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Linearized Weyl-Weyl correlator in a de Sitter breaking gauge

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We use a de Sitter breaking graviton propagator [14,15] to compute the tree-order correlator between noncoincident Weyl tensors on a locally de Sitter background. An explicit and very simple result is obtained for any spacetime dimension D, in terms of a de Sitter invariant length function and the tensor basis constructed from the metric and derivatives of this length function. Our answer does not agree with the one derived previously by Kouris [26], but that result must be incorrect because it is not transverse and lacks some of the algebraic symmetries of the Weyl tensor. Taking the coincidence limit of our result (with dimensional regularization) and contracting the indices gives the expectation value of the square of the Weyl tensor at lowest order. We propose the next order computation of this as a true test of de Sitter invariance in quantum gravity.

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I. INTRODUCTION

Students of quantum mechanics are familiar with the fact that charged particle wave functions couple to the electromagnetic vector potential, not to the field strength tensor. Hence the undifferentiated vector potential in a fixed gauge is, in some ways, observable. This point was crushingly demonstrated by the famous Aharonov-Bohm effect, in which a charged particle is made to interfere with itself in passing around a solenoid, despite the field strength being zero throughout the support of the particle's wave function [1].

Specialists in quantum field theory on curved space are engaged in a similar debate concerning inflationary gravitons. Matter fields couple to the metric, not to the curvature. There is no gauge in which this can be avoided. Hence one would think it obvious that the undifferentiated graviton field in a fixed gauge must be observable. Indeed, strenuous efforts [2–5] are under way to measure the tensor power spectrum, which is the expectation value of the conformally rescaled graviton field in transverse-traceless and synchronous gauge, taken long after the time t_k of the first horizon crossing,

$$\Delta_h^2(k) \equiv \frac{k^3}{2\pi^2} \lim_{t \gg t_k} \int d^3x e^{-i\vec{k}\cdot\vec{x}} \langle \Omega | h_{ij}^{tt}(t,\vec{x}) h_{ij}^{tt}(t,\vec{0}) | \Omega \rangle. \quad (1)$$

Mathematical physicists have for years disputed this conclusion because it conflicts with their belief in the de Sitter invariance of free gravitons on de Sitter background. (The de Sitter geometry is the most highly accelerated inflation consistent with classical stability.) The Bunch-Davies mode sum for the graviton propagator is formally de Sitter invariant, but infrared divergent. Regulating the infrared divergence breaks de Sitter invariance [6]. However, the infrared divergence is only logarithmic, so the derivatives needed to turn a graviton field into a linearized curvature render the mode sum for the linearized Weyl-Weyl correlator infrared finite and de Sitter invariant. Mathematical physicists therefore find it attractive to argue that the graviton propagator is unobservable—in spite of current efforts [2–5] to observe the tensor power spectrum (1)—and insist that only the correlator of two linearized Weyl tensors is physical. They sometimes even advance the de Sitter invariance of the Weyl-Weyl correlator as evidence that free gravitons are physically de Sitter invariant [7–9].

A digression is necessary at this stage to mention two recent insights which have dispelled decades of confusion:

(i) There is a topological obstacle that precludes adding invariant gauge fixing terms to the action on any manifold, such as de Sitter, which possesses a linearization instability [10]; and
(ii) It is incorrect to subtract off power law infrared divergences, which is what automatically happens with any analytic regularization technique, such as continuation from Euclidean de Sitter space [11].

The first point explains that there is no math error, but rather a subtle physics problem with gauge fixing in the many solutions which have been reported for the graviton propagator with a covariant gauge fixing term [12]. Attempting to ignore this problem produces provably wrong results in scalar quantum electrodynamics [13], and would do so as well in quantum gravity.

It is still possible to add noncovariant gauge fixing terms to the action, or to impose a covariant gauge exactly (as opposed to on the average with a gauge fixing term). The propagator was long ago worked out with a noncovariant gauge fixing term [14,15], and all quantum gravitational loop corrections on de Sitter have been made using this solution [16–20]. Enhancing the naive de Sitter transformation with the compensating gauge transformation needed to restore the noncovariant gauge condition reveals

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a physical breaking of de Sitter invariance [21]. The propagator has also recently been constructed in a covariant, exact gauge [22], and that solution shows explicit breaking of de Sitter invariance as well [23].

The second point of our digression explains the curious statement in the mathematical literature that exact covariant gauges are free of infrared problems except for certain discrete values of the gauge fixing parameters [24]. It has even been asserted that minimally coupled scalars with tachyonic masses are infrared finite except for the discrete values, $M^2 = -N(N+3)H^2$, where H is the Hubble parameter [7]. In fact, all tachyonic masses produce infrared divergences. The special thing about the discrete values is that one of the power law infrared divergences happens to become logarithmic for these values, and so is not automatically subtracted by the analytic regularization technique.[1]

We come now to the main point of this paper, which is to evaluate the linearized Weyl-Weyl correlator in the same noncovariant gauge [14,15] for which all existing quantum gravitational loop corrections on de Sitter background have been made [16–20]. We will demonstrate four things:

(i) That our result is both de Sitter invariant and very simple;
(ii) That the result obtained in 2001 by Kouris [26] cannot be correct because it possesses neither the algebraic symmetries of the Weyl tensor nor its transversality;
(iii) That the de Sitter invariance of our result is a trivial consequence of the derivatives needed to convert the graviton field into a linearized curvature and the disappearance of the constrained parts of the propagator from the linearized Weyl-Weyl correlator; and
(iv) That a true test of de Sitter invariance lies in evaluating the next loop order result for the coincident Weyl-Weyl correlator with its indices properly contracted.

Section II deals with the apparatus of perturbative quantum gravity on a D-dimensional de Sitter background so that dimensional regularization can be used. The actual computation is performed in Sec. III. We also discuss the discrepancy between the earlier result [26] and ours. In Sec. IV we explain what the Weyl-Weyl correlator tells one and what it does not. We also compare it to the expectation value of the stress tensor of a massless, minimally coupled scalar, both at the free level (which produces a de Sitter invariant result) and with a quartic self-interaction (which shows de Sitter breaking).

[1] Mathematical physicists occasionally ask what is wrong with the de Sitter invariant solutions one gets from subtracting off power law infrared divergences. The result is a solution to the propagator equation which is not a propagator in the sense of being the expectation value of the time-ordered product of two fields in the presence of any normalizable state. Such solutions abound, for example, $i/2$ times the sum of the advanced and retarded Green's functions [25].

II. QUANTUM FIELD THEORY ON DE SITTER

The purpose of this section is to describe the formalism for making perturbative quantum gravity computations on de Sitter background. We begin with the open conformal coordinate system which must be used if de Sitter is to fit into the larger context of inflationary cosmology. We then present the graviton propagator in our noncovariant gauge [14,15]. The section closes with a discussion of the tensor basis employed to express the linearized Weyl-Weyl correlator in a manifestly de Sitter invariant form.

A. Open conformal coordinates

We view de Sitter from the perspective of inflationary cosmology, as but a special case of the much larger class of homogeneous, isotropic, and spatially flat geometries. This means we do not want to work on the full de Sitter manifold but rather on the so-called "cosmological patch" which is spatially flat. It is convenient to use conformal coordinates $x^\mu = (\eta, \vec{x})$ with

$$-\infty < \eta < 0, \quad -\infty < x^i < +\infty \quad \text{for } i = 1, \dots, D-1. \tag{2}$$

As the name suggests, the metric in these coordinates is conformal to that of flat space,

$$ds^2 = a^2(-d\eta^2 + d\vec{x}\cdot d\vec{x}) \quad \text{where } a \equiv -\frac{1}{H\eta}. \tag{3}$$

The parameter H is known as the Hubble constant, and is related to the cosmological constant by $\Lambda = (D-1)H^2$. Although conformal coordinates do not cover the full de Sitter manifold, $\eta = \text{const}$ does represent a Cauchy surface, so information from the larger manifold can only enter the cosmological patch as initial value data.

The symmetry group of coordinate transformations which preserve the de Sitter metric plays a central role in our analysis. In open D-dimensional conformal coordinates the de Sitter group consists of $\frac{1}{2}D(D+1)$ transformations which can be arranged as follows in four parts:

(1) *Spatial translations*, which comprise $(D-1)$ transformations parametrized by a constant vector ϵ^i,

$$\eta' = \eta, \qquad x'^i = x^i + \epsilon^i. \tag{4}$$

(2) *Spatial rotations*, which comprise $\frac{1}{2}(D-1)(D-2)$ transformations parametrized by the rotation matrix R^{ij},

$$\eta' = \eta, \qquad x'^i = R^{ij}x^j. \tag{5}$$

(3) *Dilatations*, which comprise one transformation parametrized by a constant C,

$$\eta' = C\eta, \qquad x'^i = Cx^i. \tag{6}$$

(4) *Spatial special conformal transformations*, which comprise $(D-1)$ transformations parametrized by the constant vector θ^i,

$$\eta' = \frac{\eta}{1 - 2\vec{\theta}\cdot\vec{x} + \theta^2 x^\mu x_\mu}, \quad x'^i = \frac{x^i - \theta^i x^\mu x_\mu}{1 - 2\vec{\theta}\cdot\vec{x} + \theta^2 x^\mu x_\mu}. \tag{7}$$

The symmetries of cosmology are 1 and 2; symmetries 3 and 4 only appear in the de Sitter limit of maximal acceleration.

It is convenient to represent de Sitter invariant propagators between points x^μ and x'^μ using the de Sitter length function $y(x;x')$,

$$y(x;x') \equiv aa'H^2[\|\vec{x} - \vec{x}'\|^2 - (|\eta - \eta'| - i\varepsilon)^2]. \tag{8}$$

Except for the factor of $i\varepsilon$ (whose purpose is to enforce Feynman boundary conditions) the de Sitter length function can be expressed as follows in terms of the geodesic length $\ell(x;x')$ from x^μ to x'^μ:

$$y(x;x') = 4\sin^2(\tfrac{1}{2}H\ell(x;x')). \tag{9}$$

We should mention that mathematical physicists prefer a different de Sitter function $z = 1 - \frac{1}{4}y$, because it gives simpler formulas for propagators in terms of hypergeometric functions. The advantage of our length function $y(x;x')$ is that it vanishes at coincidence (that is, $x^\mu = x'^\mu$), which is quite important when renormalizing explicit loop computations.

B. The graviton propagator

We define the graviton field $h_{\mu\nu}(x)$ by conformally transforming the full metric $g_{\mu\nu}(x)$ and then subtracting off the background,

$$g_{\mu\nu}(x) \equiv a^2\tilde{g}_{\mu\nu} \equiv a^2(\eta_{\mu\nu} + \kappa h_{\mu\nu}(x)). \tag{10}$$

Here $\eta_{\mu\nu}$ is the D-dimensional, spacelike signature Minkowski metric, and $\kappa^2 \equiv 16\pi G$ is the loop counting parameter of quantum gravity. The gravitational Lagrangian is

$$\mathcal{L} \equiv \frac{1}{16\pi G}(R - (D-2)\Lambda)\sqrt{-g}. \tag{11}$$

Subtracting off a surface term and expanding in powers of the graviton field gives a form from which the perturbative interactions can be read off [14],

$$\begin{aligned}\mathcal{L} - \text{surface} = {}& \Big(\frac{D}{2} - 1\Big)Ha^{D-1}\sqrt{-\tilde{g}}\tilde{g}^{\rho\sigma}\tilde{g}^{\mu\nu}h_{\rho\sigma,\mu}h_{\nu 0} \\ &+ a^{D-2}\sqrt{-\tilde{g}}\tilde{g}^{\alpha\beta}\tilde{g}^{\rho\sigma}\tilde{g}^{\mu\nu}\Big\{\frac{1}{2}h_{\alpha\rho,\mu}h_{\nu\sigma,\beta} \\ &- \frac{1}{2}h_{\alpha\beta,\rho}h_{\sigma\mu,\nu} + \frac{1}{4}h_{\alpha\beta,\rho}h_{\mu\nu,\sigma} \\ &- \frac{1}{4}h_{\alpha\rho,\mu}h_{\beta\sigma,\nu}\Big\}.\end{aligned} \tag{12}$$

Note that $\tilde{g}^{\mu\nu}$ and $\sqrt{-\tilde{g}}$ are infinite order in the graviton field,

$$\tilde{g}^{\mu\nu} = \eta^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^{\mu\rho}h_\rho^{\nu} + O(\kappa^3), \tag{13}$$

$$\sqrt{-\tilde{g}} = 1 + \tfrac{1}{2}\kappa h + \tfrac{1}{8}\kappa^2 h^2 - \tfrac{1}{4}\kappa^2 h^{\mu\nu}h_{\mu\nu} + O(\kappa^3). \tag{14}$$

Note also that we follow the usual conventions whereby a comma denotes ordinary differentiation, $h \equiv \eta^{\mu\nu}h_{\mu\nu}$, and graviton indices are raised and lowered using the Minkowski metric, $h^\mu{}_\nu \equiv \eta^{\mu\rho}h_{\rho\nu}$ and $h^{\mu\nu} \equiv \eta^{\mu\rho}\eta^{\nu\sigma}h_{\rho\sigma}$.

The quadratic part of the invariant Lagrangian is

$$\begin{aligned}\mathcal{L}^{(2)}_{\rm inv} = \Big[&\frac{1}{2}h^{\rho\sigma,\mu}h_{\mu\sigma,\rho} - \frac{1}{2}h^{\mu\nu}{}_{,\mu}h_{,\nu} + \frac{1}{4}h^{,\mu}h_{,\mu} \\ &- \frac{1}{4}h^{\rho\sigma,\mu}h_{\rho\sigma,\mu} - \Big(\frac{D-2}{2}\Big)Hah^{0\mu}h_{,\mu}\Big]a^{D-2}.\end{aligned} \tag{15}$$

To this we add the noncovariant gauge fixing term

$$\begin{aligned}\mathcal{L}_{\rm GF} &= -\tfrac{1}{2}a^{D-2}\eta^{\mu\nu}F_\mu F_\nu, \\ F_\mu &\equiv \eta^{\rho\sigma}(h_{\mu\rho,\sigma} - \tfrac{1}{2}h_{\rho\sigma,\mu} + (D-2)Hah_{\mu\rho}\delta^0_\sigma).\end{aligned} \tag{16}$$

Note that it respects de Sitter symmetries 1–3, breaking only the spatial special conformal transformations. Because space and time are treated differently in our coordinate system and gauge, it is useful to have an expression for the purely spatial parts of the Lorentz metric and the Kronecker delta,

$$\bar{\eta}_{\mu\nu} \equiv \eta_{\mu\nu} + \delta^0_\mu\delta^0_\nu \quad \text{and} \quad \bar{\delta}^\mu_\nu \equiv \delta^\mu_\nu - \delta^\mu_0\delta^0_\nu. \tag{17}$$

The quadratic part of gauge fixed Lagrangian can be partially integrated to take the form $\frac{1}{2}h^{\mu\nu}D_{\mu\nu}{}^{\rho\sigma}h_{\rho\sigma}$, where the kinetic operator is

$$\begin{aligned}D_{\mu\nu}{}^{\rho\sigma} \equiv {}& \Big\{\frac{1}{2}\bar{\delta}_\mu{}^{(\rho}\bar{\delta}_\nu^{\sigma)} - \frac{1}{4}\eta_{\mu\nu}\eta^{\rho\sigma} - \frac{1}{2(D-3)}\delta^0_\mu\delta^0_\nu\delta^\rho_0\delta^\sigma_0\Big\}D_A \\ &+ \delta^0_{(\mu}\bar{\delta}^{(\rho}_{\nu)}\delta^{\sigma)}_0 D_B + \frac{1}{2}\Big(\frac{D-2}{D-3}\Big)\delta^0_\mu\delta^0_\nu\delta^\rho_0\delta^\sigma_0 D_C,\end{aligned} \tag{18}$$

and the three scalar differential operators are

$$D_A \equiv \partial_\mu(a^{D-2}\eta^{\mu\nu}\partial_\nu), \tag{19}$$

$$D_B \equiv \partial_\mu(a^{D-2}\eta^{\mu\nu}\partial_\nu) - (D-2)H^2a^D, \tag{20}$$

$$D_C \equiv \partial_\mu(a^{D-2}\eta^{\mu\nu}\partial_\nu) - 2(D-3)H^2a^D. \tag{21}$$

The graviton propagator in our gauge takes the form of a sum of constant index factors times scalar propagators [14,15],

$$i[_{\mu\nu}\Delta_{\rho\sigma}](x;x') = \sum_{I=A,B,C}[_{\mu\nu}T^I_{\rho\sigma}]i\Delta_I(x;x'). \tag{22}$$

The three scalar propagators invert the various scalar kinetic operators,

$$D_I \times i\Delta_I(x;x') = i\delta^D(x - x') \quad \text{for } I = A, B, C, \tag{23}$$

and we will give explicit expressions for them. The index factors are

$$[{}_{\mu\nu}T^{A}_{\rho\sigma}] = 2\bar{\eta}_{\mu(\rho}\bar{\eta}_{\sigma)\nu} - \frac{2}{D-3}\bar{\eta}_{\mu\nu}\bar{\eta}_{\rho\sigma}, \tag{24}$$

$$[{}_{\mu\nu}T^{B}_{\rho\sigma}] = -4\delta^{0}_{(\mu}\bar{\eta}_{\nu)(\rho}\delta^{0}_{\sigma)}, \tag{25}$$

$$[{}_{\mu\nu}T^{C}_{\rho\sigma}] = \frac{2}{(D-2)(D-3)}[(D-3)\delta^{0}_{\mu}\delta^{0}_{\nu} + \overline{\eta}_{\mu\nu}] \times [(D-3)\delta^{0}_{\rho}\delta^{0}_{\sigma} + \overline{\eta}_{\rho\sigma}]. \tag{26}$$

It is straightforward to verify that the graviton propagator (22) indeed inverts the gauge-fixed kinetic operator,

$$D_{\mu\nu}{}^{\rho\sigma} \times i[{}_{\rho\sigma}\Delta^{\alpha\beta}](x;x') = \delta^{(\alpha}_{\mu}\delta^{\beta)}_{\nu} i\delta^{D}(x-x'). \tag{27}$$

The A-type propagator obeys the same equation as that of a massless, minimally coupled scalar. It has long been known that no de Sitter invariant solution exists [27]. If one elects to break de Sitter invariance while preserving homogeneity and isotropy—this is known as the ''E(3)'' vacuum [28]—the solution takes the form [29]

$$i\Delta_{A}(x;x') = A(y(x;x')) + k\ln(aa'), \tag{28}$$

where the constant k is

$$k \equiv \frac{H^{D-2}}{(4\pi)^{(D/2)}}\frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})}. \tag{29}$$

The function $A(y)$ is

$$A(y) = \frac{H^{D-2}}{(4\pi)^{(D/2)}}\Big\{\Gamma\Big(\frac{D}{2}-1\Big)\Big(\frac{4}{y}\Big)^{(D/2)-1} + \frac{\Gamma(\frac{D}{2}+1)}{\frac{D}{2}-2}\Big(\frac{4}{y}\Big)^{(D/2)-2} + A_{1} - \sum_{n=1}^{\infty}\Big[\frac{\Gamma(n+\frac{D}{2}+1)}{(n-\frac{D}{2}+2)(n+1)!}\Big(\frac{y}{4}\Big)^{n-(D/2)+2} - \frac{\Gamma(n+D-1)}{n\Gamma(n+\frac{D}{2})}\Big(\frac{y}{4}\Big)^{n}\Big]\Big\}, \tag{30}$$

where the constant A_1 is

$$A_{1} = \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})}\Big\{-\psi\Big(1-\frac{D}{2}\Big) + \psi\Big(\frac{D-1}{2}\Big) + \psi(D-1) + \psi(1)\Big\}. \tag{31}$$

It should be noted that $A(y)$ obeys the differential equation

$$(4y-y^{2})A''(y) + D(2-y)A'(y) = (D-1)k. \tag{32}$$

The B-type and C-type propagators are both de Sitter invariant,

$$i\Delta_{B}(x;x') = B(y(x;x')), \quad i\Delta_{C}(x;x') = C(y(x;x')). \tag{33}$$

Rather than give the series expansion for $B(y)$ we present its relation to $A(y)$ [9],

$$B(y) = -\frac{[(4y-y^{2})A'(y) + k(2-y)]}{2(D-2)}. \tag{34}$$

For $C(y)$ it is more convenient to give the derivative [9],

$$C'(y) = A'(y) - \frac{1}{4}\Big(\frac{D-3}{D-2}\Big)[(4y-y^{2})A'(y) + k(2-y)]. \tag{35}$$

Of course our propagator breaks the 4th part of the de Sitter group (spatial special conformal transformations) because the gauge condition breaks it. However, the propagator also breaks the 3rd part of the de Sitter group (dilatations), which is preserved by the gauge condition. This is evident from the de Sitter breaking second term of the A-type propagator (28), which is needed to reproduce the famous result for the coincidence limit of the massless, minimally coupled scalar propagator [30],

$$\lim_{x\to x'} i\Delta_{A}(x;x') = \frac{H^{2}}{4\pi^{2}}\ln(a) + \text{divergent constant}. \tag{36}$$

The absence of dilatation invariance implies a physical breaking of de Sitter invariance by free gravitons. Kleppe proved this by concatenating a naive de Sitter transformation with the compensating gauge transformation needed to restore the gauge condition [21].

C. Tensor basis

Because $y(x;x')$ is de Sitter invariant, so too are covariant derivatives of it. With the metrics $g_{\mu\nu}(x)$ and $g_{\mu\nu}(x')$, the first three derivatives of $y(x;x')$ furnish a convenient basis of de Sitter invariant bi-tensors [13],

$$\frac{\partial y(x;x')}{\partial x^{\mu}} = Ha(y\delta^{0}_{\mu} + 2a'H\Delta x_{\mu}), \tag{37}$$

$$\frac{\partial y(x;x')}{\partial x'^{\nu}} = Ha'(y\delta^{0}_{\nu} - 2aH\Delta x_{\nu}), \tag{38}$$

$$\frac{\partial^{2} y(x;x')}{\partial x^{\mu}\partial x'^{\nu}} = H^{2}aa'(y\delta^{0}_{\mu}\delta^{0}_{\nu} + 2a'H\Delta x_{\mu}\delta^{0}_{\nu} - 2a\delta^{0}_{\mu}H\Delta x_{\nu} - 2\eta_{\mu\nu}). \tag{39}$$

Here and subsequently we define $\Delta x_{\mu} \equiv \eta_{\mu\nu}(x-x')^{\nu}$. Acting covariant derivatives generates more basis tensors, for example [13],

$$\frac{D^{2}y(x;x')}{Dx^{\mu}Dx^{\nu}} = H^{2}(2-y)g_{\mu\nu}(x), \qquad \frac{D^{2}y(x;x')}{Dx'^{\mu}Dx'^{\nu}} = H^{2}(2-y)g_{\mu\nu}(x'). \tag{40}$$

The contraction of any pair of the basis tensors also produces more basis tensors [13],

$$g^{\mu\nu}(x)\frac{\partial y}{\partial x^{\mu}}\frac{\partial y}{\partial x^{\nu}} = H^{2}(4y-y^{2}) = g^{\mu\nu}(x')\frac{\partial y}{\partial x'^{\mu}}\frac{\partial y}{\partial x'^{\nu}}, \tag{41}$$

$$g^{\mu\nu}(x)\frac{\partial y}{\partial x^{\nu}}\frac{\partial^{2} y}{\partial x^{\mu}\partial x'^{\sigma}} = H^{2}(2-y)\frac{\partial y}{\partial x'^{\sigma}}, \tag{42}$$

$$g^{\rho\sigma}(x')\frac{\partial y}{\partial x'^{\sigma}}\frac{\partial^2 y}{\partial x^{\mu}\partial x'^{\rho}} = H^2(2-y)\frac{\partial y}{\partial x^{\mu}}, \tag{43}$$

$$g^{\mu\nu}(x)\frac{\partial^2 y}{\partial x^{\mu}\partial x'^{\rho}}\frac{\partial^2 y}{\partial x^{\nu}\partial x'^{\sigma}} = 4H^4 g_{\rho\sigma}(x') - H^2\frac{\partial y}{\partial x'^{\rho}}\frac{\partial y}{\partial x'^{\sigma}}, \tag{44}$$

$$g^{\rho\sigma}(x')\frac{\partial^2 y}{\partial x^{\mu}\partial x'^{\rho}}\frac{\partial^2 y}{\partial x^{\nu}\partial x'^{\sigma}} = 4H^4 g_{\mu\nu}(x) - H^2\frac{\partial y}{\partial x^{\mu}}\frac{\partial y}{\partial x^{\nu}}. \tag{45}$$

The tensor structure of de Sitter breaking terms requires derivatives of the quantity $u(x;x') \equiv \ln(aa')$,

$$\frac{\partial u}{\partial x^{\mu}} = Ha\delta^0_{\mu}, \qquad \frac{\partial u}{\partial x'^{\rho}} = Ha'\delta^0_{\rho}. \tag{46}$$

Covariant derivatives of the new tensors involve some extra identities in addition to those of $y(x;x')$ [11],

$$\frac{D^2 u}{Dx^{\mu}Dx^{\nu}} = -H^2 g_{\mu\nu}(x) - \frac{\partial u}{\partial x^{\mu}}\frac{\partial u}{\partial x^{\nu}}, \qquad \frac{D^2 u}{Dx'^{\mu}Dx'^{\nu}} = -H^2 g_{\mu\nu}(x') - \frac{\partial u}{\partial x'^{\mu}}\frac{\partial u}{\partial x'^{\nu}}. \tag{47}$$

There are also some new contraction identities,

$$g^{\mu\nu}(x)\frac{\partial u}{\partial x^{\mu}}\frac{\partial u}{\partial x^{\nu}} = -H^2 = g^{\rho\sigma}(x')\frac{\partial u}{\partial x'^{\rho}}\frac{\partial u}{\partial x'^{\sigma}}, \tag{48}$$

$$g^{\mu\nu}(x)\frac{\partial u}{\partial x^{\mu}}\frac{\partial y}{\partial x^{\nu}} = -H^2\Big[y - 2 + 2\frac{a'}{a}\Big], \tag{49}$$

$$g^{\rho\sigma}(x')\frac{\partial u}{\partial x'^{\rho}}\frac{\partial y}{\partial x'^{\sigma}} = -H^2\Big[y - 2 + 2\frac{a}{a'}\Big], \tag{50}$$

$$g^{\mu\nu}(x)\frac{\partial u}{\partial x^{\mu}}\frac{\partial^2 y}{\partial x^{\nu}\partial x'^{\rho}} = -H^2\Big[\frac{\partial y}{\partial x'^{\rho}} + 2\frac{a'}{a}\frac{\partial u}{\partial x'^{\rho}}\Big], \tag{51}$$

$$g^{\rho\sigma}(x')\frac{\partial u}{\partial x'^{\rho}}\frac{\partial^2 y}{\partial x^{\mu}\partial x'^{\sigma}} = -H^2\Big[\frac{\partial y}{\partial x^{\mu}} + 2\frac{a}{a'}\frac{\partial u}{\partial x^{\mu}}\Big]. \tag{52}$$

Finally, we should explain the relation of our tensor basis to the one employed by mathematical physicists. Their literature obviously includes no mention of the de Sitter breaking tensors $\partial u/\partial x^{\mu}$ and $\partial u/\partial x'^{\mu}$, however, there are also significant differences in the de Sitter invariant sector. Our motivation for employing derivatives of the length function $y(x;x')$ is to simplify loop computations which involve derivatives of propagators. That is not a significant consideration for mathematical physicists because their literature is devoid of such computations; the only quantum gravitational loop computations so far made on de Sitter background [16–20] use our propagator. The issue of greater importance to mathematical physicists is the geometrical significance of the tensor basis. In place of $\partial y/\partial x^{\mu}$ and $\partial y/\partial x'^{\mu}$, they accordingly employ derivatives of the geodetic length function $\ell(x;x')$ (which is known as "μ" in their literature),

$$n_{\mu} \equiv \frac{\partial \ell(x;x')}{\partial x^{\mu}} = \frac{\frac{\partial y}{\partial x^{\mu}}}{H\sqrt{4y - y^2}}, \tag{53}$$

$$n_{\mu'} \equiv \frac{\partial \ell(x;x')}{\partial x'^{\mu'}} = \frac{\frac{\partial y}{\partial x'^{\mu'}}}{H\sqrt{4y - y^2}}. \tag{54}$$

(Note the mathematical physics notation in which unprimed indices belong to the tangent space at x^{μ} and primed indices belong to the x'^{μ} tangent space.) In place of the mixed second derivative $\partial^2 y/\partial x^{\mu}\partial x'^{\nu}$, mathematical physicists prefer the parallel transport matrix,

$$g_{\mu\nu'} = -\frac{1}{2H^2}\Big[\frac{\partial^2 y}{\partial x^{\mu}\partial x'^{\nu'}} + \frac{1}{4-y}\frac{\partial y}{\partial x^{\mu}}\frac{\partial y}{\partial x'^{\nu'}}\Big]. \tag{55}$$

III. DOING THE MATH

The purpose of this section is to perform the actual computation. We begin by exploiting conformal invariance to write the Weyl-Weyl correlator as a series of permutations and traces of four ordinary derivatives of the graviton propagator. We then express the index factors of the graviton propagator using the tensor basis of the previous section. The next step is to reduce the four ordinary derivatives of the various scalar propagator functions to a standard form based on the same tensor basis. The final step is to note that the standard permutations and traces remove all the noncovariant tensors, leaving only a linear combination of three de Sitter invariant tensors times exceptionally simple scalar factors. We also compare with the result of Kouris [26], and we take the coincidence limit using dimensional regularization.

A. Exploiting conformal invariance

Recall the relation (10) between the conformally transformed metric $\tilde{g}_{\mu\nu}$ and the full metric $g_{\mu\nu} = a^2\tilde{g}_{\mu\nu}$. Let $C_{\alpha\beta\gamma\delta}$ and $\tilde{C}_{\alpha\beta\gamma\delta}$ stand for the Weyl tensors constructed from each metric, with their indices raised and lowered by the appropriate metric. Because the Weyl tensor is conformally invariant with one index raised we have

$$C^{\alpha}{}_{\beta\rho\sigma} = \tilde{C}^{\alpha}{}_{\beta\rho\sigma} \Rightarrow C_{\alpha\beta\rho\sigma} = a^2\tilde{C}_{\alpha\beta\rho\sigma}. \tag{56}$$

As a consequence the correlation function of two Weyl tensors takes the form

$$\langle\Omega|C_{\alpha\beta\gamma\delta}(x)C_{\mu\nu\rho\sigma}(x')|\Omega\rangle = a^2a'^2\langle\Omega|\tilde{C}_{\alpha\beta\gamma\delta}(x)\tilde{C}_{\mu\nu\rho\sigma}(x')|\Omega\rangle. \tag{57}$$

The advantage of conformal invariance becomes apparent when we express the Weyl tensor in terms of the Riemann tensor ($R^{\rho}{}_{\sigma\mu\nu} \equiv \partial_{\mu}\Gamma^{\rho}{}_{\nu\sigma} + \Gamma^{\rho}{}_{\mu\alpha}\Gamma^{\alpha}{}_{\nu\sigma} - \mu \leftrightarrow \nu$) and its traces $R_{\mu\nu} \equiv R^{\rho}{}_{\mu\rho\nu}$ and $R \equiv g^{\mu\nu}R_{\mu\nu}$,

$$C_{\alpha\beta\gamma\delta} = R_{\alpha\beta\gamma\delta} - \frac{1}{D-2}(g_{\alpha\gamma}R_{\beta\delta} - g_{\gamma\beta}R_{\delta\alpha} + g_{\beta\delta}R_{\alpha\gamma} - g_{\delta\alpha}R_{\gamma\beta}) + \frac{1}{(D-2)(D-1)}(g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma})R. \quad (58)$$

Of course the same relation (58) gives the conformally transformed Weyl tensor in terms of the conformally transformed metrics and curvatures. But whereas the de Sitter background of $g_{\mu\nu}$ is curved, the background value of the conformally transformed metric is flat $\tilde{g}_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$. This makes it very simple to extract the linearized piece,

$$\tilde{R}_{\alpha\beta\gamma\delta}(x) = -\frac{\kappa}{2}(h_{\beta\delta,\alpha\gamma} - h_{\delta\alpha,\gamma\beta} + h_{\alpha\gamma,\beta\delta} - h_{\gamma\beta,\delta\alpha}) + \mathcal{O}(\kappa^2). \quad (59)$$

It remains to describe the index algebra needed to convert the quadruply differentiated propagator into the linearized Weyl-Weyl correlator

$$\frac{\kappa^2}{4}\partial_\alpha\partial_\gamma\partial'_\mu\partial'_\rho i[_{\beta\delta}\Delta_{\nu\sigma}](x;x') \rightarrow \langle\Omega|\tilde{C}_{\alpha\beta\gamma\delta}(x)\tilde{C}_{\mu\nu\rho\sigma}(x')|\Omega\rangle + O(\kappa^4). \quad (60)$$

We distinguish two steps:

(i) *Riemannization*, in which the linearized (and conformally transformed) Riemann-Riemann correlator is formed; and

(ii) *Weylization*, in which the traces are subtracted to give the linearized Weyl-Weyl correlator.

It is useful to define Riemannization generally for any 8-index bi-tensor "seed" with the same algebraic symmetries as the quadruply differentiated propagator on the left-hand side of (60). From expression (59) we infer

$$\mathrm{Riem}[(\mathrm{seed})_{\alpha\beta\gamma\delta\mu\nu\rho\sigma}] \equiv \mathcal{R}_{\alpha\beta\gamma\delta}{}^{\epsilon\zeta\kappa\lambda} \times \mathcal{R}_{\mu\nu\rho\sigma}{}^{\theta\phi\psi\omega} \times (\mathrm{seed})_{\epsilon\zeta\kappa\lambda\theta\phi\psi\omega}, \quad (61)$$

where

$$\mathcal{R}_{\alpha\beta\gamma\delta}{}^{\epsilon\zeta\kappa\lambda} \equiv \delta^\epsilon_\alpha\delta^\kappa_\gamma\delta^\zeta_\beta\delta^\lambda_\delta - \delta^\epsilon_\gamma\delta^\kappa_\beta\delta^\zeta_\delta\delta^\lambda_\alpha + \delta^\epsilon_\beta\delta^\kappa_\delta\delta^\zeta_\alpha\delta^\lambda_\gamma - \delta^\epsilon_\delta\delta^\kappa_\alpha\delta^\zeta_\gamma\delta^\lambda_\beta. \quad (62)$$

Weylization can be defined similarly on any 8-index bi-tensor seed with the algebraic symmetries of the product of two Riemann tensors,

$$\mathrm{Weyl}[(\mathrm{seed})_{\alpha\beta\gamma\delta\mu\nu\rho\sigma}] \equiv \mathcal{C}_{\alpha\beta\gamma\delta}{}^{\epsilon\zeta\kappa\lambda} \times \mathcal{C}_{\mu\nu\rho\sigma}{}^{\theta\phi\psi\omega} \times (\mathrm{seed})_{\epsilon\zeta\kappa\lambda\theta\phi\psi\omega}. \quad (63)$$

From expression (58) we infer

$$\mathcal{C}_{\alpha\beta\gamma\delta}{}^{\epsilon\zeta\kappa\lambda} \equiv \delta^\epsilon_\alpha\delta^\zeta_\beta\delta^\kappa_\gamma\delta^\lambda_\delta - [\eta_{\alpha\gamma}\delta^\zeta_\beta\delta^\lambda_\delta - \eta_{\gamma\beta}\delta^\zeta_\delta\delta^\lambda_\alpha + \eta_{\beta\delta}\delta^\zeta_\alpha\delta^\lambda_\gamma - \eta_{\delta\alpha}\delta^\zeta_\gamma\delta^\lambda_\beta]\frac{\eta^{\epsilon\kappa}}{D-2} + [\eta_{\alpha\gamma}\eta_{\beta\delta} - \eta_{\alpha\gamma}\eta_{\beta\delta}]\frac{\eta^{\epsilon\kappa}\eta^{\zeta\lambda}}{(D-2)(D-1)}. \quad (64)$$

The operations of Riemannization and Weylization give a simple form for the linearized Weyl-Weyl correlator,

$$\langle\Omega|C_{\alpha\beta\gamma\delta}(x)C_{\mu\nu\rho\sigma}(x')|\Omega\rangle = \frac{\kappa^2}{4}a^2a'^2\mathrm{Weyl}(\mathrm{Riem}[\partial_\alpha\partial_\gamma\partial'_\mu\partial'_\rho i[_{\beta\delta}\Delta_{\nu\sigma}](x;x')]) + O(\kappa^4). \quad (65)$$

From expression (22) for the graviton propagator, and the fact that the index factors $[_{\beta\delta}T^I_{\nu\sigma}]$ are constant in our gauge, we can write

$$a^2a'^2\partial_\alpha\partial_\gamma\partial'_\mu\partial'_\rho i[_{\beta\delta}\Delta_{\nu\sigma}](x;x') = \sum_{I=A,B,C} a^2a'^2[_{\beta\delta}T^I_{\nu\sigma}] \times \partial_\alpha\partial_\gamma\partial'_\mu\partial'_\rho i\Delta_I(x;x'). \quad (66)$$

In the next two subsections we will derive expressions for first $a^2a'^2[_{\beta\delta}T^I_{\nu\sigma}]$ and then $\partial_\alpha\partial_\gamma\partial'_\mu\partial'_\rho i\Delta_I(x;x')$.

Several comments are in order before we close this subsection. First, Riemannization is the "standard permutation" defined decades ago in a study of invariant Green's functions [31]. A result from that work which will facilitate subsequent analysis is that the Riemannization of any seed which is symmetric on the index pairs (α,γ), (β,δ), (μ,ρ), and (ν,σ) will possess all the algebraic symmetries of a Riemann tensor at each point,

$$R_{\alpha\beta\gamma\delta} = -R_{\beta\alpha\gamma\delta} = -R_{\alpha\beta\delta\gamma} = R_{\gamma\delta\alpha\beta} = R_{\alpha\gamma\beta\delta} - R_{\alpha\delta\beta\gamma}. \quad (67)$$

The second point is that the Weyl tensor possesses the additional algebraic symmetry of being traceless on any two indices, and the additional differential symmetry of being transverse,

$$D^\alpha C_{\alpha\beta\gamma\delta} = 0. \quad (68)$$

Of course it is the full covariant derivative operator that appears in (68), but the covariant derivative of the de Sitter background must annihilate the linearized Weyl-Weyl correlator. Third, every factor of the Minkowski metric in (64) is accompanied by an inverse metric, so we could just have easily expressed this tensor in terms of the de Sitter background metric,

$$\eta_{\alpha\gamma}\eta^{\epsilon\kappa} = a^2\eta_{\alpha\gamma} \times \frac{1}{a^2}\eta^{\epsilon\kappa} \equiv g_{\alpha\gamma}(x) \times g^{\epsilon\kappa}(x). \quad (69)$$

Our final point is evident in the last equation: because we no longer need the full metric $g_{\mu\nu} = a^2(\eta_{\mu\nu} + \kappa h_{\mu\nu})$, we will henceforth employ the symbol "$g_{\mu\nu}$" to denote the de Sitter background metric, $g_{\mu\nu} \equiv a^2 \eta_{\mu\nu}$.

B. Standard form for tensor structures

The de Sitter invariant part of the index factors can be written in terms of the y-basis introduced in Sec. II C. To keep the tensor factors dimensionless we employ the notation

$$\mathcal{Y}_\mu \equiv \frac{1}{H}\frac{\partial y}{\partial x^\mu}, \qquad \mathcal{Y}'_\nu \equiv \frac{1}{H}\frac{\partial y}{\partial x'^\nu} \tag{70}$$

$$\mathcal{R}_{\mu\nu}(x;x') \equiv -\frac{1}{2H^2}\frac{\partial^2 y}{\partial x^\mu \partial x'^\nu}. \tag{71}$$

The analogous dimensionless de Sitter breaking tensors are

$$\mathcal{T}_\mu \equiv \frac{1}{H}\frac{\partial u}{\partial x^\mu} = a\delta^0_\mu, \qquad \mathcal{T}'_\nu \equiv \frac{1}{H}\frac{\partial u}{\partial x'^\nu} = a'\delta^0_\nu. \tag{72}$$

The key to extracting the invariant parts of the various index factors (24)–(26) is to note that they involve purely temporal tensors such as δ^0_μ and purely spatial tensors such as $\bar{\eta}_{\mu\nu} \equiv \eta_{\mu\nu} + \delta^0_\mu \delta^0_\nu$. Of course the temporal factors can be represented using (72). The purely spatial metric can either involve two indices from the same point, or from both points. If the indices are from the same point, we can represent it using the purely spatial tangent matrix introduced in [23],

$$\begin{aligned} g^{\perp}_{\beta\delta}(x) &\equiv g_{\beta\delta}(x) + \mathcal{T}_\beta \mathcal{T}_\delta = a^2 \bar{\eta}_{\beta\delta}, \\ g^{\perp}_{\nu\sigma}(x') &\equiv g_{\nu\sigma}(x') + \mathcal{T}'_\nu \mathcal{T}'_\sigma = a'^2 \bar{\eta}_{\nu\sigma}. \end{aligned} \tag{73}$$

The case of mixed indices is given by [23],

$$\begin{aligned} \mathcal{R}^{\perp}_{\mu\nu}(x;x') &\equiv \mathcal{R}_{\mu\nu}(x;x') + \frac{1}{2}\mathcal{Y}_\mu \mathcal{T}'_\nu + \frac{1}{2}\mathcal{T}_\mu \mathcal{Y}'_\nu \\ &\quad + \frac{(2-y)}{2}\mathcal{T}_\mu \mathcal{T}'_\nu \\ &= aa'\bar{\eta}_{\mu\nu}. \end{aligned} \tag{74}$$

With these definitions the three tensor factors take the form

$$\begin{aligned} a^2 a'^2 [{}_{\beta\delta}T^A_{\nu\sigma}] &= \mathcal{R}^{\perp}_{\beta\nu}\mathcal{R}^{\perp}_{\delta\sigma} + \mathcal{R}^{\perp}_{\beta\sigma}\mathcal{R}^{\perp}_{\delta\nu} - \frac{2}{D-3} g^{\perp}_{\beta\delta}(x) g^{\perp}_{\nu\sigma}(x') \end{aligned} \tag{75}$$

$$a^2 a'^2 [{}_{\beta\delta}T^B_{\nu\sigma}] = -4\mathcal{T}_{(\beta}\mathcal{R}^{\perp}_{\delta)(\nu}\mathcal{T}'_{\sigma)} \tag{76}$$

$$\begin{aligned} a^2 a'^2 [{}_{\beta\delta}T^C_{\nu\sigma}] &= \frac{2}{(D-3)(D-2)}[(D-2)\mathcal{T}_\beta \mathcal{T}_\delta + g_{\beta\delta}] \\ &\quad \times [(D-2)\mathcal{T}'_\nu \mathcal{T}'_\sigma + g'_{\nu\sigma}]. \end{aligned} \tag{77}$$

C. Standard form for derivatives

We can perform a similar reduction for the factor $\partial_\alpha \partial_\gamma \partial'_\mu \partial'_\rho i\Delta_I(x;x')$ in (66). The B-type and C-type propagators are de Sitter invariant functions of $y(x;x')$, and taking two mixed derivatives of the A-type propagator eliminates its de Sitter breaking term. Thus, after acting these first two derivatives we can write

$$\partial_\alpha \partial_\gamma \partial'_\mu \partial'_\rho i\Delta_I(x;x') = \partial_\alpha \partial'_\mu \Big\{ I'(y) \frac{\partial^2 y}{\partial x^\gamma \partial x'^\rho} + I''(y) \frac{\partial y}{\partial x^\gamma}\frac{\partial y}{\partial x'^\rho} \Big\}. \tag{78}$$

Acting the remaining two derivatives produces noninvariant terms,

$$\begin{aligned} \partial_\alpha \partial_\gamma \partial'_\mu \partial'_\rho i\Delta_I(x;x') &= \Big[\frac{\partial^4 y}{\partial x^\alpha \partial x^\gamma \partial x'^\mu \partial x'^\rho}\Big] I'(y) + \Big[\frac{\partial^2 y}{\partial x^\alpha \partial x^\gamma}\frac{\partial^2 y}{\partial x'^\mu \partial x'^\rho} + 2\frac{\partial^2 y}{\partial x^\alpha \partial x'^{(\mu}}\frac{\partial^2 y}{\partial x'^{\rho)} \partial x^\gamma} + 2\frac{\partial y}{\partial x^{(\alpha}}\frac{\partial^3 y}{\partial x^{\gamma)} \partial x'^\mu \partial x'^\rho} \\ &\quad + 2\frac{\partial^3 y}{\partial x^\alpha \partial x^\gamma \partial x'^{(\mu}}\frac{\partial y}{\partial x'^{\rho)}}\Big] I''(y) + \Big[4\frac{\partial y}{\partial x^{(\alpha}}\frac{\partial^2 y}{\partial x^{\gamma)} \partial x'^{(\mu}}\frac{\partial y}{\partial x'^{\rho)}} + \frac{\partial y}{\partial x^\alpha}\frac{\partial y}{\partial x^\gamma}\frac{\partial^2 y}{\partial x'^\mu \partial x'^\rho} + \frac{\partial^2 y}{\partial x^\alpha \partial x^\gamma}\frac{\partial y}{\partial x'^\mu}\frac{\partial y}{\partial x'^\rho}\Big] I'''(y) \\ &\quad + \Big[\frac{\partial y}{\partial x^\alpha}\frac{\partial y}{\partial x^\gamma}\frac{\partial y}{\partial x'^\mu}\frac{\partial y}{\partial x'^\rho}\Big] I''''(y). \end{aligned} \tag{79}$$

All noninvariance comes from acting two derivatives at the same spacetime point. We can express these derivatives in standard form,

$$\begin{aligned} \frac{\partial^2 y}{\partial x^\alpha \partial x^\gamma} &= 2H^2 \Big\{\frac{a'}{a} g_{\alpha\gamma}(x) + \mathcal{T}_{(\alpha}\mathcal{Y}_{\gamma)}\Big\}, \qquad \frac{\partial^2 y}{\partial x'^\mu \partial x'^\rho} = 2H^2 \Big\{\frac{a}{a'} g_{\mu\rho}(x') + \mathcal{T}'_{(\mu}\mathcal{Y}'_{\rho)}\Big\}, \\ \frac{\partial^3 y}{\partial x^\alpha \partial x^\gamma \partial x'^\mu} &= 2H^3 \Big\{\frac{a'}{a} g_{\alpha\gamma}(x)\mathcal{T}'_\mu - 2\mathcal{T}_{(\alpha}\mathcal{R}_{\gamma)\mu}\Big\}, \qquad \frac{\partial^3 y}{\partial x^\alpha \partial x'^\mu \partial x'^\rho} = 2H^3 \Big\{\frac{a}{a'} g_{\mu\rho}(x')\mathcal{T}_\alpha - 2\mathcal{R}_{\alpha(\mu}\mathcal{T}'_{\rho)}\Big\}, \\ \frac{\partial^4 y}{\partial x^\alpha \partial x^\gamma \partial x'^\mu \partial x'^\rho} &= 4H^4 \Big\{\frac{a}{a'}\mathcal{T}_\alpha \mathcal{T}_\gamma g_{\mu\rho}(x') + \frac{a'}{a} g_{\alpha\gamma}(x)\mathcal{T}'_\mu \mathcal{T}'_\rho - 2\mathcal{T}_{(\alpha}\mathcal{R}_{\gamma)(\mu}\mathcal{T}'_{\rho)}\Big\}. \end{aligned} \tag{80}$$

D. The final result

Most of the subsequent analysis was made using the symbolic manipulation program MATHEMATICA, but it is of course advantageous to simplify even computer calculations to make them run more efficiently and transparently. It is evident that Riemannizing and then Weylizing our original seed (66) will produce a huge number of terms, many of which are permutations and traces of the same seed tensor times some function of y. Rather than process this unwieldy form all the way through Weylization, we expressed the Riemannized result as a linear combination of the rather small number of tensors which possess the algebraic symmetries of the product of two Riemann tensors. It turns out there are only nine independent invariant tensors with these symmetries [31]. There are many more noninvariant tensors, but very few of these actually occur.

A further simplification is to break the Riemannized result into those terms $\mathrm{R}^{(g)}_{\alpha\beta\gamma\delta\mu\nu\rho\sigma}$ which contain one or more factors of the de Sitter metric and those $\mathrm{R}^{(ng)}_{\alpha\beta\gamma\delta\mu\nu\rho\sigma}$ which do not,

$$\frac{\kappa^2}{4}a^2a'^2\mathrm{Riem}[\partial_\alpha\partial_\gamma\partial'_\mu\partial'_\rho i[_{\beta\delta}\Delta_{\nu\sigma}](x;x')] = \mathrm{R}^{(ng)}_{\alpha\beta\gamma\delta\mu\nu\rho\sigma} + \mathrm{R}^{(g)}_{\alpha\beta\gamma\delta\mu\nu\rho\sigma}. \qquad (81)$$

This is useful because Weylization can only change the metric terms. Of course there is the additional advantage that the number of independent tensors needed to represent the nonmetric terms is smaller. Without the metric there are only three invariant tensors with the algebraic symmetries of a product of two Riemann tensors [31]. We can represent them by Riemannizing the seeds,

$$\sigma^{(1)}_{\alpha\beta\gamma\delta\mu\nu\rho\sigma} = 2\frac{\partial^2 y}{\partial x^\alpha\partial x'^{(\mu}}\frac{\partial^2 y}{\partial x'^{\rho)}\partial x^\gamma}\times\frac{\partial^2 y}{\partial x^\beta\partial x'^{(\nu}}\frac{\partial^2 y}{\partial x'^{\sigma)}\partial x^\delta}, \qquad (82)$$

$$\sigma^{(2)}_{\alpha\beta\gamma\delta\mu\nu\rho\sigma} = 8\frac{\partial y}{\partial x^{(\alpha}}\frac{\partial^2 y}{\partial x^{\gamma)}\partial x'^{(\mu}}\frac{\partial y}{\partial x'^{\rho)}}\times\frac{\partial^2 y}{\partial x^\beta\partial x'^{(\nu}}\frac{\partial^2 y}{\partial x'^{\sigma)}\partial x^\delta}, \qquad (83)$$

$$\sigma^{(3)}_{\alpha\beta\gamma\delta\mu\nu\rho\sigma} = 2\frac{\partial y}{\partial x^\alpha}\frac{\partial y}{\partial x^\gamma}\frac{\partial y}{\partial x'^\mu}\frac{\partial y}{\partial x'^\rho}\times\frac{\partial^2 y}{\partial x^\beta\partial x'^{(\nu}}\frac{\partial^2 y}{\partial x'^{\sigma)}\partial x^\delta}. \qquad (84)$$

Although many de Sitter breaking, nonmetric tensors are conceivable, it turns out that only three occur. They derive from Riemannizing the seeds,

$$\sigma^{(4)}_{\alpha\beta\gamma\delta\mu\nu\rho\sigma} = 2\frac{\partial u}{\partial x^\alpha}\frac{\partial u}{\partial x^\gamma}\frac{\partial u}{\partial x'^\mu}\frac{\partial u}{\partial x'^\rho}\times\frac{\partial^2 y}{\partial x^\beta\partial x'^{(\nu}}\frac{\partial^2 y}{\partial x'^{\sigma)}\partial x^\delta}, \qquad (85)$$

$$\sigma^{(5)}_{\alpha\beta\gamma\delta\mu\nu\rho\sigma} = 4\frac{\partial u}{\partial x^\alpha}\frac{\partial u}{\partial x^\gamma}\frac{\partial u}{\partial x'^\mu}\frac{\partial u}{\partial x'^\rho}\times\frac{\partial y}{\partial x^{(\beta}}\frac{\partial^2 y}{\partial x^{\delta)}\partial x'^{(\nu}}\frac{\partial y}{\partial x'^{\sigma)}}, \qquad (86)$$

$$\sigma^{(6)}_{\alpha\beta\gamma\delta\mu\nu\rho\sigma} = \frac{\partial y}{\partial x^\alpha}\frac{\partial y}{\partial x^\gamma}\frac{\partial y}{\partial x'^\mu}\frac{\partial y}{\partial x'^\rho}\times\frac{\partial u}{\partial x^\beta}\frac{\partial u}{\partial x^\delta}\frac{\partial u}{\partial x'^\nu}\frac{\partial u}{\partial x'^\sigma}. \qquad (87)$$

We extracted the corresponding coefficients of seeds $\sigma^{(1)}$, $\sigma^{(2)}$, and $\sigma^{(3)}$ in $\mathrm{R}^{(ng)}_{\alpha\beta\gamma\delta\mu\nu\rho\sigma}$ and they have the wonderfully simple forms,

$$c_1 = \frac{\kappa^2}{8}A''(y), \quad c_2 = \frac{\kappa^2}{16}A'''(y), \quad c_3 = \frac{\kappa^2}{16}A''''(y). \qquad (88)$$

Even better are the results we obtained for the coefficients of the noninvariant tensors (85)–(87),

$$c_4 = -\frac{\kappa^2}{8(D-3)}[(4y-y^2)A''(y) + D(2-y)A'(y) - (D-1)k], \qquad (89)$$

$$c_5 = -\frac{\kappa^2}{8(D-3)}[(4y-y^2)A'''(y) + (D+2)(2-y)A''(y) - DA'(y)], \qquad (90)$$

$$c_6 = -\frac{\kappa^2}{8(D-3)}[(4y-y^2)A''''(y) + (D+4)(2-y)A'''(y) - 2(D+1)A''(y)]. \qquad (91)$$

Note that the coefficient c_4 is proportional to the differential equation (32) satisfied by $A(y)$, while c_5 and c_6 are proportional to its first and second derivatives, respectively. So these three coefficients vanish and we can write,

$$\mathrm{R}^{(ng)}_{\alpha\beta\gamma\delta\mu\nu\rho\sigma} = \sum_{k=1}^{3} c_k\times\mathrm{Riem}[\sigma^{(k)}_{\alpha\beta\gamma\delta\mu\nu\rho\sigma}]. \qquad (92)$$

Let us now turn to the Riemannized terms which contain one or more factors of the de Sitter metric, $\mathrm{R}^{(g)}_{\alpha\beta\gamma\delta\mu\nu\rho\sigma}$. Although the list for all possible (invariant and noninvariant) seed tensors is much longer than the first one, it turns out that they all vanish upon Weylization,

$$\mathrm{Weyl}(\mathrm{R}^{(g)}_{\alpha\beta\gamma\delta\mu\nu\rho\sigma}) = 0. \qquad (93)$$

Hence the final result is just the Weylization of (92). Expressing the seed tensors (82)–(84) in our standard, dimensionless form gives

$$\langle\Omega|C_{\alpha\beta\gamma\delta}(x)\times C_{\mu\nu\rho\sigma}(x')|\Omega\rangle = \kappa^2H^4A''(y)\times \mathrm{Weyl}(\mathrm{Riem}[[\mathcal{R}_{\alpha\mu}\mathcal{R}_{\gamma\rho}+\mathcal{R}_{\alpha\rho}\mathcal{R}_{\gamma\mu}][\mathcal{R}_{\beta\nu}\mathcal{R}_{\delta\sigma}+\mathcal{R}_{\beta\sigma}\mathcal{R}_{\delta\nu}]]) - 2\kappa^2H^4A'''(y)\times \mathrm{Weyl}(\mathrm{Riem}[\mathcal{Y}_{(\alpha}\mathcal{R}_{\gamma)(\mu}\mathcal{Y}'_{\rho)}[\mathcal{R}_{\beta\nu}\mathcal{R}_{\delta\sigma}+\mathcal{R}_{\beta\sigma}\mathcal{R}_{\delta\nu}]]) + \frac{1}{4}\kappa^2H^4A''''(y)\times \mathrm{Weyl}(\mathrm{Riem}[\mathcal{Y}_\alpha\mathcal{Y}_\gamma\mathcal{Y}'_\mu\mathcal{Y}'_\rho[\mathcal{R}_{\beta\nu}\mathcal{R}_{\delta\sigma}+\mathcal{R}_{\beta\sigma}\mathcal{R}_{\delta\nu}]]) + O(\kappa^4). \tag{94}$$

A further simplification is to express the result (94) using covariant derivatives (with respect to the de Sitter background) of the scalar propagator $i\Delta_A(x;x')$,

$$\langle\Omega|C_{\alpha\beta\gamma\delta}(x)\times C_{\mu\nu\rho\sigma}(x')|\Omega\rangle = \frac{\kappa^2}{4}\mathrm{Weyl}(\mathrm{Riem}[D_\alpha D_\gamma D'_\mu D'_\rho i\Delta_A\times[\mathcal{R}_{\beta\nu}\mathcal{R}_{\delta\sigma}+\mathcal{R}_{\beta\sigma}\mathcal{R}_{\delta\nu}]]) + O(\kappa^4). \tag{95}$$

(The flat space limit is obvious from this form.) The fact that the three algebraically independent tensor factors in expression (94) can be combined in this way is a consequence of transversality (68). Each of the three tensor factors obeys all the algebraic symmetries of a product of two Weyl tensors, but only a particular combination of all three obeys transversality.

Even more simplifications occur in $D = 4$ dimensions. For example, the general form of $A''(y)$ from definition (30) contains an infinite series,

$$A''(y) = \frac{H^{D-2}}{(4\pi)^{(D/2)}}\frac{1}{16}\Big\{\Gamma\Big(\frac{D}{2}+1\Big)\Big(\frac{4}{y}\Big)^{(D/2)+1} + \Big(\frac{D}{2}-1\Big)\Gamma\Big(\frac{D}{2}+1\Big)\Big(\frac{4}{y}\Big)^{(D/2)} + \sum_{n=1}^{\infty}\Big[\frac{(n-1)\Gamma(n+D-1)}{\Gamma(n+\frac{D}{2})}\Big(\frac{y}{4}\Big)^{n-2} - \frac{(n-\frac{D}{2}+1)\Gamma(n+\frac{D}{2}+1)}{(n+1)!}\Big(\frac{y}{4}\Big)^{n-(D/2)}\Big]\Big\}. \tag{96}$$

However, only the first two terms survive for $D = 4$,

$$\lim_{D\to 4}A''(y) = \frac{H^2}{16\pi^2}\Big\{\frac{8}{y^3}+\frac{2}{y^2}\Big\}. \tag{97}$$

E. Comparison with previous results

In 2001, Kouris reported a result for the linearized Weyl-Weyl correlator in $D = 4$ dimensions [26], derived using a de Sitter invariant propagator in a general gauge [12]. Although the reader will recall from Sec. I that all these propagators are illegitimate for one reason or another, the various problems (spurious zero modes and invalid analytic continuations in the constrained sector) should drop out of the Weyl-Weyl correlator. However, the Kouris result does not agree with ours, nor can his result be correct.

Kouris expressed his answer as a linear combination of scalar functions (given in Table I) times antisymmetrized tensor factors (the seeds for which are listed in Table II),

$$\langle\Omega|C_{abcd}(x)\times C_{a'b'c'd'}(x')|\Omega\rangle_{\mathrm{Kouris}} = \sum_{I=1}^{7}D^{(I)}\times S^{(I)}_{[ab][cd][a'b'][c'd']} + O(\kappa^4). \tag{98}$$

The problem has to do with the various algebraic and differential symmetries that the linearized Weyl-Weyl correlator must obey. We define

$$W_{abcda'b'c'd'} \equiv \sum_{I=1}^{7}D^{(I)}\times S^{(I)}_{[ab][cd][a'b'][c'd']}, \tag{99}$$

This tensor should be, and is, antisymmetric under interchange of (a,b), (c,d), (a',b'), and (c',d'). However, it must also be symmetric under the interchange of index pairs,

$$W_{abcda'b'c'd'} = W_{cdaba'b'c'd'} = W_{abcdc'd'a'b'}. \tag{100}$$

Another symmetry inherited from the Riemann tensor is

$$W_{a(bcd)a'b'c'd'} = 0 = W_{abcda'(b'c'd')}. \tag{101}$$

The result must also be traceless within any index group. That is obviously true on antisymmetric index pairs, but it must hold as well for different pairs,

$$g^{ac}W_{abcda'b'c'd'} = 0 = g^{a'c'}W_{abcda'b'c'd'}. \tag{102}$$

TABLE I. The coefficients $D^{(I)}$ of Kouris [26] expressed in terms of our de Sitter length function $y(x;x')$. Each term should be multiplied by $\frac{\kappa^2H^6}{4\pi^2}$.

I	$D^{(I)}$
1	$-12(\frac{4}{y})^3$
2	$-18(\frac{4}{y})^3 - 6(\frac{4}{y})^2$
3	$6(\frac{4}{y})^3 + 6(\frac{4}{y})^2$
4	$-3(\frac{4}{y})^3 + 3(\frac{4}{y})^2$
5	$\frac{3}{2}(\frac{4}{y})^3 + \frac{3}{2}(\frac{4}{y})^2$
6	$\frac{3}{2}(\frac{4}{y})^2$
7	$-\frac{1}{4}(\frac{4}{y})^3 + \frac{3}{4}(\frac{4}{y})^2$

TABLE II. The seed tensors $S^{(I)}_{abcda'b'c'd'}$ of Kouris [26], expressed using our standard basis tensors (70) and (71). Terms that drop after antisymmetrization have been omitted.

I	$S^{(I)}_{abcda'b'c'd'}$
1	$\frac{1}{(4y-y^2)^2}\mathcal{Y}_a\mathcal{Y}_c\mathcal{Y}'_{a'}\mathcal{Y}'_{c'}[g_{bd}g_{b'd'}-2\mathcal{R}_{bb'}\mathcal{R}_{dd'}]$
2	$\frac{1}{4y-y^2}\mathcal{Y}_a\mathcal{Y}'_{c'}\mathcal{R}_{bb'}\mathcal{R}_{cd'}[\mathcal{R}_{da'}-\frac{1}{2(4-y)}\mathcal{Y}_d\mathcal{Y}'_{a'}]$
3	$\frac{1}{4y-y^2}\mathcal{Y}_c\mathcal{Y}'_{c'}g_{bd}g_{a'd'}[\mathcal{R}_{ab'}-\frac{1}{2(4-y)}\mathcal{Y}_a\mathcal{Y}'_{b'}]$
4	$\frac{1}{4y-y^2}[g_{ac}\mathcal{Y}'_{a'}\mathcal{Y}'_{c'}\mathcal{R}_{bd'}\mathcal{R}_{db'}+\mathcal{Y}_a\mathcal{Y}_c g_{a'c'}\mathcal{R}_{b'b}\mathcal{R}_{d'd}]-\frac{1}{2(4y-y^2)}[g_{ac}g_{bd}g_{b'd'}\mathcal{Y}'_{a'}\mathcal{Y}'_{c'}+\mathcal{Y}_a\mathcal{Y}_c g_{a'c'}g_{b'd'}g_{bd}]$
5	$\mathcal{R}_{ab'}\mathcal{R}_{bc'}\mathcal{R}_{cd'}\mathcal{R}_{da'}-\frac{1}{2(4-y)}[\mathcal{R}_{ab'}\mathcal{R}_{bc'}(\mathcal{R}_{cd'}\mathcal{Y}_d\mathcal{Y}'_{a'}+\mathcal{R}_{da'}\mathcal{Y}_c\mathcal{Y}'_{d'})]+\mathcal{R}_{cd'}\mathcal{R}_{da'}(\mathcal{R}_{ab'}\mathcal{Y}_b\mathcal{Y}'_{c'}+\mathcal{R}_{bc'}\mathcal{Y}_a\mathcal{Y}'_{b'})]$ $+\frac{1}{4(4-y)^2}[\mathcal{R}_{ab'}\mathcal{R}_{cd'}\mathcal{Y}_b\mathcal{Y}_d\mathcal{Y}'_{a'}\mathcal{Y}'_{c'}+\mathcal{R}_{bc'}\mathcal{R}_{da'}\mathcal{Y}_a\mathcal{Y}_c\mathcal{Y}'_{b'}\mathcal{Y}'_{d'}]$
6	$g_{ac}g_{b'd'}\mathcal{R}_{da'}\mathcal{R}_{bc'}+g_{ac}g_{b'd'}[-\frac{1}{2(4-y)}(\mathcal{R}_{da'}\mathcal{Y}_b\mathcal{Y}'_{c'}+\mathcal{R}_{bc'}\mathcal{Y}_d\mathcal{Y}'_{a'})+\frac{1}{4(4-y)^2}\mathcal{Y}_b\mathcal{Y}_d\mathcal{Y}'_{a'}\mathcal{Y}'_{c'}]$
7	$g_{ac}g_{bd}g_{a'c'}g_{b'd'}$

None of the algebraic symmetries (100)–(102) hold, nor does the Kouris result obey transversality (68),

$$D^a W_{abcda'b'c'd'} = 0 = D^{a'} W_{abcda'b'c'd'}. \tag{103}$$

Kouris claimed to have checked (100) and (101) [26]. He does not seem to have realized that relations (102) and (103) should hold. His choice of basis tensors is also peculiar. There are nine distinct invariant tensors with the algebraic symmetries of two Riemann tensors—antisymmetry plus relations (100) and (101) [31]. However, Kouris only used the seven basis seeds listed in Table II. Enforcing tracelessness (102) should leave just three distinct tensors [31], and transversality (103) should relate the coefficients of these.

Of course our result (94) obeys (100)–(103) so it cannot agree with (98). It is not easy to compare the two results term wise because Kouris employed the geometrical tensors (53)–(55) of the mathematical physics convention. However, it is simple enough to compare those terms which contain four factors of $\mathcal{R}$. In our result (94), with the Kouris indices, these derive exclusively from the first term,

$$\kappa^2H^4A''(y)\mathrm{Riem}[[\mathcal{R}_{aa'}\mathcal{R}_{cc'}+\mathcal{R}_{ac'}\mathcal{R}_{ca'}][\mathcal{R}_{bb'}\mathcal{R}_{dd'}+\mathcal{R}_{bd'}\mathcal{R}_{db'}]]$$
$$= -16\kappa^2H^4A''(y)\Big(-\frac{1}{2H^2}\Big)^4\Big\{2\frac{\partial^2 y}{\partial x^{a]}\partial x'^{[a'}}\frac{\partial^2 y}{\partial x'^{b']}\partial x^{[c}}\frac{\partial^2 y}{\partial x^{d]}\partial x'^{[c'}}\frac{\partial^2 y}{\partial x'^{d']}\partial x^{[b}}-\frac{\partial^2 y}{\partial x^{a}\partial x'^{[a'}}\frac{\partial^2 y}{\partial x'^{b']}\partial x^{b}}\frac{\partial^2 y}{\partial x^{c}\partial x'^{[c'}}\frac{\partial^2 y}{\partial x'^{d']}\partial x^{d}}$$
$$-\frac{\partial^2 y}{\partial x^{a}\partial x'^{[c'}}\frac{\partial^2 y}{\partial x'^{d']}\partial x^{b}}\frac{\partial^2 y}{\partial x^{c}\partial x'^{[a'}}\frac{\partial^2 y}{\partial x'^{b']}\partial x^{d}}\Big\}. \tag{104}$$

The only one of Kouris's tensors which has four factors of $\mathcal{R}$ is $S^{(5)}_{abcda'b'c'd'}$. Note that in $D=4$ dimensions we can express his $I=5$ coefficient function in terms of $A''(y)$,

$$D^{(5)} = 48\kappa^2H^4\times A''(y). \tag{105}$$

Retaining only the part of $S^{(5)}_{abcda'b'c'd'}$ which contains four factors of $\mathcal{R}$ gives

$$D^{(5)}\times S^{(5)}_{[ab][cd][a'b'c][c'd']}$$
$$\rightarrow -48\kappa^2H^4A''(y)\Big(-\frac{1}{2H^2}\Big)^4\frac{\partial^2 y}{\partial x^{a]}\partial x'^{[a'}}\frac{\partial^2 y}{\partial x'^{b']}\partial x^{[c}}$$
$$\times\frac{\partial^2 y}{\partial x^{d]}\partial x'^{[c'}}\frac{\partial^2 y}{\partial x'^{d']}\partial x^{[b}}. \tag{106}$$

Although the function of y is tantalizingly close, the numerical coefficients differ even between the parts of (104) and (106) which have the same tensor structure. One also sees the absence in (106) of the final two terms of (104) which are needed to enforce symmetries (100) and (101).

Two facts about Kouris's work make us suspect that it may be resolved after correcting some minor errors:

(i) The factors of $(4-y)$—which are an artifact of the cumbersome, de Sitter invariant notation—all cancel in his final result (98); and

(ii) He claims to have checked relations (100) and (101), even though they obviously fail for the result he reported.

We accordingly consulted Kouris's advisor, A. Higuchi and he discovered that the following changes need to be made to Kouris's result[2] [32]:

[2]Starred equation numbers refer to those in [26].

(i) The tensor factor in (33)* should be

$$\Omega^{(i)}_{abcda'b'c'd} = \frac{1}{2}(S^{(i)}_{[ab][cd][a'b'][c'd']} + S^{(i)}_{[cd][ab][a'b'][c'd']} + S^{(i)}_{[ab][cd][c'd'][a'b']} + S^{(i)}_{[cd][ab][c'd'][a'b']}). \tag{107}$$

(ii) Equations (35), (38), and (39)* should read respectively,

$$S^{(2)}_{abcda'b'c'd} = \frac{1}{3} n_a n_{c'} (g_{bb'} g_{cd'} + g_{bd'} g_{cb'}) g_{da'}, \tag{108}$$

$$S^{(5)}_{abcda'b'c'd} = \frac{1}{3}(g_{ab'} g_{bc'} g_{cd'} g_{da'} + g_{aa'} g_{bb'} g_{cc'} g_{dd'}), \tag{109}$$

$$S^{(6)}_{abcda'b'c'd} = -g_{ac} g_{da'} g_{b'd'} g_{bc'}. \tag{110}$$

(iii) The factor $\frac{16G}{\pi}$ in (42)–(48)* should be replaced by $\frac{8G}{\pi}$.

When this is done, the revised Kouris result agrees with ours.

F. Coincidence limit

Even had the result of Kouris been correct, it was unregulated by virtue of being specialized to $D = 4$ dimensions. A simple but powerful application of our formalism consists of taking the coincidence limit of the Weyl-Weyl correlator using dimensional regularization. To do this we set $a' = a$, $\Delta x^\mu = 0$ and $y = 0$. It is straightforward to read off the coincidence limit of each basis tensor from (37)–(39), (70), and (71),

$$\lim_{x' \to x} \mathcal{Y}_\mu(x; x') = \frac{1}{H} \lim_{x' \to x} \frac{\partial y}{\partial x^\mu} = 0, \tag{111}$$

$$\lim_{x' \to x} \mathcal{Y}'_\nu(x; x') = \frac{1}{H} \lim_{x' \to x} \frac{\partial y}{\partial x'^\nu} = 0, \tag{112}$$

$$\lim_{x' \to x} \mathcal{R}_{\mu\nu}(x; x') = -\frac{1}{2H^2} \lim_{x' \to x} \frac{\partial^2 y}{\partial x^\mu \partial x'^\nu} = g_{\mu\nu}(x). \tag{113}$$

Hence the seed tensors $\sigma^{(2)}_{\alpha\beta\gamma\delta\mu\nu\rho\sigma}$ and $\sigma^{(3)}_{\alpha\beta\gamma\delta\mu\nu\rho\sigma}$ both vanish at coincidence and we have,

$$\langle \Omega | C_{\alpha\beta\gamma\delta}(x) \times C_{\mu\nu\rho\sigma}(x) | \Omega \rangle = 4\kappa^2 H^4 A''(0) \times \mathrm{Weyl}(\mathrm{Riem}[g_{\alpha(\mu} g_{\rho)\gamma} g_{\beta(\nu} g_{\sigma)\delta}]) + O(\kappa^4). \tag{114}$$

The coincidence limit of $A''(y)$ is also simple because we are using dimensional regularization in which any D-dependent power of zero vanishes. Hence only the $n = 2$ term of the infinite series for (96) survives,

$$A''(0) = \frac{H^{D-2}}{(4\pi)^{(D/2)}} \times \frac{1}{16} \frac{\Gamma(D+1)}{\Gamma(\frac{D}{2}+2)}. \tag{115}$$

Expanding the Weylized and Riemannized tensor factor in (114) gives

$$\begin{aligned} \mathrm{Weyl}(\mathrm{Riem}[g_{\alpha(\mu} g_{\rho)\gamma} g_{\beta(\nu} g_{\sigma)\delta}]) &= 4 g_{\alpha[\mu} g_{\nu]\beta} g_{\gamma[\rho} g_{\sigma]\delta} + 4 g_{\alpha[\rho} g_{\sigma]\beta} g_{\gamma[\mu} g_{\nu]\delta} \\ &\quad - 8 g_{\alpha][\mu} g_{\nu][\gamma} g_{\delta][\rho} g_{\sigma][\beta} \\ &\quad + \frac{24}{D-2}(g_{\alpha][\gamma} g_{\delta][\mu} g_{\nu][\rho} g_{\sigma][\beta} + g_{\alpha][\gamma} g_{\delta][\rho} g_{\sigma][\mu} g_{\nu][\beta}) \\ &\quad + \frac{24}{(D-2)(D-1)} g_{\alpha[\gamma} g_{\delta]\beta} g_{\mu[\rho} g_{\sigma]\nu}. \end{aligned} \tag{116}$$

What we are ultimately interested in is the coincident Weyl-Weyl correlator with the indices properly contracted. That is, we contract $g^{\alpha\mu} g^{\beta\nu} g^{\gamma\rho} g^{\delta\sigma}$ into (116) to obtain

$$\langle \Omega | C^{\alpha\beta\gamma\delta}(x) C_{\alpha\beta\gamma\delta}(x) | \Omega \rangle = 4(D-3)D(D+1)(D+2) A''(0) \kappa^2 H^4 + O(\kappa^4 H^8). \tag{117}$$

IV. DISCUSSION

Our result for the linearized Weyl-Weyl correlator is (94). It does not agree with what Kouris obtained [26], but that result cannot be correct because it lacks some of the algebraic symmetries of the Weyl tensor and is not transverse. Higuchi has shown how a series of corrections to Kouris's result makes it agree with ours [32]. It has also been shown that our result follows as well from the recently derived graviton propagator in de Donder gauge [33]. By taking the coincidence limit of our result (with dimensional regularization) and contracting the indices we derived an expression (117) for the expectation value of $C^{\alpha\beta\gamma\delta}(x) C_{\alpha\beta\gamma\delta}(x)$ at lowest order.

Despite the fact that our propagator shows a physical breaking of de Sitter invariance [21], the Weyl-Weyl correlator computed from it is completely de Sitter invariant at linearized order. There are different opinions about why this happened. Mathematical physicists maintain that it is because "free gravitons" are de Sitter invariant. They hold that the de Sitter breaking manifest in our propagator is merely a gauge artifact which drops out when linearized gauge invariance is enforced by going to the linearized Weyl-Weyl correlator [7,8]. We do not agree. We believe the de Sitter breaking terms dropped out because the logarithmic infrared divergence from which they derive is rendered convergent (and hence de Sitter invariant) by the derivatives needed to convert the graviton field into a linearized Weyl tensor. This was so obvious that it was noted even before the computation was begun [9].

At this point we should comment on what one learns about gravity from the linearized Weyl-Weyl correlator versus the undifferentiated propagator. The dynamical variable of gravity is the metric and, like all local force fields, it consists of three things:

(i) A pure gauge part, which fixes how we measure lengths and times;
(ii) A constrained part, which carries the gravitational response to sources of stress-energy; and
(iii) A dynamical part, which represents gravitational radiation.

In a gauge such ours [14,15], the graviton propagator contains all three of these things. By insisting on the linearized Weyl tensor in order to expunge the pure gauge part, mathematical physicists have edited out the constrained fields and they have also weighted infrared graviton modes much less strongly than ultraviolet ones. There is no question that this abandons perfectly physical and gauge invariant information. For example, the constrained part of the gauge fixed propagator provides the gravitational response to matter, which comprises all but one of the classic tests of general relativity. And the canonical weighting of graviton modes is reflected in the scale invariance of the tensor power spectrum (1).

The graviton propagator has recently been derived in the one parameter family of exact, de Sitter invariant gauges [34]. In all cases the result breaks de Sitter invariance, yet the linearized Weyl-Weyl correlator is unchanged. For essentially half of all gauge parameters, the de Sitter breaking of the constrained, spin zero sector actually results from *power law* infrared divergences. Antoniadis and Mottola long ago showed that, incorrectly insisting on a de Sitter invariant representation for these results in divergences in physical quantities such as the gravitational response to a point mass [35]. Yet all of this physical and gauge invariant information is edited out of the Weyl-Weyl correlator.

It seems clear to us that this controversy over the relevance of the gauge fixed graviton versus the linearized Weyl tensor is identical to one which was finally settled for electromagnetism by the Aharonov-Bohm effect [1]. It is a gauge invariant fact that matter fields couple to the electromagnetic vector potential, not to the field strength. This implies that the undifferentiated vector potential is itself observable in a fixed gauge. Similarly, it is a gauge invariant fact that matter—and even gravity itself—couples to the undifferentiated graviton field, not to the curvature. The same reasoning implies that the undifferentiated graviton field must be observable in a fixed gauge. Indeed, strenuous efforts [2–5] are underway to measure the tensor power spectrum (1) which is precisely such an observable. Concerns over invariance should be resolved in gravity the very same way as in gauge theories: by noting that a quantity can always be defined invariantly by specifying it in a fixed gauge. (For examples, see [36,37].)

An interesting parallel exists with the free massless, minimally coupled scalar on a nondynamical de Sitter background,

$$\mathcal{L} = -\tfrac{1}{2}\partial_\mu\varphi\partial_\nu\varphi g^{\mu\nu}\sqrt{-g}. \tag{118}$$

There is no question that this theory breaks de Sitter invariance [27,30]. If one defines things so as to preserve the homogeneity and isotropy of cosmology, then the scalar propagator is precisely the same as the spatial polarizations of our graviton field [29],

$$\langle\Omega|T[\varphi(x)\varphi(x')]|\Omega\rangle = i\Delta_A(x;x') = A(y(x;x')) + k\ln(aa'). \tag{119}$$

However, because all fields in the stress tensor are differentiated, the expectation value of the free scalar stress tensor happens to be de Sitter invariant [27],

$$\langle\Omega|T_{\mu\nu}|\Omega\rangle = (\delta^\rho_\mu\delta^\sigma_\nu - \tfrac{1}{2}g_{\mu\nu}g^{\rho\sigma})\lim_{x'\to x}\partial_\rho\partial'_\sigma i\Delta_A(x;x') = (D-2)H^2A'(0)g_{\mu\nu}. \tag{120}$$

People who believe passionately in de Sitter invariance have been known to proclaim this result as evidence that the de Sitter breaking of the scalar propagator (119) is "unphysical." However, it is nothing more nor less than the result of the de Sitter breaking infrared divergence being logarithmic, so that derivatives eliminate it.

Now add an interaction which involves undifferentiated scalars,

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi g^{\mu\nu}\sqrt{-g} - \frac{\lambda}{4!}\varphi^4\sqrt{-g} + \text{counterterms}. \tag{121}$$

Because the interacting theory contains undifferentiated scalars, the expectation value of the stress tensor shows explicit de Sitter breaking [29,38],

$$\langle\Omega|T_{\mu\nu}|\Omega\rangle = (D-2)H^2A'(0)g_{\mu\nu} - \frac{\lambda H^4}{(4\pi)^4}\Big\{\Big[2\ln^2(a) + \frac{7}{2}\ln(a)\Big]g_{\mu\nu} + \Big[\frac{4}{3}\ln(a) + \frac{13}{18}\Big]\mathcal{T}_\mu\mathcal{T}_\nu\Big\} + O(\lambda^2). \tag{122}$$

de Sitter breaking has also been exhibited for the one-particle-irreducible (1PI) 2-point function at one and two loop orders [39], and one can show generally that each additional power of λ in a 1PI function produces up to two additional de Sitter breaking factors of $\ln(a)$ [40].

The same sort of de Sitter breaking goes on *whenever* one adds interactions which involve undifferentiated scalars on nondynamical de Sitter background. Explicit, fully renormalized results exists at one and two loop orders for scalar quantum electrodynamics [41]—which shows one factor of $\ln(a)$ for each factor of the loop counting parameter e^2—and for Yukawa theory [42]—which shows one factor of $\ln(a)$ for each additional loop. Similar results have even been obtained for the nonlinear sigma model [43].

Let us now take note of the undifferentiated graviton interactions which abound in the gravitational

Lagrangian (12). Based on the known relation between interactions and infrared logarithms, one expects an additional factor of $\ln(a)$ for each extra factor of the quantum gravitational loop counting parameter κ^2 [40]. Which brings us to the observation that $\langle\Omega|C^{\alpha\beta\gamma\delta}(x)C_{\alpha\beta\gamma\delta}(x)|\Omega\rangle$ can show de Sitter breaking at order κ^4. Individual diagrams certainly make such contributions, but it might be that they all add up to zero. We propose that this be checked.

It should be noted that the operator $C^{\alpha\beta\gamma\delta}(x)C_{\alpha\beta\gamma\delta}(x)$ is a scalar, rather than a true invariant. Promoting it to an invariant requires somehow fixing the observation point x^μ, and that would inevitably involve nonlocality. However, the expectation value of $C^{\alpha\beta\gamma\delta}(x)C_{\alpha\beta\gamma\delta}(x)$ should serve as a test of the physical de Sitter invariance of the gauge fixed theory. And this quantity has a priceless advantage over invariant (and hence nonlocal) observables: *we know how to renormalize it.*

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