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Decays $\rho^- \to \eta\pi^-$ and $\tau^- \to \eta(\eta')\pi^-\nu$ in the Nambu–Jona-Lasinio model

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The widths of the decays $\rho^- \to \eta\pi^-$ and $\tau^- \to \eta(\eta')\pi^-\nu$ are calculated in the framework of the Nambu–Jona-Lasinio model. It is shown that these decays are defined by the u and d quark mass difference. It leads to the suppression of these decays in comparison with the main decay modes. In the process $\rho^- \to \eta\pi^-$ the intermediate scalar a_0^- state is taken into account. For the τ decays the intermediate states with a_0^-, $\rho^-(770)$ and $\rho^-(1450)$ mesons are used. Our estimates are compared with the results obtained in other works.

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I. INTRODUCTION

At present, the decays $\rho^- \to \eta\pi^-$ and $\tau^- \to \eta(\eta')\pi^-\nu$ are not well studied in experiments [1–3]. However, recently, a number of works devoted to the investigation of these processes in the framework of the different phenomenological models were published [4–10]. On the other hand, in [11–14] it was shown that different modes of the τ decay can be satisfactorily described in the Nambu–Jona-Lasinio (NJL) model [15–23]. In the present paper, the NJL model is used for the description of the decays $\rho^- \to \eta\pi^-$ and $\tau^- \to \eta(\eta')\pi^-\nu$. The probabilities of the transitions $\pi^0 \to \eta(\eta')$ and $\rho^-(W^-) \to a_0^-$ are calculated. These transitions are defined by the mass difference between u and d quarks and can be calculated in the framework of the NJL model without attraction of any arbitrary parameters. All calculations are performed in the mean field approximation, considering the quark loops and neglecting the meson loops. Our results will be compared with the estimates obtained in [7–10]. It is shown that the amplitudes with intermediate vector mesons dominate in the $\tau^- \to \eta\pi^-\nu$ decay.

II. THE DECAY $\rho^- \to \eta\pi^-$

For calculation of this decay we should first calculate two non-diagonal transitions $\pi^0 \to \eta$ and $\rho^- \to a_0^-$ within the NJL model. These transitions go through quark loops containing u and d quarks (see Fig. 1).

The amplitude of the transition $\pi^0 \to \eta(\eta')$ has the form

$$\epsilon_{\pi\eta(\eta')} = 2g_\pi^2((2I_1(m_d) + m_{\eta(\eta')}^2 I_2(m_d)) - (2I_1(m_u) + m_{\eta(\eta')}^2 I_2(m_u)))\frac{\epsilon_{\eta(\eta')}}{m_\pi^2 - m_{\eta(\eta')}^2}, \tag{1}$$

where m_π, m_η, $m_{\eta'}$ are masses of π, η and η' mesons, respectively, given in PDG [1]; m_u and m_d are constituent quark masses, $m_u = 280$ MeV. Using the last experimental data for the decay $\omega \to \pi\pi$ [1] we obtain $m_d - m_u \approx 3.7$ MeV. This decay was described in detail in [17]. The $\eta - \eta'$ mixing $\epsilon_\eta = \sin\bar\theta$ for the η meson and $\epsilon_{\eta'} = \cos\bar\theta$ for the η' meson. The mixing angle $\bar\theta \approx -54°$ was defined in [24]. The constant g_π and integrals $I_1(m)$, $I_2(m)$ are defined in [17]

$$g_\pi = \frac{m_u}{F_\pi}, \tag{2}$$

$$I_1(m) = -i\frac{N_c}{(2\pi)^4}\int^{\Lambda_4}\frac{d^4k}{(m^2-k^2)} = \frac{N_c}{(4\pi)^2}\left[\Lambda_4^2 - m^2\log\left(\frac{\Lambda_4^2}{m^2}+1\right)\right], \tag{3}$$

$$I_2(m) = -i\frac{N_c}{(2\pi)^4}\int^{\Lambda_4}\frac{d^4k}{(m^2-k^2)^2} = \frac{N_c}{(4\pi)^2}\left[\log\left(\frac{\Lambda_4^2}{m^2}+1\right) - \left(1+\frac{m^2}{\Lambda_4^2}\right)^{-1}\right], \tag{4}$$

where $N_c = 3$ is a number of quark colors and $\Lambda_4 \approx 1250$ MeV is a four-dimensional cutoff parameter in the standard NJL model [17].

The obtained estimates coincide with those used in [9,10]. One can see the comparison in Table I.

The transition $\rho^- \to a_0^-$ takes the form

$$\frac{\sqrt{6}}{2}(m_d - m_u)p^\mu \rho_\mu^- a_0^-, \tag{5}$$

where p is momentum of the ρ meson.

The ρ meson decay width is defined by two diagrams in Figs. 2 and 3. The first diagram describes the amplitude which contains the $\pi^0 \to \eta$ transitions in the final state

$$T_1 = g_\rho \epsilon_{\pi\eta}(p_-^\mu - p_0^\mu)\rho_\mu^- \eta\pi^-, \tag{6}$$

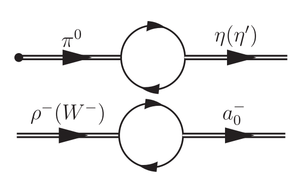

FIG. 1. $\pi^0 \to \eta(\eta')$ (top) and $\rho^-(W^-) \to a_0^-$ (bottom) transitions.

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TABLE I. Comparison of $\epsilon_{\pi\eta(\eta')}$.

$\lvert\epsilon^{\rm PR}_{\pi\eta}\rvert$ [9]	$1.34 \cdot 10^{-2}$
$\lvert\epsilon^{\rm NJL}_{\pi\eta}\rvert$	$1.55 \cdot 10^{-2}$
$\lvert\epsilon^{\rm PR}_{\pi\eta'}\rvert$ [10]	$(3 \pm 1) \cdot 10^{-3}$[10]
$\lvert\epsilon^{\rm NJL}_{\pi\eta'}\rvert$	$6.79 \cdot 10^{-3}$

where $g_\rho \approx 6.14$ is defined in [17]. The second diagram describes the amplitude containing the intermediate a_0^- meson

$$T_2 = 2Zg_\rho \frac{m_u(m_d - m_u)}{m_{a_0}^2 - m_\rho^2} \epsilon_\eta p^\mu \rho_\mu^- \eta \pi^-, \tag{7}$$

where the vertex $a_0^- \to \eta\pi^-$ is defined in [24]

$$\frac{4}{\sqrt{6}} Z g_\rho m_u \epsilon_\eta a_0^- \eta \pi^-, \tag{8}$$

$$Z = \left(1\text{–}6\frac{m_u^2}{m_{a_1}^2}\right)^{-1}, \tag{9}$$

and $m_{a_1} = 1230$ MeV is the mass of the a_1 meson [1].

Thus, for branching fractions we get

$$\mathcal{B}_1 = \epsilon_{\pi\eta}^2 \frac{\lambda^{3/2}(m_\rho^2, m_\eta^2, m_\pi^2)}{\lambda^{3/2}(m_\rho^2, m_\pi^2, m_\pi^2)} = 1.78 \cdot 10^{-5}, \tag{10}$$

$$\mathcal{B}_2 = 4Z^2\epsilon_\eta^2\left(\frac{m_u(m_d - m_u)}{m_{a_0}^2 - m_\rho^2}\right)^2 \frac{\lambda^{3/2}(m_\rho^2, m_\eta^2, m_\pi^2)}{\lambda^{3/2}(m_\rho^2, m_\pi^2, m_\pi^2)}$$
$$= 0.33 \cdot 10^{-5}, \tag{11}$$

where $\lambda(s, m_{\eta(\eta')}^2, m_\pi^2) = (s - m_{\eta(\eta')}^2 - m_\pi^2)^2 - 4m_{\eta(\eta')}^2 m_\pi^2$.

We note that in these calculations we take into account only the ground state of a_0 because the decay with the intermediate $a_0(1450)$ is suppressed by a large mass of the radial-excited meson.

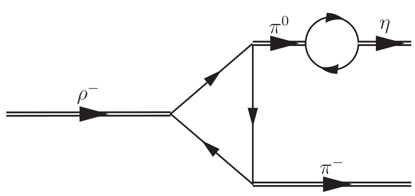

FIG. 2. The ρ^- decay with the $\pi\eta$ mixing in the final state.

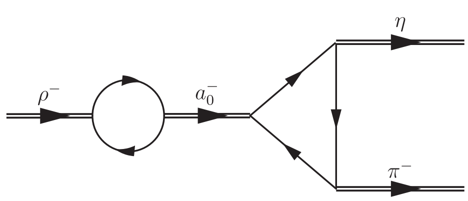

FIG. 3. The ρ^- decay though transition into the a_0^- meson.

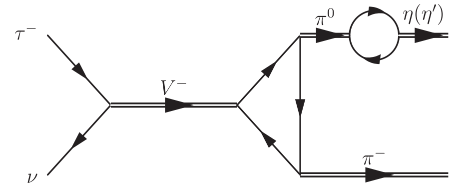

FIG. 4. The vector contribution to the τ decay [V^- includes a contact term and terms with the intermediate $\rho(770)$ and $\rho(1450)$ mesons].

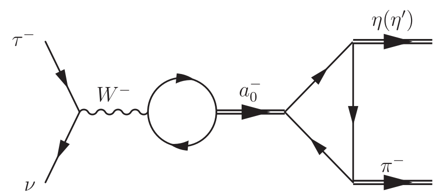

FIG. 5. The scalar contribution to τ decay.

Our estimates coincide with one taken in [7]. These estimates do not contradict known experimental limits [1,2].

III. THE DECAY $\tau^- \to \eta(\eta')\pi^-\nu$

The description of the decay $\tau \to \pi\pi\nu$ was obtained in [13] with satisfactory agreement with current experimental data.

We use the amplitude from [13] with the $\pi^0 \to \eta(\eta')$ transitions (1) in the final states (see Fig. 4)[1]

$$T_V = \epsilon_{\pi\eta(\eta')} m_\rho^2 \left(\left(1 - \frac{i\sqrt{q^2}\Gamma_\rho(p^2)}{m_\rho^2}\right) BW_\rho(p^2)\right.$$
$$\left. + \beta_\rho \frac{p^2}{m_\rho^2} BW_{\rho'}(p^2)\right)(p^\mu_{\pi^-} - p^\mu_{\eta(\eta')}) l_\mu \pi^- \eta(\eta'), \tag{12}$$

where the Breit-Wigner relation $BW_{\rho(\rho')}(p^2)$ and β_ρ parameter were defined in [13]. For the processes with the intermediate vector meson we get contributions to branching fractions

$$\mathcal{B}_V(\tau \to \eta\pi\nu) = 4.35 \cdot 10^{-6}, \tag{13}$$

$$\mathcal{B}_V(\tau \to \eta'\pi\nu) = 1.11 \cdot 10^{-8}. \tag{14}$$

The $W^- \to a_0^-$ transition takes the form

$$\frac{\sqrt{3}}{4g_\rho} g_{\rm EW} |V_{ud}| (m_d - m_u) p^\mu W_\mu^- a_0^-, \tag{15}$$

where $g_{\rm EW}$ is the electroweak constant.

The amplitude with the intermediate scalar meson (see Fig. 5) takes the form

[1] We neglect the p^2 dependence for a rough estimate.

$$T_S = 2Zm_u(m_d - m_u)\epsilon_{\eta(\eta')}(BW_{a_0}(p^2) + \beta_{a_0\eta(\eta')\pi}BW_{a'_0}(p^2))p^\mu l_\mu \pi^- \eta(\eta'), \qquad (16)$$

where $BW_{a_0(a'_0)}(p^2)$ is the Breit-Wigner formula for the $a_0(a'_0)$ meson with $m_{a_0} = 980$ MeV, $m_{a'_0} = 1474$ MeV, $\Gamma_{a'_0}(m_{a'_0}) = 265$ MeV taken from PDG [1] and $\Gamma_{a_0}(m_{a_0}) = 100$ MeV calculated from Eq. (9) which coincides with the upper PDG limit [1]. For the estimation of the contribution of the radial-excited $a_0^-(1450)$ to the τ decays we should use the extended NJL model [25–27]. The amplitudes $A_{a'_0\to\eta(\eta')\pi}$ of the $a'_0 \to \eta(\eta')\pi$ decays can be found in [27]. The transition $W^- \to a^-(1450)$ takes the form

$$C_{Wa'_0} = \frac{\sqrt{3}}{4g_\rho} g_{\rm EW}|V_{ud}|(m_d - m_u)p^\mu W_\mu^- a_0^- \left(\frac{\cos(\phi + \phi_0)}{\sin(2\phi_0)} + \Gamma\frac{\cos(\phi - \phi_0)}{\sin(2\phi_0)}\right), \qquad (17)$$

where $\phi_0 = 65.5°$ and $\phi = 72.0°$ are the mixing angles, and $\Gamma = 0.54$.

Thus, we get the $\beta_{a_0\eta(\eta')\pi}$ parameter

$$\beta_{a_0\eta(\eta')\pi} = e^{i\pi}C_{Wa'_0}\frac{\sqrt{6}}{4Z}\frac{A_{a'_0\to\eta(\eta')\pi}}{m_u}, \qquad (18)$$

where phase factor $e^{i\pi}$ is taken similarly [13]. The values $\beta_{a_0\eta\pi} = -0.24$ and $\beta_{a_0\eta'\pi} = -0.26$ do not contradict the ones given in [8,10]. The contributions to the branching fractions from the amplitude [Eq. (16)] are

$$\mathcal{B}_S(\tau \to \eta\pi\nu) = 0.37 \cdot 10^{-6}, \qquad (19)$$

$$\mathcal{B}_S(\tau \to \eta'\pi\nu) = 2.63 \cdot 10^{-8}. \qquad (20)$$

The expression for the total width is

$$\Gamma = \frac{G_f^2|V_{ud}|^2}{384\pi m_\tau^2}\int_{m^2_{\eta(\eta')}+m_\pi^2}^{m_\tau^2}\frac{ds}{s^3}\lambda^{1/2}(s, m^2_{\eta(\eta')}, m_\pi^2)(m_\tau^2 - s)^2 \times (|T_V|^2(2s + m_\tau^2)\lambda(s, m^2_{\eta(\eta')}, m_\pi^2) + |T_S|^2 3m_\tau^2(m^2_{\eta(\eta')} - m_\pi^2)^2). \qquad (21)$$

Note that there is no interference between the vector and scalar intermediate state contributions. Thus, for branchings we get

$$\mathcal{B}(\tau^- \to \eta\pi^-\nu) = 4.72 \cdot 10^{-6}, \qquad (22)$$

$$\mathcal{B}(\tau^- \to \eta'\pi^-\nu) = 3.74 \cdot 10^{-8}. \qquad (23)$$

Let us note that our estimations for scalar contributions are much less than those in previous works.

IV. CONCLUSIONS

Our calculations are in qualitative agreement with the previous theoretical estimates obtained in [7–10]. However, the NJL model allows us to describe the transitions $\pi^0 \to \eta(\eta')$ and $\rho^-(W^-) \to a_0^-$ using the same methods. As a result, we can compare the contribution of amplitudes with intermediate scalar and vector mesons from uniform positions. These calculations show that in the decays $\rho^- \to \eta\pi^-$ and $\tau^- \to \eta\pi^-\nu$ the scalar meson plays a insignificant role. However, in the decay $\tau^- \to \eta'\pi^-\nu$ the processes with intermediate a_0 and a'_0 make contributions comparable with the contributions of intermediate vector mesons.

It is worth noticing that the width of the decay $\tau^- \to a_0^-\nu$ calculated in the NJL model is close to the values obtained in [8]

$$\Gamma = \frac{G_F^2|V_{ud}|^2 m_\tau^3}{16\pi}\left(\frac{\sqrt{6}}{2}\frac{m_d - m_u}{g_\rho}\right)^2\left(1 - \frac{m^2_{a_0}}{m_\tau^2}\right)^2, \qquad (24)$$

$$\mathcal{B}(\tau^- \to a_0^-\nu) = 3.28 \cdot 10^{-6}. \qquad (25)$$

This confirms the relevance of our expression for the vertex $\tau a_0\nu$ used in Eq. (16). For the vertex $a_0 \to \eta\pi$ the expression in Eq. (9) was used. We get the amplitude [Eq. (16)] by matching these expressions through propagator of scalar a_0 meson. This contradicts the vector-dominance-model-like ansatz for the intermediate resonance used in [5,9,10]

$$\frac{\epsilon^2_{\pi\eta}M_R^2}{M_R^2 - p^2 - iM_R\Gamma_R(p^2)}. \qquad (26)$$

On the other hand, if we use this ansatz for a vector-to-scalar transition taken in [5,9,10] and calculate $\rho^- \to \eta\pi^-$ with this ansatz then we get by an order of magnitude

$$\mathcal{B} \sim \epsilon^2_{\pi\eta}\left(\frac{m^2_{a_0}}{m^2_{a_0} - m_\rho^2}\right)^2\frac{\lambda^{3/2}(m_\rho^2, m_\eta^2, m_\pi^2)}{\lambda^{3/2}(m_\rho^2, m_\pi^2, m_\pi^2)} \sim 10^{-3}. \qquad (27)$$

This estimate for the branching fraction is close to the current experimental limit [1,2] and can be tested in the near future at the high-luminosity e^+e^- colliders in Novosibirsk and Beijing, for example. Therefore, the problem of relevancy of vector-scalar transition representation can be clarified.

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