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Can Dirac fermions destroy Yang-Mills black holes?

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We study the four-dimensional Einstein-Yang-Mills black hole in the presence of a Dirac fermion field. Assuming a spherically symmetric, static, asymptotically flat black hole spacetime, we consider both massless and massive fermion fields. The $(4 + 1)$-dimensional Einstein-Yang-Mills system effectively reducing to the Einstein-Yang-Mills-Higgs-dilaton model was also taken into account. One finds that the fermion vacuum leads to the destruction of the black holes in question.

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I. INTRODUCTION AND NOTATION

Gravitational collapse is one of the most important issues of general relativity and its extension to higher dimensional spacetime connected with M/string theory schemes of unification of all known forces of nature. One expects that a newly born black hole emerging from the gravitational collapse of a massive star will settle down to the stationary axisymmetric or static spacetime. The uniqueness theorem (or classification) of nonsingular black hole solutions states that a stationary axisymmetric solution of Einstein-Maxwell (EM) electrovacuum equations is isometrically diffeomorphic to the domain of outer communication of Kerr-Newman spacetime [1].

On the other hand, the complete classification of n-dimensional charged black holes both with nondegenerate and degenerate components of the event horizon was given in Refs. [2]. Partial results for the very nontrivial case of the n-dimensional rotating black hole uniqueness theorem were provided in [3]. The aforementioned studies also comprise the case of extremal axisymmetric black holes, both in EM theory and the low-energy limit of the string theory (the so-called EMAD gravity) and supergravity theories [4]. As far as the uniqueness theorem of non-Abelian black holes is concerned, the situation is far more complicated (see [5] and references therein). It turns out that any static solution of Einstein-Yang-Mills (EYM) equations ought to either coincide with the Schwarzschild one or possess some nonvanishing Yang-Mills (YM) charges. But this was discovered not to be the case when static black hole solutions with vanishing charges were discovered [6]. They were asymptotically indistinguishable from Schwarzschild black holes. Moreover, in Ref. [7] it was shown that static black holes of the considered class with *magnetic* charge need not even be axially symmetric. In light of the above, one can remark that the non-Abelian black holes reveal considerably more composed structure compared to the EM ones.

Recently, studies of fermions in various backgrounds have attracted more attention. Exact solutions of the Dirac equation in curved spacetime may be a useful tool for investigations of physical properties of the considered spacetimes. Dirac fields were elaborated [8] in the context of the EYM background found in Ref. [9], in the near-horizon limit of the Kerr black hole [10], in Bertotti-Robinson spacetime [11], in spacetimes of black holes with nontrivial topology of the event horizon [12], in the vicinity of black holes with topological defects [13], and in the spacetime of black strings [14]. The intermediate and late-time decays of massive Dirac fermions in various black hole spacetimes were also elaborated [15–18].

Another tantalizing problem is the behavior of black holes and the surrounding matter fields. Depending on the matter model in question, black holes may allow the nontrivial fields to exist outside the event horizon. Are there any configurations of matter fields that can destroy the emerging black hole? This question was tackled in Refs. [19]. In Ref. [20] it was revealed that the Reissner-Nordström (RN) solution with both electric and magnetic charges can be destroyed in the presence of a massless Dirac fermion field. On the other hand, it was revealed [21] that the only black hole solutions of four-dimensional spinor Einstein-dilaton-Yang-Mills field equations of motion were those for which spinors vanished identically outside the black hole. This means that Dirac fermion fields either enter the black hole in question or escape to infinity. Recently, it was shown [22] that a matter configuration composed of a perfect fluid could not be at rest outside a four-dimensional black hole in asymptotically flat static spacetime.

In this paper we use the bosonization technique by which the fermionic degrees of freedom can be described by a scalar field. We shall elaborate the problem of the influence of Dirac fermion fields on YM black holes. In our considerations we assume that the black hole in question is spherically symmetric and static. We take into account the ordinary four-dimensional, static, spherically symmetric, asymptotically flat EYM black hole and the four-dimensional Einstein-Yang-Mills-Higgs-dilaton (EYMHd) system deduced from a five-dimensional EYM

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model, while the Dirac fermion field will be treated in an s-wave sector.

The organization of this paper is as follows. In Sec. II we briefly review the basic facts concerning four-dimensional EYM black holes and the Dirac fermion system. Then, we consider the massless as well as massive fermion cases and their influence on the black holes. Their asymptotical behaviors will also be discussed. Section III will be devoted to the backreaction process of Dirac fermions on YM fields. In Sec. IV we shall elaborate the $(4+1)$-EYM system which effectively reduces to EYMd field equations, where neither the matter fields nor the line element coefficients depend on the fifth dimension. We conclude our investigations in Sec. V.

II. FOUR-DIMENSIONAL EINSTEIN-YANG-MILLS BLACK HOLE AND DIRAC FERMIONS

In this section we shall focus on static spherically symmetric solutions of EYM field equations. In the case under consideration the line element can be provided by

$$ds^2 = -A^2(r)dt^2 + B^2(r)dr^2 + C^2(r)d\Omega^2, \quad (1)$$

where $d\Omega^2$ is a metric on the S^2-sphere. In what follows it will be convenient to introduce the tortoise coordinate defined as $dr_* = \frac{B}{A}dr$. Just the underlying metric yields

$$ds^2 = -A^2(r)dt^2 + A^2(r)dr_*^2 + C^2(r)d\Omega^2. \quad (2)$$

The main topic of our research will be the influence of Dirac fermion fields on the EYM black hole. Fermions under consideration will be described by the Dirac equation provided by

$$i\gamma^\mu D_\mu \psi - m\psi = 0, \quad (3)$$

where the covariant derivative D_μ implies

$$D_\mu = \nabla_\mu - i\lambda H_\mu. \quad (4)$$

λ is the gauge coupling constant, while the components of the Yang-Mills field have the forms

$$H_\mu = e^i_\mu H_i, \qquad H_i = a_i n^k \tau_k + \frac{1-K(r)}{2\lambda C}\epsilon_{ijk}n^j\tau^k, \quad (5)$$

where a_i is the *electric* and K the magnetic part of the Yang-Mills vector. n_a is the unit normal vector, while τ_a is a generator of the $SU(2)$ group. On the other hand, e^i_μ are basis one-forms defined by $g_{\mu\nu} = e^i_\mu e^j_\nu \eta_{ij}$, where η_{ij} is the metric tensor for Minkowski spacetime. The gamma Dirac matrices in a flat spacetime are defined by the relations

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad (6)$$

where σ^i are the usual Pauli matrices. It turns out that the Dirac operator takes the form

$$\gamma^\mu D_\mu = \begin{pmatrix} 0 & D^+ \\ D^- & 0 \end{pmatrix}, \quad (7)$$

where by $D^\pm$ we have denoted the following relation:

$$D^\pm = A^{-1}\partial_t - i\lambda\Big\{[\sigma^0 a_0 \pm \sigma^1 a_1]\bar{n}\cdot\bar{\tau} \pm \frac{K-1}{2\lambda C}\bar{n}\cdot\bar{\sigma}\times\bar{\tau}\Big\} + \pm\bar{\sigma}\cdot\bar{n}A^{-1}\partial_{r_*} \pm \bar{\sigma}\cdot\bar{n}\Big\{A^{-1}C^{-1}\partial_{r_*}C + \frac{1}{2}A^{-1}A^{-1}\partial_{r_*}A\Big\} \pm C^{-1}D_{S^2}. \quad (8)$$

Because of the fact that we restrict our attention to the s-wave sector, it can be spanned by two states χ_1 and $\chi_2 = \sigma^a n_a \chi_1$ which are the hedgehog spinor ansatz [23]. Moreover, they will obey the properties

$$\bar{\sigma}\cdot\bar{n}\chi_{1/2} = \chi_{2/1}, \qquad D_{S^2}\chi_{1/2} = \mp\chi_{2/1}, \quad (9)$$

$$(\bar{n}\cdot\bar{\sigma}\times\bar{\tau})\chi_{1/2} = \mp 2i\chi_{2/1}, \qquad \bar{n}(\bar{\sigma}+\bar{\tau})\chi = 0. \quad (10)$$

In terms of its components, the spinor ψ can be written as

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}, \qquad \psi_{L/R} = f_{L/R}(r_{r_*}, t)\chi_1 + g_{L/R}\chi_2. \quad (11)$$

By virtue of the above, the Lagrangian for two-component left- and right-handed spinors is

$$\mathcal{L}_F = i\bar{\psi}_R D^+\psi_R + i\bar{\Psi}_L D^-\psi_L - m\bar{\psi}_R\psi_L - m\bar{\psi}_L\psi_R. \quad (12)$$

On this account we can integrate over the angular degrees of freedom in the Dirac fermion action. From now on we will work with curved two-dimensional spacetime: $ds^2 = -A^2dt^2 + A^2dr_*^2$. Latin letters from the beginning of the alphabet will refer to the curved spacetime, i.e., $a = t, r_*$, while those from the end of the alphabet are bounded with the flat spacetime, i.e., $i = 0, 1$. In this spacetime the nonzero component of the spin connection is $\omega^{01} = \partial_{r_*}\ln A dt$, while the covariant derivative of the spinor field has the form $\nabla_a = \partial_a + \frac{1}{2}\omega^{ij}_a\sigma^i\sigma^j$. In our considerations, we introduce two-dimensional spinors $F_{L/R} = \begin{pmatrix} f_{L/R} \\ g_{L/R} \end{pmatrix}$, and we use the symmetric form of the Dirac operator $\gamma^\mu\overleftrightarrow{D}_\mu = \frac{1}{2}[\gamma^\mu\overrightarrow{D}_\mu - \gamma^\mu\overleftarrow{D}_\mu]$. It enables us to rewrite the action for fermion fields in the form

$$S_F = 4\pi\int dt\int A^2\mathcal{L}_F^2 dr_*, \quad (13)$$

as well as their Lagrangian, which implies the following:

$$\mathcal{L}_F^{(2)} = iC^2\bar{F}_R\vec{D}^+{}_{(t,r_*)}F_R - C^2\frac{K}{C}\bar{F}_R\sigma^2F_R - C^2\lambda\bar{F}_R(\sigma^ia_i)^+\sigma^1F_R + iC^2A^{-1}\partial_{r_*}\ln(\sqrt{A}C)\bar{F}_R\sigma^1F_R + iC^2\bar{F}_L\vec{D}^+{}_{(t,r_*)}F_L + C^2\frac{K}{C}\bar{F}_L\sigma^2F_L - C^2\lambda\bar{F}_L(\sigma^ia_i)^-\sigma^1F_L - iC^2A^{-1}\partial_{r_*}\ln(\sqrt{A}C)\bar{F}_L\sigma^1F_L - mC^2\bar{F}_RF_L - mC^2\bar{F}_LF_R, \tag{14}$$

where we have denoted $D^{\pm}_{(t,r_*)} = A^{-1}[\sigma^0\partial_t \pm \sigma^1\partial_{r_*}]$ and $(\sigma^ia_i)^{\pm} = \sigma^0a_0 \pm \sigma^1a_1$. We are now in a position to approach the question of the equations of motion for the above system. Namely, the Dirac equations for the two-dimensional fermions are provided by the relations

$$iD^+_{(t,r_*)}F_R - \frac{K}{C}\sigma^2F_R - (\lambda\sigma^ia_i)^+\sigma^1F_R + iA^{-1}\partial_{r_*} \times \ln(\sqrt{A}C)\sigma^1F_R + i\frac{1}{2AC^2}\partial_{r_*}C^2\sigma^1F_R - mF_L = 0, \tag{15}$$

$$iD^-_{(t,r_*)}F_L + \frac{K}{C}\sigma^2F_L - (\lambda\sigma^ia_i)^-\sigma^1F_L - iA^{-1}\partial_{r_*}\ln(\sqrt{A}C)\sigma^1F_L - i\frac{1}{2AC^2}\partial_{r_*}C^2\sigma^1F_L - mF_R = 0. \tag{16}$$

A. Massless Dirac fermions

In this subsection we shall focus our attention on massless right-handed spinors. We examine relation (15), which in two-dimensional spacetime (t, r_*) may be rewritten as follows:

$$i\sigma^a\nabla_aF_R - \lambda\sigma^aB_a\sigma^1F_R - V\sigma^2F_R + 2i\sigma^{r_*}\partial_{r_*}\ln CF_R = 0, \tag{17}$$

where we set $V = \frac{K}{C}$ with $B_a = e^i_a a_i$ being the electric part of the Yang-Mills field. It can be readily found, by the direct computation, that the term σ^ia_i will now be replaced by σ^aB_a. Further, in order to get rid of the $\partial_{r_*}\ln C$ factor, one can rescale spinors in the following way:

$$G_R \equiv i\sigma^3e^{2\int\partial_{r_*}\ln Cdr_*}F_R. \tag{18}$$

It will be useful to choose the new basis for flat gamma matrices,

$$\tilde{\gamma}^0 = -i\sigma^3, \qquad \tilde{\gamma}^1 = -\sigma^2, \tag{19}$$

$$\{\tilde{\gamma}^a, \tilde{\gamma}^b\} = 2\eta^{ab}, \qquad \eta_{00} = -1 = -\eta_{11}, \tag{20}$$

$$\tilde{\gamma}^3 = \tilde{\gamma}^0\tilde{\gamma}^1 = \sigma^1, \qquad \tilde{\gamma}_{L/R} = \tfrac{1}{2}(I \pm \tilde{\gamma}^3). \tag{21}$$

On this account equations of motion yield

$$i\tilde{\gamma}^a\nabla_aG_R + \lambda\tilde{\gamma}^aB_a\tilde{\gamma}^3G_R - V\tilde{\gamma}^3G_R = 0. \tag{22}$$

It can be deduced that they may be derived from the Lagrangian of the form

$$\mathcal{L}_{FR} = -i\bar{G}_R\tilde{\gamma}^a\nabla_aG_R - \lambda B_a\bar{G}_R\tilde{\gamma}^a\tilde{\gamma}^3G_R + V\bar{G}_R\tilde{\gamma}_LG_R - V\bar{G}_R\tilde{\gamma}_RG_R. \tag{23}$$

$\mathcal{L}_{FR}$ can be examined by means of the bosonization technique; i.e., the fermionic degrees of freedom can be described by a scalar field propagating in (t, r_*) spacetime (see, e.g., [24]). One bosonizes the above Lagrangian by the following formulas:

$$\begin{aligned} j^a_R &\equiv \bar{G}_R\tilde{\gamma}^aG_R = \frac{1}{\sqrt{\pi}}\varepsilon^{ab}\nabla_b\phi_R, \\ j^a_{3R} &\equiv \bar{G}_R\tilde{\gamma}^a\tilde{\gamma}^3G_R = \frac{1}{\sqrt{\pi}}\nabla^a\phi_R, \\ \bar{G}_R\tilde{\gamma}_LG_R &= be^{2i\sqrt{\pi}\phi_R}, \\ \bar{G}_R\tilde{\gamma}_RG_R &= b^*e^{-2i\sqrt{\pi}\phi_R}, \end{aligned} \tag{24}$$

where b and b^* are constants depending on the normalization of the current $\bar{G}\gamma_{L/R}G$. For the sake of brevity, we set $b = b^*$. Thus the bosonized Lagrangian is provided by

$$\mathcal{L}_{BR} = -\frac{1}{2}\nabla_a\phi_R\nabla^a\phi_R - \lambda B_a\frac{1}{\sqrt{\pi}}\nabla^a\phi_R + Vb(e^{2i\sqrt{\pi}\phi_R} - e^{-2i\sqrt{\pi}\phi_R}), \tag{25}$$

while the equation of motion for ϕ_R yields

$$\nabla_a\nabla^a\phi_R + \frac{\lambda}{\sqrt{\pi}}\nabla_aB^a + 4ib\sqrt{\pi}V\cos(2\sqrt{\pi}\phi_R) = 0. \tag{26}$$

Let us proceed to analyze the left-handed Dirac spinors. One can use the following substitution in order to get rid of the $\partial_{r_*}\ln C$ term,

$$G_L = i\sigma^3e^{2\int\partial_{r_*}\ln Cdr_*}F_L. \tag{27}$$

We also choose the gamma matrix basis in the form

$$\tilde{\gamma}^0 = -i\sigma^3, \quad \tilde{\gamma}^1 = +\sigma^2, \quad \tilde{\gamma}^3 = \tilde{\gamma}^0\tilde{\gamma}^1 = -\sigma^1. \tag{28}$$

Having all the above in mind, the equation of motion for G_L spinors implies

$$i\tilde{\gamma}^a\nabla_aG_L - \lambda B_a\tilde{\gamma}^a\tilde{\gamma}^3G_L - V\tilde{\gamma}^3G_L = 0. \tag{29}$$

On the other hand, the Lagrangian for G_L spinors can be written as

$$\mathcal{L}_{FL} = -i\bar{G}_L\tilde{\gamma}^a\nabla_aG_L + \lambda B_a\bar{G}_L\tilde{\gamma}^a\tilde{\gamma}^3G_L + V\bar{G}_L\tilde{\gamma}_LG_L - V\bar{G}_L\tilde{\gamma}_RG_L. \tag{30}$$

Replacing the fermionic degrees of freedom by the bosonization substitution given by the relations

$$
\begin{aligned}
j^a_L &\equiv \bar{G}_L \tilde{\gamma}^a G_L = \frac{1}{\sqrt{\pi}} \varepsilon^{ab} \nabla_b \phi_L, \\
j^a_{3L} &\equiv \bar{G}_L \tilde{\gamma}^a \tilde{\gamma}^3 G_L = \frac{1}{\sqrt{\pi}} \nabla^a \phi_L, \\
\bar{G}_L \tilde{\gamma}_L G_L &= b e^{2i\sqrt{\pi}\phi_L}, \\
\bar{G}_L \tilde{\gamma}_R G_L &= b^* e^{-2i\sqrt{\pi}\phi_L},
\end{aligned} \tag{31}
$$

we achieve the bosonized Lagrangian for the scalar fields,

$$
\mathcal{L}_{BL} = -\frac{1}{2} \nabla_a \phi_L \nabla^a \phi_L + \lambda B_a \frac{1}{\sqrt{\pi}} \nabla^a \phi_L + V b (e^{2i\sqrt{\pi}\phi_L} - e^{-2i\sqrt{\pi}\phi_L}). \tag{32}
$$

Accordingly, the equation of motion implies the following:

$$
\nabla_a \nabla^a \phi_L - \frac{\lambda}{\sqrt{\pi}} \nabla_a B^a + 4bi\sqrt{\pi} V \cos(2\sqrt{\pi}\phi_L) = 0. \tag{33}
$$

One can remark that the only difference between relations for ϕ_L and ϕ_R is the sign of the term containing B_a. However, in Ref. [25] it was pointed out that, in order to have the finite Yang-Mills black hole mass, one needs $B_a = 0$. By virtue of this argument we can readily verify that equations of motion for *right*- and *left-handed* fermion fields are identical. Therefore, in what follows, we restrict our attention to only one equation of motion.

Having in mind the arguments quoted above, we commence with the asymptotic behavior analysis of field ϕ_R. From the point of view of demanding asymptotical flatness of the black hole solution in question, one has that $g_{tt} \sim g_{rr} \sim 1$, $r \sim r_*$, and $C = r$, as the r coordinate tends to infinity. This enables us to write the underlying equation of motion in the form

$$
-\partial_t^2 \phi_R + \partial_{r_*}^2 \phi_R + 4ib\sqrt{\pi} \frac{K(\infty)}{r_*} \cos(2\sqrt{\pi}\phi_R) = 0, \tag{34}
$$

where we set $V \sim \frac{K(\infty)}{r_*}$, $K(\infty) = \pm 1$. In order to satisfy our demands about finiteness of ϕ_R as r_* tends to infinity, the last term in relation (34) vanishes. Thus, we arrive at the equation

$$
-\partial_t^2 \phi_R + \partial_{r_*}^2 \phi = 0, \tag{35}
$$

with the regular solution provided by

$$
\phi_R = d e^{-i\omega(t \pm r_*)}, \tag{36}
$$

where d is an arbitrary constant. The obtained solution is time dependent and admits the nonzero fermion current at infinity, i.e., $\nabla_j \phi_R \neq 0$.

Let us refine our study to the near-horizon geometry of a YM black hole surrounded by Dirac fermion fields. In the vicinity of the event horizon one achieves

$$
A^2 = N^2 e^{\delta(r_h)}, \qquad A^2 = N^2 e^{\delta(r_h)}, \qquad N^2 = 2\kappa(r - r_h), \tag{37}
$$

where κ is the surface gravity and $e^{\delta(r_h)}$ is a constant factor. Taking the usual change of variables $r - r_h = \rho^{-1}$, we obtain that

$$
r_* = -c\rho, \tag{38}
$$

where $c = \frac{1}{2\kappa e^{\sqrt{\delta(r_h)}}}$. Exploring these relations we can rewrite the equation of motion in the form

$$
-\partial_t^2 \phi_R + c^2 \partial_\rho^2 \phi_R + i \frac{8b\sqrt{\pi} K(r_h) \kappa e^{\sqrt{\delta(r_h)}}}{r_h \rho} \times \cos(2\sqrt{\pi}\phi_R) = 0. \tag{39}
$$

By the same reasoning as we followed in deriving Eq. (35), we conclude that, by dropping the last term in the above relation, the solution implies

$$
\phi_R = c_1 e^{-i\omega(t \pm (1/c^2)\rho)}, \tag{40}
$$

where we have set c_1 as an integration constant. Summing it all up, we conclude that the asymptotic analysis of scalar field equations being bosonized Dirac fermions leads to the time-dependent plane wave solutions. This contradicts the static nature of the considered YM black hole. On this account it is impossible to obtain a static spherically symmetric YM black hole solution surrounded by a Dirac fermion vacuum.

B. Massive Dirac fermions

In this subsection we are mainly concerned with the massive Dirac case. In what follows we take into account the following ansatze for F_L and F_R:

$$
\begin{aligned}
F_L &= i\sigma^3 F_R \equiv e^{-\int 2\partial_{r_*} \ln C dr_*} G, \\
F_R &= -i\sigma^3 e^{-\int 2\partial_{r_*} \ln C dr_*} G.
\end{aligned} \tag{41}
$$

Because of the fact that the left-handed fermions can be expressed in terms of a linear combination of the right-hand ones, we will use only the right-hand part of the original fermionic Lagrangian supplemented by the appropriate mass term. Namely, we obtain the equations of the form

$$
i\sigma^a \nabla_a F_R - \frac{K}{C} \sigma^2 F_R - \lambda \sigma^a B_a \sigma^1 F_R + 2iA^{-1} \partial_{r_*} \times \ln(C) \sigma^1 F_R - mi\sigma^3 F_R = 0. \tag{42}
$$

On this account it is customary to write the following relations for G-fermions:

$$
i\tilde{\gamma}^a \nabla_a G - V \tilde{\gamma}^3 G + \lambda \tilde{\gamma}^a B_a \tilde{\gamma}^3 G - mG = 0, \tag{43}
$$

where $\tilde{\gamma}$ are gamma matrices written in the basis which is chosen as in the massless right-handed case. We also have that $V = \frac{K}{C}$. Hence, the effective Lagrangian for G-fermions will be provided by the expression

$$\mathcal{L}_{GF} = -i\bar{G}\tilde{\gamma}^a\nabla_a G - \lambda B_a \bar{G}\tilde{\gamma}^a\tilde{\gamma}^3 G + (V+m)\bar{G}\tilde{\gamma}_L G + (m-V)\bar{G}\tilde{\gamma}_R G. \quad (44)$$

Next, as in the preceding sections, we try the bosonization scheme,

$$\begin{aligned} j^a &\equiv \bar{G}\tilde{\gamma}^a G = \frac{1}{\sqrt{\pi}}\varepsilon^{ab}\nabla_b\phi, \\ j_3^a &\equiv \bar{G}\tilde{\gamma}^a\tilde{\gamma}^3 G = \frac{1}{\sqrt{\pi}}\nabla^a\phi, \\ \bar{G}\tilde{\gamma}_L G &= be^{2i\sqrt{\pi}\phi}, \\ \bar{G}\tilde{\gamma}_R G &= b^* e^{-2i\sqrt{\pi}\phi}. \end{aligned} \quad (45)$$

When we set $b = b^*$, we are able to find the Lagrangian for the scalar field,

$$\mathcal{L}_{GB} = -\frac{1}{2}\nabla_a\phi\nabla^a\phi - \frac{\lambda}{\sqrt{\pi}}B_a\nabla^a\phi + (V+m)be^{2i\sqrt{\pi}\phi} + (m-V)be^{-2i\sqrt{\pi}\phi}, \quad (46)$$

where b is constant. On the other hand, the equation of motion for the ϕ field implies

$$\nabla_a\nabla^a\phi + \frac{\lambda}{\sqrt{\pi}}\nabla_a B^a + 2ib\sqrt{\pi}\{V[e^{2i\sqrt{\pi}\phi} + e^{-2i\sqrt{\pi}\phi}] + m[e^{2i\sqrt{\pi}\phi} - e^{-2i\sqrt{\pi}\phi}]\} = 0, \quad (47)$$

which can be rewritten in a more compact form,

$$\nabla_a\nabla^a\phi + \frac{\lambda}{\sqrt{\pi}}\nabla_a B^a + 4ib\sqrt{\pi}\{V\cos(2\sqrt{\pi}\phi) + im\sin(2\sqrt{\pi}\phi)\} = 0. \quad (48)$$

Applying the same analysis as in the preceding section leads us to the conclusion that in the near-horizon limit the bosonization field ϕ will be given in the form of a plane wave. Namely, it will be described by

$$\phi = d_1 e^{-i\omega(t\pm(1/c^2)\rho)}, \quad (49)$$

where d_1 is an integration constant while c is the same constant as in the massless case. On the other hand, in the limit when $r_* \to \infty$, the underlying equation of motion may be written as

$$-\partial_t^2\phi + \partial_{r_*}^2\phi - 4b\sqrt{\pi}m\sin(2\sqrt{\pi}\phi) = 0, \quad (50)$$

where due to the previously quoted arguments, we have omitted B_a. Because of the fact that $V \sim 1/r$, the term with the cosine function tends to zero. We attain just the form of the well-known sine-Gordon equation, the solution of which implies

$$\phi = \frac{2}{\sqrt{\pi}}\arctan\left(e^{-\sqrt{(8b\pi m)/(1-v^2)}(r_*-vt)}\right), \quad (51)$$

where v is an integration constant. From the above relation it can be inferred that the fermion current tends to zero as $r_* \to \infty$. To conclude this section, we remark that if we put $B_a = 0$, we obtain the time-dependent Dirac fermions as well as, through equations of motion, the time-dependent YM fields and the coefficients of the studied line element. All of the above contradicts our primary assumptions about the staticity of the system in question.

III. BACKREACTION ON YANG-MILLS FIELDS

If one considers a spherically symmetric spacetime being a Lorentz manifold on which the $SO(3)$ group acts like isometry in such a way that all group orbits are S^2-spheres, the spacetme in question may be locally written as a warped product of a two-dimensional Lorentz manifold and two-sphere with the standard metric on it [26–28]. It is convenient to rederive the field equations in the spherically symmetric case by varying the effective two-dimensional action. Putting in the EYM action the ansatz for the YM gauge fields and the line element describing the symmetry in question, one can obtain the two-dimensional Lagrangian, which yields

$$\mathcal{L}_{\rm YM} = -\frac{C^2}{4}f_{ab}f^{ab} - |dK|^2 - \frac{1}{2C^2}(|K|^2-1)^2, \quad (52)$$

where the covariant derivative and the strength of B_a are given by the relations

$$d = \nabla_a - iB_a, \qquad f_{ab} = \nabla_a B_b - \nabla_b B_a. \quad (53)$$

The subscript a denotes t, r_* coordinates. Equations of motion in the presence of the bosonized massless fermions are provided by

$$\nabla_a[C^2 f^{ab}] - 2|K|^2 B^b - \lambda j_{3R}^b + \lambda j_{3L}^b = 0, \quad (54)$$

$$\nabla_a\nabla^a K - \frac{2}{C^2}K(|K|^2-1) + 2KB_aB^a + \frac{b}{C}[e^{2i\sqrt{\pi}\phi_R} - e^{-2i\sqrt{\pi}\phi_R} + e^{2i\sqrt{\pi}\phi_L} - e^{-2i\sqrt{\pi}\phi_L}] = 0, \quad (55)$$

where $j_{3R}^a = \bar{G}_R\tilde{\gamma}^a\tilde{\gamma}^3 G_R$ and $j_{3L}^a = \bar{G}_L\tilde{\gamma}^a\tilde{\gamma}^3 G_L$. They can be reduced to the forms

$$\nabla_a[C^2 f^{ab}] - 2|K|^2 B^b - \frac{\lambda}{\sqrt{\pi}}[\nabla^b\phi_R - \nabla^b\phi_L] = 0, \quad (56)$$

$$\nabla_a\nabla^a K - \frac{2}{C^2}K(|K|^2-1) + 2KB_aB^a - i\frac{2b}{C}[\sin(2\sqrt{\pi}\phi_R) + \sin(2\sqrt{\pi}\phi_L)] = 0. \quad (57)$$

Having in mind Eqs. (26) and (33), in the case when $B_a = 0$, one can see that $j_{3R}^b = j_{3L}^b$. Thus, $B_a = 0$ is the trivial solution of (54). As far as the influence of fermions on YM fields is concerned, from relation (57) it can be inferred that the last term is not equal to zero, and this leads to the conclusion that fermions have influence on the magnetic part of YM fields. By virtue of the above, we observe that there is a nonzero fermion contribution to the magnetic part of the Yang-Mills field. Since ϕ_L and ϕ_R are time dependent, the magnetic part of YM fields, K, is also time dependent, which in turn means that the coefficients of the line element in question also depend on time. In the case when $B_a \neq 0$, from Eqs. (26) and (33), one can find fermion currents j^b_{3R}, j^b_{3L}. On the other hand, inspection of Eq. (54) reveals the fact that the fermion current causes the nontrivial solution for B_a. Consequently, the black hole in question has both magnetic and electric charges. One gets a dyonic black hole. But this in turn leads to the infiniteness of the mass of the considered object [25].

As was remarked, taking into account massive fermions, one has that $F_L = i\sigma^3 F_R$. This fact reduces the number of independent fields; i.e., instead of ϕ_L and ϕ_R we have only one, field ϕ, satisfying the relation

$$\nabla_a(C^2 f^{ab}) - 2|K|^2 B^b - \frac{\lambda}{\sqrt{\pi}} \nabla^b \phi = 0. \tag{58}$$

Hence, we get the black hole with both electric and magnetic charges (the dyon). On the other hand, this dyonic black hole cannot have finite mass [25].

The above analysis reveals the fact that the presence of the Dirac fermion field leads to the destruction of a static YM black hole solution. First of all, the scalar which we get in the bosonization process is time dependent. This in turn causes the magnetic part of the YM fields as well as the coefficients of the line element in question to depend also on time. This destroys our assumption about staticity of the considered black hole. These conclusions are true both for massless and massive Dirac fermions. Second, we can readily see that the presence of the Dirac fermion fields will provide the existence of the electric part of the YM fields, which in turn leads to the infiniteness of black hole mass.

IV. FIVE-DIMENSIONAL YANG-MILLS BLACK HOLE

Studies of spherically symmetric solutions in higher dimensional theories reveal two ways of research. One is connected with the assumption that we have spherically symmetric solutions in n dimensions (this attitude is important in high-energy problems), and the other is when one assumes that solutions are spherically symmetric only in a four-dimensional manifold. The second approach is importance from the point of view of the present Universe. These ideas were explored in Refs. [29], where $(4 + 1)$-dimensional EYM systems were elaborated. It turns out that the EYM system reduces to an effective four-dimensional EYMHd model. The $(4 + n)$-dimensional case was considered in Ref. [30], with the assumption that all of the n-dimensional fields are not dependent on the extra dimensions. The solutions were spherically symmetric in four dimensions, while the additional dimensions were bounded with a Ricci flat manifold.

Now, we shall proceed to elaborate $(4 + 1)$-dimensional EYM theory, where both the matter fields and the line element coefficients are not dependent on the fifth coordinate. Let us suppose that the five-dimensional metric and five-dimensional field are parametrized as follows:

$$\begin{aligned} {}^{(5)}ds^2 &= g_{MN} dx^N dx^M \\ &= e^{-\xi}[-A^2 dt^2 + B^2 dr^2 + C^2 d\Omega^2] + e^{2\xi}(dx^5)^2, \end{aligned} \tag{59}$$

$$H^a_M dx^M = H^a_\mu dx^\mu + \Phi^a dx^5, \tag{60}$$

where $M, N = t, r, \theta, \phi, x^5$, a is a group index, and H_M is the five-dimensional Yang-Mills field. On the other hand, H_μ denotes the four-dimensional Yang-Mills field components and ξ plays the role of the dilaton [29]. The above relations allow us to attain an effective four-dimensional EYMd theory for which the Lagrangian after compactification yields

$$\begin{aligned} \mathcal{L}_4 &= a_1 R - a_2 \nabla_\mu \xi \nabla^\mu \xi - \tfrac{1}{4} e^{\xi} F^a_{\mu\nu} F^{a\mu\nu} \\ &\quad - \tfrac{1}{2} e^{-2\xi} D_\mu \Phi^a D^\mu \Phi^a, \end{aligned} \tag{61}$$

where a_1 and a_2 are constants depending on the five-dimensional gravitational constant, while the covariant derivative of the Higgs field in the adjoint representation implies

$$D_\mu \Phi^a = \nabla_\mu \Phi^a + \varepsilon_{abc} H^b_\mu \Phi^c. \tag{62}$$

Let us assume further that $\Phi = \nu Y(r) \tau^i n_i$, where ν is the expectation value of the Higgs field. The matter Lagrangian written in (t, r_*) coordinates is given by

$$\begin{aligned} \mathcal{L}_{2-dim} &= -a_2 C^2 \nabla_a \xi \nabla^a \xi - \frac{C^2}{4} e^{\xi} f_{ab} f^{ab} - e^{\xi}(dK)^2 \\ &\quad - \frac{1}{2C^2} e^{\xi}(|K|^2 - 1)^2 - \frac{C^2}{2} e^{-2\xi} \nabla_a Y \nabla^a Y \\ &\quad - e^{-2\xi}|K|^2 Y^2. \end{aligned} \tag{63}$$

As far as the five-dimensional fermions are concerned, after dimensional reduction their action yields

$$S_{F_4} = \int \sqrt{-{}^{(4)}g}\, d^4x [i \bar{\psi} \gamma^\mu \overleftrightarrow{D}_\mu \psi + \lambda_2 \bar{\psi} \gamma^5 e^{-2\xi} \Phi_i \tau^i \psi], \tag{64}$$

where

$$\gamma^5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}.$$

One can observe that they gain mass by coupling to the five-dimensional component of the YM field Φ^i. Namely, it implies

$$-m[\bar{\psi}_R\psi_L+\bar{\psi}_L\psi_R]\to\lambda_2 e^{-2\xi}\bar{\psi}_R\Phi^i\tau^i\psi_L - \lambda_2 e^{-2\xi}\bar{\psi}_L\Phi^i\tau^i\psi_R, \quad (65)$$

where λ_2 is the fermion coupling constant and τ^i are $SU(2)$ generators. On the other hand, Φ^i resembles Higgs fields in a four-dimensional background. Massless fermions will be described by the same equations, and therefore we restrict our attention to the massive case. Further, we represent $\psi_{L/R}$ in terms of $\chi_{1/2}$,

$$\lambda_2 e^{-2\xi}\bar{\psi}_R\Phi^i\tau^i\psi_L = -C^2\lambda_2\nu Y e^{-2\xi}\bar{F}_R\sigma^1 F_L. \quad (66)$$

Expressing fermions as two-dimensional ones and integrating over the angles, we get the effective two-dimensional Lagrangian given by (14) with the following substitutions:

$$-m\bar{F}_R F_L \to -\lambda_2\nu e^{-2\xi}Y\bar{F}_R\sigma^1 F_L, \qquad -m\bar{F}_L F_R \to \lambda_2\nu e^{-2\xi}Y\bar{F}_L\sigma^1 F_R. \quad (67)$$

Bosonization of the Dirac fermion fields will take place as in the preceding sections. Namely, we introduce G fermions $F_L = i\sigma^3 F_R$ and $F_R = -i\sigma^3 e^{-\int 2\partial_{r_*}\ln(C)dr_*}G$, where the two-dimensional base of the gamma matrices is chosen as follows:

$$\tilde{\gamma}^0 = -i\sigma^3, \qquad \tilde{\gamma}^1 = -\sigma^2, \qquad \tilde{\gamma}^3 = \tilde{\gamma}^0\tilde{\gamma}^1. \quad (68)$$

This leads us to the equation of motion of the form

$$i\tilde{\gamma}^a\nabla_a G - V'\tilde{\gamma}^3 G + \lambda\tilde{\gamma}^a B_a\tilde{\gamma}^3 G = 0, \quad (69)$$

where we set $V' = \frac{K}{C} + \lambda_2\nu e^{-2\xi}Y$. They can be derived from the effective Lagrangian provided by

$$\mathcal{L}_{GF} = -i\bar{G}\tilde{\gamma}^a\nabla_a G - \lambda\bar{G}\tilde{\gamma}^a B_a\tilde{\gamma}^3 G + V'\bar{G}\tilde{\gamma}_L G - V'\bar{G}\tilde{\gamma}_R G. \quad (70)$$

It can be easily verified that using the following bosonization formulas,

$$j^a = \bar{G}\tilde{\gamma}^a G = \frac{1}{\sqrt{\pi}}\varepsilon^{ab}\nabla_b\phi, \quad j_3^a = \bar{G}\tilde{\gamma}^a\tilde{\gamma}^3 G = \frac{1}{\sqrt{\pi}}\nabla^a\phi, \quad \bar{G}\tilde{\gamma}_L G = be^{2i\sqrt{\pi}\phi}, \quad \bar{G}\tilde{\gamma}_R G = b^* e^{-2i\sqrt{\pi}\phi}, \quad (71)$$

and setting $b = b^*$, the bosonized Lagrangian becomes

$$\mathcal{L}_{GB} = -\frac{1}{2}\nabla_a\phi\nabla^a\phi - \frac{\lambda}{\sqrt{\pi}}B_a\nabla^a\phi + V'b[e^{2i\sqrt{\pi}\phi} - e^{-2i\sqrt{\pi}\phi}]. \quad (72)$$

Consequently, the equation of motion for the scalar field ϕ is given by

$$\nabla_a\nabla^a\phi + \frac{\lambda}{\sqrt{\pi}}\nabla_a B^a + 2ib\sqrt{\pi}V'[e^{2i\sqrt{\pi}\phi} + e^{-2i\sqrt{\pi}\phi}] = 0, \quad (73)$$

or in a more compact form, it implies

$$\nabla_a\nabla^a\phi + \frac{\lambda}{\sqrt{\pi}}\nabla_a B^a + 4ib\sqrt{\pi}V'\cos(2\sqrt{\pi}\phi) = 0. \quad (74)$$

A. Asymptotic analysis of equations of motions

Taking the same change of variable as in the massless case, namely, $r_* \sim -\rho$, and noticing that $A^2 \sim \rho^{-1}$, we see that in the near-horizon limit, ϕ is given by the plane wave solution. For the finiteness of the black hole mass, we assume that $B_a = 0$; then one arrives at

$$\phi = e_1 e^{-i\omega(t\pm(1/c^2)\rho)}, \quad (75)$$

where e_1 is an arbitrary constant while c^2 is the same as in the previously studied massless case. Thus, if we demand $B_a = 0$, the regular solution for ϕ must be time dependent.

Now we will look for a solution in the $r_* \to \infty$ limit. Namely, one has that

$$-\partial_t^2\phi + \partial_{r_*}^2\phi + 4ib\sqrt{\pi}m_0\cos(2\sqrt{\pi}\phi) = 0, \quad (76)$$

where $m_0 = \lambda_2\nu Y_0 e^{-2\xi_0}$, and Y_0 and ξ_0 are the asymptotic values of the Higgs and dilaton fields, respectively. One can rewrite ϕ as $\phi(r_*) = \phi_a(r_*) - \frac{1}{2\sqrt{\pi}}\frac{\pi}{2}$, which leads us to the following form of the equations of motion,

$$-\partial_t^2\phi_a + \partial_{r_*}^2\phi_a - 4ib\sqrt{\pi}m_0\sin(2\sqrt{\pi}\phi_a) = 0. \quad (77)$$

As we can see, this is the sine-Gordon equation with complex coefficients. Its solution yields

$$\phi_a = \frac{2}{\sqrt{\pi}}\arctan\left(e^{-\sqrt{(8i\pi bm_0)/(1-v^2)}(r_*-vt)}\right), \quad (78)$$

where ν is an integration constant.

Note that ϕ tends to the constant value $-\frac{\sqrt{\pi}}{4}$ as r_* goes to infinity. But ϕ itself is not a physical quantity. Physical quantities are fermion fields, and they are given by derivatives of ϕ. In conclusion, in both cases of massive fermions (normal and Higgs generated mass) fermionic currents decay at infinity. The analysis of the fermion backreaction on the Yang-Mills field goes along the same line as in the pure four-dimensional case. Conclusions are qualitatively the same. Namely, time-dependent fermion fields lead to the destruction of the static ansatz for the Yang-Mills black hole (massive and massless cases), or massive fermions lead to the appearance of the nonzero electric part of the YM field.

V. CONCLUSIONS

In our paper we have considered the influence of the Dirac fermion field on an EYM black hole. One takes into account two cases, i.e., the four-dimensional YM black hole [25] and the black hole in the five-dimensional generalization of YM theory. In the latter case the five-dimensional theory reduces to the four-dimensional EYMHd model. In both cases we elaborated $SU(2)$ YM theory and treated the Dirac fermion in the s-wave sector. Assuming a spherically symmetric, static, asymptotically flat black hole spacetime, we bosonized fermion fields and studied the equations of motion for the obtained scalar fields.

In a massless fermion sector we arrived at two scalar fields, ϕ_L and ϕ_R, corresponding, respectively, to the left- and right-handed Dirac fermions. The action governing the scalar fields differs only by the sign in the term connected with the electric part of the YM field. Because of the fact that the finite mass YM black hole configuration exists only when the electric part of the YM field is equal to zero, the resulting equations of motion for both scalar fields are identical. The analysis of $\phi_{L/R}$ fields in the near-horizon and near-infinity limits reveals the fact that they are given by plane wave solutions. This in turn leads to the time dependence of the magnetic part of the YM field as well as, through EYM field equations, to the time dependence of the considered line element coefficients.

The situation is slightly different in the massive fermion case. First of all, one should recognize two kinds of mass terms emerging in our considerations. Namely, the *ordinary* mass term $m\bar{\psi}\psi$ appears in four-dimensional spacetime, and the mass term connected with the Higgs field $\lambda\Phi\bar{\psi}\psi$ appears in the five-dimensional case, which reduces effectively to the four-dimensional EYMHd theory. Before bosonization we use the simplifying assumption that the right- and left-handed parts of the Dirac fermion field are connected through the transformation $F_R = i\sigma^3 F_L$. This allows us to express the Dirac fermion field by only one scalar field ϕ. The next step is to analyze the behavior of the ϕ field in the near-horizon and near-infinity limits. It turns out that in the near-horizon limit the solution is described by a plane wave. This conclusion is true for both aforementioned masses. On the other hand, as far as the near-infinity limit is concerned, equations of motion for the *ordinary* mass term $m\bar{\psi}\psi$ reduce to the sine-Gordon type of equation. This equation has a time-dependent decaying solution as one approaches the near-infinity limit (the so-called *antikink* solution). In the case of the *Higgs generated* mass term $\lambda\Phi\bar{\psi}\psi$, the equations in question can also be changed to the sine-Gordon type of equation but with a complex coefficient. It can be found that the solution reduces to the decaying oscillation function plus a nonzero constant term. Moreover, the presence of the Dirac fermion field currents will cause a nontrivial value of the electric part of the YM fields, which in turn leads to the infiniteness of the black hole mass.

Summing it all up, we remark that the presence of the Dirac fermion field (fermion vacuum) in the spacetime of a magnetically charged, spherically symmetric, static, asymptotically flat YM black hole will lead to the destruction of this black object. This will happen by destroying the static ansatz for the black hole with both massive and massless Dirac fermions. An asymptotical analysis of the behavior of the fermion fields suggests that the massless fermions are propagating in the whole spacetime, while massive Dirac fermions are confined to the near-horizon region. This conclusion remains true for the effectively reduced five-dimensional YM theory. In order to have a static YM black hole, one should have Dirac fermions which are constant in the domain of outer communication of the black hole in question, or Dirac fermion fields ought to enter the black hole (massive Dirac field) or escape to infinity (massless Dirac fermions). This is the same conclusion as conceived in Refs. [21,31].

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