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Derivation of an Abelian effective model for instanton chains in 3D Yang-Mills theory

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In this work, we derive a recently proposed Abelian model to describe correlated monopoles, center vortices, and dual fields in three-dimensional $SU(2)$ Yang-Mills theory. Following recent polymer techniques, special care is taken to obtain the end-to-end probability for a single interacting center vortex, which constitutes a key ingredient to represent the ensemble integration.

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I. INTRODUCTION

Correlated monopoles and center vortices are believed to play a relevant role in accommodating the different properties of the confining string in Yang-Mills theories, receiving support from lattice simulations [1–6]. In fact, scenarios based on either monopoles or closed center vortices are only partially successful to describe the expected behavior of the potential between quarks (for a review, see Ref. [7]). At asymptotic distances, this potential should be linear, depend on the representation of the subgroup $Z(N)$ of $SU(N)$ (N-ality), and display stringy behavior (a $1/R$ Lüscher term). At intermediate scales, it should posses Casimir scaling.

Monopoles can be seen as defects that arise when implementing the Abelian projection [8]. The Cho-Faddeev-Niemi representation (CFN) can also be used to associate monopoles with defects of the local color frame used to decompose the gauge fields [9–13]. An important issue is how to deal with unphysical objects such as Dirac strings (or world sheets) when charged fields are present. This has been studied in Ref. [14], using the CFN representation of $SU(2)$ Yang-Mills theory. There, we showed how to decouple Dirac strings in the partition function of the theory, only leaving the effect of their borders where monopoles are placed.

In Ref. [15], the possible frame defects were extended to describe not only monopoles but also center vortices, correlated or not. In Ref. [16], this procedure has been related with the usual manner to introduce thin center vortices in the continuum, providing a natural way to discuss the stability of center vortices. In this framework, we also discussed the relationship between large dual transformations in three- and four-dimensional Yang-Mills theories and phases where Wilson surfaces can be either decoupled or become integration variables [17]. This is relevant for the problem of confinement, a phase where the surface whose border is the Wilson loop becomes observable.

In these scenarios, one of the difficulties is how to deal with the integration over an ensemble of extended objects, after considering a phenomenological parametrization of their properties, such as stiffness, interactions with dual fields, and interactions between them. This is particularly severe in four-dimensional theories where center vortices generate two-dimensional extended world surfaces. However, in three dimensions center vortices generate worldlines, so these ensembles are naturally associated with a second quantized field theory. For this reason, we were able to propose in Refs. [15,17], following heuristic arguments, an Abelian effective model to describe the large distance behavior of the three-dimensional (3D) $SU(2)$ Yang-Mills theory (for a non-Abelian version, see Ref. [18]). This model corresponds to a generalization of the t' Hooft model [19]; it includes a coupling with the dual field that can be defined in order to represent the off-diagonal sector. This coupling is essential to relate the possible vortex phases with enabled or disabled large dual transformations, and discuss in this framework the observability of Wilson surfaces [17].

The aim of this article is presenting a careful derivation of this effective model, after parametrizing some intrinsic physical properties that these objects could have. One of the fundamental ingredients will be the adoption of recent techniques borrowed from polymer physics [20], where the extended objects are also one dimensional. The polymer formulation of field theory in Euclidean spacetime [21] has also been used to study the magnetic component of the Yang-Mills plasma due to monopoles [22], which in four-dimensional spacetime are stringlike objects.

In this work, we present a detailed derivation of the equation for the end-to-end probability for a center vortex worldline, including the effect of interactions. This probability can be thought of as a Green's function that depends on the position and orientation at the worldline boundaries, where (monopolelike) instantons are placed. In the limit of semiflexible polymers, a reduced Green's function for a complex vortex field minimally coupled to the dual field is obtained. This constitutes a key ingredient to derive the above-mentioned effective model.

In Sec. II, we briefly review how to use the CFN decomposition to include vortices as defects of the local color frame. In Sec. III, we rewrite the ensemble for correlated monopoles and center vortices in terms of the weight for a single interacting vortex, while in Sec. IV we derive the associated Fokker-Planck equation. In Sec. V, we combine the previous results to obtain the generalized effective model. Finally, in Sec. VI we present our conclusions.

II. CORRELATED INSTANTONS AND CENTER VORTICES

The starting point is the $SU(2)$ Yang-Mills (YM) action defined in three-dimensional Euclidean spacetime,

$$S_{\rm YM} = \frac{1}{2}\int d^3x \,{\rm tr}(F_{\mu\nu}F_{\mu\nu}), \qquad F_{\mu\nu} = F^a_{\mu\nu}T^a. \quad (1)$$

The generators of $SU(2)$ can be expressed in terms of Pauli matrices $T^a = \tau^a/2$, $a = 1, 2, 3$, and the field strength in terms of the gauge fields A^a_μ,

$$\vec{F}_{\mu\nu} = \partial_\mu\vec{A}_\nu - \partial_\nu\vec{A}_\mu + g\vec{A}_\mu\times\vec{A}_\nu, \qquad \vec{A}_\mu = A^a_\mu\hat{e}_a,$$
$$\vec{F}_{\mu\nu} = F^a_{\mu\nu}\hat{e}_a, \quad (2)$$

where $\hat{e}_a$ is the canonical basis in color space, $a = 1, 2, 3$.

Following Ref. [15], in order to separate the perturbative sector from the sector of topological defects, we can use the CFN representation,

$$\vec{A}_\mu = A_\mu\hat{n} - \frac{1}{g}\hat{n}\times\partial_\mu\hat{n} + X^1_\mu\hat{n}_1 + X^2_\mu\hat{n}_2 \quad (3)$$

to parametrize not only monopoles but also (correlated or uncorrelated) center vortices as defects of the local color frame $\hat{n}_a$ ($\hat{n}_3 \equiv \hat{n}$). The frame can be given in terms of a mapping $S \in SU(2)$,

$$ST_aS^{-1} = \hat{n}_a\cdot\vec{T}. \quad (4)$$

Defining $C^a_\mu = -\frac{1}{2g}\epsilon^{abc}\hat{n}_b\cdot\partial_\mu\hat{n}_c$ and renaming,

$$A_\mu = \mathcal{A}^3_\mu - C^3_\mu, \qquad X^1_\mu = \mathcal{A}^1_\mu, \qquad X^2_\mu = \mathcal{A}^2_\mu, \quad (5)$$

the parametrization (3) can be also written in a more symmetrical form,

$$\vec{A}_\mu = \vec{A}_\mu(\vec{\mathcal{A}}, S) = (\mathcal{A}^a_\mu - C^a_\mu)\hat{n}_a, \quad (6)$$

where the vector field $\vec{\mathcal{A}}_\mu$ represents a perturbative sector. For a particular class of frame defects, as discussed in Ref. [16], this form coincides with the usual manner to introduce closed thin center vortices in the continuum [23]. The advantage of the ansatz (6) is that frame defects can also represent monopoles concatenated by pairs of center vortices to form chains [15].

When considering the CFN decomposition, or the symmetrical form (6), there is an overcounting of degrees of freedom to be fixed. In fact, as shown in Refs. [9–11], when the local color frame contains no defects, we can write

$$\vec{A}_\mu\cdot\vec{T} = S\vec{\mathcal{A}}_\mu\cdot\vec{T}S^{-1} + \frac{i}{g}S\partial_\mu S^{-1}, \quad (7)$$

$$\vec{\mathcal{A}}_\mu = \mathcal{A}^1_\mu\hat{e}_1 + \mathcal{A}^2_\mu\hat{e}_2 + \mathcal{A}^3_\mu\hat{e}_3. \quad (8)$$

Therefore, in this case the overcounting simply amounts to introducing additional gauge transformations, parametrized by a regular S, on top of the vector field $\vec{\mathcal{A}}_\mu$. However, it is important to underline that when describing center vortices in 3D, the mapping S is discontinuous on some surfaces where it jumps by a center element. As a consequence, the ansatz in (6) can no longer be written as a gauge transformation (see Refs. [15–18]).

To avoid overcounting, two types of fixings will be needed, and two types of BRST transformations should be introduced (for a discussion in terms of the CFN variables, see Ref. [24]). From the previous discussion it is clear that when considering a local frame containing defects, a proper gauge fixing will end up with the usual perturbative sector plus a topological sector associated with the frame defects. In this regard, it is important to underline that lattice results point to the idea that center vortices would be stabilized by generating a finite radius. In the continuum, a possible stabilization of the thin defects by generating a finite size could only occur as a quantum effect that cannot be accessed by means of a simple inspection of the Yang-Mills classical action, which contains no finite size classical vortex solutions. Some progress on this direction has been achieved in Ref. [16], where we showed how a natural definition of thick center objects tends to eliminate the well-known Savvidy-Nielsen-Olesen instability problem of magnetic backgrounds.

In this article, having in mind the previous discussion, instead of pursuing a nonperturbative definition of the integration measure for Yang-Mills theories, a difficult task due to well-known Gribov copies [25], we will adopt the procedure given in Ref. [15]. In that reference, an effective partition function has been considered (see Appendix),

$$Z_{\rm YM} = \int[\mathcal{D}\lambda][\mathcal{D}\Psi]\Delta_{\rm FP}[\vec{\mathcal{A}}]\delta[f(\vec{\mathcal{A}})]\times e^{-S_c-\int d^3x(1/2)\lambda_\mu\lambda_\mu}$$
$$\times e^{i\int d^3x[\lambda_\mu k_\mu + \mathcal{A}_\mu(\epsilon_{\mu\nu\rho}\partial_\nu\lambda_\rho - J_\mu)]}Z_{v,m}[\lambda], \quad (9)$$

where a nonperturbative sector of magnetic defects is parametrized in $Z_{v,m}[\lambda]$, while the remaining part of the path integral is the standard one. That is, S_c is the action for the off-diagonal gluons $\Phi_\mu = (\mathcal{A}^1_\mu + i\mathcal{A}^2_\mu)/\sqrt{2}$,

$$S_c = \int d^3x[\bar{\Lambda}^\mu\Lambda^\mu - i(\bar{\Lambda}^\mu\epsilon^{\mu\nu\rho}\partial_\nu\Phi_\rho + \Lambda^\mu\epsilon^{\mu\nu\rho}\partial_\nu\bar{\Phi}_\rho)], \quad (10)$$

linearized by means of an auxiliary field Λ_μ. We also have $k_\mu = \frac{g}{2i}\epsilon_{\mu\nu\rho}(\bar{\Phi}_\nu\Phi_\rho - \Phi_\nu\bar{\Phi}_\rho)$, while the off-diagonal current,

$$J^\mu = ig\epsilon^{\mu\nu\rho}\bar{\Lambda}_\nu\Phi_\rho - ig\epsilon^{\mu\nu\rho}\Lambda_\nu\bar{\Phi}_\rho \quad (11)$$

is minimally coupled to the diagonal gluon field $\mathcal{A}_\mu \equiv \mathcal{A}^3_\mu$. The measure $[\mathcal{D}\Psi]$ integrates over gluon and auxiliary fields. In addition, the Faddeev-Popov determinant implements the maximal Abelian gauge [see Eqs. (A8) and (A9)]. Note that in Eq. (9) we have the implicit constraint,

$$J^c_\mu = \epsilon_{\mu\nu\rho}\partial_\nu\lambda_\rho, \tag{12}$$

that is, the topologically conserved current associated with λ_μ describes the off-diagonal sector.

If the factor $Z_{v,m}$ were absent, $Z_{\rm YM}$ would correspond to the usual perturbative representation of the Yang-Mills partition function, where the Faddeev-Popov procedure is well defined. The model in Eq. (9) has been obtained by initially considering Yang-Mills field configurations $\vec{A}_\mu(\vec{\mathcal{A}}, S)$ containing frame defects on top of the perturbative sector $\vec{\mathcal{A}}_\mu$. By means of large gauge transformations, the frames can be deformed so that the direction $\hat{n}_3$ is the one associated with monopolelike singularities. The directions $\hat{n}_1$, $\hat{n}_2$ then describe the corresponding stringlike singularities (Dirac worldlines and center vortices). In this process, the dual field strength tensor gains a singular part concentrated on the frame defects,

$$\vec{\mathcal{F}}_\mu(A) = [\mathcal{F}^a_\mu(\mathcal{A}) - \mathcal{F}^a_\mu(C)]\hat{n}_a, \tag{13}$$

$$\mathcal{F}^a_\mu(C) = -d_\mu\delta^{a3}, \tag{14}$$

only along the Cartan subalgebra, and minimally coupled to λ_μ. Our choice of frame defects can be thought of as a fixing for the large gauge transformations. As an example, for a monopole/antimonopole pair correlated with center vortices, we have

$$\begin{aligned} d_\mu &= d^{(1)}_\mu + d^{(2)}_\mu, \\ d^{(k)}_\mu &= \frac{2\pi}{g}\int ds\frac{dx^{(k)}_\mu}{ds}\delta^{(3)}(x - x^\alpha(s)). \end{aligned} \tag{15}$$

Here, $x^{(k)}(s)$, $k = 1, 2$, is a pair of open center vortex worldlines with the same boundaries at x^+, x^-, where the monopole and antimonopole are located, so that it is verified,

$$\partial_\mu d^{(k)}_\mu = \frac{2\pi}{g}(\delta^{(3)}(x - x^+) - \delta^{(3)}(x - x^-)). \tag{16}$$

The possibility that magnetic defects could become relevant objects is represented by introducing a measure where the thin objects gain dimensional properties, leading to a nontrivial ensemble average. These considerations motivate the proposal,

$$Z_{v,m}[\lambda] = \int[\mathcal{D}m][\mathcal{D}v]e^{-S_d}e^{i\int d^3x\lambda_\mu d_\mu}, \tag{17}$$

where the action for the defects S_d is phenomenological in origin. It will be parametrized by taking the simplest properties observable thick center vortices could have. These can be divided into noninteracting and interacting parts according to

$$S_d = S^0_d + S^{\rm int}_d. \tag{18}$$

At large distances, for every vortex localized around a worldline $x(s)$ we will consider a term in S^0_d proportional to the vortex length and the first corrections due to curvature,

$$\int_0^L ds\bigg[\mu + \frac{1}{2\kappa}\dot{u}\cdot\dot{u}\bigg], \qquad u = \frac{dx}{ds}, \qquad u\cdot u = 1, \tag{19}$$

that is, a vortex tension and stiffness, respectively, Different phases will be associated with different parameter choices. In $S^{\rm int}_d$, we will parametrize scalar vortex-vortex interactions that could be relevant to stabilize the vortex matter. In the partition function (17), there is also an interaction with the dual vector field λ_μ, an effect that is already present when center vortices are thin. Each center vortex will contribute to $\int d^3x\lambda_\mu d_\mu$ with a term,

$$\frac{2\pi}{g}\int_0^L ds\frac{dx}{ds}\cdot\lambda. \tag{20}$$

In the next section, we present a detailed description of the measure $[\mathcal{D}m][\mathcal{D}v]$, as well as the action S_d, needed to integrate over the ensemble of monopole chains.

III. ENSEMBLE OF MONOPOLE CHAINS

To start handling the ensemble integration over defects, we write the partition function for the monopole chains in the form,

$$\begin{aligned} Z_{v,m} = \sum_n\int[\mathcal{D}m]_n[\mathcal{D}v]_n e^{-[S^0_d+S^{\rm int}_d]} \\ \times e^{i(2\pi/g)\sum_{k=1}^{2n}\int_0^{L_k}ds\dot{x}^{(k)}_\alpha\lambda_\alpha(x^{(k)})}, \end{aligned} \tag{21}$$

$$S^0_d = \sum_{k=1}^{2n}\int_0^{L_k}ds\bigg[\mu + \frac{1}{2\kappa}\dot{u}^{(k)}_\alpha\dot{u}^{(k)}_\alpha\bigg], \tag{22}$$

$$S^{\rm int}_d = \frac{1}{2}\sum_{k,k'}\int_0^{L_k}\int_0^{L'_k}dsds'V(x^{(k)}(s), x^{(k')}(s')). \tag{23}$$

The integer n denotes the number of monopole/antimonopole pairs. Center vortices are attached in pairs to the previous pointlike objects, so that for a given realization of defects, with a given n, the number of attached center vortex worldlines is $2n$. In the previous formula, these stringlike objects have been denoted by $x^{(k)}(s)$, $k = 1, \ldots, 2n$. For each center vortex, s denotes the associated arc length parameter running from 0 to L_k, the total length of the worldline. In terms of the tangent vector $u^{(k)}(s) = \dot{x}^{(k)}(s)$, the defining condition for this parameter is $u^{(k)}_\alpha u^{(k)}_\alpha = 1$, where α is summed over $\alpha = 1, 2, 3$ (no summation over k).

In Eqs. (22) and (23), we have phenomenological dimensional parameters. The first term in S^0_d describes tensile center vortices, the second one is associated with their stiffness, while $S_d^{\rm int}$ represents scalar interactions among them. Introducing the density,

$$\rho(x) = \sum_k \int_0^{L_k} ds \delta(x - x^{(k)}(s)), \tag{24}$$

the vortex-vortex interaction term contributes to the integrand of Eq. (21) with a factor,

$$e^{-(1/2)\int d^3x d^3x' \rho(x) V(x,x') \rho(x')} = \int [D\phi] e^{-S_\phi} e^{-\int d^3x \rho(x)\phi(x)}. \tag{25}$$

Here, we have introduced a representation in terms of an auxiliary field $\phi(x)$ whose associated action is given by

$$S_\phi = -\frac{1}{2}\int d^3x d^3x' \phi(x) V^{-1}(x,x')\phi(x'), \tag{26}$$

where

$$\int d^3y V^{-1}(x,y) V(y,z) = \delta(x-z). \tag{27}$$

In particular, $V(x-y) = (1/\zeta)\delta(x-y)$ corresponds to a contact interaction. Then, using Eq. (25), the partition function can be written as

$$Z_{v,m} = \int [\mathcal{D}\phi] e^{-S_\phi} \sum_n Z_n,$$
$$Z_n = \int [\mathcal{D}m]_n [\mathcal{D}v]_n \exp\Big[\sum_{k=1}^{2n} \int_0^{L_k} ds\Big(i\frac{2\pi}{g}\dot{x}_\alpha^{(k)} \lambda_\alpha(x^{(k)}) - \phi(x^{(k)})\Big) - S_d^0\Big]. \tag{28}$$

For a given n, the measure $[\mathcal{D}m]_n = \xi^{2n} d^3x_1 \dots d^3x_{2n}$ represents the integral over the positions of the $2n$ monopoles and antimonopoles. In order to match the dimensions of the different terms, the parameter ξ will have dimension of mass. As we will see in the next section, it permits counting in the continuum the number of open center vortices.

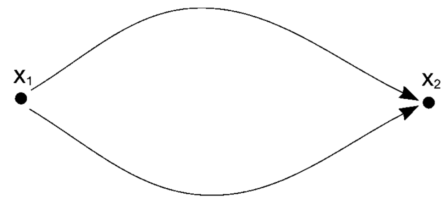

FIG. 1. Monopole/antimonopole correlated with a pair of center vortices.

For a given realization of positions of the monopolelike instantons, the $[\mathcal{D}v]_n$ integration measure includes the sum on the different inequivalent manners to join them with center vortices, with the associated symmetry factor. In addition, for each one of the $2n$ center vortices, this measure contains the path integral over all center vortex worldlines $[Dx^{(k)}(s)]$ with fixed extrema, and fixed length L_k, followed by the integral over the lengths $\int_0^\infty dL_k$.

Then, from Eq. (28), it becomes clear that all possible terms in the partition function depend on a fundamental building block, namely, the weight for a center vortex with fixed endpoints,

$$Q(x, x_0) = \int_0^\infty dL e^{-\mu L} q(x, x_0, L),$$
$$q(x, x_0, L) = \int [Dx(s)] \times e^{-\int_0^L ds[(1/2\kappa)\dot{u}_\alpha \dot{u}_\alpha + \phi(x(s)) - i(2\pi/g) u_\alpha(s)\lambda_\alpha(x(s))]}, \tag{29}$$

where $[Dx(s)]$ represents the integral over all possible paths $x(s)$ with fixed length L, and extrema at x_0 and x.

For a monopole/antimonopole pair (Fig. 1), we have the contribution:

$$Z_1 = \frac{1}{2!}\int d^3x_1 d^3x_2 \xi^2 [Q^2_{x_2,x_1} + Q^2_{x_1,x_2}]. \tag{30}$$

For two monopole/antimonopole pairs, we have six different manners to distribute the monopoles and antimonopoles at positions x_1, x_2, x_3 and x_4. These fixed boundaries

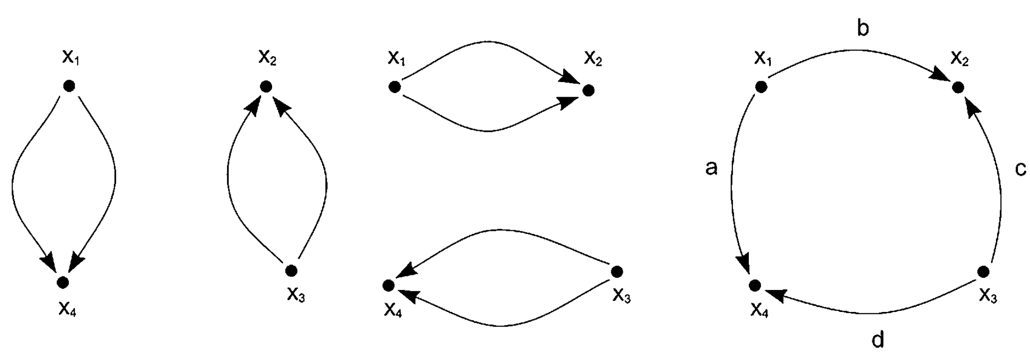

FIG. 2. Different manners to correlate two given monopole/antimonopole pairs with center vortices.

can be linked in three different forms: two disconnected and one connected (Fig. 2).

Note that for the connected configurations we have to consider some symmetry aspects. We can generate a new contribution by interchanging the vortices a, b, as well as the vortices c, d, that is, we have $2!.2! = 4$ manners to realize a given connected configuration. Then, for two pairs the contribution is

$$Z_2 = \frac{1}{4!}\int d^3x_1 d^3x_2 d^3x_3 d^3x_4 \xi^4 [Q^2_{x_4,x_1} Q^2_{x_2,x_3} + Q^2_{x_2,x_1} Q^2_{x_4,x_3} + 4Q_{x_2,x_1} Q_{x_4,x_1} Q_{x_2,x_3} Q_{x_4,x_3} + \text{permutations}]. \tag{31}$$

We can continue analyzing the different terms in the expansion to obtain all the terms from a functional generator as follows,

$$1 + Z_1 + Z_2 + \ldots = \Big\{1 + \int d^3x_1 I\Big(\frac{\delta}{\delta C(x_1)}\Big) + \frac{1}{2!}\int d^3x_1 d^3x_2 I\Big(\frac{\delta}{\delta C(x_1)}\Big) I\Big(\frac{\delta}{\delta C(x_2)}\Big) + \frac{1}{3!}\int d^3x_1 d^3x_2 d^3x_3 I\Big(\frac{\delta}{\delta C(x_1)}\Big) I\Big(\frac{\delta}{\delta C(x_2)}\Big) I\Big(\frac{\delta}{\delta C(x_3)}\Big) + \frac{1}{4!}\int d^3x_1 d^3x_2 d^3x_3 d^3x_4 I\Big(\frac{\delta}{\delta C(x_1)}\Big) I\Big(\frac{\delta}{\delta C(x_2)}\Big) I\Big(\frac{\delta}{\delta C(x_3)}\Big) I\Big(\frac{\delta}{\delta C(x_4)}\Big) + \ldots\Big\} \times e^{-\int d^3x d^3y \bar{K}(x) Q(x,y) K(y)}|_{C=0}. \tag{32}$$

Here, we have defined the operator,

$$I\Big(\frac{\delta}{\delta C(x)}\Big) = \xi\Big(\Big[\frac{\delta}{\delta \bar{K}(x)}\Big]^2 + \Big[\frac{\delta}{\delta K(x)}\Big]^2\Big), \tag{33}$$

where $C(x)$ represents the set of sources $K(x)$, $\bar{K}(x)$. This can be verified by performing the functional derivatives and evaluating at $K(x) = 0$, $\bar{K}(x) = 0$. In other words, we can write

$$Z_{v,m} = \int [\mathcal{D}\phi] e^{-S_\phi} e^{\int d^3x I(\delta/\delta C(x))} \times e^{-\int d^3x d^3y \bar{K}(x) Q(x,y) K(y)}|_{C=0}. \tag{34}$$

Then, it becomes clear that in order to obtain an effective vortex theory, it is essential to have a simple field representation for the Q-dependent factor, thus enabling the possibility of performing the path integral over ϕ.

IV. STATISTICAL WEIGHT FOR A SINGLE CENTER VORTEX

The discussion about how to represent the path integration over a stringlike object with stiffness is not simple even in the noninteracting case. It is usually done relying on the assumption that stiffness is equivalent to an effective monomer size in the random chain calculation, as it tends to locally straighten the chain, which is supported after cumbersome calculations of different momenta for the associated probability distributions [26]. For noninteracting random chains, the end-to-end probability is given by

$$q_N(x, x_0) = \prod_{n=1}^{N}\Big[\int (d^3\Delta x_n) \frac{1}{4\pi a^2} \delta(|\Delta x_n| - a)\Big] \times \delta\Big(x - x_0 - \sum_{n=1}^{N} \Delta x_n\Big),$$

which for large N behaves like

$$q_N(x, x_0) \approx \Big(\frac{3}{2\pi N a^2}\Big)^{3/2} \exp\Big[-\frac{3(x - x_0)^2}{2Na^2}\Big]. \tag{35}$$

Note that the continuum limit with $Na = L$ cannot be implemented here. However, by considering the above-mentioned effective monomer size $a \to a_{\text{eff}}$, and replacing $Na^2/3 \to L/\alpha$, $\alpha = 3/a_{\text{eff}}$, results

$$q(x, x_0, L) = \Big(\frac{\alpha}{2\pi L}\Big)^{3/2} e^{-(\alpha/2L)(x-x_0)^2} = \int \frac{d^3k}{(2\pi)^3} e^{-(L/2\alpha)k^2} e^{ik\cdot(x-x_0)}. \tag{36}$$

Then, integrating over the different lengths weighted by $e^{-\mu L}$, as is well known, $Q(x, x_0)$ turns out to be the Green's function for a free field theory,

$$Q(x, x_0) = 2\alpha \int \frac{d^3k}{(2\pi)^3} \frac{e^{ik\cdot(x-x_0)}}{k^2 + m^2}, \qquad m^2 = 2\alpha\mu, \tag{37}$$

$$(-\nabla^2 + m^2) Q(x, x_0) = 2\alpha\delta(x - x_0). \tag{38}$$

Now, we would like to present a careful extension of this property, as controlled as possible, to the case where scalar ϕ interactions and vector λ interactions are present, as is the case of our path integral over a single center vortex in Eq. (29). In this case, the momenta of the distribution for general external sources cannot be computed in a closed form, nor an explicit expression for the random chain integration is available. A manner to overcome this difficulty is noting that we are only interested in obtaining a field representation for $Q(x, x_0)$. Then, we can follow recent techniques [20] for semiflexible interacting polymers, adapted to the fixed extrema and variable length stringlike objects we have in $Q(x, x_0)$. The desired representation will be obtained from

$$Q(x, x_0) = \int_0^\infty dL e^{-\mu L} \int \frac{d^2u_0}{4\pi} \frac{d^2u}{4\pi} q(x, u, x_0, u_0, L), \tag{39}$$

where $q(x, u, x_0, u_0, L)$ is the correlator for center vortices with fixed length, positions and tangent vectors at the edges, where monopoles are placed (see Fig. 3). The differentials d^2u_0, d^2u integrate on the unit sphere S^2 and are normalized such that $\int d^2u_0 = \int d^2u = 4\pi$.

The dimensions of $Q(x, x_0)$ in Eq. (38), or its interacting version, is $[\text{length}]^{-2}$. Then, considering in Eqs. (30) and (31) a parameter ξ with mass dimensions, the quantity $\xi Q(x, x_0) d^3x$ becomes a dimensionless weight associated with open center vortices of any size L ending at a small volume d^3x around x, giving that the initial point is at x_0. This parameter plays a similar role to the fugacity parameter needed to describe the monopole plasma in compact QED(3), and derive a dual action exhibiting confinement [27]. In that case, the monopolelike instantons are associated with unobservable Dirac strings so that the ensemble integration is only concerned with the monopoles. In order to match the dimensions, this requires a fugacity parameter with dimension L^{-3} (a monopole density), that multiplied by d^3x leads to a dimensionless quantity counting the number of monopoles inside d^3x. In our model, monopoles and antimonopoles are not joined by unobservable Dirac strings but by pairs of center vortices, assumed to be observable and relevant objects. For this reason, our parameter ξ has dimensions L^{-1}, to be able to combine partial weights of the form $\xi Q d^3x$ and obtain the contribution Z_n originated from those chains containing n monopole/antimonopole pairs.

In order to generate a discretized version of $q(x, u, x_0, u_0, L)$, let us consider the recursive relation,

$$q_{j+1}(x, u, x_0, u_0) = e^{-\Delta L \omega(x,u)} \int d^3x' d^2u' \psi(u - u') \times \delta(x - x' - u\Delta L) q_j(x', u', x_0, u_0), \tag{40}$$

together with the initial condition,

$$q_0(x, u, x_0, u_0) = \delta(x - x_0)\delta(u - u_0). \tag{41}$$

Here, $\delta(u - u_0)$ is defined on S^2, and we have considered a general angular distribution $\psi(u - u')$ and interaction $\omega(x, u)$. Using $j = 0$, together with Eq. (41), we get

$$q_1(x, u, x_0, u_0) = e^{-\Delta L \omega(x,u)} \psi(u - u_0) \delta(x - x_0 - u\Delta L). \tag{42}$$

Continuing the iteration, it is easy to see that for $j = N - 1$, we will have

$$q_N(x, u, x_0, u_0) = \int d^3x_1 d^2u_1 \dots d^3x_{N-1} d^2u_{N-1} e^{-\Delta L[\omega(x_1,u_1)+\dots\omega(x_{N-1},u_{N-1})+\omega(x,u)]} \times \psi(u_1 - u_0) \dots \psi(u_{N-1} - u_{N-2}) \psi(u - u_{N-1}) \times \delta(x_1 - x_0 - u_1\Delta L) \dots \times \delta(x_{N-1} - x_{N-2} - u_{N-1}\Delta L) \delta(x - x_{N-1} - u\Delta L). \tag{43}$$

Defining $x = x_N$, $u = u_N$, we can rewrite Eq. (43) in a more compact form,

$$q_N(x, u, x_0, u_0) = \int d^3x_1 d^2u_1 \dots d^3x_{N-1} d^2u_{N-1} \times e^{-\Delta L \sum_{i=1}^{N} \omega(x_i,u_i)} \times \prod_{j=0}^{N-1} \psi(u_{j+1} - u_j) \times \delta(x_{j+1} - x_j - u_{j+1}\Delta L). \tag{44}$$

On the other hand, from Eqs. (29) and (39), the correlators $q(x, x_0, L)$ and $q(x, u, x_0, u_0, L)$ satisfy

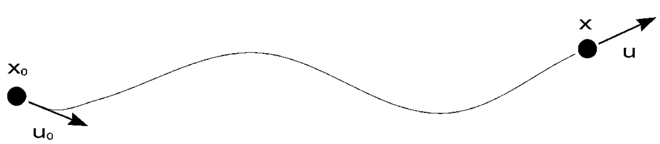

FIG. 3. Interacting center vortices with fixed length, and orientations at the endpoints, define the weight $q(x, u, x_0, u_0, L)$.

$$q(x, x_0, L) = \int \frac{d^2u_0}{4\pi} \frac{d^2u}{4\pi} q(x, u, x_0, u_0, L), \tag{45}$$

so that the latter can be written in the form

$$q(x, u, x_0, u_0, L) = \int [Dx(s)]_u e^{-\int_0^L ds[(1/2\kappa)\dot{u}_\alpha \dot{u}_\alpha + \phi(x(s)) - i(2\pi/g) u_\alpha(s) \lambda_\alpha(x(s))]}, \tag{46}$$

where $[Dx(s)]_u$ represents the integral over all possible paths $x(s)$ with fixed length L, fixed extrema $x(0) = x_0$, $x(L) = x$, and fixed initial and final tangent vectors, $\dot{x}(0) = u_0$, $\dot{x}(L) = u$.

Then, by using in Eq. (44)

$$\psi(u - u') = \mathcal{N} e^{-(1/2\kappa)\Delta L (u - u'/\Delta L)^2}, \tag{47}$$

$$\omega(x, u) = \left[\phi(x) - i\frac{2\pi}{g} u \cdot \lambda(x)\right], \tag{48}$$

a discretization of $q(x, u, x_0, u_0, L)$ by N "monomers" is clearly obtained. In particular, the factors $\psi(u_{j+1} - u_j)$

together with the constraints $\delta(x_{j+1} - x_j - u_{j+1}\Delta L)$, will reproduce the contribution of stiffness in Eq. (46).

In other words, we have $q(x, u, x_0, u_0, L) = \lim_{N\to\infty} q_N(x, u, x_0, u_0)$, with $L = N\Delta L$. In addition, from Eq. (40), we have

$$q_{N+1}(x, u, x_0, u_0) = e^{-\Delta L \omega(x,u)} \int d^3x' d^2u' \psi(u - u') \times q_N(x - u\Delta L, u', x_0, u_0), \quad (49)$$

so that for large N, after expanding both members in $\Delta L = L/N$, and using that $\psi(u - u')$ is localized, to expand $q_N(x - u\Delta L, u', x_0, u_0)$ around $u' \approx u$, the following diffusion equation is obtained (see Ref. [28]),

$$\begin{aligned}\partial_L q(x, u, x_0, u_0, L) &= \left[\frac{\kappa}{2}\nabla_u^2 - \omega(x, u) - u \cdot \nabla_x\right] \times q(x, u, x_0, u_0, L) \\ &= \left[\frac{\kappa}{2}\nabla_u^2 - \phi(x) - u \cdot D_x\right] \times q(x, u, x_0, u_0, L).\end{aligned} \quad (50)$$

Here, ∇_u^2 is the Laplacian on the unit sphere, $D_x = \nabla_x - i\frac{2\pi}{g}\lambda(x)$, and Eq. (41) implies that this equation has to be solved with the condition,

$$q(x, u, x_0, u_0, 0) = \delta(x - x_0)\delta(u - u_0). \quad (51)$$

In the process of obtaining $Q(x, x_0)$ from $q(x, u, x_0, u_0, L)$, the integrals in Eq. (39) can be organized as follows. We will initially integrate over d^2u_0 to obtain the reduced Green's function,

$$q(x, u, x_0, L) = \int \frac{d^2u_0}{4\pi} q(x, u, x_0, u_0, L), \quad (52)$$

which after integrating both members in Eqs. (50) and (51), satisfies

$$\partial_L q(x, u, x_0, L) = \left[\frac{\kappa}{2}\nabla_u^2 - \phi(x) - u \cdot D_x\right] q(x, u, x_0, L), \quad (53)$$

$$q(x, u, x_0, 0) = \delta(x - x_0). \quad (54)$$

Next, by integrating over the different lengths, we obtain

$$Q(x, u, x_0) = \int_0^\infty dL e^{-\mu L} q(x, u, x_0, L). \quad (55)$$

This function verifies

$$\begin{aligned}\left[\frac{\kappa}{2}\nabla_u^2 - \phi(x) - u \cdot D_x\right] Q(x, u, x_0) &= \int_0^\infty dL e^{-\mu L} \partial_L q(x, u, x_0, L) \\ &= \int_0^\infty dL \partial_L [e^{-\mu L} q(x, u, x_0, L)] \\ &\quad + \mu \int_0^\infty dL e^{-\mu L} q(x, u, x_0, L) \\ &= -q(x, u, x_0, 0) + \mu Q(x, u, x_0),\end{aligned} \quad (56)$$

that is,

$$\left[-\frac{\kappa}{2}\nabla_u^2 + \phi(x) + u \cdot D_x + \mu\right] Q(x, u, x_0) = \delta(x - x_0). \quad (57)$$

Finally, we can obtain $Q(x, x_0)$ from

$$Q(x, x_0) = \int \frac{d^2u}{4\pi} Q(x, u, x_0). \quad (58)$$

In other words, $Q(x, x_0)$ is given by the zeroth component $\mathcal{Q}_0$ of a u expansion of $Q(x, u, x_0)$ in terms of different angular momenta,

$$Q(x, u, x_0) = \sum_{l=0} \mathcal{Q}_l(x, u, x_0), \qquad Q(x, x_0) = \mathcal{Q}_0(x, x_0). \quad (59)$$

We can also use the expansion,

$$u \cdot D_x Q(x, u, x_0) = \sum_{l=0} u \cdot D_x \mathcal{Q}_l = \sum_{l=0} \mathcal{R}_l, \quad (60)$$

$$\begin{aligned}\mathcal{R}_0 &= [u \cdot D_x \mathcal{Q}_1]_0, \\ \mathcal{R}_1 &= [u \cdot D_x \mathcal{Q}_0 + u \cdot D_x \mathcal{Q}_2]_1, \\ \mathcal{R}_2 &= [u \cdot D_x \mathcal{Q}_1 + u \cdot D_x \mathcal{Q}_3]_2 \ldots\end{aligned} \quad (61)$$

to obtain

$$[\phi(x) + \mu]\mathcal{Q}_0 + \mathcal{R}_0 = \delta(x - x_0), \quad (62)$$

and for $l \neq 0$,

$$\frac{1}{f_l(x)} \mathcal{Q}_l + \mathcal{R}_l = 0, \qquad f_l(x) = \left[\phi(x) + \mu + \frac{l(l+1)\kappa}{2}\right]^{-1}. \quad (63)$$

Then, we have

$$\begin{aligned}\mathcal{R}_0 = [u \cdot D_x \mathcal{Q}_1]_0 &= -[u \cdot D_x (f_1 \mathcal{R}_1)]_0 \\ &= -[\mathcal{R}_1 u \cdot \nabla_x f_1 + f_1 u \cdot D_x \mathcal{R}_1]_0,\end{aligned} \quad (64)$$

and as in the second line of Eq. (61), $[u \cdot D_x \mathcal{Q}_0]_1 = u \cdot D_x \mathcal{Q}_0$, we obtain

$$\mathcal{R}_0 = -[(u \cdot D_x \mathcal{Q}_0)(u \cdot \nabla_x f_1) + f_1 (u \cdot D_x)^2 \mathcal{Q}_0]_0 - [[u \cdot D_x \mathcal{Q}_2]_1 (u \cdot \nabla_x f_1) + f_1 u \cdot D_x [u \cdot D_x \mathcal{Q}_2]_1]_0. \quad (65)$$

Now, if the $\mathcal{Q}_l$ components with momentum $l \geq 2$ are supposed to be small (semiflexible limit), we get

$$\mathcal{R}_0 \approx -[D_\alpha \mathcal{Q}_0 \partial_\beta f_1 + f_1 D_\alpha D_\beta \mathcal{Q}_0][u_\alpha u_\beta]_0, \quad (66)$$

and decomposing the tensor into a traceless symmetric ($l = 2$) and scalar ($l = 0$) part, $u_\alpha u_\beta = (u_\alpha u_\beta - \frac{1}{3}\delta_{\alpha\beta}) + \frac{1}{3}\delta_{\alpha\beta}$, we get

$$\mathcal{R}_0 \approx -\tfrac{1}{3}[\partial_\alpha f_1 D_\alpha \mathcal{Q}_0 + f_1 D^2 \mathcal{Q}_0], \quad (67)$$

and replacing in Eq. (62),

$$-\tfrac{1}{3}[\partial_\alpha f_1 D_\alpha \mathcal{Q}_0 + f_1 D^2 \mathcal{Q}_0] + [\phi(x) + \mu]\mathcal{Q}_0 = \delta(x - x_0), \quad (68)$$

$$f_1(x) = [\phi(x) + \mu + \kappa]^{-1}. \quad (69)$$

Therefore, for κ much larger than μ and the mass scales associated with ϕ, we finally obtain the approximated differential equation,

$$\left[-\frac{1}{3\kappa} D^2 + \phi(x) + \mu\right]\mathcal{Q}_0 = \delta(x - x_0). \quad (70)$$

V. EFFECTIVE FIELD THEORY

As a consequence of the calculations presented in the previous section, we see that the Q-dependent factor in the partition function $Z_{v,m}$ in Eq. (34) can be expressed in terms of a complex field v,

$$e^{-\int d^3x d^3y \bar{K}(x) Q(x,y) K(y)} = \det\hat{O} \int [\mathcal{D}v][\mathcal{D}\bar{v}] \times e^{-S_v - \int d^3x \sqrt{3\kappa}[\bar{K}v + \bar{v}K]}, \quad (71)$$

whose action is given by

$$S_v = \int d^3x \bar{v}\, \hat{O}\, v, \qquad \hat{O} = [-D^2 + \tilde{\phi}(x) + m^2], \quad (72)$$

$$\tilde{\phi}(x) = 3\kappa\phi(x), \qquad m^2 = 3\kappa\mu. \quad (73)$$

Therefore, we obtain

$$Z_{v,m} = \int [\mathcal{D}\phi] e^{-S_\phi} e^{\int d^3x I(\delta/\delta C(x))} \det\hat{O} \int [\mathcal{D}v] \times [\mathcal{D}\bar{v}] e^{-S_v - \int d^3x \sqrt{3\kappa}[\bar{K}v + \bar{v}K]}|_{C=0}$$
$$= \int [\mathcal{D}v][\mathcal{D}\bar{v}] \int [\mathcal{D}\phi] e^{-S_\phi} \times \det\hat{O} e^{-S_v - \int d^3x 3\kappa\xi[v^2 + \bar{v}^2]}, \quad (74)$$

$$S_\phi = -\frac{\zeta}{2} \int d^3x \phi^2(x). \quad (75)$$

Now, in order to obtain the effective theory, we still have to perform the functional integration over ϕ. However, the determinant is ϕ dependent, so that a closer look to this object is necessary. As usual, we can write $\det\hat{O} = e^{\mathrm{tr}\ln\hat{O}}$ and note that $\mathrm{tr}\ln\hat{O} = F[\tilde{\phi}, \lambda]$ is a functional that must be symmetric under the transformation $\lambda_\mu \to \lambda_\mu + \partial_\mu\omega$. As there is no parity symmetry breaking, F must depend on λ_μ through the combination $\epsilon_{\mu\nu\rho}\partial_\nu\lambda_\rho$. That is, we can write

$$F[\tilde{\phi}, \lambda] = F_\phi[\tilde{\phi}] + F_\lambda[\epsilon\partial\lambda] + F_{\mathrm{int}}[\tilde{\phi}, \epsilon\partial\lambda]. \quad (76)$$

In order to organize a derivative expansion, containing local terms, we can initially suppose $\mu, \kappa \neq 0$.

In the functional F_ϕ, it is important to underline that the auxiliary field $\phi(x)$ is related to the vortex density $\rho(x)$ through the exact relation,

$$\langle\phi(x)\rangle = \zeta\langle\rho(x)\rangle, \quad (77)$$

where ζ controls the strength of the vortex contact interaction introduced in Sec. III. Here, we have used the identity $\int \mathcal{D}\phi \frac{\delta}{\delta\phi(x)}[e^{-S_\phi}\sum_n Z_n] = 0$, and defined

$$\langle f(x)\rangle = \int \mathcal{D}\phi e^{-S_\phi} \sum_n \int [\mathcal{D}m]_n [\mathcal{D}v]_n \times f(x) \times e^{[\sum_k \int_0^{L_k} ds (i(2\pi/g)\dot{x}_\alpha^{(k)} \lambda_\alpha(x^{(k)}(s)) - \phi(x^{(k)}(s))) - S_d^0]}. \quad (78)$$

As the vortex density $\rho(x)$ in Eq. (24) is a sum of δ distributions, the field $\phi(x)$ would be expected to contain high-momenta Fourier modes which may invalidate the derivative expansion. However, lattice calculations have shown that center vortices posses a finite size [29], with a density scaling that ensures they survive the continuum limit [2]. If this information is included in the form of a smearing of the delta distributions in $\rho(x)$, when center vortices proliferate, a derivative expansion can then be justified. Before proceeding, we would like to discuss how this smearing could be originated. This is an important issue that would also serve as a basis for the formulation of random surface models for four-dimensional $SU(3)$ Yang-Mills theory in the continuum [30].

In this case, the consideration of monopoles and center vortices as relevant configurations depends on their stability. This can be studied by computing quantum fluctuations around them and analyzing if the effective action contains an imaginary part or not. In the affirmative case, the probability to stay in a state containing such an initial configuration would be smaller than 1, thus signaling instability. In Refs. [31–33], thick center vortex background fields have been analyzed to one loop showing they are unstable. This is an example of the well-known Savvidy-Nielsen-Olesen instability of magnetic backgrounds. On the other hand, we have shown in Ref. [16] that a natural definition of a thick center vortex object, as a diagonal deformation of the thin center vortex introduced in the continuum in Ref. [23], improves the situation regarding the instability problem in 3D or four-dimensional spacetimes with $SU(2)$ and $SU(3)$ gauge groups. For these objects, besides the usual coupling to a center vortex background with gyromagnetic factor 2, quantum fluctuations also couple to a frame defect with gyromagnetic factor 1, thus changing the analysis of bound states in the fluctuation operator.

The above-mentioned deformation corresponds to replacing the frame-dependent fields C^a_μ in Eq. (6) by more regular quantities. In this process, the field strengths $\mathcal{F}^a_\mu(C)$, concentrated on the frame defects, would be replaced by expressions localized on a finite radius, thus providing the necessary smearing of the delta distributions in the vortex density $\rho(x)$ to validate a derivative expansion of the functional $F_\phi[\tilde{\phi}] = \ln\det[-\nabla^2 + m^2 + \tilde{\phi}(x)]$.

This expansion will start with the effective potential term, containing no derivatives of $\tilde{\phi}$,

$$F_\phi[\tilde{\phi}] = -\int d^3x U_{\rm eff}(\tilde{\phi}) + \ldots, \tag{79}$$

$$\begin{aligned}-U_{\rm eff}(\tilde{\phi}) &= \int \frac{d^3k}{(2\pi)^3} \ln[(k^2 + m^2 + \tilde{\phi}(x)] \\ &= \int \frac{d^3k}{(2\pi)^3} \ln[k^2 + m^2] \\ &\quad + \int \frac{d^3k}{(2\pi)^3} \ln\left[1 + \frac{\tilde{\phi}}{k^2 + m^2}\right] \\ &= A + B\tilde{\phi} - \frac{\tilde{\phi}^2}{2} I_0 + \frac{\tilde{\phi}^3}{3} I_1 - \frac{\tilde{\phi}^4}{4} I_2 + \cdots,\end{aligned} \tag{80}$$

where A and $B = \int \frac{d^3k}{(2\pi)^3} \frac{1}{k^2+m^2}$ are divergent, and I_n, $n = 0, 1, \ldots$ are convergent and given by

$$\begin{aligned} I_n &= \int \frac{d^3k}{(2\pi)^3} \left[\frac{1}{k^2 + m^2}\right]^{n+2} \\ &= \frac{1}{|m|^{2n+1}} \int \frac{d^3u}{(2\pi)^3} \left[\frac{1}{1 + u^2/3}\right]^{n+2}. \end{aligned} \tag{81}$$

The functional $F_\lambda[\epsilon\partial\lambda] = \ln\det[-D^2 + m^2]$ depends on the combination $(1/g)\lambda_\mu$, with mass dimensions. Its dominant term is quadratic on this combination, that is, a Maxwell term of the form $\int d^3x \frac{1}{2|m|g^2} f_\mu f_\mu$, with $f_\mu = \epsilon_{\mu\nu\rho}\partial_\nu\lambda_\rho$. In addition, we note that in the effective potential for $\tilde{\phi}$, the quadratic part also behaves like $\sim 1/|m|$, while nonquadratic terms are accompanied by powers $1/|m|^{2n+1}$, $n \geq 1$. Any gauge and Lorentz invariant local term mixing $\tilde{\phi}$ and $(1/g)f_\mu$ will also contain these higher-order powers. For these reasons, if we work up to order $1/|m|$, the interaction term $F_{\rm int}$ can be disregarded, and we are led to evaluate a quadratic ϕ integral in Eq. (74).

After including a linear term in S_ϕ and renormalizing the ϕ sector so as to maintain the vortex density $\rho(x)$ fixed, [cf. Eq. (77)], the path integral over ϕ can be done by the replacement,

$$e^{-S_\phi + {\rm Tr}\ln\hat{O}} \to \exp\int d^3x\left[b\tilde{\phi} + \frac{a}{2}\tilde{\phi}^2 + \frac{1}{2|m|g^2} f^2\right], \tag{82}$$

where $a = \zeta/(3\kappa)^2 - I_0$, and we have maintained the dominant terms in a large m expansion. Completing the square, now we can perform the integral of the ϕ-dependent part in Eq. (74). Therefore, the final expression for the partition function of correlated monopoles and center vortices turns out to be

$$Z_{v,m} = \mathcal{N} \int [\mathcal{D}v][\mathcal{D}\bar{v}] e^{-\int d^3x\{\bar{v}[-D^2+m^2]v + 3\kappa\xi[v^2+\bar{v}^2] + (1/2a)(\bar{v}v-b)^2 + (1/2|m|g^2)f^2\}}. \tag{83}$$

The derivation of this partition function is the main result of our work.

Now, combining Eq. (83) with Eq. (9), we obtain the model proposed in Refs. [15,17], where the nonperturbative sector of correlated monopoles and center vortices are represented by an effective vortex field,

$$\begin{aligned} Z^{\rm eff}_{\rm YM} &= \int [\mathcal{D}\lambda][\mathcal{D}\Psi][\mathcal{D}v][\mathcal{D}\bar{v}] e^{-S_c} \\ &\quad \times e^{-\int d^3x\{(1/2)\lambda_\mu\lambda_\mu - i\lambda_\mu k_\mu + iA_\mu(J^c_\mu - \epsilon_{\mu\nu\rho}\partial_\nu\lambda_\rho) + \bar{v}[-D^2+m^2]v + 3\kappa\xi[v^2+\bar{v}^2] + (1/2a)(\bar{v}v-b)^2 + (1/2|m|g^2)f^2\}}. \end{aligned} \tag{84}$$

This result contains a number of dimensional parameters. For instance, $m^2 = 3\kappa\mu$ and $3\kappa\xi$ combine the effect of stiffness with the center vortex tension μ and with ξ, a parameter related to the presence of open center vortices attached in pairs to monopoles. Note that larger positive κ values mean that larger values of $\dot{u}$ may be realized [cf. Eq. (29)], that is, the chain tends to be more flexible ($1/\kappa$ measures typical radius of curvature). In addition, the parameter a contains information about the effective vortex-vortex interactions. It is also important to note that the $m^2\bar{v}v$ term could be rewritten by means of the redefinition $b \to b - 3\kappa\mu a$, up to a constant term in the action. Therefore, for positive a (needed to stabilize the vortex field), a case where κ and μ have opposite signs would tend to induce a vortex condensate. Considering the smearing proposed in [16], i.e., the diagonal deformation of a thin center vortex, a classical contribution to the action for the defects S_d in Eq. (21) could be computed. At large distances, this contribution will certainly contain a term proportional to the vortex size plus the effect of curvature. However, obtaining the parameters μ and κ corrected by the effect of quantum fluctuations would be a more difficult task. This investigation is in progress and the corresponding results will be presented elsewhere.

Finally, by keeping the relevant terms when performing the path integral over the $[\mathcal{D}\Psi]$ sector (see Ref. [34]), the model in Eq. (84) can be further reduced to

$$Z^{\rm eff} = \int [\mathcal{D}\lambda][\mathcal{D}v][\mathcal{D}\bar{v}] e^{-\int d^3x \{(1/2) f_\mu \hat{K} f_\mu + (\gamma/2)\lambda_\mu \lambda_\mu + \bar{v}[-D^2+m^2]v + 3\kappa\xi[v^2+\bar{v}^2] + (1/2a)(\bar{v}v-b)^2\}}. \tag{85}$$

Here, $\hat{K}$ is a differential operator that depends on the Laplacian ∂^2, and contains a Maxwell term, $\hat{K} = \frac{1}{|m|g^2} + \ldots$, originated from the determinant in $F_\lambda[\epsilon\partial\lambda]$. This is a fundamental ingredient for the discussion of Abelian dominance as due to a mass gap for the dual field λ_μ (see Ref. [15]).

The vortex sector in Eqs. (84) and (85) corresponds to a generalization of the 't Hooft model [19] where an additional coupling with the dual field λ_μ has naturally arisen from the calculation. On the other hand, if an ensemble of closed center vortices were considered, instead of correlated monopoles and center vortices, the $U(1)$-symmetry breaking term $v^2 + \bar{v}^2$ would be absent. This can be seen from the initial representation for the ensemble of defects in Eq. (28). In the case of closed center vortices (placed at $\mathcal{C}_k$), Z_n can be written in terms of the $U(1)$-gauge-invariant quantities $\oint_{\mathcal{C}_k} dx_\alpha \lambda_\alpha(x)$ (see also Refs. [15,17]). This would correspond to a generalization of Cornwall's model for closed vortices [35], now coupled with the vector field λ_μ. If this field acquires a large mass gap, we would make contact with the previous proposals.

The interesting point regarding (84) and (85) is that it allows relating the different vortex phases with enabled or disabled large dual transformations [17], leading to decoupling of the Wilson surfaces or turning them variables to be integrated together with the other fields, respectively.

VI. CONCLUSIONS

In this article, we have considered three-dimensional $SU(2)$ Yang-Mills theory, and followed polymer techniques to derive a field representation of the partition function for stringlike center vortices with monopolelike instantons at their borders. For this aim, we have assumed some phenomenological properties such as a vortex stiffness and vortex-vortex interactions. In addition, vortices naturally interact with the vector field λ_μ that can be defined in Yang-Mills theories, and that can be thought of as a dual field describing the off-diagonal charged sector.

In $SU(2)$, center vortices and monopoles carry magnetic charge $2\pi/g$ and $4\pi/g$, respectively, so that configurations in the ensemble are formed by pairs of vortices attached to monopoles and antimonopoles. Initially, we have been able to write the ensemble integration in terms of a building block $Q(x, x_0)$, the weight to be ascribed to the path integral over a center vortex with fixed endpoints and variable length. Then, the obtention of the effective theory becomes subject to the possibility of representing $Q(x, x_0)$ as a vortex field correlator. In the noninteracting case, the field representation of the end-to-end probability for a single stiff polymer is originated from the knowledge of the momenta for this distribution, that permits to associate it with a random chain with an effective monomer size. In the interacting case, we had to adopt more recent techniques developed to study wormlike chains in terms of a Fokker-Plank equation, describing a diffusion $q(x, u, x_0, u_0, L)$ not only in x space (the final end point), but also in u space (the final orientation). After integrating over the lengths, initial and final orientations, we obtained an equation for $Q(x, x_0)$, that can be approximated by disregarding components with angular momenta $l \geq 2$ in the u expansion of $q(x, u, x_0, u_0, L)$. In Ref. [36], a similar approximation has been implemented for the noninteracting string with stiffness, after associating it with the evolution of a "rigid body" in the tangent space. This can be justified for semiflexible vortices, as for long chains the probability distribution for the final orientation is expected to be nearly isotropic.

As a result of the approximation, the weight $Q(x, x_0)$ turns out to be the Green's function for a Klein-Gordon-type operator $\hat{O}$ where the usual derivative is replaced by a covariant one, that contains the dual vector field λ_μ. Finally, by representing this Green's function by means of a complex vortex field, and analyzing the dominant terms originated from the functional determinant $\det\hat{O}$, we were able to perform the ϕ integration, thus obtaining in a controlled manner a recently proposed effective Abelian model [15,17] for three-dimensional $SU(2)$ Yang-Mills theory. In this model, the coupling with the dual vector field is essential to relate the possible phases of the vortex sector with enabled or disabled large dual transformations, thus permitting the decoupling, or not, of the Wilson surface appearing in the Petrov-Diakonov representation of the Wilson loop [17]. This formalism could be extended to accommodate new symmetries such as isospin, and to obtain effective field theories for more complex systems containing extended objects.

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APPENDIX A

It is well known, due to the presence of Gribov copies in covariant gauges [25], a precise definition of the path-integral measure for Yang-Mills theories in the nonperturbative regime is a difficult task. A proposal to eliminate infinitesimal copies has been implemented in [37] showing that, in the infrared regime, the path integral is dominated by configurations near the Gribov horizon. In addition, it has been shown that topological magnetic objects proliferate in that region [38–40]. With this motivation in mind, in this article we follow an heuristic procedure where nonperturbative physics is initially parametrized in terms of defects of a local color frame, describing correlated monopolelike instantons and center vortices.

We can start with the more symmetrical form in Eq. (6), and define the partition function,

$$Z_{\rm YM} = \int [\mathcal{D}\vec{\mathcal{A}}][\mathcal{D}S] e^{-S_{\rm YM}[A]}. \tag{A1}$$

The field strength tensor for $\vec{A} = \vec{A}[\vec{\mathcal{A}}, S]$ differs from that associated with the vector field $\vec{\mathcal{A}}_\mu$ by a term localized on the frame defects [16,18]. As a consequence, the Yang-Mills action can be written as

$$\begin{aligned} S_{\rm YM}[A] &= \int d^3x \frac{1}{4} \vec{F}^2_{\mu\nu}[A] \\ &= \int d^3x \frac{1}{4} [F^a_{\mu\nu}(\vec{\mathcal{A}}) - F^a_{\mu\nu}(\vec{C})]^2, \end{aligned} \tag{A2}$$

where $F^a_{\mu\nu}(\vec{\mathcal{A}})$ is the field strength tensor for $\mathcal{A}^a_\mu$.

A regular gauge transformation $\vec{A}^U_\mu$ amounts to changing the frame $\hat{n}_a$ defined by S in Eq. (4), to a frame $\hat{n}'_a$ defined by US, leaving $\vec{\mathcal{A}}_\mu$ fixed. But, as the frames are local, besides the usual overcounting due to gauge symmetry, we also have infinite manners to write one and the same $\vec{A}_\mu$,

$$\vec{A}_\mu(\vec{\mathcal{A}}, S) = \vec{A}_\mu(\vec{\mathcal{A}}^{\tilde{U}}, S\tilde{U}^{-1}). \tag{A3}$$

In the perturbative sector $\vec{\mathcal{A}}_\mu$, it is natural to use the Faddeev-Popov procedure, introducing an identity in the form

$$1 = \Delta_{\rm FP}[\vec{\mathcal{A}}] \int [\mathcal{D}\tilde{U}] \delta[f(\vec{\mathcal{A}}^{\tilde{U}})]. \tag{A4}$$

As usual, we have $\Delta_{\rm FP}[\vec{\mathcal{A}}] = \Delta_{\rm FP}[\vec{\mathcal{A}}^{\tilde{U}}]$, so we can write

$$\begin{aligned} Z_{\rm YM} = \int [\mathcal{D}\tilde{U}] \int [\mathcal{D}\vec{\mathcal{A}}] \\ \times [\mathcal{D}S] \delta[f(\vec{\mathcal{A}}^{\tilde{U}})] \Delta_{\rm FP}[\vec{\mathcal{A}}^{\tilde{U}}] e^{-S_{\rm YM}[A]}. \end{aligned} \tag{A5}$$

The Yang-Mills action depends on the fields $\vec{A}_\mu(\vec{\mathcal{A}}, S)$, and using the property (A3), they can be rewritten as $\vec{A}_\mu(\vec{\mathcal{A}}^{\tilde{U}}, S\tilde{U}^{-1})$. Then, performing the change $\vec{\mathcal{A}}^{\tilde{U}} \to \vec{\mathcal{A}}$, $S\tilde{U}^{-1} \to S$, after factoring the group volumes we get

$$Z_{\rm YM} = \int [\mathcal{D}\vec{\mathcal{A}}][\mathcal{D}S] \delta[f(\vec{\mathcal{A}})] \Delta_{\rm FP}[\vec{\mathcal{A}}] e^{-S_{\rm YM}[A]}. \tag{A6}$$

It is important to stress that the gauge has not been fixed yet. All we have done is fixing the redundancy originated from the many different manners to write one and the same vector field $\vec{A}_\mu$. This will lead to a well-defined integration over the vector field $\vec{\mathcal{A}}_\mu$. All the points of the gauge orbits are still present, S and US give the same contribution because of the gauge invariance of $S_{\rm YM}$. If all the mappings S, defining frames containing defects, were replaced by just a regular sector, then in that case the ansatz $\vec{A}_\mu$ would simply correspond to a regular gauge transformation of $\vec{\mathcal{A}}_\mu$: $\vec{A}_\mu = \vec{\mathcal{A}}^S_\mu$. Then, in Eq. (A6), we would have $S_{\rm YM}[\vec{A}] = S_{\rm YM}[\vec{\mathcal{A}}]$, the group volumes $\int[\mathcal{D}S]$ could be factored out, and the representation would correspond to the usual perturbative definition of the path integral. In general, because of gauge invariance, for a given sector where S describes a frame with a given distribution of defects, the group volumes could be also factored out. After implementing a gauge-fixing procedure to put in evidence the group volumes, we would be left with an ensemble integration over correlated monopoles and center vortices. It could also occur that the thin objects that were initially considered as frame defects in (A5) become thick objects due to the effect of quantum fluctuations, with a thickness given by the scale provided by $\Lambda_{\rm QCD}$. This situation is in fact observed in lattice simulations where a thickness of the order of 1 fm has been observed. Of course, determining the correct ensemble in Yang-Mills theories is the difficult part of the problem of confinement, this is outside the scope of this article.

Here, we will assume possible ensembles parametrized by the inclusion of an action S_d localized on the frame defects, containing possible relevant terms and associated dimensional parameters. For thick objects, this would correspond to an effective manner to deal with the center vortex thickness at large distances. That is, introducing a color-valued auxiliary field λ^a_μ to deal with a first-order version of the Yang-Mills action in Eqs. (A2) and (A5), we propose the effective partition function,

$$Z_{\rm YM} = \int [\mathcal{D}\mathcal{A}][\mathcal{D}\lambda][\mathcal{D}m][\mathcal{D}v] e^{-S_d} \delta[f(\vec{\mathcal{A}})]\Delta_{\rm FP}[\vec{\mathcal{A}}] \times e^{-\int d^3x [(1/2)\lambda^a_\mu \lambda^a_\mu - i\lambda^a_\mu [\mathcal{F}^a_\mu(\mathcal{A}) - \mathcal{F}^a_\mu(C)]]},$$

$$\mathcal{F}^a_\mu = \frac{1}{2}\epsilon_{\mu\nu\rho} F^a_{\nu\rho} = \epsilon_{\mu\nu\rho}\partial_\nu \mathcal{A}^a_\rho + \frac{g}{2}\epsilon_{\mu\nu\rho}\epsilon^{abc}\mathcal{A}^b_\mu \mathcal{A}^c_\nu. \tag{A7}$$

If in this representation we consider an ensemble where the defects are such that $\mathcal{F}^a_\mu(C)$ points along the third direction in color space, $\mathcal{F}^a_\mu(C) = -\delta^{a3} d_\mu$, then defining $\lambda^3_\mu \equiv \lambda_\mu$, $\Lambda_\mu \equiv (\lambda^1_\mu + i\lambda^2_\mu)/\sqrt{2}$, we obtain Eqs. (9) and (17). For any thick center vortex localized around a worldline $x(s)$, the first relevant terms in S_d are expected to be associated with the vortex length, μL, and curvature, $\int_0^L ds \frac{1}{\kappa}\ddot{x}\cdot\ddot{x}$ (s is the arc length parameter). The dimensional constants μ and κ describe vortex tension and stiffness, respectively. Larger values of κ correspond to more flexible chains. As seen in Sec. V, vortex-vortex scalar contact interactions in S_d could also be relevant to stabilize vortex matter.

Here, we have considered a general condition to fix the perturbative sector. To make contact with Ref. [15], the conditions,

$$\partial_\mu \mathcal{A}_\mu = 0, \qquad \mathcal{D}_\mu \Phi_\mu = 0, \qquad \bar{\mathcal{D}}_\mu \bar{\Phi}_\mu = 0, \tag{A8}$$

$$\mathcal{D}_\mu = \partial_\mu + ig\mathcal{A}_\mu, \qquad \mathcal{A}_\mu \equiv \mathcal{A}^3_\mu, \tag{A9}$$

should be adopted, together with the usual representation of the Faddeev-Popov determinant in terms of ghost fields.

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