z)^{-1} = 127.916 \pm 0.015$, and the weak mixing angle is $\sin^2\theta_W(M_Z) = 0.23116 \pm 0.00013$ [27]. With the help of these input parameters, one can determine the gauge coupling constants $g_s^2 = 4\pi\alpha_s$, $g^2 = 4\pi\alpha/\sin^2\theta_W$, and $g' = g\tan\theta_W$ at M_Z.

(iii) The four parameters of quark flavor mixing and CP violation in the modified Wolfenstein parametrization are $\lambda = 0.2253 \pm 0.0007$, $A = 0.808^{+0.022}_{-0.015}$, $\bar{\rho} = 0.132^{+0.022}_{-0.014}$, and $\bar{\eta} = 0.341 \pm 0.013$ [27]. These values, together with the values of six quark masses, allow us to reconstruct the quark Yukawa coupling matrices $Y_{\rm u}$ and $Y_{\rm d}$ at M_Z. The RGEs of $Y_{\rm u}$ and $Y_{\rm d}$ can therefore help us to run the quark masses and flavor mixing parameters to a much higher energy scale.

(iv) The allowed ranges of three lepton flavor mixing angles are $30.6° \leq \theta_{12} \leq 36.8°$, $35.7° \leq \theta_{23} \leq 53.1°$, and $1.8° \leq \theta_{13} \leq 12.1°$ [32], and the allowed ranges of two neutrino mass-squared differences are 6.99×10^{-5} eV$^2 \leq \delta m^2 \leq 8.18 \times 10^{-5}$ eV^2 and 2.06×10^{-3} eV$^2 \leq |\Delta m^2| \leq 2.67 \times 10^{-3}$ eV^2 [32]. For simplicity, we only take the best-fit values $\theta_{12} = 33.6°$, $\theta_{23} = 40.4°$, and $\theta_{13} = 8.3°$ together with $\delta m^2 = 7.58 \times 10^{-5}$ eV^2 and $|\Delta m^2| = 2.35 \times 10^{-3}$ eV^2 as the inputs at M_Z in our numerical calculations. In particular, the value of θ_{13} taken above is essentially consistent with the latest Daya Bay [33] and RENO [34] results. The unknown CP-violating phases in the lepton sector are all assumed to be zero. In view of the fact that the absolute neutrino mass scale is also unknown, we shall only consider the normal mass hierarchy with $m_1 = 0.001$ eV and $m_1 < m_2 \ll m_3$ at M_Z for illustration. For the same reason, only the one-loop RGE for neutrino masses is considered. It is then possible to reconstruct the charged-lepton Yukawa coupling matrix Y_l and the effective neutrino coupling matrix κ at M_Z from the given lepton masses and flavor mixing parameters [35].[1]

For a complete list of the RGEs to be used in our numerical analysis, we refer the reader to Ref. [15] and the references therein. Note that we retain the two-loop RGEs for the charged-lepton and quark masses because a full set of three-loop RGEs has been lacking and the main uncertainties of running quark masses at high-energy scales come from the input values of quark masses at the low-energy scales.

Tables I and II summarize our numerical results for the running quark and charged-lepton masses at some typical

[1]Here we simply assume that finite neutrino masses are generated from the effective dimension-five operator $\kappa(\bar{\ell}_{\rm L}H)(H^T\ell^c_{\rm L})$ [12] that can be obtained after integrating out the relevant heavy degrees of freedom from a full theory at a superhigh-energy scale.

TABLE II. Running charged-lepton masses at some typical energy scales in the SM, including the Higgs mass $M_H \simeq 125$ GeV and the corresponding cutoff scale $\Lambda_{\rm VS} \simeq 4 \times 10^{12}$ GeV, where the uncertainties of $m_l(\mu)$ are determined by those of M_l. Note that the pole masses M_l, rather than the running masses $m_l(M_l)$, are given in the last row for comparison.

μ	$m_e(\mu)$ (MeV)	$m_\mu(\mu)$ (MeV)	$m_\tau(\mu)$ (MeV)
$m_c(m_c)$	$0.495473903 \pm 0.000000013$	$104.4617350^{+0.0000059}_{-0.0000060}$	1774.62 ± 0.16
$m_b(m_b)$	$0.493099926 \pm 0.000000013$	$103.9961602^{+0.0000059}_{-0.0000060}$	1767.02 ± 0.16
M_W	$0.486845781^{+0.000000013}_{-0.000000012}$	$102.7721083 \pm 0.0000059$	$1747.05^{+0.15}_{-0.16}$
M_Z	$0.486570154^{+0.000000012}_{-0.000000013}$	$102.7181337^{+0.0000059}_{-0.0000058}$	$1746.17^{+0.15}_{-0.16}$
M_H	$0.485858771^{+0.000000013}_{-0.000000012}$	$102.5788227^{+0.0000058}_{-0.0000059}$	1743.89 ± 0.16
$m_t(m_t)$	$0.485285152^{+0.000000012}_{-0.000000013}$	$102.4664851^{+0.0000059}_{-0.0000058}$	1742.06 ± 0.16
1 TeV	$0.489535765^{+0.000000013}_{-0.000000012}$	$103.3441945 \pm 0.0000059$	1756.81 ± 0.16
$\Lambda_{\rm VS}$	$0.484511554^{+0.000000012}_{-0.000000013}$	$102.2835586^{+0.0000058}_{-0.0000059}$	1738.82 ± 0.16
M_l	$0.510998910 \pm 0.000000013$	105.658367 ± 0.0000040	1776.82 ± 0.16

energy scales, respectively. Different from the previous works, here the scales characterized by the Higgs mass M_H and the vacuum stability cutoff $\Lambda_{\rm VS}$ are taken into account for the first time. The values of the fermion masses at M_H will be used to calculate the branching ratios of some important Higgs decay modes in Sec. IV, and those at $\Lambda_{\rm VS}$ are expected to be useful for building possible flavor models beyond the SM.

In studying the running behaviors of 12 fermion masses above M_Z, we have used the inputs at M_Z to numerically solve the RGEs of the Yukawa coupling matrices $Y_{\rm u}$, $Y_{\rm d}$, Y_l, and the effective neutrino coupling matrix κ, as well as the two-loop RGEs of the quartic Higgs coupling $\lambda(\mu)$ and gauge couplings at $\mu \geq M_Z$. After $Y_{\rm u}$, $Y_{\rm d}$, Y_l, and κ are diagonalized, one can obtain the running quark masses $m_q(\mu) = y_q(\mu)v/\sqrt{2}$ (for $q = u$, c, t and d, s, b), the running charged-lepton masses $m_l(\mu) = y_l(\mu)v/\sqrt{2}$ (for $l = e$, μ, τ), and the running neutrino masses $m_i(\mu) = \kappa_i(\mu)v^2/2$ (for $i = 1, 2, 3$). The corresponding quark and lepton flavor mixing parameters can simultaneously be achieved. For simplicity, let us define $R_f(\mu) \equiv m_f(\mu)/m_f(M_Z)$, where the subscript f runs over the mass-eigenstate indices of six quarks and six leptons. We find that $R_u(\mu) \approx R_d(\mu) \approx R_s(\mu) \approx R_c(\mu) \approx R_e(\mu) \approx R_\mu(\mu) \approx 1$ holds to a good degree of accuracy if μ is below the cutoff scale $\Lambda_{\rm VS}$. So we only plot the numerical results of $R_t(\mu)$, $R_b(\mu)$, and $R_\tau(\mu)$ in Fig. 2. The ratios $R_i(\mu)$ for three neutrino masses are shown in Fig. 3. Some discussions are in order.

(1) The mass ratios $R_f(\mu)$ are not very sensitive to the quartic Higgs coupling $\lambda(\mu)$ or equivalently the Higgs mass M_H simply because the latter enters the RGEs of fermion masses only at the two-loop level. As observed in Ref. [15], there exists a maximum for the charged-lepton masses around $\mu \sim 10^6$ GeV, while the quark masses monotonously decrease as the energy scale increases. In view of the vacuum instability problem discussed in Sec. II, we argue that the evolution of fermion masses above the cutoff scale $\Lambda_{\rm VS}$ might not be meaningful anymore. We expect that some kind of new physics takes effect around $\Lambda_{\rm VS}$ and thus modifies the RGEs of the SM.

(2) In most cases the running behaviors of three neutrino masses are neither sensitive to their absolute values nor sensitive to their mass hierarchies [36]. Only when the neutrino masses are assumed to be nearly degenerate, the running effects of neutrino mass and mixing parameters are likely to be somewhat significant. But the dependence of $m_i(\mu)$ on $\lambda(\mu)$ or M_H is quite evident simply because the effective neutrino coupling matrix κ receives the one-loop corrections from the quartic Higgs interaction [15,36].

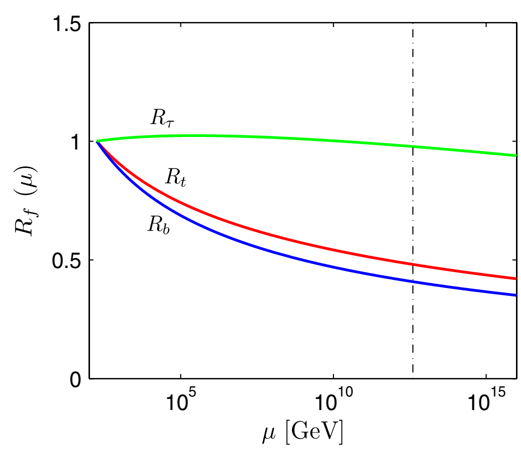

FIG. 2 (color online). The running behaviors of $R_t(\mu)$, $R_b(\mu)$, and $R_\tau(\mu)$ with respect to the energy scale μ in the SM, where the vertical dashed line indicates the cutoff scale $\Lambda_{\rm VS} \simeq 4 \times 10^{12}$ GeV as required by the vacuum stability for $M_H \simeq 125$ GeV.

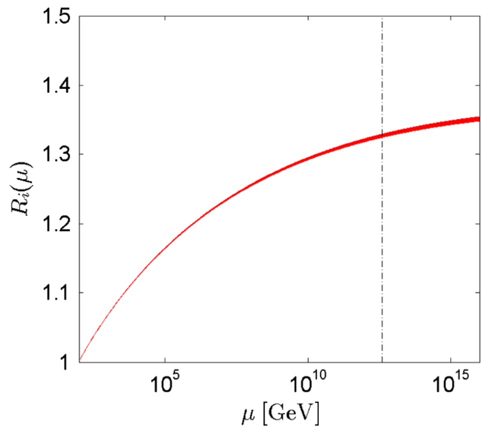

FIG. 3 (color online). The evolution of $R_i(\mu)$ with respect to the energy scale μ, where the red band corresponds to the variation of the Higgs mass in the range $M_H \simeq (124 \cdots 126)$ GeV, and the vertical dashed line indicates the cutoff scale $\Lambda_{\rm VS} \simeq 4 \times 10^{12}$ GeV. Note that $R_1(\mu) \simeq R_2(\mu) \simeq R_3(\mu)$ holds to an excellent degree of accuracy.

For simplicity, we skip the numerical illustration of the running behaviors of quark and lepton flavor mixing parameters in this paper.

IV. BRANCHING RATIOS OF THE HIGGS DECAYS

The present ATLAS and CMS experiments are mainly sensitive to the Higgs boson via its decay channels $H \to \gamma\gamma$, $H \to b\bar{b}$, $H \to \tau^+\tau^-$, $H \to W^+W^-(2l2\nu)$, and $H \to ZZ(4l, 2l2\nu, 2l2q, 2l2\tau)$, where $l = e$ or μ and ν denote the neutrinos of any flavors [3]. Which channel is dominant depends crucially on the Higgs mass. If $M_H \lesssim 135$ GeV holds, the decay mode $H \to b\bar{b}$ is expected to have the largest branching ratio, and if the Higgs mass is slightly heavier, the decay mode $H \to W^+W^-$ will surpass the others [16].

We first consider the leptonic $H \to l^+l^-$ decays where l runs over e, μ or τ. Including the one-loop electroweak corrections, the decay width of $H \to l^+l^-$ is given by [37]

$$\Gamma_l = \frac{G_{\rm F} M_H}{4\sqrt{2}\pi} M_l^2 \left(1 - \frac{4M_l^2}{M_H^2}\right)^{3/2} (1 + \delta_{\rm QED} + \delta_{\rm W}), \quad (6)$$

where $G_{\rm F}$ is the Fermi constant, $\delta_{\rm QED} = 9\alpha[3 - 2\ln(M_H^2/M_l^2)]/(12\pi)$, and

$$\delta_{\rm W} = \frac{G_{\rm F}}{8\sqrt{2}\pi^2}\left\{7M_t^2 + M_W^2\left(\frac{3}{\sin^2\theta_W}\ln\cos^2\theta_W - 5\right) - M_Z^2\left[3(1 - 4\sin^2\theta_W)^2 - \frac{1}{2}\right]\right\}. \quad (7)$$

Note that the large logarithmic term $\ln(M_H^2/M_l^2)$ in $\delta_{\rm QED}$ can be absorbed in the running mass of l at the scale of M_H, which has been given in Table II.

Now we turn to $H \to q\bar{q}$ where q runs over u, d, s, c, or b for 114 GeV $\lesssim M_H \lesssim$ 141 GeV. Since the decay rates of $H \to u\bar{u}$, $d\bar{d}$, and $s\bar{s}$ are highly suppressed by the corresponding quark masses, we are mainly interested in the decay rates of $H \to c\bar{c}$ and $H \to b\bar{b}$. Up to the three-loop QCD corrections [38],

$$\Gamma_q = \frac{3G_{\rm F}M_H}{4\sqrt{2}\pi} m_q^2(M_H)(\delta_{\rm QCD} + \delta_t), \quad (8)$$

where

$$\delta_{\rm QCD} = 1 + 5.67\left(\frac{\alpha_s(M_H)}{\pi}\right) + 29.14\left(\frac{\alpha_s(M_H)}{\pi}\right)^2 + 41.77\left(\frac{\alpha_s(M_H)}{\pi}\right)^3 \quad (9)$$

and

$$\delta_t = \left(\frac{\alpha_s(M_H)}{\pi}\right)^2\left[1.57 - \frac{2}{3}\ln\left(\frac{M_H^2}{M_t^2}\right) + \frac{1}{9}\ln^2\left(\frac{m_q^2(M_H)}{M_H^2}\right)\right]. \quad (10)$$

Note that the running quark masses $m_q(M_H)$ and the strong coupling constant $\alpha_s(M_H)$ are useful here to absorb the large logarithmic terms.

A detailed discussion about the important two-body decay modes $H \to \gamma\gamma$, $H \to W^+W^-$, $H \to ZZ$, $H \to gg$, and $H \to t\bar{t}$ can be found in Ref. [39]. For simplicity, here we do not elaborate the relevant analytical results but do a numerical recalculation based on the updated particle masses at M_H. In order to compute the branching ratios of the above decay channels, we implement the latest version of the program HDECAY [40] and update the input parameters according to our Tables I and II together with Ref. [27]. Some comments are in order.

(i) The pole masses of the charged leptons (i.e., M_l), instead of their running masses $m_l(M_H)$, have been used as the input parameters in the program HDECAY. This treatment is consistent with the formula of Γ_l given in Eq. (6). If one chooses to use the running masses $m_l(M_H)$ in the numerical calculation, then the formula of the decay rates in Eq. (6) should be changed accordingly.

(ii) The one-loop pole masses of c and b quarks are used as the input parameters in the program HDECAY because they must be consistent with the corresponding parton distribution function when the production of the Higgs boson in a hadron collider (e.g., the LHC) is taken into account [41]. In our numerical calculations, we start from the values of $m_c(m_c)$ and $m_b(m_b)$ [27] and then evaluate the pole masses M_c and M_b as precisely as possible by using the relevant four-loop RGEs and three-loop matching conditions [15]. Hence we obtain the pole masses $M_c = 1.84$ GeV and $M_b = 4.92$ GeV, as given in Table I.

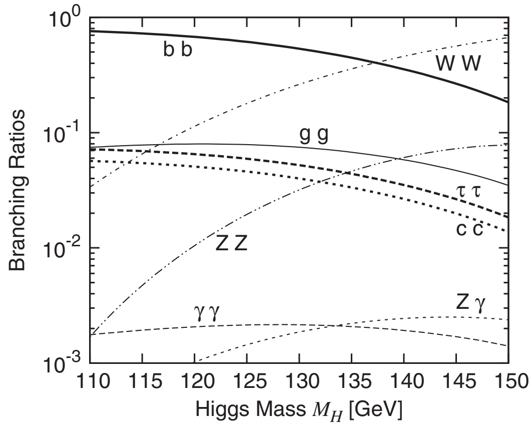

FIG. 4. The branching ratios of two-body Higgs decays versus the Higgs mass M_H. The thick lines stand for the dominant $H \to f\bar{f}$ modes: $b\bar{b}$ (solid line), $\tau^+\tau^-$ (dashed line), and $c\bar{c}$ (dotted line); and the thin lines denote $H \to gg$ (solid line), $\gamma\gamma$ (dashed line), $Z\gamma$ (dotted line), W^+W^- (dotted-dashed line), and ZZ (double-dotted dashed line).

Our numerical results for the branching ratios of $H \to b\bar{b}$, $c\bar{c}$, and $\tau^+\tau^-$ decays are shown in Fig. 4 where the branching ratios of $H \to \gamma\gamma$, gg, W^+W^-, ZZ, and $Z\gamma$ decays are also plotted for comparison. Note that the branching ratios of $H \to s\bar{s}$ and $\mu^+\mu^-$ decays are only of $\mathcal{O}(10^{-4})$ for M_H to lie in the range 110 GeV $\leq M_H \leq$ 150 GeV, and thus they are not taken into account in Fig. 4.

V. SUMMARY

In view of the recent results from the ATLAS and CMS experiments, which have shown quite strong evidence for the existence of the Higgs boson, we have examined the impact of the Higgs mass on the vacuum stability in the SM by means of the two-loop RGEs. We find that $M_H \simeq 125$ GeV leads us to an interesting cutoff scale $\Lambda_{\rm VS} \sim 10^{12}$ GeV, as required by the vacuum stability. Some kind of new physics is therefore expected to take effect around $\Lambda_{\rm VS}$. In other words, $\Lambda_{\rm VS}$ characterizes the end of the gauge desert and the beginning of a new physics oasis.

We have argued that possible new physics responsible for the vacuum stability of the SM might also be able to help solve the flavor puzzles. Hence we have recalculated the running fermion masses up to the cutoff scale $\Lambda_{\rm VS}$ by inputting $M_H = 125$ GeV and the updated values of other SM parameters into the full set of the two-loop RGEs for the quartic Higgs coupling, the Yukawa couplings, and the gauge couplings. In particular, the values of lepton and quark masses at $\mu = M_H$ and $\Lambda_{\rm VS}$ are obtained for the first time. As a by-product, the branching ratios of some important two-body Higgs decay modes, such as $H \to b\bar{b}$, $H \to \tau^+\tau^-$, $H \to \gamma\gamma$, $H \to W^+W^-$, and $H \to ZZ$, have been recalculated with the help of the new values of relevant particle masses obtained at M_H. Our numerical results should be very useful for model building and flavor physics.

We reiterate that a discovery of the Higgs boson at the LHC will pave the way for us to confirm the Yukawa interactions between the Higgs field and the fermion fields. That will be a crucial step towards understanding the origin of fermion masses, flavor mixing, and *CP* violation either within or beyond the SM. This point is especially true for testing the seesaw mechanisms, which attribute the tiny masses of three known neutrinos to the presence of some unknown heavy degrees of freedom via the Yukawa interactions. We believe that a new era of flavor physics is coming to the surface.

ACKNOWLEDGMENTS

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