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Anomalous conformal currents, shadow fields, and massive AdS fields

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In the framework of the gauge invariant approach involving Stueckelberg and auxiliary fields, a totally symmetric arbitrary spin anomalous conformal current and shadow field in flat space-time of dimension greater than or equal to 4 are studied. Gauge invariant differential constraints for such anomalous conformal current and shadow field and realization of global conformal symmetries are obtained. A gauge invariant two-point vertex of the arbitrary spin anomalous shadow field is also obtained. In the Stueckelberg gauge frame, the two-point gauge invariant vertex becomes the standard two-point vertex of CFT. A light-cone gauge two-point vertex of the arbitrary spin anomalous shadow field is derived. The AdS/CFT correspondence for arbitrary spin anomalous conformal current and shadow field and the respective normalizable and non-normalizable modes of massive arbitrary spin AdS field is studied. The AdS field is considered in modified de Donder gauge which simplifies considerably the study of AdS/CFT correspondence. We show that on-shell leftover gauge symmetries of bulk massive field are related to gauge symmetries of boundary anomalous conformal current and shadow field, while the modified de Donder gauge condition for a bulk massive field is related to differential constraints for boundary anomalous conformal current and shadow field.

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I. INTRODUCTION

We start with a brief recall of some basic notions of CFT. In the space-time of dimension $d \geq 4$, fields of CFT can be separated into two groups: conformal currents and shadow fields. This is to say that a field having Lorentz algebra spin s, $s \geq 1$, and conformal dimension $\Delta = s + d - 2$ is referred to as a conformal current with canonical dimension, while a field having Lorentz algebra spin s, $s \geq 1$, and conformal dimension $\Delta > s + d - 2$ is referred to as an anomalous conformal current. Accordingly, a field having Lorentz algebra spin s, $s \geq 1$, and conformal dimension $\Delta = 2 - s$ is referred to as a shadow field with canonical dimension,[1] while a field having Lorentz algebra spin s, $s \geq 1$, and conformal dimension $\Delta < 2 - s$ is referred to as an anomalous shadow field.

In Refs. [3,4], we developed the gauge invariant formulation of the conformal currents and shadow fields having the canonical conformal dimensions. We recall that, in the framework of AdS/CFT correspondence, such conformal currents and shadow fields are related to massless AdS fields. In Ref. [5], we extended our approach to the case of low spin-s, $s = 1, 2$, anomalous conformal currents and shadow fields. The purpose of this paper is to develop a gauge invariant approach to a bosonic totally symmetric arbitrary spin-s anomalous conformal current and shadow field. In the framework of AdS/CFT correspondence, a spin-s anomalous conformal current and shadow field are related to the spin-s massive AdS field. Massive totally symmetric spin-s AdS fields with even $s \geq 4$ constitute the leading Regge trajectory of AdS string theory. Therefore, an extension of our approach to the case of arbitrary s is important. Our approach to an anomalous conformal current and shadow field is summarized as follows.

(i) Starting with the field content of an anomalous conformal current (and anomalous shadow field) in the standard CFT, we introduce Stueckelberg fields and auxiliary fields. In other words, we extend the space of fields entering the standard CFT.

(ii) On the extended space of fields entering our approach, we introduce differential constraints, gauge transformations, and conformal algebra transformations. The differential constraints are required to be invariant under the gauge transformations and the conformal algebra transformations.

(iii) The gauge symmetries and the differential constraints allow us to match our approach and the standard CFT. Namely, by imposing gauge conditions to exclude the Stueckelberg fields and by solving differential constraints to exclude the auxiliary fields, we obtain formulation of anomalous conformal current and shadow field in the standard CFT.

Besides the gauge invariant approach, we discuss the anomalous conformal current and shadow field by using Stueckelberg gauge and light-cone gauge conditions. The reasons for discussing these two gauge conditions are as follows.

(i) The Stueckelberg gauge reduces our approach to the standard formulation of CFT. This is to say that use of the Stueckelberg gauge allows us to demonstrate

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[1]Shadow fields having the canonical dimension are used to build conformal invariant equations of motion and Lagrangian formulations for conformal fields in Ref. [1]. An interesting discussion of shadow field dualities can be found in Ref. [2].

how the standard approach to anomalous conformal current and shadow field is connected with our gauge invariant approach.

(ii) Studying of CFT in the light-cone gauge frame is motivated by the conjectured duality of the supersymmetric Yang-Mills theory and AdS superstring theory [6]. We expect, by analogy with flat space, that a quantization of type IIB Green-Schwarz AdS superstring [7] will be straightforward only in the light-cone gauge [8–10]. Therefore we think that, from the stringy perspective of AdS/CFT correspondence, the light-cone approach to CFT deserves to be understood better. In this respect, we note that our approach provides quick access to the light-cone gauge formulation of CFT. This implies that our approach gives easy access to the studying of AdS/CFT correspondence in light-cone gauge. This seems to be important for the future application of our approach to the studying of string/gauge theory duality.

We use our gauge invariant formulation of CFT for the studying of AdS/CFT correspondence between the arbitrary spin massive AdS field and the corresponding arbitrary spin boundary anomalous conformal current and shadow field. Namely, we show that non-normalizable modes of the arbitrary spin-s massive AdS field are related to the arbitrary spin-s anomalous shadow field, while normalizable modes of the arbitrary spin-s massive AdS field are related to the arbitrary spin-s anomalous conformal current. We recall that, in earlier literature, the AdS/CFT correspondence between non-normalizable modes of massive spin-1 and spin-2 AdS fields and the corresponding spin-1 and spin-2 anomalous shadow fields was studied in Refs. [11,12]. The AdS/CFT correspondence for the spin-s massive AdS field with $s > 2$ and the corresponding spin-s anomalous conformal current and shadow field has not been considered in the earlier literature. Our treatment of AdS/CFT correspondence is summarized as follows.

(i) We exploit the CFT adapted gauge invariant approach to massive AdS fields and the modified de Donder gauge obtained in Refs. [13,14]. The modified de Donder gauge leads to the simple decoupled bulk equations of motion which are easily solved. We show that the two-point gauge invariant vertex of the arbitrary spin-s anomalous shadow field does indeed emerge from the massive arbitrary spin-s AdS field action when it is evaluated on the solution of the Dirichlet problem. Throughout this paper the AdS field action evaluated on the solution of the Dirichlet problem is referred to as the effective action.

(ii) The number of boundary gauge fields involved in our approach to the anomalous conformal current (or anomalous shadow field) coincides with the number of gauge fields involved in the CFT adapted formulation of the massive AdS field in Ref. [14].

(iii) The number of gauge transformation parameters involved in our approach to the anomalous conformal current (or anomalous shadow field) coincides with the number of gauge transformation parameters involved in the CFT adapted gauge invariant formulation of the massive AdS field in Ref. [14].

(iv) The modified de Donder gauge for the massive bulk field is related to the differential constraint for the boundary anomalous conformal current (or anomalous shadow field).

(v) On-shell leftover gauge symmetries of massive bulk field are related to the gauge symmetries of the boundary anomalous conformal current (or anomalous shadow field).

Our paper is organized as follows.

In Sec. II, we summarize our notation and conventions.

Section III is devoted to the gauge invariant formulation of the arbitrary spin-s anomalous conformal current. We discuss gauge symmetries and the realization of global conformal algebra symmetries on the space of gauge fields we use for the description of the anomalous conformal current. We demonstrate how our gauge invariant approach is related to the standard CFT. Also, using our approach, we obtain a light-cone gauge description of the arbitrary spin-s anomalous conformal current.

In Sec. IV, we extend results in Sec. III to the case of the arbitrary spin-s anomalous shadow field. Also, we find the gauge invariant two-point vertex for the arbitrary spin-s anomalous shadow field. We discuss the two-point vertex in the Stueckelberg gauge frame and in the light-cone gauge frame.

In Sec. V, we discuss the two-point current-shadow field interaction vertex.

In Sec. VI, we review the CFT adapted gauge invariant approach to the massive arbitrary spin AdS field. Because the use of the modified de Donder gauge makes our study of AdS/CFT correspondence for arbitrary spin fields similar to the one for the scalar field, we briefly review the AdS/CFT correspondence for the scalar field.

Section VII is devoted to the study of AdS/CFT correspondence between normalizable modes of the massive spin-s AdS field and the spin-s anomalous conformal current, while, in Sec. VIII, we study the AdS/CFT correspondence between non-normalizable modes of the massive spin-s AdS field and the spin-s anomalous shadow field.

Section IX summarizes our conclusions and suggests directions for future research.

In the Appendix, we present some details of matching of the bulk and boundary conformal boost symmetries.

II. PRELIMINARIES

A. Notation

We use the following conventions. The Cartesian coordinates in d-dimensional flat space-time are denoted by x^a, while derivatives with respect to x^a are denoted by ∂_a,

$\partial_a \equiv \partial/\partial x^a$. The vector indices of the Lorentz algebra $so(d-1,1)$ take the values a, b, c, and $e = 0, 1, \ldots, d-1$. We use the mostly positive flat metric tensor η^{ab} and, to simplify our expressions, we drop η_{ab} in scalar products: $X^aY^a \equiv \eta_{ab}X^aY^b$. Creation operators α^a, α^z, and ζ and the respective annihilation operators $\bar\alpha^a$, $\bar\alpha^z$, and $\bar\zeta$ are referred to as oscillators.[2] Commutation relations of the oscillators, the vacuum $|0\rangle$, and Hermitian conjugation rules are defined as

$$[\bar\alpha^a, \alpha^b] = \eta^{ab}, \qquad [\bar\alpha^z, \alpha^z] = 1, \qquad [\bar\zeta, \zeta] = 1, \tag{2.1}$$

$$\bar\alpha^a|0\rangle = 0, \qquad \bar\alpha^z|0\rangle = 0, \qquad \bar\zeta|0\rangle = 0, \tag{2.2}$$

$$\alpha^{a\dagger} = \bar\alpha^a, \qquad \alpha^{z\dagger} = \bar\alpha^z, \qquad \zeta^\dagger = \bar\zeta. \tag{2.3}$$

The oscillators α^a and $\bar\alpha^a$ transform in the vector representation of the Lorentz algebra, while the oscillators α^z, ζ, $\bar\alpha^z$, and $\bar\zeta$ transform in the scalar representation of the Lorentz algebra $so(d-1,1)$. Throughout this paper we use operators constructed out of the derivatives, coordinates, and the oscillators,

$$\Box \equiv \partial^a\partial^a, \qquad x\partial \equiv x^a\partial^a, \qquad x^2 \equiv x^ax^a, \tag{2.4}$$

$$\alpha\partial \equiv \alpha^a\partial^a, \qquad \bar\alpha\partial \equiv \bar\alpha^a\partial^a, \tag{2.5}$$

$$\alpha^2 \equiv \alpha^a\alpha^a, \qquad \bar\alpha^2 \equiv \bar\alpha^a\bar\alpha^a, \tag{2.6}$$

$$N_\alpha \equiv \alpha^a\bar\alpha^a, \qquad N_z \equiv \alpha^z\bar\alpha^z, \qquad N_\zeta \equiv \zeta\bar\zeta, \tag{2.7}$$

$$M^{ab} \equiv \alpha^a\bar\alpha^b - \alpha^b\bar\alpha^a, \tag{2.8}$$

$$\Pi^{[1,2]} \equiv 1 - \alpha^2\frac{1}{2(2N_\alpha + d)}\bar\alpha^2, \tag{2.9}$$

$$\boldsymbol{\mu} \equiv 1 - \tfrac{1}{4}\alpha^2\bar\alpha^2, \tag{2.10}$$

$$\tilde C^a \equiv \alpha^a - \alpha^2\frac{1}{2N_\alpha + d - 2}\bar\alpha^a, \tag{2.11}$$

$$\bar C^a_\perp \equiv \bar\alpha^a - \tfrac{1}{2}\alpha^a\bar\alpha^2, \tag{2.12}$$

$$\begin{aligned} r_\zeta &\equiv \left(\frac{(s + \frac{d-4}{2} - N_\zeta)(\kappa - s - \frac{d-4}{2} + N_\zeta)(\kappa + 1 + N_\zeta)}{2(s + \frac{d-4}{2} - N_\zeta - N_z)(\kappa + N_\zeta - N_z)(\kappa + N_\zeta - N_z + 1)}\right)^{1/2}, \\ r_z &\equiv \left(\frac{(s + \frac{d-4}{2} - N_z)(\kappa + s + \frac{d-4}{2} - N_z)(\kappa - 1 - N_z)}{2(s + \frac{d-4}{2} - N_\zeta - N_z)(\kappa + N_\zeta - N_z)(\kappa + N_\zeta - N_z - 1)}\right)^{1/2}, \end{aligned} \tag{2.13}$$

where parameter κ appearing in (2.13) is defined in (3.4). Throughout the paper the notation $\lambda \in [n]_2$ implies that $\lambda = -n, -n+2, -n+4, \ldots, n-4, n-2, n$:

$$\lambda \in [n]_2 \Rightarrow \lambda = -n, -n+2, -n+4, \ldots, n-4, n-2, n. \tag{2.14}$$

Often, we use the following set of scalar, vector, and totally symmetric tensor fields of the Lorentz algebra $so(d-1,1)$:

$$\phi_\lambda^{a_1\cdots a_{s'}}, \qquad s' = 0, 1, \ldots, s, \qquad \lambda \in [s - s']_2, \tag{2.15}$$

where $\phi_\lambda^{a_1\cdots a_{s'}}$ is a rank-s' totally symmetric traceful tensor field of the Lorentz algebra $so(d-1,1)$. To illustrate the field content given in (2.15), we use shortcut $\phi_\lambda^{s'}$ for the field $\phi_\lambda^{a_1\cdots a_{s'}}$ and note that fields in (2.15) can be represented as

$$\begin{array}{ccccccccc} & & & & \phi_0^s & & & & \\ & & & \phi_{-1}^{s-1} & & \phi_1^{s-1} & & & \\ & & \cdots & & \cdots & & \cdots & & \\ & \phi_{1-s}^1 & & \phi_{3-s}^1 & \cdots & \phi_{s-3}^1 & & \phi_{s-1}^1 & \\ \phi_{-s}^0 & & \phi_{2-s}^0 & & \cdots & & \phi_{s-2}^0 & & \phi_s^0. \end{array} \tag{2.16}$$

Our conventions for the light-cone frame are as follows. The space-time coordinates are decomposed as $x^a = x^+$, x^-, and x^i, where the coordinates in $\pm$ directions are defined as $x^\pm = (x^{d-1} \pm x^0)/\sqrt{2}$ and x^+ is taken to be a light-cone time. Vector indices of the $so(d-2)$ algebra take values $i, j = 1, \ldots, d-2$. We use the following conventions for the derivatives:

$$\partial^i = \partial_i \equiv \partial/\partial x^i, \qquad \partial^\pm = \partial_\mp \equiv \partial/\partial x^\mp. \tag{2.17}$$

B. Global conformal symmetries

In d-dimensional flat space-time, the conformal algebra $so(d,2)$ consists of translation generators P^a, dilatation generator D, conformal boost generators K^a, and generators of the $so(d-1,1)$ Lorentz algebra J^{ab}. We use the following nontrivial commutators of the conformal algebra:

[2] We use oscillators to handle the many indices appearing for tensor fields (discussion of oscillator formulation can be found in Refs. [15,16]).

$$[D,P^a]=-P^a,\quad [P^a,J^{bc}]=\eta^{ab}P^c-\eta^{ac}P^b,$$
$$[D,K^a]=K^a,\quad [K^a,J^{bc}]=\eta^{ab}K^c-\eta^{ac}K^b,$$
$$[P^a,K^b]=\eta^{ab}D-J^{ab},\quad [J^{ab},J^{ce}]=\eta^{bc}J^{ae}+3\text{ terms}. \tag{2.18}$$

Let ϕ denote the anomalous conformal current (or the anomalous shadow field) in the d-dimensional flat space-time. Under the action of conformal algebra, the ϕ transforms as

$$\delta_{\hat{G}}\phi = \hat{G}\phi, \tag{2.19}$$

where the realization of the conformal algebra generators $\hat{G}$ on space of ϕ is given by

$$P^a = \partial^a, \tag{2.20}$$

$$J^{ab} = x^a\partial^b - x^b\partial^a + M^{ab}, \tag{2.21}$$

$$D = x\partial + \Delta, \tag{2.22}$$

$$K^a = K^a_{\Delta,M} + R^a, \tag{2.23}$$

$$K^a_{\Delta,M} \equiv -\tfrac{1}{2}x^2\partial^a + x^a D + M^{ab}x^b. \tag{2.24}$$

In relations (2.21), (2.22), and (2.23), Δ is an operator of conformal dimension, while M^{ab} is a spin operator of the Lorentz algebra,

$$[M^{ab},M^{ce}] = \eta^{bc}M^{ae} + 3\text{ terms}. \tag{2.25}$$

The spin operator of the Lorentz algebra is well known for the arbitrary spin anomalous conformal current and the shadow field [see (2.8)]. In general, operator R^a appearing in (2.23) depends on the derivatives and does not depend on the space-time coordinates.[3] In the standard CFT, the operator R^a is equal to zero, while, in the gauge invariant approach to the anomalous conformal current and shadow field we develop in this paper, the operator R^a is nontrivial. This is to say that, in the framework of our gauge invariant approach, the complete description of the conformal current and shadow field requires, among other things, finding the operator R^a.

III. ARBITRARY SPIN ANOMALOUS CONFORMAL CURRENT

A. Gauge invariant formulation

Field content.—To develop a gauge invariant formulation of the arbitrary spin-s anomalous conformal current in flat space of dimension $d \ge 4$ we use the following fields:

$$\phi^{a_1\cdots a_{s'}}_{\mathrm{cur},\lambda},\qquad s'=0,1,\ldots,s,\qquad \lambda\in[s-s']_2. \tag{3.1}$$

We note that

(i) In (3.1), the fields $\phi_{\mathrm{cur},\lambda}$ and $\phi^a_{\mathrm{cur},\lambda}$ are the respective scalar and vector fields of the Lorentz algebra, while the field $\phi^{a_1\cdots a_{s'}}_{\mathrm{cur},\lambda}$, $s'>1$, is the rank-s' totally symmetric traceful tensor field of the Lorentz algebra $so(d-1,1)$. Using shortcut $\phi^{s'}_\lambda$ for the field $\phi^{a_1\cdots a_{s'}}_{\mathrm{cur},\lambda}$, fields in (3.1) can be represented as in (2.16).

(ii) The tensor fields $\phi^{a_1\cdots a_{s'}}_{\mathrm{cur},\lambda}$ with $s'\ge 4$ satisfy the double-tracelessness constraint

$$\phi^{aabba_5\cdots a_{s'}}_{\mathrm{cur},\lambda}=0,\qquad s'=4,5,\ldots,s. \tag{3.2}$$

(iii) The fields $\phi^{a_1\cdots a_{s'}}_{\mathrm{cur},\lambda}$ have the following conformal dimensions:

$$\Delta(\phi^{a_1\cdots a_{s'}}_{\mathrm{cur},\lambda})=\frac{d}{2}+\kappa+\lambda. \tag{3.3}$$

(iv) In the framework of AdS/CFT correspondence, κ is related to the mass parameter m of the spin-s massive field in AdS_{d+1} as

$$\kappa\equiv\sqrt{m^2+\Big(s+\frac{d-4}{2}\Big)^2}. \tag{3.4}$$

In order to obtain the gauge invariant description in an easy-to-use form we use the oscillators and introduce a ket-vector $|\phi_{\mathrm{cur}}\rangle$ defined by

$$|\phi_{\mathrm{cur}}\rangle=\sum_{s'=0}^{s}|\phi^{s'}_{\mathrm{cur}}\rangle,$$
$$|\phi^{s'}_{\mathrm{cur}}\rangle=\sum_{\lambda\in[s-s']_2}\frac{\zeta^{(s-s'+\lambda)/2}\alpha_z^{(s-s'-\lambda)/2}\alpha^{a_1}\cdots\alpha^{a_{s'}}}{s'!\sqrt{(\frac{s-s'+\lambda}{2})!(\frac{s-s'-\lambda}{2})!}}\phi^{a_1\cdots a_{s'}}_{\mathrm{cur},\lambda}|0\rangle. \tag{3.5}$$

From (3.5), we see that the ket-vector $|\phi_{\mathrm{cur}}\rangle$ is a degree-s homogeneous polynomial in the oscillators α^a, α^z, and ζ, while the ket-vector $|\phi^{s'}_{\mathrm{cur}}\rangle$ is a degree-s' homogeneous polynomial in the oscillators α^a, i.e., these ket-vectors satisfy the relations

$$(N_\alpha+N_z+N_\zeta-s)|\phi_{\mathrm{cur}}\rangle=0, \tag{3.6}$$

$$(N_\alpha-s')|\phi^{s'}_{\mathrm{cur}}\rangle=0. \tag{3.7}$$

In terms of the ket-vector $|\phi_{\mathrm{cur}}\rangle$, double-tracelessness constraint (3.2) takes the form[4]

[3]For the anomalous conformal currents and shadow fields considered in this paper, the operator R^a is independent of the derivatives. Dependence of the operator R^a on the derivatives appears in the ordinary-derivative approach to conformal fields (see Refs. [17–19]).

[4]In this paper we adapt the formulation in terms of the double-traceless gauge fields [20]. To develop the gauge invariant approach one can use unconstrained gauge fields studied in Ref. [21]. Discussion of other gauge fields which seem to be the most suitable for the theory of interacting fields can be found, e.g., in [22].

$$(\bar{\alpha}^2)^2|\phi_{\rm cur}\rangle = 0. \quad (3.8)$$

Differential constraint.—We find the following differential constraint for the anomalous conformal current:

$$\bar{C}_{\rm cur}|\phi_{\rm cur}\rangle = 0, \quad (3.9)$$

$$\bar{C}_{\rm cur} = \bar{\alpha}\partial - \tfrac{1}{2}\alpha\partial\bar{\alpha}^2 - \bar{e}_{1,\rm cur}\Pi^{[1,2]} + \tfrac{1}{2}e_{1,\rm cur}\bar{\alpha}^2, \quad (3.10)$$

$$e_{1,\rm cur} = \zeta r_\zeta \Box + \alpha^z r_z, \qquad \bar{e}_{1,\rm cur} = -r_\zeta\bar{\zeta} - r_z\bar{\alpha}^z\Box, \quad (3.11)$$

where the operators $\Pi^{[1,2]}$, r_ζ, and r_z are defined in (2.9) and (2.13). One can make sure that constraint (3.9) is invariant under gauge transformation and conformal algebra transformations which we discuss below.

Gauge symmetries.—We now discuss gauge symmetries of the anomalous conformal current. To this end we introduce the following gauge transformation parameters:

$$\xi_{{\rm cur},\lambda}^{a_1\cdots a_{s'}}, \quad s' = 0,1,\ldots,s-1, \quad \lambda\in[s-1-s']_2. \quad (3.12)$$

We note that

(i) In (3.12), the gauge transformation parameters $\xi_{{\rm cur},\lambda}$ and $\xi_{{\rm cur},\lambda}^a$ are the respective scalar and vector fields of the Lorentz algebra, while the gauge transformation parameter $\xi_{{\rm cur},\lambda}^{a_1\cdots a_{s'}}$, $s' > 1$ is the rank-s' totally symmetric tensor field of the Lorentz algebra $so(d-1,1)$.

(ii) The gauge transformation parameters $\xi_{{\rm cur},\lambda}^{a_1\cdots a_{s'}}$ with $s'\geq 2$ satisfy the tracelessness constraint,

$$\xi_{{\rm cur},\lambda}^{aaa_3\cdots a_{s'}} = 0, \qquad s' = 2,3,\ldots,s-1. \quad (3.13)$$

(iii) The gauge transformation parameters $\xi_{{\rm cur},\lambda}^{a_1\cdots a_{s'}}$ have the conformal dimensions

$$\Delta(\xi_{{\rm cur},\lambda}^{a_1\cdots a_{s'}}) = \frac{d}{2} + \kappa + \lambda - 1. \quad (3.14)$$

Now, as usual, we collect the gauge transformation parameters in a ket-vector $|\xi_{\rm cur}\rangle$ defined by

$$|\xi_{\rm cur}\rangle = \sum_{s'=0}^{s-1}|\xi_{\rm cur}^{s'}\rangle,$$

$$|\xi_{\rm cur}^{s'}\rangle = \sum_{\lambda\in[s-1-s']_2}\frac{\zeta^{(s-1-s'+\lambda)/2}\alpha_z^{(s-1-s'-\lambda)/2}\alpha^{a_1}\cdots\alpha^{a_{s'}}}{s'!\sqrt{(\frac{s-1-s'+\lambda}{2})!(\frac{s-1-s'-\lambda}{2})!}} \times \xi_{{\rm cur},\lambda}^{a_1\cdots a_{s'}}|0\rangle. \quad (3.15)$$

The ket-vectors $|\xi_{\rm cur}\rangle$, $|\xi_{\rm cur}^{s'}\rangle$ satisfy the algebraic constraints

$$(N_\alpha + N_z + N_\zeta - s + 1)|\xi_{\rm cur}\rangle = 0, \quad (3.16)$$

$$(N_\alpha - s')|\xi_{\rm cur}^{s'}\rangle = 0, \quad (3.17)$$

which tell us that $|\xi_{\rm cur}\rangle$ is a degree-$(s-1)$ homogeneous polynomial in the oscillators α^a, α^z, and ζ, while the ket-vector $|\xi_{\rm cur}^{s'}\rangle$ is a degree-s' homogeneous polynomial in the oscillators α^a. In terms of the ket-vector $|\xi_{\rm cur}\rangle$, tracelessness constraint (3.13) takes the form

$$\bar{\alpha}^2|\xi_{\rm cur}\rangle = 0. \quad (3.18)$$

Gauge transformation can entirely be written in terms of $|\phi_{\rm cur}\rangle$ and $|\xi_{\rm cur}\rangle$. This is to say that gauge transformation takes the form

$$\delta|\phi_{\rm cur}\rangle = G_{\rm cur}|\xi_{\rm cur}\rangle, \quad (3.19)$$

$$G_{\rm cur} \equiv \alpha\partial - e_{1,\rm cur} - \alpha^2\frac{1}{2N_\alpha + d - 2}\bar{e}_{1,\rm cur}, \quad (3.20)$$

where $e_{1,\rm cur}$, $\bar{e}_{1,\rm cur}$ are given in (3.11). As already said, constraint (3.9) is invariant under gauge transformation (3.19).

Realization of conformal algebra symmetries.—To complete the gauge invariant formulation of the spin-s anomalous conformal current we provide realization of the conformal algebra symmetries on space of the ket-vector $|\phi_{\rm cur}\rangle$. All that is needed is to fix the operators M^{ab}, Δ, and R^a and insert these operators into (2.20)–(2.23). Realization of the spin operator M^{ab} on ket-vector $|\phi_{\rm cur}\rangle$ (3.5) is given in (2.8), while realization of the operator Δ,

$$\Delta_{\rm cur} = \frac{d}{2} + \nu, \qquad \nu \equiv \kappa + N_\zeta - N_z, \quad (3.21)$$

can be read from (3.3). In the gauge invariant formulation, finding the operator R^a provides the real difficulty. We find the following realization of the operator R^a on space of $|\phi_{\rm cur}\rangle$:

$$R_{\rm cur}^a = -2\zeta r_\zeta((\nu+1)\bar{\alpha}^a - \bar{C}_\perp^a) - 2\Big(\nu\tilde{C}^a + \alpha^2\frac{1}{2N_\alpha + d - 2}\bar{C}_\perp^a\Big)r_z\bar{\alpha}^z, \quad (3.22)$$

where the operators $\tilde{C}^a$, $\bar{C}_\perp^a$ are given in (2.11) and (2.12), while the operators r_ζ, r_z are defined in (2.13).

We derived the differential constraint, gauge transformation, and the realization of the operator $R_{\rm cur}^a$ by generalizing our results for the spin-s conformal current with the canonical dimension which we obtained in Ref. [3]. It is worthwhile to note that results in this section can also be obtained by using the tractor approach in Refs. [23,24].[5] Our constraint (3.9) and gauge transformation (3.19) can be matched with the ones in Ref. [23] by using appropriate field redefinitions. The basis of the fields we use in this paper turns out to be more convenient for the study of

[5]In mathematical literature, an interesting discussion of the tractor approach can be found in Ref. [25].

AdS/CFT correspondence. To summarize, our fields (3.1) can be written as a tractor rank-s tensor field subject to a Thomas-D divergence type constraint in Ref. [23]. A similar construction was used to describe the spin-s massive bulk field in Ref. [23]. Note that, in our approach, we use our fields (3.1) for the discussion of the spin-s anomalous conformal current.

B. Stueckelberg gauge frame

We proceed with the discussion of the spin-s anomalous conformal current in the Stueckelberg gauge frame. To this end we note that the Stueckelberg gauge frame is achieved through the use of differential constraint (3.9) and the Stueckelberg gauge condition. From (3.19), we see that a field defined by

$$\bar{\alpha}^z \Pi^{[1,2]} |\phi_{\rm cur}\rangle \qquad (3.23)$$

transforms as a Stueckelberg field. Therefore this field can be gauged away via Stueckelberg gauge fixing,

$$\bar{\alpha}^z \Pi^{[1,2]} |\phi_{\rm cur}\rangle = 0, \qquad (3.24)$$

where $\Pi^{[1,2]}$ is given in (2.9). Using gauge condition (3.24) and differential constraint (3.9), we find the relations

$$\bar{\alpha}^2 |\phi_{\rm cur}^{s'}\rangle = 0, \qquad (3.25)$$

$$|\phi_{\rm cur}^{s'}\rangle = X_{\rm cur} \zeta^{s-s'} (\bar{\alpha}\partial)^{s-s'} |\phi_{\rm cur}^{s}\rangle,$$
$$X_{\rm cur} \equiv \frac{(-)^{s-s'}}{(s-s')!} \Big(\frac{2^{s-s'} \Gamma(\kappa - s - \frac{d-4}{2}) \Gamma(\kappa + s - s')}{\Gamma(\kappa - s' - \frac{d-4}{2}) \Gamma(\kappa)} \Big)^{1/2}. \qquad (3.26)$$

Equation (3.25) tells us that all fields $\phi_{{\rm cur},\lambda}^{a_1\cdots a_{s'}}$ with $s' \geq 2$ become traceless. From Eq. (3.26), we learn that the fields $\phi_{{\rm cur},\lambda}^{a_1\cdots a_{s'}}$ with $\lambda \neq s - s'$ become equal to zero, while the fields $\phi_{{\rm cur},\lambda}^{a_1\cdots a_{s'}}$ with $\lambda = s - s'$ and $s' = 0, 1, \ldots, s-1$ are expressed in terms of the rank-s traceless tensor field $\phi_{{\rm cur},0}^{a_1\cdots a_s}$,

$$\phi_{{\rm cur},\lambda}^{a_1\cdots a_{s'}} = 0, \quad \hbox{for } \lambda \neq s - s', \qquad (3.27)$$

$$\phi_{{\rm cur},s-s'}^{a_1\cdots a_{s'}} = \sqrt{(s-s')!} X_{\rm cur} \partial^{b_1} \cdots \partial^{b_{s-s'}} \phi_{{\rm cur},0}^{b_1\cdots b_{s-s'} a_1 \cdots a_{s'}}. \qquad (3.28)$$

We recall that, in the standard CFT, the spin-s anomalous conformal current is described by the traceless field $\phi_{{\rm cur},0}^{a_1\cdots a_s}$. Thus we see that by making use of the gauge symmetry and differential constraint we reduce the field content of our approach to the one in the standard CFT. In other words, use of the gauge symmetry and differential constraint allows us to match our approach and the standard formulation of the spin-s anomalous conformal current.[6] To summarize, our gauge invariant approach is equivalent to the standard one.

C. Light-cone gauge frame

We now discuss the spin-s anomalous conformal current in the light-cone gauge frame. To this end we note that, for the anomalous conformal current, the light-cone gauge frame is achieved through the use of differential constraint (3.9) and the light-cone gauge condition. Using gauge symmetry of the spin-s anomalous conformal current (3.19), we impose the light-cone gauge on the $|\phi_{\rm cur}\rangle$,

$$\bar{\alpha}^+ \Pi^{[1,2]} |\phi_{\rm cur}\rangle = 0, \qquad (3.29)$$

where $\Pi^{[1,2]}$ is given in (2.9). Using gauge (3.29) and differential constraint (3.9), we find

$$|\phi_{\rm cur}\rangle = \exp\Big(-\frac{\alpha^+}{\partial^+} (\bar{\alpha}^i \partial^i - \bar{e}_{1{\rm cur}}) \Big) |\phi_{\rm cur}^{\rm l.c.}\rangle, \qquad (3.30)$$

$$\bar{\alpha}^i \bar{\alpha}^i |\phi_{\rm cur}^{\rm l.c.}\rangle = 0, \qquad (3.31)$$

where a light-cone ket-vector $|\phi_{\rm cur}^{\rm l.c.}\rangle$ is obtained from $|\phi_{\rm cur}\rangle$ (3.5) by equating $\alpha^+ = \alpha^- = 0$,

$$|\phi_{\rm cur}^{\rm l.c.}\rangle \equiv |\phi_{\rm cur}\rangle|_{\alpha^+ = \alpha^- = 0}. \qquad (3.32)$$

We see that we are left with light-cone fields

$$\phi_{{\rm cur},\lambda}^{i_1\cdots i_{s'}}, \qquad s' = 0, 1, \ldots, s, \qquad \lambda \in [s - s']_2, \qquad (3.33)$$

which are traceless tensor fields of $so(d-2)$ algebra, $\phi_{{\rm cur},\lambda}^{iii_3\cdots i_{s'}} = 0$. These fields constitute the field content of the light-cone gauge frame. Note that, in contrast to the Stueckelberg gauge frame, all fields $\phi_{{\rm cur},\lambda}^{i_1\cdots i_{s'}}$ are not equal to zero. Also note that, in contrast to the gauge invariant approach, the fields (3.33) are not subject to any differential constraint.

IV. ARBITRARY SPIN ANOMALOUS SHADOW FIELD

A. Gauge invariant formulation

Field content.—To discuss the gauge invariant formulation of the arbitrary spin-s anomalous shadow field in flat space of dimension $d \geq 4$ we use the following fields:

$$\phi_{{\rm sh},\lambda}^{a_1\cdots a_{s'}}, \qquad s' = 0, 1, \ldots, s, \qquad \lambda \in [s - s']_2. \qquad (4.1)$$

We note that

[6] We note that, as in the standard CFT, our currents can be considered either as composite operators or as fundamental field degrees of freedom. At the group theoretical level we study in this paper, this distinction does not matter. Methods for building conformal currents as composite operators are discussed in Ref. [26].

(i) In (4.1), the fields $\phi_{\mathrm{sh},\lambda}$ and $\phi_{\mathrm{sh},\lambda}^a$ are the respective scalar and vector fields of the Lorentz algebra, while the field $\phi_{\mathrm{sh},\lambda}^{a_1\cdots a_{s'}}$, $s'>1$, is the rank-s' totally symmetric traceful tensor field of the Lorentz algebra $so(d-1,1)$. Using shortcut $\phi_\lambda^{s'}$ for the field $\phi_{\mathrm{sh},\lambda}^{a_1\cdots a_{s'}}$, fields in (4.1) can be represented as in (2.16).

(ii) The tensor fields $\phi_{\mathrm{sh},\lambda}^{a_1\cdots a_{s'}}$ with $s'\geq 4$ satisfy the double-tracelessness constraint

$$\phi_{\mathrm{sh},\lambda}^{aabba_5\cdots a_{s'}}=0, \qquad s'=4,5,\ldots,s. \tag{4.2}$$

(iii) The fields $\phi_{\mathrm{sh},\lambda}^{a_1\cdots a_{s'}}$ have the following conformal dimensions:

$$\Delta(\phi_{\mathrm{sh},\lambda}^{a_1\cdots a_{s'}})=\frac{d}{2}-\kappa+\lambda, \tag{4.3}$$

(iv) In the framework of AdS/CFT correspondence, κ is related to the mass parameter m of the spin-s massive field in AdS_{d+1} as in (3.4).

In order to obtain the gauge invariant description in an easy-to-use form we use the oscillators and introduce a ket-vector $|\phi_{\mathrm{sh}}\rangle$ defined by

$$|\phi_{\mathrm{sh}}\rangle=\sum_{s'=0}^{s}|\phi_{\mathrm{sh}}^{s'}\rangle,$$

$$|\phi_{\mathrm{sh}}^{s'}\rangle=\sum_{\lambda\in[s-s']_2}\frac{\zeta^{(s-s'-\lambda)/2}\alpha_z^{(s-s'+\lambda)/2}\alpha^{a_1}\cdots\alpha^{a_{s'}}}{s'!\sqrt{(\frac{s-s'+\lambda}{2})!(\frac{s-s'-\lambda}{2})!}}\phi_{\mathrm{sh},\lambda}^{a_1\cdots a_{s'}}|0\rangle. \tag{4.4}$$

From (4.4), we see that the ket-vector $|\phi_{\mathrm{sh}}\rangle$ is a degree-s homogeneous polynomial in the oscillators α^a, α^z, and ζ, while the ket-vector $|\phi_{\mathrm{sh}}^{s'}\rangle$ is a degree-s' homogeneous polynomial in the oscillators α^a, i.e., these ket-vectors satisfy the relations

$$(N_\alpha+N_z+N_\zeta-s)|\phi_{\mathrm{sh}}\rangle=0, \tag{4.5}$$

$$(N_\alpha-s')|\phi_{\mathrm{sh}}^{s'}\rangle=0. \tag{4.6}$$

In terms of the ket-vector $|\phi_{\mathrm{sh}}\rangle$, double-tracelessness constraint (4.2) takes the form

$$(\bar{\alpha}^2)^2|\phi_{\mathrm{sh}}\rangle=0. \tag{4.7}$$

Differential constraint.—We find the following differential constraint for the anomalous shadow field:

$$\bar{C}_{\mathrm{sh}}|\phi_{\mathrm{sh}}\rangle=0, \tag{4.8}$$

$$\bar{C}_{\mathrm{sh}}=\bar{\alpha}\partial-\tfrac{1}{2}\alpha\partial\bar{\alpha}^2-\bar{e}_{1,\mathrm{sh}}\Pi^{[1,2]}+\tfrac{1}{2}e_{1,\mathrm{sh}}\bar{\alpha}^2, \tag{4.9}$$

$$e_{1,\mathrm{sh}}=\zeta r_\zeta+\alpha^z r_z\Box, \qquad \bar{e}_{1,\mathrm{sh}}=-r_\zeta\bar{\zeta}\Box-r_z\bar{\alpha}^z, \tag{4.10}$$

where the operators $\Pi^{[1,2]}$, r_ζ, and r_z are defined in (2.9) and (2.13). One can make sure that constraint (4.8) is invariant under gauge transformation and conformal algebra transformations which we discuss below.

Gauge symmetries.—We now discuss gauge symmetries of the anomalous shadow field. To this end we introduce the following gauge transformation parameters:

$$\xi_{\mathrm{sh},\lambda}^{a_1\cdots a_{s'}}, \qquad s'=0,1,\ldots,s-1, \qquad \lambda\in[s-1-s']_2. \tag{4.11}$$

We note that

(i) In (4.11), the gauge transformation parameters $\xi_{\mathrm{sh},\lambda}$ and $\xi_{\mathrm{sh},\lambda}^a$ are the respective scalar and vector fields of the Lorentz algebra, while the gauge transformation parameter $\xi_{\mathrm{sh},\lambda}^{a_1\cdots a_{s'}}$, $s'>1$ is the rank-s' totally symmetric tensor field of the Lorentz algebra $so(d-1,1)$.

(ii) The gauge transformation parameters $\xi_{\mathrm{sh},\lambda}^{a_1\cdots a_{s'}}$ with $s'\geq 2$ satisfy the tracelessness constraint

$$\xi_{\mathrm{sh},\lambda}^{aaa_3\cdots a_{s'}}=0, \qquad s'=2,3,\ldots,s-1. \tag{4.12}$$

(iii) The gauge transformation parameters $\xi_{\mathrm{sh},\lambda}^{a_1\cdots a_{s'}}$ have the conformal dimensions

$$\Delta(\xi_{\mathrm{sh},\lambda}^{a_1\cdots a_{s'}})=\frac{d}{2}-\kappa+\lambda-1. \tag{4.13}$$

Now, as usual, we collect the gauge transformation parameters in a ket-vector $|\xi_{\mathrm{sh}}\rangle$ defined by

$$|\xi_{\mathrm{sh}}\rangle=\sum_{s'=0}^{s-1}|\xi_{\mathrm{sh}}^{s'}\rangle,$$

$$|\xi_{\mathrm{sh}}^{s'}\rangle=\sum_{\lambda\in[s-1-s']_2}\frac{\zeta^{(s-1-s'-\lambda)/2}\alpha_z^{(s-1-s'+\lambda)/2}\alpha^{a_1}\cdots\alpha^{a_{s'}}}{s'!\sqrt{(\frac{s-1-s'+\lambda}{2})!(\frac{s-1-s'-\lambda}{2})!}}\times\xi_{\mathrm{sh},\lambda}^{a_1\cdots a_{s'}}|0\rangle. \tag{4.14}$$

The ket-vectors $|\xi_{\mathrm{sh}}\rangle$, $|\xi_{\mathrm{sh}}^{s'}\rangle$ satisfy the algebraic constraints

$$(N_\alpha+N_z+N_\zeta-s+1)|\xi_{\mathrm{sh}}\rangle=0, \tag{4.15}$$

$$(N_\alpha-s')|\xi_{\mathrm{sh}}^{s'}\rangle=0, \tag{4.16}$$

which tell us that $|\xi_{\mathrm{sh}}\rangle$ is a degree-$(s-1)$ homogeneous polynomial in the oscillators α^a, α^z, and ζ, while the ket-vector $|\xi_{\mathrm{sh}}^{s'}\rangle$ is a degree-s' homogeneous polynomial in the oscillators α^a. In terms of the ket-vector $|\xi_{\mathrm{sh}}\rangle$, tracelessness constraint (4.12) takes the form

$$\bar{\alpha}^2|\xi_{\mathrm{sh}}\rangle=0. \tag{4.17}$$

Gauge transformation can entirely be written in terms of $|\phi_{\mathrm{sh}}\rangle$ and $|\xi_{\mathrm{sh}}\rangle$. This is to say that gauge transformation takes the form

$$\delta|\phi_{\mathrm{sh}}\rangle=G_{\mathrm{sh}}|\xi_{\mathrm{sh}}\rangle, \tag{4.18}$$

$$G_{\rm sh} = \alpha\partial - e_{1,{\rm sh}} - \alpha^2 \frac{1}{2N_\alpha + d - 2}\bar{e}_{1,{\rm sh}}, \qquad (4.19)$$

where $e_{1,{\rm sh}}$, $\bar{e}_{1,{\rm sh}}$ are given in (4.10). Constraint (4.8) is invariant under gauge transformation (4.18).

Realization of conformal algebra symmetries.—To complete the gauge invariant formulation of the spin-s anomalous shadow field we provide realization of the conformal algebra symmetries on space of the ket-vector $|\phi_{\rm sh}\rangle$. All that is required is to fix the operators M^{ab}, Δ, and R^a and then insert these operators into (2.20)–(2.23). Realization of the spin operator M^{ab} on ket-vector $|\phi_{\rm sh}\rangle$ (4.4) is given in (2.8), while realization of the operator Δ,

$$\Delta_{\rm sh} = \frac{d}{2} - \nu, \qquad \nu = \kappa + N_\zeta - N_z, \qquad (4.20)$$

can be read from (4.3). Realization of the operator R^a on space of $|\phi_{\rm sh}\rangle$, which we find, is given by

$$R^a_{\rm sh} = 2\alpha^z r_z((\nu - 1)\bar{\alpha}^a + \bar{C}^a_\perp) + 2\Big(\nu\tilde{C}^a - \alpha^2\frac{1}{2N_\alpha + d - 2}\bar{C}^a_\perp\Big) r_\zeta\bar{\zeta}, \qquad (4.21)$$

where the operators $\tilde{C}^a$, $\bar{C}^a_\perp$ are given in (2.11) and (2.12), while the operators r_ζ, r_z are defined in (2.13).

Two-point gauge invariant vertex.—We now discuss a two-point vertex for the spin-s anomalous shadow field. This is to say that we find the following gauge invariant two-point vertex:

$$\Gamma = \int d^dx_1 d^dx_2 \Gamma_{12}, \qquad (4.22)$$

$$\Gamma_{12} = \frac{1}{2}\langle\phi_{\rm sh}(x_1)|\frac{\boldsymbol{\mu} f_\nu}{|x_{12}|^{2\nu+d}}|\phi_{\rm sh}(x_2)\rangle, \qquad (4.23)$$

$$f_\nu = \frac{\Gamma(\nu + \frac{d}{2})\Gamma(\nu + 1)}{4^{\kappa-\nu}\Gamma(\kappa + \frac{d}{2})\Gamma(\kappa + 1)}, \qquad (4.24)$$

$$\nu = \kappa + N_\zeta - N_z, \qquad (4.25)$$

$$|x_{12}|^2 \equiv x^a_{12}x^a_{12}, \qquad x^a_{12} = x^a_1 - x^a_2, \qquad (4.26)$$

where $\boldsymbol{\mu}$ is given in (2.10). Vertex Γ (4.22) is invariant under gauge transformation of the anomalous shadow field $|\phi_{\rm sh}\rangle$ (4.18) provided this anomalous shadow field satisfies differential constraint (4.8). The vertex is obviously invariant under the Poincaré algebra and dilatation symmetries. We make sure that vertex (4.22) is invariant under the conformal boost transformations.

To illustrate structure of the vertex Γ_{12} we note that, in terms of the tensor fields $\phi^{a_1\cdots a_{s'}}_{{\rm sh},\lambda}$, vertex Γ_{12} (4.23) can be represented as

$$\Gamma_{12} = \sum_{s'=0}^{s}\sum_{\lambda\in[s-s']_2}\Gamma^{s'}_{12,\lambda}, \qquad (4.27)$$

$$\Gamma^{s'}_{12,\lambda} = \frac{w_\lambda}{2s'!|x_{12}|^{2\kappa-2\lambda+d}}\Big(\phi^{a_1\cdots a_{s'}}_{{\rm sh},\lambda}(x_1)\phi^{a_1\cdots a_{s'}}_{{\rm sh},\lambda}(x_2) - \frac{s'(s'-1)}{4}\phi^{aaa_3\cdots a_{s'}}_{{\rm sh},\lambda}(x_1)\phi^{bba_3\cdots a_{s'}}_{{\rm sh},\lambda}(x_2)\Big), \qquad (4.28)$$

$$w_\lambda \equiv \frac{\Gamma(\kappa - \lambda + \frac{d}{2})\Gamma(\kappa - \lambda + 1)}{4^\lambda\Gamma(\kappa + \frac{d}{2})\Gamma(\kappa + 1)}. \qquad (4.29)$$

We note that the kernel of the vertex Γ is connected with a two-point correlation function of the anomalous conformal current. In the framework of our approach, the anomalous conformal current is described by gauge fields (3.1) subject to differential constraints. In order to discuss the correlation function of the spin-s anomalous conformal current in a proper way, we can impose a gauge condition on the gauge fields given in (3.1). Recall that we have considered the spin-s anomalous conformal current by using the Stueckelberg and light-cone gauge frames. Obviously, the two-point correlation function of the anomalous conformal current in the Stueckelberg and light-cone gauge frames is obtained from the two-point vertex Γ taken in the respective Stueckelberg and light-cone gauge frames. To this end we proceed by discussing the anomalous shadow field in the Stueckelberg and light-cone gauge frames.

B. Stueckelberg gauge frame

We now discuss the spin-s anomalous shadow field in the Stueckelberg gauge frame. We note that the Stueckelberg gauge frame can be achieved through the use of a differential constraint given in (4.8) and the Stueckelberg gauge condition. From (4.18), we see that a field defined by

$$\bar{\zeta}\Pi^{[1,2]}|\phi_{\rm sh}\rangle \qquad (4.30)$$

transforms as a Stueckelberg field. Therefore this field can be gauged away via a Stueckelberg gauge fixing,

$$\bar{\zeta}\Pi^{[1,2]}|\phi_{\rm sh}\rangle = 0. \qquad (4.31)$$

Using this gauge condition and differential constraint (4.8), we find the following relations:

$$\bar{\alpha}^2|\phi^{s'}_{\rm sh}\rangle = 0, \qquad (4.32)$$

$$|\phi^{s'}_{\rm sh}\rangle = X_{\rm sh}\alpha_z^{s-s'}(\bar{\alpha}\partial)^{s-s'}|\phi^s_{\rm sh}\rangle,$$
$$X_{\rm sh} \equiv \frac{(-)^{s-s'}}{(s-s')!}\Big(\frac{2^{s-s'}\Gamma(\kappa + \frac{d-2}{2} + s')\Gamma(\kappa+1)}{\Gamma(\kappa + s + \frac{d-2}{2})\Gamma(\kappa - s + 1 + s')}\Big)^{1/2}. \qquad (4.33)$$

Equation (4.32) tells us that all fields $\phi_{\mathrm{sh},\lambda}^{a_1\cdots a_{s'}}$ with $s'\geq 2$ become traceless. From Eq. (4.33), we learn that the fields $\phi_{\mathrm{sh},\lambda}^{a_1\cdots a_{s'}}$ with $\lambda\neq s-s'$ become equal to zero, while the fields $\phi_{\mathrm{sh},\lambda}^{a_1\cdots a_{s'}}$ with $\lambda = s-s'$ and $s'=0,1,\ldots,s-1$ are expressed in terms of the rank-s traceless tensor field $\phi_{\mathrm{sh},0}^{a_1\cdots a_s}$,

$$\phi_{\mathrm{sh},\lambda}^{a_1\cdots a_{s'}} = 0, \quad \text{for } \lambda\neq s-s', \tag{4.34}$$

$$\phi_{\mathrm{sh},s-s'}^{a_1\cdots a_{s'}} = \sqrt{(s-s')!}X_{\mathrm{sh}}\partial^{b_1}\cdots\partial^{b_{s-s'}}\phi_{\mathrm{sh},0}^{b_1\cdots b_{s-s'}a_1\cdots a_{s'}}. \tag{4.35}$$

We recall that, in the standard CFT, the spin-s anomalous shadow field is described by the traceless rank-s tensor field $\phi_{\mathrm{sh},0}^{a_1\cdots a_s}$. Thus we see that making use of the gauge symmetry and differential constraint we reduce the field content of our approach to the one in the standard CFT. In other words, use of the gauge symmetry and differential constraint allows us to match our approach and the standard formulation of the anomalous shadow field. To summarize, our gauge invariant approach is equivalent to the standard one.

We now discuss the Stueckelberg gauge-fixed two-point vertex of the anomalous shadow field. In other words, we are going to connect our vertex (4.22) with the one in the standard CFT. To do that we note that the vertex of the standard CFT is obtained from our gauge invariant vertex (4.22) by plugging a solution to the differential constraint (4.33) into (4.22). Doing so, we find the following two-point density (up to the total derivative) in the Stueckelberg gauge frame:

$$\Gamma_{12}^{\text{Stuck g fram}} = k_s\Gamma_{12}^{\text{stand}}, \tag{4.36}$$

$$\Gamma_{12}^{\text{stand}} = s!\langle\phi_{\mathrm{sh}}^s(x_1)|\mathbf{O}_{12}|\phi_{\mathrm{sh}}^s(x_2)\rangle, \tag{4.37}$$

$$\mathbf{O}_{12} \equiv \sum_{n=0}^{s}\frac{(-)^n 2^n}{n!}\frac{(\alpha x_{12})^n(\bar\alpha x_{12})^n}{|x_{12}|^{2\kappa+d+2n}}, \tag{4.38}$$

$$k_s \equiv \frac{2\kappa+2s+d-2}{2s!(2\kappa+d-2)}, \tag{4.39}$$

where $\alpha x_{12}=\alpha^a x_{12}^a$, $\bar\alpha x_{12}=\bar\alpha^a x_{12}^a$, and $\Gamma_{12}^{\text{stand}}$ in (4.36) and (4.37) stands for the two-point vertex of the spin-s anomalous shadow field in the standard CFT. Equation (4.37) provides an oscillator representation for the $\Gamma_{12}^{\text{stand}}$. In terms of the tensor field $\phi_{\mathrm{sh},0}^{a_1\cdots a_s}$, vertex $\Gamma_{12}^{\text{stand}}$ (4.37) can be represented in the commonly used form,

$$\Gamma_{12}^{\text{stand}} = \phi_{\mathrm{sh},0}^{a_1\cdots a_s}(x_1)\frac{O_{12}^{a_1b_1}\cdots O_{12}^{a_sb_s}}{|x_{12}|^{2\kappa+d}}\phi_{\mathrm{sh},0}^{b_1\cdots b_s}(x_2), \tag{4.40}$$

$$O_{12}^{ab} \equiv \eta^{ab} - \frac{2x_{12}^a x_{12}^b}{|x_{12}|^2}. \tag{4.41}$$

From (4.36), we see that our gauge invariant vertex Γ_{12} considered in the Stueckelberg gauge frame coincides, up to normalization factor k_s, with the two-point vertex in the standard CFT. In Sec. III B, we have demonstrated that, in the Stueckelberg gauge frame, we are left with the rank-s traceless tensor field $\phi_{\mathrm{cur},0}^{a_1\cdots a_s}$. The two-point correlation function of this tensor field is defined by the kernel of vertex Γ^{stand} (4.40).

C. Light-cone gauge frame

We proceed with discussion of the anomalous shadow field in the light-cone gauge frame. For the anomalous shadow field, the light-cone gauge frame can be achieved through the use of differential constraint (4.8) and the light-cone gauge condition. Using gauge symmetry of the spin-s anomalous shadow field (4.18), we impose the light-cone gauge on the $|\phi_{\mathrm{sh}}\rangle$,

$$\bar\alpha^{\,+}\Pi^{[1,2]}|\phi_{\mathrm{sh}}\rangle = 0, \tag{4.42}$$

where $\Pi^{[1,2]}$ is given in (2.9). Using gauge condition (4.42) and differential constraint (4.8), we obtain

$$|\phi_{\mathrm{sh}}\rangle = \exp\Big(-\frac{\alpha^+}{\partial^+}(\bar\alpha^i\partial^i - \bar e_{1\mathrm{sh}})\Big)|\phi_{\mathrm{sh}}^{\mathrm{l.c.}}\rangle, \tag{4.43}$$

$$\bar\alpha^{\,i}\bar\alpha^i|\phi_{\mathrm{sh}}^{\mathrm{l.c.}}\rangle = 0, \tag{4.44}$$

where a light-cone ket-vector $|\phi_{\mathrm{sh}}^{\mathrm{l.c.}}\rangle$ is obtained from $|\phi_{\mathrm{sh}}\rangle$ (4.4) by equating $\alpha^+=\alpha^-=0$,

$$|\phi_{\mathrm{sh}}^{\mathrm{l.c.}}\rangle \equiv |\phi_{\mathrm{sh}}\rangle|_{\alpha^+=\alpha^-=0}. \tag{4.45}$$

We see that we are left with light-cone fields

$$\phi_{\mathrm{sh},\lambda}^{i_1\cdots i_{s'}}, \qquad s'=0,1,\ldots,s, \qquad \lambda\in[s-s']_2, \tag{4.46}$$

which are traceless tensor fields of $so(d-2)$ algebra, $\phi_{\mathrm{sh},\lambda}^{iii_3\cdots i_{s'}}=0$. These fields constitute the field content of the light-cone gauge frame. Note that, in contrast to the Stueckelberg gauge frame, all fields $\phi_{\mathrm{sh},\lambda}^{i_1\cdots i_{s'}}$ are not equal to zero. Also note that, in contrast to the gauge invariant approach, fields (4.46) are not subject to any differential constraint. Using (4.43) in (4.23) leads to light-cone gauge-fixed vertex

$$\Gamma_{12}^{\mathrm{l.c.}} = \frac{1}{2}\langle\phi_{\mathrm{sh}}^{\mathrm{l.c.}}(x_1)|\frac{f_\nu}{|x_{12}|^{2\nu+d}}|\phi_{\mathrm{sh}}^{\mathrm{l.c.}}(x_2)\rangle, \tag{4.47}$$

where f_ν is defined in (4.24).

To illustrate the structure of vertex $\Gamma_{12}^{\mathrm{l.c.}}$ (4.47) we note that, in terms of the fields $\phi_{\mathrm{sh},\lambda}^{i_1\cdots i_{s'}}$, the vertex can be represented as

$$\Gamma_{12}^{\mathrm{l.c.}} = \sum_{s'=0}^{s}\sum_{\lambda\in[s-s']_2}\Gamma_{12,\lambda}^{s'\mathrm{l.c.}}, \tag{4.48}$$

$$\Gamma_{12,\lambda}^{s'\mathrm{l.c.}} = \frac{w_\lambda}{2s'!|x_{12}|^{2\kappa-2\lambda+d}}\phi_{\mathrm{sh},\lambda}^{i_1\cdots i_{s'}}(x_1)\phi_{\mathrm{sh},\lambda}^{i_1\cdots i_{s'}}(x_2), \tag{4.49}$$

where w_λ is given in (4.29). We see that, as in the case of the gauge invariant vertex, light-cone vertex (4.48) is diagonal with respect to the light-cone fields $\phi_{\mathrm{sh},\lambda}^{i_1\cdots i_{s'}}$. Note however that, in contrast to the gauge invariant vertex, the light-cone vertex is constructed out of the light-cone fields which are not subject to any differential constraints.

Thus, we see that our gauge invariant vertex does indeed provide easy and quick access to the light-cone gauge vertex. Namely, all that is needed to obtain light-cone gauge vertex (4.48) is to remove traces of the tensor fields $\phi_{\mathrm{sh},\lambda}^{a_1\cdots a_{s'}}$ and replace the $so(d-1,1)$ Lorentz algebra vector indices appearing in gauge invariant vertex (4.23) by the respective vector indices of the $so(d-2)$ algebra.

The kernel of the light-cone vertex gives the two-point correlation function of the spin-s anomalous conformal current taken to be in the light-cone gauge. Defining two-point correlation functions of the fields $\phi_{\mathrm{cur},\lambda}^{i_1\cdots i_{s'}}$ as the second functional derivative of Γ with respect to the shadow fields $\phi_{\mathrm{sh},-\lambda}^{i_1\cdots i_{s'}}$, we obtain the following correlation functions:

$$\langle\phi_{\mathrm{cur},\lambda}^{i_1\cdots i_{s'}}(x_1),\phi_{\mathrm{cur},\lambda}^{j_1\cdots j_{s'}}(x_2)\rangle = \frac{w_{-\lambda}}{|x_{12}|^{2\kappa+2\lambda+d}}\Pi^{i_1\cdots i_{s'};j_1\cdots j_{s'}}, \tag{4.50}$$

where w_λ is defined in (4.29) and $\Pi^{i_1\cdots i_{s'};j_1\cdots j_{s'}}$ stands for the projector on the traceless rank-s' tensor field of the $so(d-2)$ algebra. An explicit form of the projector may be found, e.g., in Ref. [27].

V. TWO-POINT CURRENT-SHADOW FIELD INTERACTION VERTEX

We now briefly discuss the two-point current-shadow field interaction vertex. In our approach, this interaction vertex is determined by requiring that

(i) the vertex is invariant under both gauge transformations of the anomalous conformal current and the shadow field;

(ii) the vertex is invariant under conformal algebra transformations.

We find the following vertex:

$$\mathcal{L} = \langle\phi_{\mathrm{cur}}|\boldsymbol{\mu}|\phi_{\mathrm{sh}}\rangle, \tag{5.1}$$

where $\boldsymbol{\mu}$ is given in (2.10). We note that, under gauge transformation of the anomalous conformal current (3.19), the variation of vertex (5.1) takes the form (up to total derivative)

$$\delta_{\xi_{\mathrm{cur}}}\mathcal{L} = -\langle\xi_{\mathrm{cur}}|\bar{C}_{\mathrm{sh}}|\phi_{\mathrm{sh}}\rangle. \tag{5.2}$$

From (5.2), we see that the vertex $\mathcal{L}$ is invariant under gauge transformation of the anomalous conformal current provided the anomalous shadow field satisfies differential constraint (4.8). Next, we note that under gauge transformation of the anomalous shadow field (4.18) the gauge variation of vertex (5.1) takes the form (up to a total derivative)

$$\delta_{\xi_{\mathrm{sh}}}\mathcal{L} = -\langle\xi_{\mathrm{sh}}|\bar{C}_{\mathrm{cur}}|\phi_{\mathrm{cur}}\rangle. \tag{5.3}$$

From (5.3), we see that the vertex $\mathcal{L}$ is invariant under gauge transformation of the anomalous shadow field provided the anomalous conformal current satisfies differential constraint (3.9).

Using the realization of the conformal algebra symmetries obtained in Secs. III and IV, we make sure that vertex $\mathcal{L}$ (5.1) is invariant under the conformal algebra transformations.

VI. ADS/CFT CORRESPONDENCE: PRELIMINARIES

We now study the AdS/CFT correspondence for a free arbitrary spin massive AdS field and a boundary arbitrary spin anomalous conformal current and shadow field. To study the AdS/CFT correspondence we use the gauge invariant CFT adapted formulation of the massive AdS field and the modified de Donder gauge condition found in Ref. [14].[7] We emphasize that it is the use of our massive gauge fields and the modified de Donder gauge condition that leads to the decoupled gauge-fixed equations of motion and surprisingly simple Lagrangian.[8] The use of our massive gauge fields and the modified de Donder gauge condition makes the study of AdS/CFT correspondence for the arbitrary spin-s massive AdS field similar to the one for the spin-0 massive AdS field. Owing these properties of our massive gauge fields and the modified de Donder gauge condition, the computation of effective action is considerably simplified. Perhaps this is the main advantage of our approach.

In our approach to the AdS/CFT correspondence, we have gauge symmetries not only at the AdS side but also at the boundary CFT. Also, we note that the modified de Donder gauge condition turns out to be invariant under on-shell leftover gauge symmetries of the massive AdS field. This is to say that, in the framework of our approach, the study of AdS/CFT correspondence implies the matching of

[7] Applications of the standard de Donder gauge to the various problems of massless fields can be found in Ref. [28]. Recent interesting discussion of a modified de Donder gauge can be found in Ref. [29]. We believe that our modified de Donder gauge will also be useful for better understanding of various aspects of AdS/QCD correspondence which are discussed, e.g., in Ref. [30].

[8] Our massive gauge fields are obtained from gauge fields used in a gauge invariant approach to massive fields in Ref. [31] by the invertible transformation which is described in the appendix of Ref. [14]. Discussion of interesting methods for solving AdS field equations of motion without gauge fixing can be found in Ref. [32].

(i) a modified de Donder gauge condition for the bulk massive field and the corresponding differential constraint for the boundary anomalous conformal current and shadow field;
(ii) on-shell leftover gauge symmetries of the bulk massive field and the corresponding gauge symmetries of the boundary anomalous conformal current and shadow field;
(iii) on-shell global symmetries of the bulk massive field and the corresponding global symmetries of the boundary anomalous conformal current and shadow field;
(iv) AdS field action evaluated on the solution of the AdS massive field equations of motion with the Dirichlet problem corresponding to the boundary anomalous shadow field and the boundary two-point gauge invariant vertex for the anomalous shadow field.

As we have already said, to discuss the AdS/CFT correspondence for the bulk arbitrary spin massive AdS field and the boundary arbitrary spin anomalous conformal current and shadow field we use the CFT adapted gauge invariant Lagrangian and the modified de Donder gauge condition for the arbitrary spin massive AdS field found in Ref. [14]. We begin therefore with the presentation of our result in Ref. [14].

A. CFT adapted approach to massive arbitrary spin AdS field

In AdS_{d+1} space, the massive spin-s field is described by the following scalar, vector, and totally symmetric tensor fields of the $so(d)$ algebra[9]:

$$\phi_\lambda^{a_1\cdots a_{s'}}, \qquad s'=0,1,\ldots,s, \qquad \lambda\in[s-s']_2. \tag{6.1}$$

Using shortcut $\phi_\lambda^{s'}$ for the field $\phi_\lambda^{a_1\cdots a_{s'}}$, fields in (6.1) can be represented as in (2.16). The fields $\phi_\lambda^{a_1\cdots a_{s'}}$ with $s'\geq 4$ are double traceless,

$$\phi_\lambda^{aabba_5\cdots a_{s'}}=0, \qquad s'=4,5,\ldots,s. \tag{6.2}$$

In order to obtain the gauge invariant description in an easy-to-use form we use the oscillators and introduce a ket-vector $|\phi\rangle$ defined by

$$|\phi\rangle=\sum_{s'=0}^{s}|\phi^{s'}\rangle,$$

$$|\phi^{s'}\rangle=\sum_{\lambda\in[s-s']_2}\frac{\zeta^{(s-s'+\lambda)/2}\alpha_z^{(s-s'-\lambda)/2}\alpha^{a_1}\cdots\alpha^{a_{s'}}}{s'!\sqrt{(\frac{s-s'+\lambda}{2})!(\frac{s-s'-\lambda}{2})!}}\phi_\lambda^{a_1\cdots a_{s'}}|0\rangle. \tag{6.3}$$

From (6.3), we see that the ket-vector $|\phi\rangle$ is the degree-s homogeneous polynomial in the oscillators α^a, α^z, and ζ, while the ket-vector $|\phi^{s'}\rangle$ is the degree-s' homogeneous polynomial in the oscillators α^a. In terms of the ket-vector $|\phi\rangle$, double-tracelessness constraint (6.2) takes the form

$$(\bar\alpha^2)^2|\phi\rangle=0. \tag{6.4}$$

Using the Poincaré parametrization of AdS_{d+1} space

$$ds^2=\frac{1}{z^2}(dx^adx^a+dzdz), \tag{6.5}$$

we present CFT adapted gauge invariant action and Lagrangian [14],

$$S=\int d^dxdz\mathcal{L}, \tag{6.6}$$

$$\mathcal{L}=\tfrac{1}{2}\langle\partial^a\phi|\boldsymbol{\mu}|\partial^a\phi\rangle+\tfrac{1}{2}\langle\mathcal{T}_{\nu-(1/2)}\phi|\boldsymbol{\mu}|\mathcal{T}_{\nu-(1/2)}\phi\rangle -\tfrac{1}{2}\langle\bar C\phi||\bar C\phi\rangle, \tag{6.7}$$

where we use the notation

$$\bar C=\bar\alpha\partial-\tfrac{1}{2}\alpha\partial\bar\alpha^2-\bar e_1\Pi^{[1,2]}+\tfrac{1}{2}e_1\bar\alpha^2, \tag{6.8}$$

$$-e_1=\zeta r_\zeta\mathcal{T}_{-\nu-(1/2)}+\alpha^z r_z\mathcal{T}_{\nu-(1/2)}, \tag{6.9}$$

$$-\bar e_1=\mathcal{T}_{\nu+(1/2)}r_\zeta\bar\zeta+\mathcal{T}_{-\nu+(1/2)}r_z\bar\alpha^z, \tag{6.10}$$

$$\mathcal{T}_\nu=\partial_z+\frac{\nu}{z}, \tag{6.11}$$

$$\nu=\kappa+N_\zeta-N_z, \tag{6.12}$$

and $\Pi^{[1,2]}$, $\boldsymbol{\mu}$, r_ζ, r_z, and κ are given in (2.9), (2.10), (2.13), and (3.4), respectively.

To discuss gauge symmetries of Lagrangian (6.7) we introduce the gauge transformation parameters,

$$\xi_\lambda^{a_1\cdots a_{s'}}, \quad s'=0,1,\ldots,s-1, \quad \lambda\in[s-1-s']_2, \tag{6.13}$$

which are scalar, vector, and totally symmetric tensor fields of the $so(d)$ algebra. The gauge transformation parameters with $s'\geq 2$ are traceless, $\xi_\lambda^{aaa_3\cdots a_{s'}}=0$.

As usual, we collect the gauge transformation parameters in a ket-vector $|\xi\rangle$ defined by

$$|\xi\rangle=\sum_{s'=0}^{s-1}|\xi^{s'}\rangle,$$

$$|\xi^{s'}\rangle=\sum_{\lambda\in[s-1-s']_2}\frac{\zeta^{(s-1-s'+\lambda)/2}\alpha_z^{(s-1-s'-\lambda)/2}\alpha^{a_1}\cdots\alpha^{a_{s'}}}{s'!\sqrt{(\frac{s-1-s'+\lambda}{2})!(\frac{s-1-s'-\lambda}{2})!}} \times\xi_\lambda^{a_1\cdots a_{s'}}|0\rangle. \tag{6.14}$$

We see that $|\xi\rangle$ is a degree-$(s-1)$ homogeneous polynomial in the oscillators α^a, α^z, and ζ, while the ket-vector

[9]From now on we use, unless otherwise specified, the Euclidean signature.

$|\xi^{s'}\rangle$ is the degree-s' homogeneous polynomial in the oscillators α^a.

Lagrangian (6.7) is invariant under the gauge transformation (up to total derivative)

$$\delta|\phi\rangle = G|\xi\rangle, \tag{6.15}$$

$$G \equiv \alpha\partial - e_1 - \alpha^2 \frac{1}{2N_\alpha + d - 2}\bar{e}_1. \tag{6.16}$$

Global AdS symmetries in CFT adapted approach.—As is well known relativistic symmetries of fields in AdS_{d+1} space are described by the $so(d,2)$ algebra. Global symmetries of anomalous conformal currents and shadow fields in d-dimensional space are also described by the $so(d,2)$ algebra. We have discussed global symmetries of anomalous conformal currents and shadow fields by using the conformal basis of the $so(d,2)$ algebra in (2.18). Therefore, for studying the AdS/CFT correspondence, it is convenient to realize the relativistic symmetries of fields in AdS_{d+1} space by using also the conformal basis of the $so(d,2)$ algebra. To achieve the conformal basis realization of bulk $so(d,2)$ symmetries we use the Poincaré parametrization of AdS space (6.5).[10] In the Poincaré parametrization, the $so(d,2)$ algebra transformation of arbitrary spin massive AdS field $|\phi\rangle$ is given by $\delta_{\hat{G}}|\phi\rangle = \hat{G}|\phi\rangle$, where the realization of the $so(d,2)$ algebra generators $\hat{G}$ on space of $|\phi\rangle$ takes the form

$$P^a = \partial^a, \tag{6.17}$$

$$J^{ab} = x^a\partial^b - x^b\partial^a + M^{ab}, \tag{6.18}$$

$$D = x\partial + \Delta, \qquad \Delta = z\partial_z + \frac{d-1}{2}, \tag{6.19}$$

$$K^a = K^a_{\Delta,M} + R^a, \tag{6.20}$$

$$K^a_{\Delta,M} = -\tfrac{1}{2}x^2\partial^a + x^a D + M^{ab}x^b, \tag{6.21}$$

$$R^a = R^a_{(0)} + R^a_{(1)}, \tag{6.22}$$

$$R^a_{(0)} = z\tilde{C}^a(r_\zeta\bar{\zeta} + r_z\bar{\alpha}^z) - z(\zeta r_\zeta + \alpha^z r_z)\bar{\alpha}^a, \tag{6.23}$$

$$R^a_{(1)} = -\tfrac{1}{2}z^2\partial^a, \tag{6.24}$$

and the operator $\tilde{C}^a$ is given in (2.11).

Modified de Donder gauge.—Gauge invariant equations of motion obtained from Lagrangian (6.7) take the form

$$\boldsymbol{\mu}\,\Box_\nu|\phi\rangle - C\bar{C}|\phi\rangle = 0, \tag{6.25}$$

$$\Box_\nu \equiv \Box + \partial_z^2 - \frac{1}{z^2}\Big(\nu^2 - \frac{1}{4}\Big), \tag{6.26}$$

$$C \equiv \alpha\partial - \tfrac{1}{2}\alpha^2\bar{\alpha}\partial - e_1\Pi^{[1,2]} + \tfrac{1}{2}\bar{e}_1\alpha^2, \tag{6.27}$$

where $\bar{C}$ and ν are given in (6.8) and (6.12). Note that, for the derivation of (6.25), we use the relations $C^\dagger = -\bar{C}$,

$$\mathcal{T}^\dagger_{\nu-(1/2)}\mathcal{T}_{\nu-(1/2)} = -\partial_z^2 + \frac{1}{z^2}\Big(\nu^2 - \frac{1}{4}\Big). \tag{6.28}$$

The modified de Donder gauge is defined to be

$$\bar{C}|\phi\rangle = 0, \qquad \text{modified de Donder gauge}, \tag{6.29}$$

where $\bar{C}$ is given in (6.8). Using this gauge in (6.25) leads to the simple gauge-fixed equations of motion,

$$\Box_\nu|\phi\rangle = 0, \tag{6.30}$$

i.e., the gauge-fixed equations turn out to be decoupled.

We note that the modified de Donder gauge and gauge-fixed equations have on-shell leftover gauge symmetry. This is to say that modified de Donder gauge (6.29) and gauge-fixed equation (6.30) are invariant with respect to the gauge transformation given in (6.15) provided the gauge transformation parameter satisfies the following equation:

$$\Box_\nu|\xi\rangle = 0. \tag{6.31}$$

B. AdS/CFT correspondence for spin-0 field

As we have already said, the use of our massive gauge fields and the modified de Donder gauge makes the study of AdS/CFT correspondence for the arbitrary spin-s massive AdS field similar to the one for the spin-0 massive AdS field. Therefore, for the reader's convenience, we now briefly recall the AdS/CFT correspondence for the scalar massive AdS field.

AdS/CFT correspondence for normalizable modes of spin-0 massive AdS field and spin-0 conformal current.[11] The action of the massive scalar field in AdS_{d+1} background takes the form

$$S = \int d^dx dz \mathcal{L}, \tag{6.32}$$

$$\mathcal{L} = \tfrac{1}{2}\sqrt{|g|}(g^{\mu\rho}\partial_\mu\Phi\partial_\rho\Phi + m^2\Phi^2). \tag{6.33}$$

Using the canonically normalized field ϕ defined by relation $\Phi = z^{(d-1)/2}\phi$, we represent Lagrangian (6.33) as (up to total derivative)

$$\mathcal{L} = \tfrac{1}{2}|d\phi|^2 + \tfrac{1}{2}|\mathcal{T}_{\nu-(1/2)}\phi|^2, \tag{6.34}$$

[10] We note that, in our approach, only $so(d-1,1)$ symmetries are realized manifestly. Symmetries of the $so(d,2)$ algebra could be realized manifestly by using the framework of the ambient space approach (see, e.g., Refs. [33–35]).

[11] Also, see Ref. [36].

$$\mathcal{T}_\nu \equiv \partial_z + \frac{\nu}{z}, \qquad \nu = \sqrt{m^2 + \frac{d^2}{4}}. \tag{6.35}$$

Note that only for the spin-0 massive AdS field the ν takes the form given in (6.35). For the spin-s massive AdS field with $s > 0$ the ν is given in (6.12). The equation of motion obtained from Lagrangian (6.34) is given by

$$\Box_\nu \phi = 0, \tag{6.36}$$

where $\Box_\nu$ is defined in (6.26). The normalizable solution of Eq. (6.36) takes the form

$$\phi(x, z) = U_\nu^{\rm sc} \phi_{\rm cur}(x), \tag{6.37}$$

$$U_\nu^{\rm sc} \equiv h_\nu \sqrt{zq} J_\nu(zq) q^{-\nu-(1/2)}, \tag{6.38}$$

$$h_\nu \equiv 2^\nu \Gamma(\nu + 1), \qquad q^2 \equiv \Box, \tag{6.39}$$

where J_ν stands for the Bessel function. The asymptotic behavior of solution (6.37) is given by

$$\phi(x, z) \stackrel{z\to 0}{\longrightarrow} z^{\nu+(1/2)} \phi_{\rm cur}(x). \tag{6.40}$$

From (6.40), we see that the field $\phi_{\rm cur}$ is indeed the asymptotic boundary value of the normalizable solution.

For the case of the scalar field, there are no gauge symmetries and gauge conditions to be matched. All that is needed to complete the AdS/CFT correspondence is to match the bulk global symmetries of the AdS field $\phi(x, z)$ and the respective boundary global symmetries of the current $\phi_{\rm cur}(x)$. Realization of the global symmetries on the AdS and CFT sides is given in Eqs. (6.17)–(6.24) and Eqs. (2.20)–(2.23), respectively. Obviously, the Poincaré symmetries match automatically. Introducing the notation $D_{\rm AdS}$ and $D_{\rm CFT}$ for the respective realizations of D symmetry on bulk fields (6.19) and boundary conformal currents (2.22), we get the relation

$$D_{\rm AdS} \phi(x, z) = U_\nu^{\rm sc} D_{\rm CFT} \phi_{\rm cur}(x), \tag{6.41}$$

where $D_{\rm CFT}$ corresponding to the conformal spin-0 current $\phi_{\rm cur}$ is obtained from (2.22) by using $\Delta = \frac{d}{2} + \nu$ with ν given in (6.35). From (6.41), we see that D symmetries of $\phi(x,z)$ and $\phi_{\rm cur}(x)$ also match. Finally, using $R^a \phi_{\rm cur}(x) = 0$ and taking into account that, for the spin-0 AdS field, $R^a_{(0)} \phi(x, z) = 0$, we see that the K^a symmetries also match.

AdS/CFT correspondence for non-normalizable modes of spin-0 massive AdS field and spin-0 shadow field.—As shown in Ref. [37], the non-normalizable solution of Eq. (6.36) with the Dirichlet problem corresponding to the boundary shadow field $\phi_{\rm sh}(x)$ can be presented as

$$\phi(x, z) = \sigma \int d^d y G_\nu(x - y, z) \phi_{\rm sh}(y), \tag{6.42}$$

$$G_\nu(x, z) = \frac{c_\nu z^{\nu+(1/2)}}{(z^2 + |x|^2)^{\nu+(d/2)}}, \tag{6.43}$$

$$c_\nu \equiv \frac{\Gamma(\nu + \frac{d}{2})}{\pi^{d/2}\Gamma(\nu)}. \tag{6.44}$$

For later use, we introduce the normalization factor σ in relation (6.42). We recall that the commonly used value of σ is achieved by setting $\sigma = 1$. The asymptotic behaviors of Green function (6.43) and solution (6.42) are given by

$$G_\nu(x, z) \stackrel{z\to 0}{\longrightarrow} z^{-\nu+(1/2)} \delta^d(x), \tag{6.45}$$

$$\phi(x, z) \stackrel{z\to 0}{\longrightarrow} z^{-\nu+(1/2)} \sigma \phi_{\rm sh}(x). \tag{6.46}$$

Equation (6.46) tells us that solution (6.42) has indeed asymptotic behavior corresponding to the shadow field.

Taking into account (6.32), (6.34), and (6.36), we find the well-known expression for the effective action,[12]

$$-S_{\rm eff} = \int d^d x \mathcal{L}_{\rm eff}|_{z\to 0}, \tag{6.47}$$

$$\mathcal{L}_{\rm eff} = \tfrac{1}{2} \phi \mathcal{T}_{\nu-(1/2)} \phi. \tag{6.48}$$

Using the solution of the Dirichlet problem (6.42) in (6.47) and (6.48), we get the effective action

$$-S_{\rm eff} = \nu c_\nu \sigma^2 \int d^d x_1 d^d x_2 \frac{\phi_{\rm sh}(x_1)\phi_{\rm sh}(x_2)}{|x_{12}|^{2\nu+d}}. \tag{6.49}$$

Plugging the commonly used value of σ, $\sigma = 1$, into (6.49), we get the properly normalized effective action found in Refs. [38,39]. An interesting novelty of our computation of the effective action is that we use the Fourier transform of the Green function. For the details of our computation, see Appendix C in Ref. [4].

VII. ADS/CFT CORRESPONDENCE FOR NORMALIZABLE MODES OF MASSIVE ADS FIELD AND ANOMALOUS CONFORMAL CURRENT

We now consider the AdS/CFT correspondence for the spin-s massive AdS field and spin-s anomalous conformal current. We begin with the discussion of the normalizable solution of Eq. (6.30). The normalizable solution of Eq. (6.30) is given by

$$|\phi(x, z)\rangle = U_\nu |\phi_{\rm cur}(x)\rangle, \tag{7.1}$$

$$U_\nu \equiv h_\kappa (-)^{N_z} \sqrt{zq} J_\nu(zq) q^{-\nu-(1/2)}, \tag{7.2}$$

$$h_\kappa \equiv 2^\kappa \Gamma(\kappa + 1), \qquad q^2 \equiv \Box, \tag{7.3}$$

where we do not show explicitly the dependence of U_ν on z, q, and the parameter κ defined in (3.4). The asymptotic behavior of solution (7.1) takes the form

[12]As usual, since the solution of the Dirichlet problem (6.42) tends to zero as $z \to \infty$, we ignore the contribution to $S_{\rm eff}$ when $z = \infty$.

$$|\phi(x,z)\rangle \stackrel{z\to 0}{\longrightarrow} z^{\nu+(1/2)} \frac{2^\kappa \Gamma(\kappa+1)}{2^\nu \Gamma(\nu+1)} (-)^{N_z} |\phi_{\rm cur}(x)\rangle. \qquad (7.4)$$

From (7.4), we see that $|\phi_{\rm cur}\rangle$ is indeed a boundary value of the normalizable solution. In the right-hand side of Eq. (7.1), we use the notation $|\phi_{\rm cur}\rangle$, because we are going to demonstrate that this boundary value is indeed the gauge field appearing in the gauge invariant formulation of the spin-s anomalous conformal current in Sec. III. Namely, we are going to prove the following statements:

(i) For normalizable solution (7.1), modified de Donder gauge condition (6.29) leads to the differential constraint of the spin-s anomalous conformal current given in (3.9).

(ii) On-shell leftover gauge transformation (6.15) of normalizable solution (7.1) leads to the gauge transformation of the spin-s anomalous conformal current given in (3.19).[13]

(iii) On-shell bulk $so(d,2)$ symmetries of the normalizable solution (7.1) amount to boundary $so(d,2)$ conformal symmetries of the spin-s anomalous conformal current.

To prove these statements we use the following relations for the operator U_ν:

$$\mathcal{T}_{\nu-(1/2)} U_\nu = U_{\nu-1}, \qquad (7.5)$$

$$\mathcal{T}_{-\nu-(1/2)} U_\nu = -U_{\nu+1}\Box, \qquad (7.6)$$

$$\mathcal{T}_{-\nu+(1/2)}(zU_\nu) = -zU_{\nu+1}\Box + 2U_\nu, \qquad (7.7)$$

$$\Box_\nu (zU_{\nu+1}) = 2U_\nu, \qquad (7.8)$$

which, in turn, can be derived by using the following textbook identities for the Bessel function:

$$\mathcal{T}_\nu J_\nu = J_{\nu-1}, \qquad \mathcal{T}_{-\nu} J_\nu = -J_{\nu+1}. \qquad (7.9)$$

Matching of bulk modified de Donder gauge and boundary constraint.—We now demonstrate how a differential constraint for the anomalous conformal current (3.9) is obtained from modified de Donder gauge condition (6.29). Using (7.5) and (7.6), we find the important relations

$$e_1 U_\nu = U_\nu e_{1,\rm cur}, \qquad \bar{e}_1 U_\nu = U_\nu \bar{e}_{1,\rm cur}. \qquad (7.10)$$

Acting with operator $\bar{C}$ (6.8) on solution $|\phi\rangle$ (7.1) and using (7.10), we obtain the relation

$$\bar{C}|\phi(x,z)\rangle = U_\nu \bar{C}_{\rm cur}|\phi_{\rm cur}(x)\rangle, \qquad (7.11)$$

where $\bar{C}_{\rm cur}$ is given in (3.10). From (7.11), we see that our modified de Donder gauge condition (6.29) leads indeed to the differential constraint for the anomalous conformal current given in (3.9).

Matching of bulk and boundary gauge symmetries.—We now show how gauge transformation of the anomalous conformal current (3.19) is related to the on-shell leftover gauge transformation of the massive AdS field (6.15). To this end we note that on-shell leftover gauge transformation of the massive AdS field is obtained from (6.15) by plugging a gauge transformation parameter that satisfies Eq. (6.31) into (6.15). The normalizable solution of the equation for the gauge transformation parameter (6.31) takes the form

$$|\xi(x,z)\rangle = U_\nu |\xi_{\rm cur}(x)\rangle, \qquad (7.12)$$

where U_ν is given in (7.2). On the one hand, plugging (7.12) into (6.15) and using (7.10), we find that bulk on-shell leftover gauge transformation takes the form

$$\delta|\phi(x,z)\rangle = U_\nu G_{\rm cur}|\xi_{\rm cur}(x)\rangle. \qquad (7.13)$$

On the other hand, Eq. (7.1) leads to

$$\delta|\phi(x,z)\rangle = U_\nu \delta|\phi_{\rm cur}(x)\rangle. \qquad (7.14)$$

Comparing (7.13) and (7.14), we conclude that bulk and boundary gauge symmetries do indeed match.

Matching of bulk and boundary global symmetries.—Now we are going to demonstrate the matching of the $so(d,2)$ algebra generators for the bulk massive AdS field in (6.17), (6.18), (6.19), and (6.20) and the ones for the boundary anomalous conformal current in (2.20)–(2.23). Representation for generators of the $so(d,2)$ algebra given in (6.17), (6.18), (6.19), and (6.20) is valid for the gauge invariant theory of AdS fields. Note however that the modified de Donder gauge respects the Poincaré and dilatation symmetries, but breaks the conformal boost symmetries (K^a symmetries). This implies that realization for generators P^a, J^{ab}, and D given in (6.17), (6.18), and (6.19) is still valid for the gauge-fixed AdS fields, while realization for the conformal boost generator K^a given in (6.20) must be modified to restore K^a symmetries of the gauge-fixed AdS fields. We begin with matching of the Poincaré and dilatation symmetries. Matching of the Poincaré symmetries is obvious: from (2.20), (2.21), (6.17), and (6.18), we see that bulk and boundary generators of the Poincaré algebra, P^a, J^{ab}, coincide. Next, we consider the dilatation generator D. To match the dilatation symmetries we need an explicit form of the solution for AdS field equations of motion in (7.1). Using the notation $D_{\rm AdS}$ and $D_{\rm CFT}$ for the respective bulk dilatation generator in (6.19) and boundary dilatation generator in (2.22), we get the relation

$$D_{\rm AdS}|\phi(x,z)\rangle = U_\nu D_{\rm CFT}|\phi_{\rm cur}(x)\rangle, \qquad (7.15)$$

where $D_{\rm CFT}$ corresponding to $|\phi_{\rm cur}\rangle$ is obtainable from (2.22) and the conformal dimension operator given in (3.21). From (7.15), we see that the generators $D_{\rm AdS}$ and $D_{\rm CFT}$ match.

[13]Note that the gauge transformation given in (6.15) is an off-shell gauge transformation. The on-shell leftover gauge transformation is obtained from gauge transformation (6.15) by using a gauge transformation parameter which satisfies Eq. (6.31).

We now match the K^a symmetries. As we noted above, our modified de Donder gauge breaks the K^a symmetries. This can be seen as follows. Using realization of K^a transformations in (6.20), we find that the gauge-fixed massive AdS field satisfies the relation

$$\bar{C}K^a|\phi\rangle = -2\bar{C}^a_\perp|\phi\rangle, \tag{7.16}$$

which tells us that the modified de Donder gauge condition, $\bar{C}|\phi\rangle = 0$, is not invariant under K^a symmetries that are described by generator K^a in (6.20). Therefore, to restore the K^a symmetries of the gauge-fixed AdS field theory, we should modify the generator K^a in (6.20). We modify the generator K^a by using the standard procedure. Namely, we add compensating gauge transformations to maintain the K^a symmetries. This is to say that, in order to find improved $K^a_{\rm impr}$ transformations we start with the generic global K^a transformations (6.20) supplemented by the suitable compensating gauge transformation

$$K^a_{\rm impr}|\phi\rangle = K^a|\phi\rangle + G|\xi^{K^a}\rangle, \tag{7.17}$$

where G is given in (6.16) and $|\xi^{K^a}\rangle$ stands for the parameter of the compensating gauge transformation. Using the relation

$$\bar{C}G|\xi^{K^a}\rangle = \Box_\nu|\xi^{K^a}\rangle \tag{7.18}$$

and (7.16), we find

$$\bar{C}K^a_{\rm impr}|\phi\rangle = -2\bar{C}^a_\perp|\phi\rangle + \Box_\nu|\xi^{K^a}\rangle. \tag{7.19}$$

Requiring the improved $K^a_{\rm impr}$ transformations to maintain the modified de Donder gauge condition,

$$\bar{C}K^a_{\rm impr}|\phi\rangle = 0, \tag{7.20}$$

we get the equation for $|\xi^{K^a}\rangle$,

$$\Box_\nu|\xi^{K^a}\rangle - 2\bar{C}^a_\perp|\phi\rangle = 0. \tag{7.21}$$

From (7.21), we see that the compensating gauge transformation parameter $|\xi^{K^a}\rangle$ should satisfy the nonhomogeneous second-order differential equation. Plugging the normalizable solution of Eq. (7.1) into (7.21), we see that Eq. (7.21) leads to

$$\Box_\nu|\xi^{K^a}(x,z)\rangle = 2U_\nu\bar{C}^a_\perp|\phi_{\rm cur}(x)\rangle. \tag{7.22}$$

Using identity (7.8), the solution to Eq. (7.22) is found to be

$$|\xi^{K^a}(x,z)\rangle = zU_{\nu+1}\bar{C}^a_\perp|\phi_{\rm cur}(x)\rangle. \tag{7.23}$$

Plugging (7.1) and (7.23) into (7.17), we check that the improved $K^a_{\rm impr}$ transformations of the spin-s massive AdS field lead to the conformal boost transformations of the spin-s anomalous conformal current given in (2.19) and (2.23) with operator $R^a_{\rm cur}$ defined in (3.22) (for details, see the Appendix).

VIII. ADS/CFT CORRESPONDENCE FOR NON-NORMALIZABLE MODES OF MASSIVE ADS FIELD AND ANOMALOUS SHADOW FIELD

We now discuss the AdS/CFT correspondence for the bulk spin-s massive AdS field and the boundary spin-s anomalous shadow field. We begin with an analysis of the non-normalizable solution of Eq. (6.30). Because gauge-fixed equation of motion (6.30) is similar to the one for the scalar AdS field (6.36) we can simply apply the result in Sec. VI. This is to say that the solution of Eq. (6.30) with the Dirichlet problem corresponding to the spin-s anomalous shadow field takes the form

$$|\phi(x,z)\rangle = \sigma_\nu\int d^dyG_\nu(x-y,z)|\phi_{\rm sh}(y)\rangle, \tag{8.1}$$

$$\sigma_\nu \equiv \frac{2^\nu\Gamma(\nu)}{2^\kappa\Gamma(\kappa)}(-)^{N_z}, \tag{8.2}$$

where the Green function is given in (6.43).

Using asymptotic behavior of the Green function G_ν (6.45), we find the asymptotic behavior of our solution

$$|\phi(x,z)\rangle \stackrel{z\to 0}{\longrightarrow} z^{-\nu+(1/2)}\sigma_\nu|\phi_{\rm sh}(x)\rangle. \tag{8.3}$$

From this expression, we see that solution (8.1) has indeed asymptotic behavior corresponding to the spin-s anomalous shadow field.[14] In the right-hand side of (8.1) we use the notation $|\phi_{\rm sh}\rangle$, because we are going to show that this boundary value is indeed the gauge field entering the gauge invariant formulation of the spin-s anomalous shadow field in Sec. IV. Namely, we are going to prove the following statements:

(i) For solution (8.1), modified de Donder gauge condition (6.29) leads to a differential constraint of the anomalous shadow field given in (4.8).

(ii) On-shell leftover gauge transformation (6.15) of solution (8.1) leads to gauge transformation of the spin-s anomalous shadow field given in (4.18).

(iii) On-shell global $so(d,2)$ symmetries of solution (8.1) become global $so(d,2)$ conformal symmetries of the spin-s anomalous shadow field.

(iv) Action (6.6) evaluated on solution (8.1) coincides, up to a normalization factor, with a boundary two-point gauge invariant vertex for the anomalous shadow field given in (4.22).

Below we demonstrate how these statements can be proved by using the following relations for the Green function $G_\nu \equiv G_\nu(x-y,z)$:

$$\mathcal{T}_{-\nu+(1/2)}G_{\nu-1} = -2(\nu-1)G_\nu, \tag{8.4}$$

[14] Since solution (8.1) has nonintegrable asymptotic behavior (8.3), such solution is sometimes referred to as the non-normalizable solution.

$$\mathcal{T}_{\nu+(1/2)} G_{\nu+1} = \frac{1}{2\nu} \Box G_\nu, \qquad (8.5)$$

$$\Box_\nu (z G_{\nu-1}) = -4(\nu-1) G_\nu. \qquad (8.6)$$

Matching of bulk modified de Donder gauge and boundary differential constraint.—We note that it is the choice of normalization factor σ_ν in (8.2) that allows us to match the bulk modified de Donder gauge and boundary differential constraint. Namely, the normalization factor σ_ν (8.2) is uniquely determined by the following two requirements:

(i) The factor σ_ν is normalized to be

$$\sigma_\nu = 1, \quad \text{for } N_z = 0, \quad N_\zeta = 0. \qquad (8.7)$$

(ii) The modified de Donder gauge condition for AdS field $|\phi\rangle$ (6.29) should lead to the differential constraint for the shadow field $|\phi_{\rm sh}\rangle$ (4.8).

We note that the choice of normalization condition (8.7) is a matter of convenience. This commonly used condition implies the following normalization of asymptotic behavior of solution (8.1) for the leading rank-s tensor field $\phi_0^{a_1\cdots a_s}$ in (6.1):

$$\phi_0^{a_1\cdots a_s}(x,z) \overset{z\to 0}{\longrightarrow} z^{-\kappa+(1/2)} \phi_{{\rm sh},0}^{a_1\cdots a_s}(x), \qquad (8.8)$$

where $\phi_{{\rm sh},0}^{a_1\cdots a_s}$ is the leading rank-s tensor field in (4.1).

Using (8.4) and (8.5) we find the relations

$$e_1 \sigma_\nu G_\nu = \sigma_\nu G_\nu e_{1,{\rm sh}}, \qquad \bar{e}_1 \sigma_\nu G_\nu = \sigma_\nu G_\nu \bar{e}_{1,{\rm sh}}, \qquad (8.9)$$

where Laplace operator $\Box$ appearing in $e_{1,{\rm sh}}$, $\bar{e}_{1,{\rm sh}}$ is acting on the Green function G_ν. Acting with operator $\bar{C}$ (6.8) on solution (8.1) and using (8.9), we find the relation

$$\bar{C}|\phi\rangle = \sigma_\nu \int d^d y G_\nu(x-y,z) \bar{C}_{\rm sh} |\phi_{\rm sh}(y)\rangle. \qquad (8.10)$$

From this relation, we see that the modified de Donder gauge for the spin-s massive AdS field (6.29) and differential constraint for the spin-s anomalous shadow field given in (4.8) match.

Matching of bulk and boundary gauge symmetries.—We now show how gauge transformation of the anomalous shadow field (4.18) is obtained from the on-shell leftover gauge transformation of the massive AdS field (6.15). To this end we note that the corresponding on-shell leftover gauge transformation of the massive AdS field is obtained from (6.15) by plugging the non-normalizable solution of the equation for gauge transformation parameter (6.31) into (6.15). The non-normalizable solution of Eq. (6.31) is given by

$$|\xi(x,z)\rangle = \sigma_\nu \int d^d y G_\nu(x-y,z) |\xi_{\rm sh}(y)\rangle, \qquad (8.11)$$

where σ_ν is given in (8.2). We now note that, on the one hand, plugging (8.11) into (6.15) and using (8.9), we can cast the on-shell leftover gauge transformation of $|\phi\rangle$ into the form

$$\delta|\phi\rangle = \sigma_\nu \int d^d y G_\nu(x-y,z) G_{\rm sh} |\xi_{\rm sh}(y)\rangle. \qquad (8.12)$$

On the other hand, making use of relation (8.1), we get

$$\delta|\phi\rangle = \sigma_\nu \int d^d y G_\nu(x-y,z) \delta|\phi_{\rm sh}(y)\rangle. \qquad (8.13)$$

Comparing (8.12) with (8.13), we conclude that the on-shell leftover gauge symmetries of the solution of the Dirichlet problem for the spin-s massive AdS field are indeed related to gauge symmetries of the spin-s anomalous shadow field.

Matching of bulk and boundary global symmetries.—Matching of global symmetries can be demonstrated by using the procedure we described for the spin-s anomalous conformal current in Sec. VII. Therefore, to avoid repetition, let us briefly discuss only relevant details. The matching of bulk and boundary symmetries of the Poincaré algebra is straightforward. To match bulk and boundary dilatation symmetries all that we needed are the solution for the bulk field in (8.1), the bulk dilatation operator (6.19), and the conformal dimension operator for the spin-s anomalous shadow field given in (4.20). To match conformal boost symmetries we introduce improved bulk $K_{\rm impr}^a$ transformations with a compensating gauge transformation parameter that satisfies Eq. (7.21) with $|\phi\rangle$ as in (8.1). Using Eq. (8.6), we find that the solution to Eq. (7.21) with $|\phi\rangle$ as in (8.1) is given by

$$|\xi^{K^a}(x,z)\rangle = -z\sigma_{\nu-1} \int d^d y G_{\nu-1}(x-y,z) \bar{C}_\perp^a |\phi_{\rm sh}(y)\rangle. \qquad (8.14)$$

Using (8.1) and (8.14), we check that improved bulk $K_{\rm impr}^a$ transformations lead to K^a transformations of the spin-s anomalous shadow field given in (2.23) and (4.21).

Matching of effective action and boundary two-point vertex.—To find the effective action we follow the standard procedure. Namely, we plug a non-normalizable solution of the bulk equation of motion with the Dirichlet problem corresponding to the boundary anomalous shadow field (8.1) into the bulk action (6.6). We proceed as follows. Using gauge invariant equation of motion (6.25) in (6.6), we get the following expression for the effective action:

$$S_{\rm eff} = -\int d^d x \mathcal{L}_{\rm eff}|_{z\to 0}, \qquad (8.15)$$

$$\mathcal{L}_{\rm eff} = \tfrac{1}{2}\langle\phi|\boldsymbol{\mu}\mathcal{T}_{\nu-(1/2)}|\phi\rangle + \tfrac{1}{2}\langle\phi|Y\bar{C}|\phi\rangle,$$
$$Y \equiv \tfrac{1}{2}\alpha^2(r_\zeta\bar{\zeta} + r_z\bar{\alpha}^z) - \zeta r_\zeta - \alpha^z r_z. \qquad (8.16)$$

From (8.16), it is clear that the use of modified de Donder gauge condition (6.29) considerably simplifies the expression for $\mathcal{L}_{\rm eff}$,

$$\mathcal{L}_{\rm eff}|_{\bar{C}|\phi\rangle=0} = \tfrac{1}{2}\langle\phi|\boldsymbol{\mu}\mathcal{T}_{\nu-(1/2)}|\phi\rangle. \qquad (8.17)$$

Thus we see that it is the use of the modified de Donder gauge that leads to $\mathcal{L}_{\rm eff}$ for massive arbitrary spin AdS field (8.17) which has the same form as $\mathcal{L}_{\rm eff}$ for scalar field (6.48). Therefore, to find $S_{\rm eff}$ for the massive arbitrary spin AdS field we can use results for the scalar field. Namely, all that remains to obtain the effective action is to plug solution (8.1) into (8.15) and (8.17) and use the general formula given in (6.49). Doing so, we get[15]

$$-S_{\rm eff} = 2\kappa c_\kappa \Gamma, \tag{8.18}$$

where κ and c_κ are given in (3.4) and (6.44), respectively, while Γ stands for the gauge invariant two-point vertex of the spin-s anomalous shadow field given in (4.22) and (4.23).

To summarize, using the modified de Donder gauge for the computation of the spin-s massive AdS field action on the solution of equations of motion with the Dirichlet problem corresponding to the boundary anomalous shadow field, we obtain the gauge invariant two-point vertex of the spin-s anomalous shadow field. It is the matching of the bulk leftover on-shell gauge symmetries of solution to the Dirichlet problem and bulk global symmetries and the respective boundary gauge symmetries of the anomalous shadow field and boundary global symmetries that explains why the effective action of the AdS massive field coincides, up to a normalization factor, with the gauge invariant two-point vertex for the boundary anomalous shadow field.

In the literature, the effective action is expressed in terms of the two-point vertex taken in the Stueckelberg gauge frame $\Gamma^{\rm stand}$ (4.40). To express our result in terms of $\Gamma^{\rm stand}$, we use relations (4.36) and (4.39) to represent our result (8.18) as

$$-S_{\rm eff} = \frac{\kappa(2\kappa + 2s + d - 2)}{s!(2\kappa + d - 2)} c_\kappa \Gamma^{\rm stand}. \tag{8.19}$$

The following remarks are in order:

(i) For the particular values $s = 1$ and $s = 2$, our results for the normalization factor in front of $\Gamma^{\rm stand}$ (8.19) coincide with the ones obtained in Refs. [11,12], respectively. Thus, our results agree with the previously reported results for the particular values $s = 1$, 2 and give the normalization factor for arbitrary values of s.

(ii) Using (8.19) and taking into account the expression for $\Gamma^{\rm stand}$ given in (4.22) and (4.40), we see that the AdS/CFT correspondence for the massive spin-s AdS field leads to the two-point correlation function of the spin-s anomalous conformal current with the conformal dimension given by

$$\Delta = \frac{d}{2} + \sqrt{m^2 + \left(s + \frac{d-4}{2}\right)^2}. \tag{8.20}$$

According to the AdS/CFT correspondence this conformal dimension should be equal to lowest energy value E_0 for the massive spin-s AdS field with mass parameter m. Comparing Δ (8.20) with E_0 found in Ref. [40] [see Eq. (5.74) in Ref. [40]] we see that $E_0 = \Delta$.

(iii) The fact that the effective action of the massive AdS field is proportional to the two-point vertex of the anomalous shadow field is expected because of the conformal symmetry. Note however that for the systematical study of AdS/CFT correspondence it is important to know the normalization factor in front of $\Gamma^{\rm stand}$ (8.19).

(iv) In the massless limit, $m \to 0$, our result for $S_{\rm eff}$ (8.19) agrees with the previously reported results in literature. Computation of $S_{\rm eff}$ for spin-1 and spin-2 massless fields can be found in the respective Refs. [39,41]. Computation of $S_{\rm eff}$ for the arbitrary spin-s massless fields can be found in Ref. [4]. The study of AdS/CFT correspondence for massless fields in a light-cone gauge frame can be found in Refs. [42–44] (see also Ref. [45]).

(v) The effective action given in (8.18) is gauge invariant, while the effective action given in (8.19) is obtained from the one in (8.18) by using the Stueckelberg gauge frame. One of the advantages of our approach is that our approach gives the possibility to study the effective action by using other gauge conditions which might be preferable in various applications. For example, in the light-cone gauge frame, the effective action given in (8.18) takes the form

$$-S_{\rm eff} = 2\kappa c_\kappa \Gamma^{(\rm l.c.)}, \tag{8.21}$$

where light-cone gauge vertex $\Gamma^{(\rm l.c.)}$ is given in (4.22) and (4.47). It is Eq. (8.21) that seems to be interesting for the studying the duality of light-cone gauge type IIB Green-Schwarz AdS superstring and the corresponding CFT.

IX. CONCLUSIONS

In this paper, we extended our gauge invariant approach to CFT initiated in Ref. [3] to the studying of arbitrary spin anomalous conformal currents and shadow fields. We recall that, in the framework of string/gauge theory duality, the anomalous conformal currents and shadow fields are related to massive fields of AdS string theory. We note that all Lorentz covariant approaches to string field theory involve large amounts of Stueckelberg fields (see, e.g., Ref. [46]). Because our approach to anomalous conformal currents and shadow fields also involves Stueckelberg

[15] All that is needed for the derivation of (8.18) is to make the replacement $\sigma \to \sigma_\nu$ in the formula for scalar field (6.49) and note the easily derived algebraic relation $\nu c_\nu \sigma_\nu^2 = \kappa c_\kappa f_\nu$, where f_ν is defined in (4.24).

fields we believe that our approach will be helpful in understanding string/gauge theory duality better.

We obtained the gauge invariant vertex for the anomalous shadow field which, in the framework of AdS/CFT correspondence, is related to AdS field action evaluated on the solution of the Dirichlet problem. Our gauge invariant vertex provides quick and easy access to the light-cone gauge vertex. Because one expects that the quantization of AdS superstring is straightforward only in the light-cone gauge we believe that our light-cone gauge vertex will also be helpful in various studies of AdS/CFT duality. Our results have a number of the following interesting applications and generalizations.

(i) In this paper, we studied bosonic anomalous conformal currents and shadow fields. It would be interesting to extend our approach to the study of AdS/CFT correspondence for arbitrary spin fermionic anomalous conformal currents and shadow fields and related arbitrary spin massive fermionic fields [47].

(ii) In this paper, we studied the AdS/CFT correspondence by using a CFT adapted approach to massive AdS fields developed in Ref. [14]. In the last years, new interesting approaches to massive AdS fields were developed (see, e.g., Ref. [48]). It would be interesting to apply these new approaches to the study of AdS/CFT correspondence for massive AdS fields.

(iii) An extension of our approach to the case of 3-point and 4-point gauge invariant vertices of anomalous shadow fields will give us the possibility to study various applications of our approach along the lines of Ref. [49].

(iv) The idea of arranging d-dimensional conformal physics in $d+2$-dimensional multiplets was extensively studied in Ref. [50]. Obviously, the use of the methods developed in Ref. [50] will be very useful for studying the AdS/CFT correspondence.

(v) The Becchi-Rouet-Stora-Tyutin (BRST) approach is one of the powerful methods of modern quantum field theory (see, e.g., Ref. [51]). Obviously, an extension of the BRST approach to the case of anomalous conformal currents and shadow fields will provide new interesting possibilities for studying the CFT.

(vi) Mixed-symmetry fields have extensively been studied in the last years (see, e.g., Ref. [52]). Needless to say, that generalization of our approach to the mixed-symmetry conformal currents and shadow fields could be of some interest.

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APPENDIX: MATCHING OF BULK AND BOUNDARY CONFORMAL BOOST SYMMETRIES

In this Appendix, we demonstrate matching of the improved $K^a_{\rm impr}$ transformations of the normalizable modes of massive AdS field (7.17) and the K^a transformations of the boundary anomalous conformal current given in (2.19) and (2.23), with operator $R^a_{\rm cur}$ defined in (3.22). Matching of improved $K^a_{\rm impr}$ transformations of non-normalizable modes of the massive AdS field and boundary K^a transformations of anomalous shadow fields can be demonstrated in a quite similar way.

We start with the realization of the improved $K^a_{\rm impr}$ transformations on space of normalizable modes of the massive gauge-fixed AdS field given by [see (7.17)]

$$K^a_{\rm impr}|\phi_{\rm norm}\rangle = K^a_{\rm AdS}|\phi_{\rm norm}\rangle + G_{\rm AdS}|\xi^{K^a}_{\rm norm}\rangle, \qquad \text{(A1)}$$

where, in this Appendix, the normalizable solution $|\phi\rangle$ in (7.1) is denoted by $|\phi_{\rm norm}\rangle$, while the normalizable solution for the compensating gauge transformation parameter $|\xi^{K^a}\rangle$ in (7.23) is denoted by $|\xi^{K^a}_{\rm norm}\rangle$. Also, in this Appendix, the generic generator of K^a symmetries in (6.20) is denoted by $K^a_{\rm AdS}$, while the gauge transformation operator G in (6.16) is denoted by $G_{\rm AdS}$. Our purpose is to demonstrate that the improved $K^a_{\rm impr}$ transformations of the normalizable modes of the massive gauge-fixed AdS field become K^a transformations of the anomalous conformal current. Namely, we are going to prove the relation

$$K^a_{\rm impr}|\phi_{\rm norm}\rangle = U_\nu K^a_{\rm CFT}|\phi_{\rm cur}\rangle, \qquad \text{(A2)}$$

where $K^a_{\rm CFT}$ stands for the realization of the conformal boost generator on space of the anomalous conformal current given in (2.23) and (3.22).

In order to prove relation (A2) we represent the operator $K^a_{\rm AdS}$ (6.20) as

$$K^a_{\rm AdS} = K^a_{\Delta_{\rm AdS}} + R^a_{(1)} + M^{ab}x^b + R^a_{(0)}, \qquad \text{(A3)}$$

$$K^a_{\Delta_{\rm AdS}} \equiv -\tfrac{1}{2}x^2\partial^a + x^a D_{\rm AdS}, \qquad \text{(A4)}$$

where operators $D_{\rm AdS}$, $R^a_{(0)}$, and $R^a_{(1)}$ are given in (6.19), (6.23), and (6.24), respectively. Next, we note the relations

$$(K^a_{\Delta_{\rm AdS}} + R^a_{(1)})|\phi_{\rm norm}\rangle = U_\nu K^a_{\Delta_{\rm cur}}|\phi_{\rm cur}\rangle, \qquad \text{(A5)}$$

$$(M^{ab}x^b + R^a_{(0)})|\phi_{\rm norm}\rangle + G_{\rm AdS}|\xi^{K^a}_{\rm norm}\rangle$$
$$= U_\nu(M^{ab}x^b + R^a_{\rm cur})|\phi_{\rm cur}\rangle, \qquad \text{(A6)}$$

where

$$K^a_{\Delta_{\rm cur}} \equiv -\tfrac{1}{2}x^2\partial^a + x^a D_{\rm cur}, \qquad \text{(A7)}$$

$$D_{\rm cur} \equiv x\partial + \Delta_{\rm cur}, \qquad \text{(A8)}$$

while $\Delta_{\rm cur}$ and $R^a_{\rm cur}$ are given in (3.21) and (3.22), respectively. Using (A5) and (A6), we see that Eq. (A2) does indeed hold.

We now comment on the derivation of relations (A5) and (A6). These relations are obtained by using the following general formulas:

$$(K^a_{\Delta_{\rm AdS}} + R^a_{(1)})U_\nu = U_\nu(K^a_{\Delta_{\rm cur}} + x^a z\partial_z) - h_\kappa(-)^{N_z} q^{-\nu-(3/2)}\partial^a(\partial_q Z_\nu(qz))z\partial_z, \tag{A9}$$

$$(M^{ab}x^b + R^a_{(0)})U_\nu + G_{\rm AdS}(zU_{\nu+1}\bar{C}^a_\perp) \approx U_\nu(M^{ab}x^b + R^a_{\rm cur}), \tag{A10}$$

where $Z_\nu(z) \equiv \sqrt{z}J_\nu(z)$, while q is defined in (7.3). In (A10) and relation (A11), the sign $\approx$ implies that relations (A10) and (A11) are valid only on space of the anomalous conformal current $|\phi_{\rm cur}\rangle$. Recall that $|\phi_{\rm cur}\rangle$ is subject to differential constraint (3.9). We now see that, by applying relations (A9) and (A10) to the anomalous conformal current $|\phi_{\rm cur}\rangle$, we obtain the respective relations (A5) and (A6).

For the reader's convenience, we write down the helpful formulas to be used for the derivation of relation (A10),

$$M^{ab}x^bU_\nu \approx U_\nu M^{ab}x^b + zU_{\nu+1}(-G_{\rm cur}\bar{C}^a_\perp - e_{1,\rm cur}\bar{\alpha}^a + \tilde{C}^a\bar{e}_{1,\rm cur}), \tag{A11}$$

$$R^a_{(0)}U_\nu = zU_{\nu+1}(\alpha^z r_z\bar{\alpha}^a + \tilde{C}^a r_\zeta\bar{\zeta}) - zU_{\nu-1}(\zeta r_\zeta\bar{\alpha}^a + \tilde{C}^a r_z\bar{\alpha}^z), \tag{A12}$$

$$G_{\rm AdS}(zU_{\nu+1}\bar{C}^a_\perp) = zU_{\nu+1}G_{\rm cur}\bar{C}^a_\perp + U_\nu\Big(2\zeta r_\zeta - 2\alpha^2\frac{1}{2N_\alpha + d - 2}r_z\bar{\alpha}^z\Big)\bar{C}^a_\perp, \tag{A13}$$

$$zU_{\nu-1} + z\Box U_{\nu+1} = 2\nu U_\nu. \tag{A14}$$

In turn, relation (A11) is obtained by using the identity

$$M^{ab}\partial^b + G_{\rm cur}\bar{C}^a_\perp = \alpha^a\bar{C}_{\rm cur} - e_{1,\rm cur}\bar{\alpha}^a + \tilde{C}^a\bar{e}_{1,\rm cur} \tag{A15}$$

and differential constraint (3.9).

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