Physical equivalence between the covariant and physical graviton two-point functions in de Sitter spacetime

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It is known that the covariant graviton two-point function in de Sitter spacetime is infrared-divergent for some choices of gauge parameters. On the other hand, it is also known that there are no infrared divergences requiring an infrared cutoff for the physical graviton two-point function for this spacetime in the transverse-traceless-synchronous gauge in the global coordinate system. We show in this paper that the covariant graviton Wightman two-point function with two gauge parameters is equivalent to the physical one in the global coordinate system in the sense that they produce the same two-point function of any local gauge-invariant tensor linear in the graviton field such as the linearized Weyl tensor. This confirms the fact, pointed out decades ago, that the infrared divergences of the graviton two-point function in the covariant gauge for some choices of gauge parameters are a gauge artifact in the sense that they do not contribute to the Wightman two-point function of any local gauge-invariant tensor field in linearized theory.

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I. INTRODUCTION

Infrared (IR) divergences of graviton two-point functions have been a matter of contention for over two decades. There are two separate issues which are sometimes mistakenly thought to be related. One issue is the IR divergences of the physical graviton two-point function in the transverse-traceless-synchronous gauge in conformally flat coordinates [1–3]. (See, e.g. Refs. [4,5], for recent works on this issue.) The other is the IR divergences of the covariant gauge for some choices of gauge parameters [6,7]. (There is also the issue of large-distance growth of the two-point function, which will not be discussed in this paper.) Since linearized gravity has gauge invariance, it is important to determine whether or not these IR divergences are a gauge artifact. One of the reasons why the research community has not reached a consensus about this question seems to be that, when it is asserted that some IR divergences are a gauge artifact, their precise definition is not made sufficiently clear.

The main purpose of this paper is to clarify in what sense the IR divergences of the graviton Wightman two-point function in the covariant gauge for some choices of gauge parameters are a gauge artifact. (Below, by a two-point function, we mean a Wightman two-point function unless otherwise stated.) This is in fact an old result of Allen [8]. We add to this result by showing that the covariant graviton two-point function with any choice of gauge parameters is physically equivalent to the physical one in the transverse-traceless-synchronous gauge in global coordinates [9], which suffers no IR divergences. This will also imply that the two-point function of any local gauge-invariant tensor field linear in the graviton field evaluated in the covariant gauge is independent of gauge parameters as expected.

Miao, Tsamis and Woodard [10] find that the covariant two-point function corresponding to an *IR-finite* choice of gauge parameters [8,11], the "strictly enforced" de Donder gauge, is IR divergent in the Poincaré patch of de Sitter spacetime, which is the spatially flat expanding half of this spacetime. We confirm, however, that IR divergences of the two-point function for a tachyonic scalar field, which is partly responsible for the breaking of de Sitter invariance in Ref. [10], are absent in global de Sitter spacetime. We also find no IR divergences in the tensor sector of the two-point function. Thus, the covariant two-point function constructed using the mode-sum method agrees with the IR-finite two-point function in the Euclidean approach also in the de Donder gauge. (This gauge should probably be avoided in perturbation theory in any case because the corresponding two-point function behaves rather badly at large separation.)

We emphasize that this paper has nothing to say about interacting theory. In particular, we do not couple the covariant graviton two-point function even to an external stress-energy tensor field. Thus, in this paper, the covariant two-point function is regarded as a graviton correlator and is shown to be equivalent to the physical one in Ref. [9] as such. If the gravitons are coupled to an external

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stress-energy tensor, for example, there will be nonlocal interaction terms in the physical gauge of Ref. [9] similar to the Coulomb-interaction term in QED in the Coulomb gauge (see, e.g. Ref. [12]), and an explicit demonstration of the equivalence between the physical and covariant gauges would be rather nontrivial.

In linearized gravity, the two-point function of the graviton field $h_{ab}(x)$ has no physical meaning by itself because this theory has gauge invariance under the gauge transformation,

$$\delta h_{ab}(x) = \nabla_a \Lambda_b(x) + \nabla_b \Lambda_a(x), \tag{1.1}$$

where $\Lambda_a(x)$ is any vector field. Here, the covariant derivative is the one compatible with the background de Sitter metric, $g_{ab}(x)$. One can find tensor fields at x which are linear in h_{ab} and are invariant under this gauge transformation. An example of such a tensor field is the linearized Weyl tensor $W_{abcd}(x) = \tilde{W}_{[ab][cd]}(x)$, where

$$\tilde{W}_{abcd}(x) = \nabla_c \nabla_b h_{ad}(x) + H^2 g_{ad}(x) h_{cb}(x). \tag{1.2}$$

Here, the constant H is the Hubble constant of de Sitter spacetime. (See Ref. [13] for conditions for a local tensor field to be gauge-invariant.) The two-point function of $W_{abcd}(x)$ evaluated in the covariant gauge can be found in Ref. [14].

Now, suppose that a graviton two-point function $\Delta_{aba'b'}(x, x') = \langle 0|h_{ab}(x) h_{a'b'}(x')|0\rangle$ can be written as

$$\Delta_{aba'b'}(x, x') = \tilde{\Delta}_{aba'b'}(x, x') + \nabla_{(a} Q_{b)a'b'}(x, x') + \nabla_{(a'} Q_{|ab|b')}(x, x'), \tag{1.3}$$

for some $Q_{aa'b'}(x, x')$ and $Q_{aba'}(x, x')$. (In this paper, we use the convention of Ref. [15] that primed indices are associated with point x' and unprimed indices with point x.) Then, the two-point function of a local gauge-invariant tensor field linear in h_{ab} will be the same whether one uses $\Delta_{aba'b'}(x, x')$ or $\tilde{\Delta}_{aba'b'}(x, x')$ as the graviton two-point function. This motivates the following definition: we say that the two graviton two-point functions, $\Delta_{aba'b'}(x, x')$ and $\tilde{\Delta}_{aba'b'}(x, x')$, are *physically equivalent in linearized gravity* if Eq. (1.3) is satisfied for some $Q_{aa'b'}(x, x')$ and $Q_{aba'}(x, x')$, which are not required to be bounded.

A more precise formulation of the graviton two-point function would correspond to defining it in the smeared form as

$$D(f^{(1)}, f^{(2)}) = \int d^4x \sqrt{-g(x)} \int d^4x' \sqrt{-g(x')} f^{(1)ab}(x) \times f^{(2)a'b'}(x') \Delta_{aba'b'}(x, x'), \tag{1.4}$$

where $f^{(1)ab}(x)$ and $f^{(2)a'b'}(x')$ are smooth, compactly supported and divergence-free symmetric tensor fields in de Sitter spacetime. Thus, the two-point function D would be defined as a functional on the space of pairs of smooth, compactly supported and divergence-free symmetric tensor fields. In such a definition, the functions $\Delta_{aba'b'}(x, x')$ and $\tilde{\Delta}_{aba'b'}(x, x')$ satisfying Eq. (1.3) can be regarded as two representatives of the same two-point function D. [It can be shown that there are "sufficiently many" smooth, compactly supported and divergence-free symmetric tensor fields for characterizing the gauge-invariant content of the graviton two-point function as in Eq. (1.4).]

Now suppose that a graviton two-point function $\Delta_{abab'}(x, x')$ has an IR cutoff ϵ and that it is divergent in the limit $\epsilon \to 0$. If it is physically equivalent in linearized gravity to $\Delta_{aba'b'}(x, x')$ which is not IR-divergent, then the two-point function of a local gauge-invariant tensor field will not depend on ϵ, i.e. will not be IR-divergent. What we show in this paper is that the covariant graviton two-point function for any choice of gauge parameters is physically equivalent in linearized gravity to the graviton two-point function in the transverse-traceless-synchronous gauge in global coordinates, which is IR-finite [9]. This will imply that the IR divergences of the covariant two-point function for a certain gauge choice can be said to be a gauge artifact in linearized gravity in the sense that the divergences will not manifest themselves in the two-point function of any local gauge-invariant tensor field linear in the graviton field, confirming and clarifying the claim in Ref. [8].

The rest of the paper is organized as follows. In Sec. II, we summarize some properties of the solutions to the free-field equations which we will need later for scalar, vector and symmetric tensor fields in de Sitter spacetime. We leave the explicit expressions of these solutions to Appendix A. In Sec. III, we review the physical two-point function in the transverse-traceless-synchronous gauge in global coordinates. In Sec. IV, we find all solutions to the field equation in the covariant gauge with two parameters. In Sec. V, we describe the quantization of linearized gravity in the covariant gauge in de Sitter spacetime. Then, we construct the covariant two-point function using the mode-sum method and show that it is equivalent to the physical two-point function of Ref. [9]. In Sec. VI, we summarize the results in this paper. We give explicit expressions for solutions to the free-field equations in Appendix A. Appendix B contains a technical result used in Sec. V. In Appendix C, the scalar two-point function, including the tachyonic case, is constructed by the mode-sum method in global de Sitter spacetime. We also show how this IR-finite two-point function can be recovered in Poincaré patch by subtracting the IR divergences. In Appendix D, we explicitly show that the covariant two-point function constructed in this paper is the same as that obtained in the Euclidean approach [11] for spacelike-separated points. We use the metric signature $- + + +$ and let $\hbar = c = 1$ and take the metric of de Sitter spacetime to be

$$ds^2 = -dt^2 + \cosh^2 t d\Omega^2, \tag{1.5}$$

where $d\Omega^2$ is the line element on the unit 3-sphere (S^3), throughout this paper. Thus, we choose units such that the

Hubble constant is 1. A point x in this spacetime has coordinates $(t, \mathbf{x})$, where $\mathbf{x}$ is a point on S^3.

II. SOLUTIONS TO FREE-FIELD EQUATIONS

In this section, we summarize some known properties of the solutions to the free-field equations for spin 0, 1 and 2 of arbitrary mass in de Sitter spacetime following Ref. [16]. We present the explicit solutions in Appendix A. First, we recall that the scalar, transverse vector and transverse-traceless tensor spherical harmonics on S^3, which we denote by $Y^{(0\ell\sigma)}$, $Y_i^{(1\ell\sigma)}$ and $Y_{ij}^{(2\ell\sigma)}$, are orthonormal eigenfunctions of the Laplace-Beltrami operator $\tilde{\nabla}^2$ on S^3 satisfying

$$-\tilde{\nabla}^2 Y^{(0\ell\sigma)} = \ell(\ell+2)Y^{(0\ell\sigma)}, \qquad \ell = 0, 1, 2, \ldots, \tag{2.1}$$

$$-\tilde{\nabla}^2 Y_i^{(1\ell\sigma)} = [\ell(\ell+2) - 1]Y_i^{(1\ell\sigma)}, \qquad \ell = 1, 2, 3, \ldots, \tag{2.2}$$

$$-\tilde{\nabla}^2 Y_{ij}^{(2\ell\sigma)} = [\ell(\ell+2) - 2]Y_{ij}^{(2\ell\sigma)}, \qquad \ell = 2, 3, 4, \ldots, \tag{2.3}$$

where σ represents all labels other than ℓ (see, e.g. Refs. [17,18]).

Let us start with the solutions to the scalar field equation,

$$(-\Box + \mu^2)\phi = 0. \tag{2.4}$$

(The solutions we present here are valid for $\mu^2 > 0$ and for most negative values of μ^2.) We can choose the solutions to be proportional to $Y^{(0\ell\sigma)}$. We denote the "positive-frequency" solutions which determine the Bunch-Davies (or Euclidean) vacuum [19,20] proportional to $Y^{(0\ell\sigma)}$ by $\phi^{(\mu^2;\ell\sigma)}(x)$. (We mean by positive-frequency solutions the coefficient functions of annihilation operators when the field is quantized.) They are

$$\phi^{(\mu^2;\ell\sigma)}(x) \propto (\cosh t)^{-1} \mathrm{P}_{L_0+1}^{-(\ell+1)}(i \sinh t) Y^{(0\ell\sigma)}(\mathbf{x}), \tag{2.5}$$

with $L_0 = -\frac{3}{2} + \sqrt{\frac{9}{4} - \mu^2}$, where $\mathrm{P}_{L_0+1}^{-(\ell+1)}(z)$ are the Legendre functions of the first kind given in terms of Gauss's hypergeometric function as

$$\mathrm{P}_{L_0+1}^{-(\ell+1)}(z) = \frac{1}{(\ell+1)!}\left(\frac{1-z}{1+z}\right)^{(\ell+1)/2} \times F\left(-L_0 - 1, L_0 + 2; \ell + 2; \frac{1-z}{2}\right). \tag{2.6}$$

These solutions and their complex conjugates, $\overline{\phi^{(\mu^2;\ell\sigma)}}$, form a complete set of solutions to Eq. (2.4).

We define the Klein-Gordon (KG) inner product for two solutions $\phi^{(1)}$ and $\phi^{(2)}$ to Eq. (2.4) as follows:

$$\langle \phi^{(1)}, \phi^{(2)} \rangle_{\mathrm{KG}} = i \int_\Sigma d\Sigma_a [\overline{\phi^{(1)}} \nabla^a \phi^{(2)} - (\nabla^a \overline{\phi^{(1)}})\phi^{(2)}], \tag{2.7}$$

where $d\Sigma_a = d\Sigma n_a$ with n^a being the future pointing unit normal vector to the Cauchy surface Σ. We normalize the solutions $\phi^{(\mu^2;\ell\sigma)}$ by requiring

$$\langle \phi^{(\mu^2;\ell\sigma)}, \phi^{(\mu^2;\ell'\sigma')} \rangle_{\mathrm{KG}} = \delta^{\ell\ell'}\delta^{\sigma\sigma'}. \tag{2.8}$$

The orthogonality follows from that of the spherical harmonics $Y^{(0\ell\sigma)}$ on S^3. We also note that $\phi^{(\mu^2;\ell\sigma)}$ are orthogonal to $\overline{\phi^{(\mu^2;\ell'\sigma')}}$ with respect to the Klein-Gordon inner product.

We write the field equation for a transverse vector field A_a satisfying $\nabla_a A^a = 0$ as

$$(-\Box + 3 + \mu^2)A_a = 0. \tag{2.9}$$

The gauge-invariant equation, $\nabla^b(\nabla_a A_b - \nabla_b A_a) = 0$, is equivalent to $(-\Box + 3)A_a = 0$, i.e. the $\mu = 0$ case of Eq. (2.9). This can readily be seen by recalling that $R_{abcd} = g_{ac}g_{bd} - g_{ad}g_{bc}$. We will be particularly interested in the case $\mu^2 = -6$, which is equivalent to $\nabla^b(\nabla_a A_b + \nabla_b A_a) = 0$.

There are two classes of solutions to Eq. (2.9). We introduce a label m to distinguish between these classes. The positive-frequency solutions will be denoted $A_a^{(\mu^2;m\ell\sigma)}$. Those with $m = 0$ have the time component given by

$$A_0^{(\mu^2;0\ell\sigma)} \propto (\cosh t)^{-2} \mathrm{P}_{L_1+1}^{-(\ell+1)}(i \sinh t) Y^{(0\ell\sigma)}(\mathbf{x}), \qquad \ell \geq 1, \tag{2.10}$$

where

$$L_1 = -\tfrac{3}{2} + \sqrt{\tfrac{1}{4} - \mu^2}. \tag{2.11}$$

The space components, $A_i^{(\mu^2;0\ell\sigma)}$, are obtained by postulating $A_i^{(\mu^2;0\ell\sigma)} = f_\ell(t)\tilde{\nabla}_i Y^{(0\ell\sigma)}$, where $\tilde{\nabla}_i$ is the covariant derivative on S^3, and solving the equation $\nabla^a A_a = 0$ for $f_\ell(t)$. (This equation cannot be solved for $\ell = 0$. Hence, there are no solutions with $\ell = 0$.) The solutions with $m = 1$ have $A_0^{(\mu^2;1\ell\sigma)} = 0$ and

$$A_i^{(\mu^2;1\ell\sigma)} \propto \mathrm{P}_{L_1+1}^{-(\ell+1)}(i \sinh t) Y_i^{(\ell\sigma)}(\mathbf{x}), \qquad \ell \geq 1. \tag{2.12}$$

We define the Klein-Gordon inner product for two transverse solutions $A_a^{(1)}$ and $A_a^{(2)}$ to Eq. (2.9) as

$$\langle A^{(1)}, A^{(2)} \rangle_{\mathrm{KG}} = i \int_\Sigma d\Sigma_a [\overline{A^{(1)b}} \nabla^a A_b^{(2)} - (\nabla^a \overline{A^{(1)b}}) A_b^{(2)}]. \tag{2.13}$$

Any two solutions with different sets of quantum numbers m, ℓ and σ are orthogonal to each other with respect to this inner product. For $-6 < \mu^2 < 0$, the positive-frequency solutions with $m = 0$ have negative norm with respect to this inner product, whereas the $m = 1$ solutions have positive norm. We normalize these solutions for $\ell \geq 2$ by requiring

$$\langle A^{(\mu^2;m\ell\sigma)}, A^{(\mu^2;m'\ell'\sigma')}\rangle_{\rm KG} = (-1)^{m+1}\delta^{mm'}\delta^{\ell\ell'}\delta^{\sigma\sigma'}. \quad (2.14)$$

The solutions $A_a^{(\mu^2;m,\ell=1,\sigma)}$ become Killing vectors in the limit $\mu^2 \to -6$. This implies that the Klein-Gordon inner product vanishes for these solutions because if $\xi^{(1)a}$ and $\xi^{(2)a}$ are Killing vectors, then

$$\int_\Sigma d\Sigma_a(\xi^{(1)b}\nabla^a\xi_b^{(2)} - \xi^{(2)b}\nabla^a\xi_b^{(1)}) = \int_\Sigma d\Sigma_a\nabla_b(\xi^{(1)a}\xi^{(2)b} - \xi^{(1)b}\xi^{(2)a}) = 0 \quad (2.15)$$

by the generalized Stokes theorem, which states that for any antisymmetric tensor F^{ab},

$$\int_\Sigma d\Sigma_a\nabla_b F^{ab} = 0. \quad (2.16)$$

For this reason, we normalize the $\ell = 1$ solutions as

$$\langle A^{(\mu^2;m,\ell=1,\sigma)}, A^{(\mu^2;m',\ell=1,\sigma')}\rangle_{\rm KG} = (-1)^{m+1}(\mu^2+6)\delta^{mm'}\delta^{\sigma\sigma'}. \quad (2.17)$$

We write the field equation for a transverse-traceless tensor field H_{ab} satisfying $\nabla^b H_{ab} = 0$ and $H^a{}_a = 0$ as

$$(-\Box + 2 + M^2)H_{ab} = 0. \quad (2.18)$$

The $M = 0$ case corresponds to linearized gravity. There are three classes of solutions distinguished by the label $m = 0, 1, 2$. We write the positive-frequency solutions as $H_{ab}^{(M^2;m\ell\sigma)}$. Those with $m = 0$ have

$$H_{00}^{(M^2;0\ell\sigma)} \propto (\cosh t)^{-3}{\rm P}_{L_2+1}^{-(\ell+1)}(i\sinh t)Y^{(0\ell\sigma)}(\mathbf{x}), \qquad \ell \geq 2, \quad (2.19)$$

where

$$L_2 = -\tfrac{3}{2} + \sqrt{\tfrac{9}{4} - M^2}. \quad (2.20)$$

The other components are obtained by postulating that $H_{0i} = f_\ell(t)\tilde{\nabla}_i Y^{(0\ell\sigma)}$, $H_{ij} = g_\ell^{(1)}(t)\tilde{\nabla}_i\tilde{\nabla}_j Y^{(0\ell\sigma)} + g_\ell^{(2)}(t)\tilde{\eta}_{ij}Y^{(0\ell\sigma)}$, where $\tilde{\eta}_{ij}$ is the metric on S^3, and solving the equations $\nabla^b H_{ab} =$ and $H^a{}_a = 0$ for the functions $f_\ell(t)$, $g_\ell^{(1)}(t)$ and $g_\ell^{(2)}(t)$. This is not possible if $\ell = 0$ or 1 in Eq. (2.19). The positive-frequency solutions with $m = 1$ have $H_{00}^{(M^2;1\ell\sigma)} = 0$ and

$$H_{0i}^{(M^2;1\ell\sigma)} \propto (\cosh t)^{-1}{\rm P}_{L_2+1}^{-(\ell+1)}(i\sinh t)Y_i^{(1\ell\sigma)}(\mathbf{x}), \qquad \ell \geq 2. \quad (2.21)$$

Then, we postulate that $H_{ij} = f_\ell(t)\tilde{\nabla}_{(i}Y_{j)}^{(1\ell\sigma)}$ and solve $\nabla^a H_{ab} = 0$ for $f_\ell(t)$. This is not possible if $\ell = 1$ in Eq. (2.21). Finally, the positive-frequency solutions with $m = 2$ have $H_{00}^{(M^2;2\ell\sigma)} = H_{0i}^{(M^2;2\ell\sigma)} = 0$ and

$$H_{ij}^{(M^2;2\ell\sigma)} \propto \cosh t\,{\rm P}_{L_2+1}^{-(\ell+1)}(i\sinh t)Y_{ij}^{(2\ell\sigma)}(\mathbf{x}), \qquad \ell \geq 2. \quad (2.22)$$

We define the Klein-Gordon inner product for two transverse-traceless solutions $H_{ab}^{(1)}$ and $H_{ab}^{(2)}$ to Eq. (2.18) as

$$\langle H^{(1)}, H^{(2)}\rangle_{\rm KG} = i\int_\Sigma d\Sigma_a[\overline{H^{(1)bc}}\nabla^a H_{bc}^{(2)} - (\nabla^a\overline{H^{(1)bc}})H_{bc}^{(2)}]. \quad (2.23)$$

We can normalize the $m = 2$ solutions as

$$\langle H^{(M^2;2\ell\sigma)}, H^{(M^2;2\ell'\sigma')}\rangle_{\rm KG} = 2\delta^{\ell\ell'}\delta^{\sigma\sigma'}. \quad (2.24)$$

The factor of 2 here is for later convenience. For $M = 0$, i.e. for linearized gravity, Eq. (2.18) is satisfied by $H_{ab} = \nabla_{(a}A_{b)}^{(-6;m\ell\sigma)}$. Indeed, one finds, using the associated Legendre equation (A17) and the lowering and raising differential operators, Eqs. (A18) and (A19),

$$H_{ab}^{(0;m\ell\sigma)} = \nabla_a A_b^{(-6;m\ell\sigma)} + \nabla_b A_a^{(-6;m\ell\sigma)}, \quad m = 0, 1, \quad \ell \geq 2, \quad (2.25)$$

after choosing a phase factor for $H_{ab}^{(0;m\ell\sigma)}$ appropriately [21]. Now, if $H_{ab}^{(1)}$ is any solution to Eq. (2.18) with $M^2 = 0$ and if $H_{ab}^{(2)} = \nabla_{(a}A_{b)}^{(2)}$ with $(\Box + 3)A_a^{(2)} = 0$ so that $H_{ab}^{(2)}$ is a solution to Eq. (2.18), then we find

$$\langle H^{(1)}, H^{(2)}\rangle_{\rm KG} = i\int_\Sigma d\Sigma_a\nabla_b[\overline{H^{(1)bc}}\nabla^a A_c^{(2)} - \overline{H^{(1)ac}}\nabla^b A_c^{(2)} + (\nabla^b\overline{H^{(1)ac}} - \nabla^a\overline{H^{(1)bc}})A_c^{(2)}] = 0 \quad (2.26)$$

by the generalized Stokes theorem. Thus,

$$\langle H^{(0;m\ell\sigma)}, H^{(0;m'\ell'\sigma')}\rangle_{\rm KG} = 0, \qquad m, m' = 0, 1. \quad (2.27)$$

It is also known that the solutions $H_{ab}^{(M^2;0\ell\sigma)}$ have negative norm if $0 < M^2 < 2$ [22,23], whereas the solutions $H^{(M^2;1\ell\sigma)}$ have positive norm if $M^2 > 0$. For these reasons, and since we will be interested in the $M \to 0$ limit, we normalize the solutions with $m = 0, 1$ as

$$\langle H^{(M^2;m\ell\sigma)}, H^{(M^2;m'\ell'\sigma')}\rangle_{\rm KG} = (-1)^{m+1}2M^2\delta^{mm'}\delta^{\ell\ell'}\delta^{\sigma\sigma'}. \quad (2.28)$$

III. PHYSICAL GRAVITON TWO-POINT FUNCTION

The Lagrangian for free gravitons in de Sitter spacetime can be written as

$$\mathcal{L}_{\rm inv} = \sqrt{-g}[\tfrac{1}{2}\nabla_a h^{ac}\nabla^b h_{bc} - \tfrac{1}{4}\nabla_a h_{bc}\nabla^a h^{bc} + \tfrac{1}{4}(\nabla^a h - 2\nabla^b h^a{}_b)\nabla_a h - \tfrac{1}{2}(h_{ab}h^{ab} + \tfrac{1}{2}h^2)], \quad (3.1)$$

with $h \equiv h^a{}_a$. The corresponding field equation is

$$\begin{aligned} L_{ab}^{(\mathrm{inv})cd}h_{cd} \equiv& \tfrac{1}{2}[-\Box h_{ab} + \nabla_a\nabla_c h^c{}_b + \nabla_b\nabla_c h^c{}_a - \nabla_a\nabla_b h \\ &+ g_{ab}\Box h - g_{ab}\nabla_c\nabla_d h^{cd}] + h_{ab} + \tfrac{1}{2}g_{ab}h \\ =& 0. \end{aligned} \tag{3.2}$$

It is well-known that the gauge degrees of freedom can be used to impose the conditions $\nabla^b h_{ab} = 0$ and $h = 0$ (see, e.g. Ref. [21]). Then Eq. (3.2) becomes

$$(\Box - 2)h_{ab} = 0. \tag{3.3}$$

This equation is Eq. (2.18) with $M = 0$. Thus, its solutions are given by Eqs. (A8)–(A15) with $M = 0$.

We have seen that the solutions $H_{ab}^{(0;m\ell\sigma)}$, $m = 0, 1$, are gauge solutions [see Eq. (2.25)]. Hence, only the solutions $H_{ab}^{(0;2\ell\sigma)}$ represent physical excitations. Retaining only these solutions corresponds to the synchronous transverse-traceless gauge, $h_{0a} = 0$, $\tilde{\nabla}_j h^j{}_i = 0$ and $h^i{}_i = 0$. Quantization of the field h_{ab} in this gauge, which we sometimes call the physical gauge, proceeds as follows. First, we find the canonical conjugate momentum current p^{abc} as

$$p^{abc} = \frac{1}{\sqrt{-g}}\frac{\partial\mathcal{L}}{\partial\nabla_a h_{bc}} = -\frac{1}{2}\nabla^a h^{bc}, \tag{3.4}$$

where we have used the conditions $\nabla^b h_{ab} = 0$ and $h^a{}_a = 0$. We note that the field equation (3.2) can be given as

$$\nabla_a p^{abc} - \frac{1}{\sqrt{-g}}\frac{\partial\mathcal{L}_{\mathrm{inv}}}{\partial h_{bc}} = 0. \tag{3.5}$$

We define the symplectic product between two solutions $h_{ab}^{(1)}$ and $h_{ab}^{(2)}$ to this equation as follows:

$$(h^{(1)}, h^{(2)})_{\mathrm{symp}} = -i\int_\Sigma d\Sigma_a(\overline{h_{bc}^{(1)}}p^{(2)abc} - \overline{p^{(1)abc}}h_{bc}^{(2)}), \tag{3.6}$$

where $p^{(1)abc}$ is obtained by substituting $h_{ab} = h_{ab}^{(1)}$ into Eq. (3.4) and similarly for $p^{(2)abc}$. This product is independent of the Cauchy surface Σ thanks to Eqs. (3.4) and (3.5). Then, we find

$$\begin{aligned} (H^{(0;2\ell\sigma)}, H^{(0;2\ell'\sigma')})_{\mathrm{symp}} &= \tfrac{1}{2}\langle H^{(0;2\ell\sigma)}, H^{(0;2\ell'\sigma')}\rangle_{\mathrm{KG}} \\ &= \delta^{\ell\ell'}\delta^{\sigma\sigma'}. \end{aligned} \tag{3.7}$$

We expand the quantum field h_{ab} in the synchronous transverse-traceless gauge as

$$h_{ab}(x) = \sum_{\ell=2}^{\infty}\sum_\sigma[a_{\ell\sigma}H_{ab}^{(0;2\ell\sigma)}(x) + a_{\ell\sigma}^\dagger\overline{H_{ab}^{(0;2\ell\sigma)}(x)}]. \tag{3.8}$$

We then impose the commutation relations $[a_{\ell\sigma}, a_{\ell'\sigma'}^\dagger] = \delta_{\ell\ell'}\delta_{\sigma\sigma'}$ with all other commutators vanishing. We define the Bunch-Davies vacuum state $|0\rangle$ by requiring that $a_{\ell\sigma}|0\rangle = 0$ for all ℓ and σ. Then, the Wightman two-point function is readily found as

$$\begin{aligned} \Delta_{aba'b'}^{(\mathrm{phys})}(x, x') &= \langle 0|h_{ab}(x)h_{a'b'}(x')|0\rangle \\ &= \sum_{\ell=2}^{\infty}\sum_\sigma H_{ab}^{(0;2\ell\sigma)}(x)\overline{H_{a'b'}^{(0;2\ell\sigma)}(x')}. \end{aligned} \tag{3.9}$$

This two-point function vanishes if any of the indices is "0". The space components can be found, using Eq. (A9) with $M = 0$, as

$$\begin{aligned} &\Delta_{iji'j'}^{(\mathrm{phys})}(x, x') \\ &= \sum_{\ell=2}^{\infty}(\ell-1)!(\ell+2)!\cosh t\cosh t' \mathrm{P}_1^{-(\ell+1)}(i\sinh t) \\ &\quad\times\overline{\mathrm{P}_1^{-(\ell+1)}(i\sinh t')}\sum_\sigma Y_{ij}^{(2\ell\sigma)}(\mathbf{x})\overline{Y_{i'j'}^{(2\ell\sigma)}}(\mathbf{x}'), \end{aligned} \tag{3.10}$$

where $x = (t, \mathbf{x})$ and $x' = (t', \mathbf{x}')$. This series can be summed in a closed form. The result of this summation can be found in Ref. [9], in which it was shown that there are no IR divergences for this two-point function in the sense that it is well-defined without any infrared cutoff.

IV. SOLUTIONS TO THE FIELD EQUATION IN THE COVARIANT GAUGE

If we add a covariant gauge-fixing term in the Lagrangian, there will be solutions to the graviton field equations in addition to those given by (A8)–(A15) with $M = 0$. In this section, we describe all solutions including these additional solutions to the graviton field equation in the covariant gauge. These solutions will be used in the next section to find the two-point function.

The Lagrangian in the covariant gauge is

$$\mathcal{L} = \mathcal{L}_{\mathrm{inv}} + \mathcal{L}_{\mathrm{gf}}, \tag{4.1}$$

where $\mathcal{L}_{\mathrm{inv}}$ is given by Eq. (3.1) and where

$$\mathcal{L}_{\mathrm{gf}} = -\frac{\sqrt{-g}}{2\alpha}\left(\nabla_a h^{ab} - \frac{1+\beta}{\beta}\nabla^b h\right)\left(\nabla^c h_{cb} - \frac{1+\beta}{\beta}\nabla_b h\right). \tag{4.2}$$

We require that $\alpha > 0$ for now. We also assume $\beta > 0$, but most of our results will be valid also for most negative values of β. The Euler-Lagrange field equation derived from $\mathcal{L}_{\mathrm{inv}} + \mathcal{L}_{\mathrm{gf}}$ is

$$L_{ab}^{(\mathrm{inv})cd}h_{cd} + L_{ab}^{(\mathrm{gf})cd}h_{cd} = 0, \tag{4.3}$$

where $L_{ab}^{(\mathrm{inv})cd}$ is defined by Eq. (3.2) and where

$$L^{(\mathrm{gf})cd}_{ab}h_{cd} = -\frac{1}{2\alpha}\left[\nabla_a\left(\nabla_c h^c{}_b - \frac{1+\beta}{\beta}\nabla_b h\right) + \nabla_b\left(\nabla_c h^c{}_a - \frac{1+\beta}{\beta}\nabla_a h\right)\right] + \frac{1+\beta}{\alpha\beta}g_{ab}\nabla_d\left(\nabla_c h^{cd} - \frac{1+\beta}{\beta}\nabla^d h\right). \tag{4.4}$$

Let us first find the solutions of the form

$$h^{(S)}_{ab} = \nabla_a\nabla_b B + g_{ab}\Psi. \tag{4.5}$$

By substituting this expression into Eq. (4.3), we find

$$\nabla_a\nabla_b X + g_{ab}Y = 0, \tag{4.6}$$

where

$$X = \frac{1}{\alpha\beta}[(\Box - 3\beta)B + (4 - \alpha\beta + 3\beta)\Psi], \tag{4.7}$$

$$Y = -\frac{1+\beta}{\alpha\beta^2}\Box(\Box - 3\beta)B + \left[1 - \frac{4(1+\beta)^2}{\alpha\beta^2} + \frac{1+\beta}{\alpha\beta}\right]\Box\Psi + 3\Psi. \tag{4.8}$$

This calculation is simplified by noting that $\nabla_a\nabla_b B$ does not contribute to $L^{(\mathrm{inv})cd}_{ab}h^{(S)}_{cd}$ due to gauge invariance. Equation (4.6) is obviously satisfied if $X = Y = 0$. These equations can be simplified by solving the equation $X = 0$ for $(\Box - 3\beta)B$ and substituting the result into the equation $Y = 0$. Thus, the equations $X = Y = 0$ can readily be shown to be equivalent to

$$(\Box - 3\beta)B + [4 - (\alpha - 3)\beta]\Psi = 0, \tag{4.9}$$

$$(\Box - 3\beta)\Psi = 0. \tag{4.10}$$

The following solutions and their complex conjugates form a complete set of solutions to Eqs. (4.9) and (4.10):

$$B^{(S1;\ell\sigma)} = \phi^{(3\beta;\ell\sigma)}, \tag{4.11}$$

$$\Psi^{(S1;\ell\sigma)} = 0 \tag{4.12}$$

and

$$B^{(S2;\ell\sigma)} = -[4 - (\alpha - 3)\beta]\frac{\partial}{\partial\mu^2}\phi^{(\mu^2;\ell\sigma)}\Big|_{\mu^2=3\beta}, \tag{4.13}$$

$$\Psi^{(S2;\ell\sigma)} = \phi^{(3\beta;\ell\sigma)}, \tag{4.14}$$

where $\partial/\partial\mu^2$ denotes the first derivative with respect to μ^2 (rather than the second derivative with respect to μ). Equation (4.9) can be verified for the solutions $(B^{(S2;\ell\sigma)}, \Psi^{(S2;\ell\sigma)})$ by noting

$$\frac{\partial}{\partial\mu^2}(-\Box + \mu^2)\phi^{(\mu^2;\ell\sigma)}\Big|_{\mu^2=3\beta} = 0. \tag{4.15}$$

Note that the mass of these modes are β-dependent [8,11]. In particular, if $\beta < 0$, then they are tachyonic because their mass squared is $\mu^2 = 3\beta < 0$. Unfortunately, the familiar de Donder gauge condition, $\nabla^b h_{ab} - \frac{1}{2}\nabla_a h = 0$, corresponds to $\mu^2 = 3\beta = -6$ (and $\alpha \to 0$). Thus, these modes are tachyonic for the de Donder gauge [10,24]. The gauge chosen by Antoniadis and Mottola [7], $\nabla^b h_{ab} - \frac{1}{4}\nabla_a h = 0$, corresponds to $3\beta = -4$. This choice has an additional problem: the scalar field theory suffers IR divergences if $\mu^2 = -k(k+3)$ for $k = 0, 1, 2, \ldots$ [8]. [This fact can readily be seen from Eq. (D7).] This is the cause of the IR divergences in the Antoniadis-Mottola gauge. These problems can easily be avoided by requiring $\beta > 0$.

Although the de Donder gauge ($3\beta = -6$) does not lead to IR divergences in the sense that the two-point function is finite without an IR cutoff, there are IR divergences in its expansion in terms of momentum eigenfunctions in the spatially flat coordinate system [10]. These divergences are due to the growth of the two-point function for large separation, which renders the momentum expansion ill-defined. However, it is possible to regularize the IR divergences in such a way that one recovers the finite two-point function when the regulator is removed, as we show in Appendix C.

Let us write the solutions to Eq. (4.3) corresponding to Eqs. (4.11), (4.12), (4.13), and (4.14) as

$$S^{(1;\ell\sigma)}_{ab} = \nabla_a\nabla_b B^{(S1;\ell\sigma)}, \tag{4.16}$$

$$S^{(2;\ell\sigma)}_{ab} = \nabla_a\nabla_b B^{(S2;\ell\sigma)} + g_{ab}\Psi^{(S2;\ell\sigma)}. \tag{4.17}$$

We show next that any solution h_{ab} to Eqs. (4.3) can be decomposed as $h_{ab} = h^{(T)}_{ab} + h^{(S)}_{ab}$, where $h^{(S)}_{ab}$ is a linear combination of the solutions $S^{(A;\ell\sigma)}_{ab}$, $A = 1, 2$, and their complex conjugates and where $\nabla^a\nabla^b h^{(T)}_{ab} = 0$ and $g^{ab}h^{(T)}_{ab} = 0$. For this purpose, it is sufficient to show that for any given solution h_{ab} to Eq. (4.3), one can construct scalar fields B and Ψ satisfying Eqs. (4.9) and (4.10) such that the field $h^{(S)}_{ab} = \nabla_a\nabla_b B + g_{ab}\Psi$ satisfies $\nabla^a h_{ab} = \nabla^a h^{(S)}_{ab}$ and $g^{ab}h_{ab} = g^{ab}h^{(S)}_{ab}$. To do so, for any solution h_{ab} to Eq. (4.3), we define

$$\Phi(h) = -\frac{1}{\alpha}\left(\nabla_a\nabla_b h^{ab} - \frac{1+\beta}{\beta}\Box h\right). \tag{4.18}$$

Then, by taking the divergence of Eq. (4.3) twice, we find

$$(\Box - 3\beta)\Phi(h) = 0. \tag{4.19}$$

This calculation is made easier by noting that the tensor field $L^{(\mathrm{inv})cd}_{ab}h_{cd}$ is divergence-free (the background Bianchi identity). Next, by taking the trace of Eq. (4.3) and using Eq. (4.19), we find

$$(\Box - 3\beta)h + [4 - (\alpha - 3)\beta]\Phi(h) = 0. \tag{4.20}$$

Now, we define

$$h^{(S)}_{ab} = \nabla_a\nabla_b B(h) + g_{ab}\Psi(h), \tag{4.21}$$

where

$$B(h) = \frac{1}{3\beta}\left(h - \frac{\alpha-3}{3}\Phi(h)\right), \tag{4.22}$$

$$\Psi(h) = \frac{1}{3\beta}\Phi(h). \tag{4.23}$$

Then, one can readily see that Eqs. (4.19) and (4.20) imply Eqs. (4.9) and (4.10). Thus, $h^{(S)}_{ab}$ is a solution to Eq. (4.3). Moreover, we find

$$g^{ab}h^{(S)}_{ab} = \Box B(h) + 4\Psi(h) = h \tag{4.24}$$

and

$$\nabla^a\nabla^b h^{(S)}_{ab} - \frac{1+\beta}{\beta}g^{ab}\Box h^{(S)}_{ab} = \nabla_a\nabla_b h^{ab} - \frac{1+\beta}{\beta}\Box h, \tag{4.25}$$

and, hence,

$$\nabla^a\nabla^b h^{(S)}_{ab} = \nabla^a\nabla^b h_{ab}. \tag{4.26}$$

Thus, any h_{ab} satisfying Eq. (4.3) can be written as $h_{ab} = h^{(T)}_{ab} + h^{(S)}_{ab}$, where $\nabla^a\nabla^b h^{(T)}_{ab} = 0$ and $g^{ab}h^{(T)}_{ab} = 0$.

Our next task is to construct all solutions to Eq. (4.3) satisfying $\nabla^a\nabla^b h^{(T)}_{ab} = 0$ and $g^{ab}h^{(T)}_{ab} = 0$. Equation (4.3) becomes

$$\begin{aligned} L^{(T)cd}_{ab}h^{(T)}_{cd} \equiv & -\frac{1}{2}\Box h^{(T)}_{ab} \\ & +\frac{1}{2}\left(1-\frac{1}{\alpha}\right)(\nabla_a\nabla_c h^{(T)c}_b + \nabla_b\nabla_c h^{(T)c}{}_a) + h^{(T)}_{ab} \\ = & \, 0. \end{aligned} \tag{4.27}$$

We show that a complete set of solutions $h^{(T)}_{ab}$ is given by

$$E^{(1;m\ell\sigma)}_{ab} \equiv H^{(0;m\ell\sigma)}_{ab}, \qquad m = 0, 1, 2, \qquad \ell \geq 2, \tag{4.28}$$

$$\begin{aligned} E^{(2;m\ell\sigma)}_{ab} & \equiv \lim_{M\to 0}\frac{1}{M^2}(\nabla_a A^{(\mu^2;m\ell\sigma)}_b + \nabla_b A^{(\mu^2;m\ell\sigma)}_a - H^{(M^2;m\ell\sigma)}_{ab}) \\ & = 2\alpha\frac{\partial}{\partial\mu^2}\nabla_{(a}A^{(\mu^2;m\ell\sigma)}_{b)}\bigg|_{\mu^2=-6} \\ & \quad - \frac{\partial}{\partial M^2}H^{(M^2;m\ell\sigma)}_{ab}\bigg|_{M^2=0}, \qquad m = 0, 1, \qquad \ell \geq 1, \end{aligned} \tag{4.29}$$

and their complex conjugates, with the identification

$$\mu^2 = \alpha M^2 - 6. \tag{4.30}$$

We have defined $H^{(M^2;m,\ell=1,\sigma)}_{ab} = 0$ in the second equation. The second equality in Eq. (4.29) follows from Eq. (2.25), which is valid also for $\ell = 1$.

One can readily see that $h^{(T)}_{ab} = E^{(1;m\ell\sigma)}_{ab}$ and their complex conjugates give a complete set of solutions to Eq. (4.27) under a stronger condition $\nabla^b h^{(T)}_{ab} = 0$. The tensor fields $E^{(2;m\ell\sigma)}_{ab}$ and their complex conjugates are also solutions (under the original condition $\nabla^a\nabla^b h^{(T)}_{ab} = 0$) because both $h^{(M^2)}_{ab} = \nabla_{(a}A^{(\mu^2;m\ell\sigma)}_{b)}$ and $H^{(M^2;m\ell\sigma)}_{ab}$ are solutions to the massive equation [25]

$$L^{(T)cd}_{ab}h^{(M^2)}_{cd} + \tfrac{1}{2}M^2(h^{(M^2)}_{ab} - g_{ab}h^{(M^2)c}_c) = 0. \tag{4.31}$$

Then, what is left to do is show that for any solution $h^{(T)}_{ab}$ of Eq. (4.27), we can find a linear combination $h^{(W)}_{ab}$ of $E^{(2;m\ell\sigma)}_{ab}$ and their complex conjugates such that $C_a(h^{(T)}) \equiv \nabla^b h^{(T)}_{ab} = \nabla^b h^{(W)}_{ab}$. This can be done as follows. By taking the divergence of Eq. (4.27), we find

$$(\Box + 3)C_a(h^{(T)}) = 0. \tag{4.32}$$

A complete set of solutions to this equation is given by $C_a = A^{(-6;m\ell\sigma)}_a$, $m = 0, 1$, $\ell \geq 1$, and their complex conjugates. Now, since

$$\nabla^b(\nabla_a A^{(\mu^2;m\ell\sigma)}_b + \nabla_b A^{(\mu^2;m\ell\sigma)}_a) = \alpha M^2 A^{(\mu^2;m\ell\sigma)}_a \tag{4.33}$$

and $H^{(M^2;m\ell\sigma)}_{ab}$ are divergence-free, we find

$$\nabla^b E^{(2;m\ell\sigma)}_{ab} = \alpha A^{(-6;m\ell\sigma)}_a. \tag{4.34}$$

Hence, if $C_a(h^{(T)}) = \nabla^b h^{(T)}_{ab} = A^{(-6;m\ell\sigma)}_a$, then we have $\nabla^b h^{(W)}_{ab} = C_a(h^{(T)})$ by setting $h^{(W)}_{ab} = \alpha^{-1}E^{(2;m\ell\sigma)}_{ab}$. It is clear that a similar construction works if $h^{(T)}_{ab}$ is any linear combination of $A^{(-6;m\ell\sigma)}_a$ and their complex conjugates.

Thus, we have constructed a complete set of solutions to Eq. (4.3), and these solutions are given by Eqs. (4.16), (4.17), (4.28), and (4.29) and their complex conjugates.

V. THE TWO-POINT FUNCTION IN THE COVARIANT GAUGE

In this section, we compute the Wightman two-point function for the quantized linearized-gravity field h_{ab} and show that it is physically equivalent to the physical two-point function of Ref. [9] in linearized gravity.

We define the momentum current conjugate to the field h_{ab} by

$$p^{abc} \equiv \frac{1}{\sqrt{-g}}\frac{\partial\mathcal{L}}{\partial(\nabla_a h_{bc})} = p^{abc}_{\text{inv}} + p^{abc}_{\text{gf}}, \tag{5.1}$$

where

$$\begin{aligned} p^{abc}_{\text{inv}} = & -\tfrac{1}{2}\nabla^a h^{bc} + \tfrac{1}{2}(g^{ab}\nabla_d h^{dc} + g^{ac}\nabla_d h^{db} - g^{bc}\nabla_d h^{da}) \\ & - \tfrac{1}{4}(g^{ab}\nabla^c h + g^{ac}\nabla^b h) + \tfrac{1}{2}g^{bc}\nabla^a h, \end{aligned} \tag{5.2}$$

$$p_{\rm gf}^{abc} = -\frac{1}{2\alpha} g^{ab}\left(\nabla_d h^{dc} - \frac{1+\beta}{\beta}\nabla^c h\right) - \frac{1}{2\alpha} g^{ac}\left(\nabla_d h^{db} - \frac{1+\beta}{\beta}\nabla^b h\right) + \frac{1+\beta}{\alpha\beta} g^{bc}\left(\nabla_d h^{da} - \frac{1+\beta}{\beta}\nabla^a h\right). \quad (5.3)$$

Then the Euler-Lagrange equation (4.3) can be written as

$$\nabla_a p^{abc} - \frac{1}{\sqrt{-g}}\frac{\partial \mathcal{L}}{\partial h_{bc}} = 0. \quad (5.4)$$

The equal-time commutation relations on a $t = \text{const}$ Cauchy surface are then given by

$$[h_{ab}(t,\mathbf{x}), h_{cd}(t,\mathbf{x}')] = [p^{0ab}(t,\mathbf{x}), p^{0cd}(t,\mathbf{x}')] = 0, \quad (5.5)$$

$$[h_{ab}(t,\mathbf{x}), p^{0cd}(t,\mathbf{x}')] = \frac{\sqrt{-g(x)}}{2}(\delta_a^c\delta_b^d + \delta_a^d\delta_b^c)\delta(\mathbf{x},\mathbf{x}'), \quad (5.6)$$

where $\delta(\mathbf{x},\mathbf{x}')$ is defined by

$$\int_{S^3} d^3\mathbf{x} f(\mathbf{x})\delta(\mathbf{x},\mathbf{x}') = f(\mathbf{x}'). \quad (5.7)$$

Here, the $d^3\mathbf{x}$ is the coordinate volume element. That is, if θ_1, θ_2 and θ_3 are the coordinates on S^3, then $d^3\mathbf{x} = d\theta_1 d\theta_2 d\theta_3$.

For any two solutions $h_{ab}^{(1)}$ and $h_{ab}^{(2)}$ to the Euler-Lagrange equation (4.3), we define the symplectic product by

$$(h^{(1)}, h^{(2)})_{\rm symp} = -i\int_\Sigma d\Sigma_a(\overline{h_{bc}^{(1)}} p^{(2)abc} - \overline{p^{(1)abc}} h_{bc}^{(2)}), \quad (5.8)$$

where

$$p^{(1)abc} \equiv \frac{\partial \mathcal{L}}{\partial(\nabla_a h_{bc})}\bigg|_{h_{ab}=h_{ab}^{(1)}} \quad (5.9)$$

on a Cauchy surface Σ, and similarly for $p^{(2)abc}$. This symplectic product is independent of the Cauchy surface Σ because the current $\overline{h_{bc}^{(1)}} p^{(2)abc} - \overline{p^{(1)abc}} h_{bc}^{(2)}$ is conserved [26,27]. If $h_{ab}^{(1)}$ and $h_{ab}^{(2)}$ are transverse-traceless positive-frequency solutions from Sec. III, then this symplectic product agrees with Eq. (3.6).

Now, we can expand the quantum field h_{ab} using the solutions found in Sec. IV as follows:

$$\begin{aligned} h_{ab}(x) = &\sum_{\ell=2}^{\infty}\sum_\sigma a_{\ell\sigma}^{(TT)} H_{ab}^{(0;2\ell\sigma)}(x) \\ &+ \sum_{m=0}^{1}\left[\sum_{\ell=2}^{\infty}\sum_\sigma a_{m\ell\sigma}^{(T1)} E_{ab}^{(1;m\ell\sigma)}(x) + \sum_{\ell=1}^{\infty}\sum_\sigma a_{m\ell\sigma}^{(T2)} E_{ab}^{(2;m\ell\sigma)}(x)\right] \\ &+ \sum_{\ell=1}^{\infty}\sum_\sigma [a_{\ell\sigma}^{(S1)} S_{ab}^{(1\ell\sigma)}(x) + a_{\ell\sigma}^{(S2)} S_{ab}^{(2\ell\sigma)}(x)] + \text{H.c.} \end{aligned} \quad (5.10)$$

Let us denote the symplectic product among these solutions as follows:

$$M_{AB}^{(G;m)} = (E^{(A;m\ell\sigma)}, E^{(B;m\ell\sigma)})_{\rm symp}, \quad A,B = 1,2, \quad m = 0,1, \quad \ell \geq 2, \quad (5.11)$$

$$M^{(G1;m)} = (E^{(2;m,\ell=1,\sigma)}, E^{(2;m,\ell=1,\sigma)})_{\rm symp}, \qquad m = 0,1, \quad (5.12)$$

$$M_{AB}^{(S)} = (S^{(A;\ell\sigma)}, S^{(B;\ell\sigma)})_{\rm symp}, \qquad A,B = 1,2. \quad (5.13)$$

[It turns out that these matrix elements are independent of ℓ and σ. We have already seen that $(H^{(0;2\ell\sigma)}, H^{(0;2\ell'\sigma')})_{\rm symp} = \delta^{\ell\ell'}\delta^{\sigma\sigma'}$.] In Appendix B, we show that $S_{ab}^{(A;\ell\sigma)}$ are orthogonal to the solutions $E_{ab}^{(A;m\ell\sigma)}$ with respect to the symplectic product (5.8). Then, it is not difficult to show that the equal-time commutation relations (5.5) and (5.6) imply

$$[a_{\ell\sigma}^{(TT)}, a_{\ell'\sigma'}^{(TT)\dagger}] = \delta_{\ell\ell'}\delta_{\sigma\sigma'}, \quad (5.14)$$

$$[a_{m1\sigma}^{(T2)}, a_{m'1\sigma'}^{(T2)\dagger}] = (M^{(G1;m)})^{-1}\delta_{mm'}\delta_{\sigma\sigma'}, \quad (5.15)$$

$$[a_{m\ell\sigma}^{(TA)}, a_{m'\ell'\sigma'}^{(TB)\dagger}] = (M^{(G;m)})_{AB}^{-1}\delta_{mm'}\delta_{\ell\ell'}\delta_{\sigma\sigma'}, \quad (5.16)$$

$$[a_{\ell\sigma}^{(SA)}, a_{\ell'\sigma'}^{(SB)\dagger}] = (M^{(S)})_{AB}^{-1}\delta_{\ell\ell'}\delta_{\sigma\sigma'}. \quad (5.17)$$

(See, e.g. Ref. [28].) Then the Wightman two-point function for the Bunch-Davies vacuum can be given as follows:

$$\begin{aligned} \langle 0|h_{ab}(x)h_{a'b'}(x')|0\rangle &= \Delta_{aba'b'}(x,x') \\ &= \Delta_{aba'b'}^{\rm (phys)}(x,x') + \Delta_{aba'b'}^{(G)}(x,x') + \Delta_{aba'b'}^{(S)}(x,x'), \end{aligned} \quad (5.18)$$

where $\Delta_{aba'b'}^{\rm (phys)}(x,x')$ is the physical two-point function discussed in Sec. III and where

$$\Delta^{(G)}_{aba'b'}(x,x') = \sum_{m=0}^{1}\sum_{\sigma}(M^{(G1;m)})^{-1} \times E^{(2;m,\ell=1,\sigma)}_{ab}(x)\overline{E^{(2;m,\ell=1,\sigma)}_{a'b'}(x')} + \sum_{m=0}^{1}\sum_{\ell=2}^{\infty}\sum_{\sigma}(M^{(G;m)})^{-1}_{AB} \times E^{(A;m\ell\sigma)}_{ab}(x)\overline{E^{(B;m\ell\sigma)}_{a'b'}(x')}, \tag{5.19}$$

$$\Delta^{(S)}_{aba'b'}(x,x') = \sum_{\ell=0}^{\infty}\sum_{\sigma}(M^{(S)})^{-1}_{AB}S^{(A;\ell\sigma)}_{ab}(x)\overline{S^{(B;\ell\sigma)}_{a'b'}(x')}. \tag{5.20}$$

Here, the summation over A and B is understood. Thus, all we need to do is find the matrix elements of the symplectic product defined by Eqs. (5.11), (5.12), and (5.13).

First, we compute $M^{(G;m)}_{AB}$ for $\ell \geq 2$ and $M^{(G1;m)}$ and find $\Delta^{(G)}_{aba'b'}(x,x')$ defined by Eq. (5.19). Let us define the invariant and gauge-fixing parts of the symplectic product as follows:

$$(h^{(1)},h^{(2)})_{\rm inv} = -i\int_\Sigma d\Sigma_a(\overline{h^{(1)}_{bc}}p^{(2)abc}_{\rm inv} - \overline{p^{(1)abc}_{\rm inv}}h^{(2)}_{bc}), \tag{5.21}$$

$$(h^{(1)},h^{(2)})_{\rm gf} = -i\int_\Sigma d\Sigma_a(\overline{h^{(1)}_{bc}}p^{(2)abc}_{\rm gf} - \overline{p^{(1)abc}_{\rm gf}}h^{(2)}_{bc}). \tag{5.22}$$

It is well-known that if $h^{(k)}_{ab} = \nabla_a A^{(k)}_b + \nabla_b A^{(k)}_a$ for some $A^{(k)}_a$, $k=1,2$, then $(h^{(1)},h^{(2)})_{\rm inv} = 0$ (see, e.g. Ref. [29]). Now, the solutions $E^{(1;m\ell\sigma)}_{ab}$ are of this form for $m=0,1$ and are divergence-free and traceless. This implies that $p^{abc}_{\rm gf} = 0$ for these solutions, and, hence,

$$M^{(G;m)}_{11} = (E^{(1;m\ell\sigma)},E^{(1;m\ell\sigma)})_{\rm inv} = 0, \qquad \ell \geq 2. \tag{5.23}$$

Next, we examine $M^{(G;m)}_{22}$. We write Eq. (4.29) as

$$E^{(2;m\ell\sigma)}_{ab} = \lim_{M\to 0}\frac{1}{M^2}(K^{(M^2;m\ell\sigma)}_{ab} - H^{(M^2;m\ell\sigma)}_{ab}), \tag{5.24}$$

where

$$K^{(M^2;m\ell\sigma)}_{ab} = \nabla_a A^{(\mu^2;m\ell\sigma)}_b + \nabla_b A^{(\mu^2;m\ell\sigma)}_a, \tag{5.25}$$

with $\mu^2 = \alpha M^2 - 6$ [see Eq. (4.30)]. We have

$$(H^{(M^2;m\ell\sigma)},H^{(M^2;m'\ell'\sigma')})_{\rm symp} = (-1)^{m+1}M^2\delta_{mm'}\delta_{\ell\ell'}\delta_{\sigma\sigma'} \tag{5.26}$$

for $m, m' = 0, 1$ from Eq. (2.24) because the symplectic product (5.8) is half the Klein-Gordon inner product (2.23) for these solutions. The symplectic product for the modes $K^{(M^2;m\ell\sigma)}_{ab}$ can be found as follows. First, since these are of pure-gauge form, we have

$$(K^{(M^2;m\ell\sigma)},K^{(M^2;m'\ell'\sigma')})_{\rm inv} = 0. \tag{5.27}$$

Hence,

$$(K^{(M^2;m\ell\sigma)},K^{(M^2;m'\ell'\sigma')})_{\rm symp} = (K^{(M^2;m\ell\sigma)},K^{(M^2;m'\ell'\sigma')})_{\rm gf}. \tag{5.28}$$

For $h_{ab} = K^{(M^2;m\ell\sigma)}_{ab}$, we find

$$p^{abc}_{\rm gf} = -\tfrac{1}{2}M^2(g^{ab}A^{(\mu^2;m\ell\sigma)c} + g^{ac}A^{(\mu^2;m\ell\sigma)b}). \tag{5.29}$$

Using this equation and the equality

$$A^c\nabla_c A'^a - A'^c\nabla_c A^a = \nabla_c(A'^a A^c - A^a A'^c) \quad \text{if } \nabla_c A^c = \nabla_c A'^c = 0 \tag{5.30}$$

and the generalized Stokes theorem, we obtain

$$(K^{(M^2;m\ell\sigma)},K^{(M^2;m'\ell'\sigma')})_{\rm symp} = -M^2\langle A^{(\mu^2;m\ell\sigma)},A^{(\mu^2;m'\ell'\sigma')}\rangle_{\rm KG} = (-1)^m M^2\delta_{mm'}\delta_{\ell\ell'}\delta_{\sigma\sigma'}. \tag{5.31}$$

Equation (5.26) and this equation together with the fact that there are no cross terms [28], i.e. $(K^{(M^2;m\ell\sigma)},H^{(M^2;m'\ell'\sigma')}) = 0$, imply

$$M^{(G;m)}_{22} = 0. \tag{5.32}$$

Finally, Eq. (5.31) and the fact that there are no cross terms lead to

$$M^{(G;m)}_{12} = M^{(G;m)}_{21} = (-1)^m. \tag{5.33}$$

The first line of Eq. (5.31) is valid for $\ell = 1$ as well, but in this case, $H^{(M^2;m,\ell=1,\sigma)}_{ab} = 0$ in Eq. (5.24). Hence, we find, noting Eq. (4.30),

$$M^{(G1;m)} = (-1)^m\lim_{M\to 0}\frac{\mu^2+6}{M^2} = (-1)^m\alpha. \tag{5.34}$$

Clearly, the inverse of the matrix $M^{(G;m)}_{AB}$ is itself, and $(M^{(G1;m)})^{-1} = (-1)^m\alpha^{-1}$. Hence, from Eq. (5.19), we find

$$\Delta^{(G)}_{aba'b'}(x,x') = \alpha^{-1}\sum_{m=0}^{1}\sum_{\sigma}(-1)^m E^{(2;m,\ell=1,\sigma)}_{ab}(x) \times \overline{E^{(2;m,\ell=1,\sigma)}_{a'b'}(x')} + \sum_{m=0}^{1}\sum_{\ell=2}^{\infty}\sum_{\sigma}(-1)^m\Big[E^{(1;m\ell\sigma)}_{ab}(x)\overline{E^{(2;m\ell\sigma)}_{a'b'}(x')} + E^{(2;m\ell\sigma)}_{ab}(x)\overline{E^{(1;m\ell\sigma)}_{a'b'}(x')}\Big]. \tag{5.35}$$

Now, we define the vector two-point function with squared mass μ^2 as

$$\begin{aligned}\Delta^{(V;\mu^2)}_{aa'}(x,x') &\equiv \sum_{m=0}^{1}\sum_{\ell=1}^{\infty}\sum_{\sigma}\langle A^{(\mu^2;m\ell\sigma)},A^{(\mu^2;m\ell\sigma)}\rangle^{-1}_{\mathrm{KG}}A^{(\mu^2;m\ell\sigma)}_a(x)\overline{A^{(\mu^2;m\ell\sigma)}_{a'}(x')}\\ &= (\mu^2+6)^{-1}\sum_{m=0}^{1}\sum_{\sigma}(-1)^{m+1}A^{(\mu^2;m,\ell=1,\sigma)}_a(x)\overline{A^{(\mu^2;m,\ell=1,\sigma)}_{a'}(x')}\\ &\quad+\sum_{m=0}^{1}\sum_{\ell=2}^{\infty}\sum_{\sigma}(-1)^{m+1}A^{(\mu^2;m\ell\sigma)}_a(x)\overline{A^{(\mu^2;m\ell\sigma)}_{a'}(x')}.\end{aligned}\tag{5.36}$$

Let us also write

$$\Delta^{(V;(1)\mu^2)}_{aa'}(x,x')\equiv\frac{\partial}{\partial\mu^2}\Delta^{(V;\mu^2)}_{aa'}(x,x')\tag{5.37}$$

and define

$$U_{aa'b'}(x,x')\equiv\sum_{m=0}^{1}\sum_{\ell=2}^{\infty}\sum_{\sigma}A^{(-6;m\ell\sigma)}_a(x)\times\frac{\partial}{\partial M^2}\overline{H^{(M^2;m\ell\sigma)}_{a'b'}(x')}|_{M^2=0}.\tag{5.38}$$

Then, by Eqs. (4.28), (4.29), and (4.25), we find

$$\begin{aligned}\Delta^{(G)}_{aba'b'}(x,x') &= -2\alpha\lim_{\mu^2\to-6}[\nabla_{(a}\nabla_{|a'|}\Delta^{(V;(1)\mu^2)}_{b)b'}(x,x')\\ &\quad+\nabla_{(a}\nabla_{|b'|}\Delta^{(V;(1)\mu^2)}_{b)a'}(x',x)]\\ &\quad+2\nabla_{(a}U_{b)a'b'}(x,x')+2\nabla_{(a'}\overline{U_{b')ab}(x',x)}.\end{aligned}\tag{5.39}$$

We have used

$$\lim_{\mu^2\to-6}(\mu^2+6)^{-1}\nabla_{(a}A^{(\mu^2;m,\ell=1,\sigma)}_{b)}=\frac{\partial}{\partial\mu^2}\nabla_{(a}A^{(\mu^2;m,\ell=1,\sigma)}_{b)}|_{\mu^2=-6},\tag{5.40}$$

which is true because $\nabla_{(a}A^{(-6;m,\ell=1,\sigma)}_{b)}=0$. Thus, $\Delta^{(G)}_{aba'b'}$ is of pure-gauge form, i.e. is equivalent to 0 in linearized gravity.

Next, we find the matrix $M^{(S)}_{AB}$ for the solutions $S^{(A;\ell\sigma)}_{ab}$, $A=1,2$, given by Eqs. (4.16) and (4.17) and use it to find $\Delta^{(S)}_{aba'b'}(x,x')$ defined by Eq. (5.20). We first express the symplectic product of two solutions

$$S^{(k)}_{ab}=\nabla_a\nabla_bB^{(k)}+g_{ab}\Psi^{(k)},\qquad k=1,2,\tag{5.41}$$

in terms of the Klein-Gordon inner product (2.7). Let us write the conjugate momentum current for the solutions $S^{(k)}_{ab}$ as

$$p^{(k)abc}=p^{(B,k)abc}_{\mathrm{inv}}+p^{(\Psi,k)abc}_{\mathrm{inv}}+p^{(k)abc}_{\mathrm{gf}},\tag{5.42}$$

where $p^{(B,k)abc}_{\mathrm{inv}}$ and $p^{(\Psi,k)abc}_{\mathrm{inv}}$ are the contribution of $\nabla_b\nabla_cB^{(k)}$ and $g_{bc}\Psi^{(k)}$, respectively, to $p^{(k)abc}_{\mathrm{inv}}$ defined by Eq. (5.2). The tensor $p^{(k)abc}_{\mathrm{gf}}$ is defined by Eq. (5.3). As noted after Eq. (5.22), we have

$$\int_\Sigma d\Sigma_a[\nabla_b\nabla_c\overline{B^{(1)}}p^{(B,2)abc}_{\mathrm{inv}}-\overline{p^{(B,1)abc}_{\mathrm{inv}}}\nabla_b\nabla_cB^{(2)}]=0.\tag{5.43}$$

Hence,

$$\begin{aligned}(S^{(1)},S^{(2)})_{\mathrm{symp}} &= -i\int_\Sigma d\Sigma_a[\nabla_b\nabla_c\overline{B^{(1)}}(p^{(\Psi,2)abc}_{\mathrm{inv}}+p^{(2)abc}_{\mathrm{gf}})\\ &\quad-(\overline{p^{(\Psi,1)abc}_{\mathrm{inv}}}+\overline{p^{(1)abc}_{\mathrm{gf}}})\nabla_b\nabla_cB^{(2)}]\\ &\quad-i\int_\Sigma d\Sigma_a[g_{bc}\overline{\Psi^{(1)}}p^{(2)abc}-\overline{p^{(1)abc}}g_{bc}\Psi^{(2)}].\end{aligned}\tag{5.44}$$

By straightforward calculations, we find

$$p^{(\Psi,k)abc}_{\mathrm{inv}}+p^{(k)abc}_{\mathrm{gf}}=-\frac{1}{\beta}g^{bc}\nabla^a\Psi^{(k)},\tag{5.45}$$

$$g_{bc}p^{(k)abc}=-\frac{4}{\beta}\nabla^a\Psi^{(k)}-3\nabla^aB^{(k)}.\tag{5.46}$$

By substituting these equations into Eq. (5.44) and using the field equation (4.9) satisfied by $B^{(1)}$ and $B^{(2)}$, we find

$$\begin{aligned}(S^{(1)},S^{(2)})_{\mathrm{symp}} &= 3(\langle B^{(1)},\Psi^{(2)}\rangle_{\mathrm{KG}}+\langle\Psi^{(1)},B^{(2)}\rangle_{\mathrm{KG}})\\ &\quad+(\alpha-3)\langle\Psi^{(1)},\Psi^{(2)}\rangle_{\mathrm{KG}}.\end{aligned}\tag{5.47}$$

Note that $\langle B^{(1)},\Psi^{(2)}\rangle_{\mathrm{KG}}$, $\langle\Psi^{(1)},B^{(2)}\rangle_{\mathrm{KG}}$ and $\langle\Psi^{(1)},\Psi^{(2)}\rangle_{\mathrm{KG}}$ are not conserved individually, though $(S^{(1)},S^{(2)})_{\mathrm{symp}}$ is. The symplectic product for the solutions $S^{(A;\ell\sigma)}_{ab}$, $A=1,2$, given by Eqs. (4.11), (4.12), (4.13), and (4.14) is then

$$M^{(S)}_{11}=0,\qquad M^{(S)}_{12}=3,\qquad M^{(S)}_{22}=\alpha-3.\tag{5.48}$$

We have used

$$\left\langle\phi^{(\mu^2;\ell\sigma)},\frac{\partial}{\partial\mu^2}\phi^{(\mu^2;\ell\sigma)}\right\rangle_{\mathrm{KG}}+\left\langle\frac{\partial}{\partial\mu^2}\phi^{(\mu^2;\ell\sigma)},\phi^{(\mu^2;\ell\sigma)}\right\rangle_{\mathrm{KG}}=\frac{\partial}{\partial\mu^2}\langle\phi^{(\mu^2;\ell\sigma)},\phi^{(\mu^2;\ell\sigma)}\rangle_{\mathrm{KG}}=0\tag{5.49}$$

in computing $M^{(S)}_{22}$. The inverse of the matrix $M^{(S)}_{AB}$ is given by

$$(M^{(S)})^{-1}_{11} = \tfrac{1}{9}(3-\alpha), \quad (M^{(S)})^{-1}_{12} = \tfrac{1}{3}, \quad (M^{(S)})^{-1}_{22} = 0. \tag{5.50}$$

Hence, defining the two-point function for the scalar field with mass μ and its μ^2-derivative as

$$\Delta_{\mu^2}(x,x') = \sum_{\ell=0}^{\infty}\sum_{\sigma}\phi^{(\mu^2;\ell\sigma)}(x)\overline{\phi^{(\mu^2;\ell\sigma)}(x')}, \tag{5.51}$$

$$\Delta^{(1)}_{\mu^2}(x,x') = -\frac{\partial}{\partial\mu^2}\Delta_{\mu^2}(x,x'), \tag{5.52}$$

and substituting Eq. (5.50) into Eq. (5.20), we find

$$\Delta^{(S)}_{aba'b'}(x,x') = \nabla_a\nabla_b\nabla_{a'}\nabla_{b'}\left\{\frac{3-\alpha}{9}\Delta_{3\beta}(x,x') + \frac{1}{3}[4-(\alpha-3)\beta]\Delta^{(1)}_{3\beta}(x,x')\right\} + \frac{1}{3}(g_{ab}\nabla_{a'}\nabla_{b'} + g_{a'b'}\nabla_a\nabla_b)\Delta_{3\beta}(x,x'). \tag{5.53}$$

This is clearly of pure-gauge form and generalizes the result obtained in Ref. [30], where $\Delta^{(S)}_{aba'b'}(x,x')$ for the cases with $(\alpha-3)\beta = 4$ was found.

Thus, we have shown that the two-point function $\Delta_{aba'b'}(x,x')$ in the covariant gauge given by Eq. (5.18) is equivalent to $\Delta^{(\mathrm{phys})}_{aba'b'}(x,x')$ in linearized gravity because $\Delta^{(G)}_{aba'b'}(x,x')$ and $\Delta^{(S)}_{aba'b'}(x,x')$ given by Eqs. (5.39) and (5.53) are of pure-gauge form. Notice that the $\alpha\to 0$ limit of $\Delta_{aba'b'}(x,x')$ is well-defined and de Sitter-covariant unless $3\beta = -k(k+3)$, where k is a non-negative integer. (In the $\alpha\to 0$ limit, the gauge condition $\nabla^b h_{ab} - \frac{1+\beta}{\beta}\nabla_a h = 0$ is strictly enforced on h_{ab}.) Thus, we disagree with the authors of Ref. [10], who claim that de Sitter invariance is broken in the case $\alpha = 0$ and $\beta = -2$.

One of the main observations in Ref. [10] is that the scalar two-point function $\Delta_{\mu^2}(x,x')$, which appears in the scalar part $\Delta^{(s)}_{aba'b'}$ of the graviton two-point function, is IR-divergent for all negative μ^2. This is true if this two-point function is constructed in the Poincaré patch of de Sitter spacetime in momentum expansion. However, no IR divergences are encountered in the mode-sum construction of $\Delta_{\mu^2}(x,x')$ in global de Sitter spacetime as shown in Appendix C. (We also show in this appendix that the IR-finite two-point function is recovered even in the Poincaré patch by appropriate subtraction.)

Finally, we write $\Delta^{(\mathrm{phys})}_{aba'b'} + \Delta^{(G)}_{aba'b'}$ in a covariant form. We first define $\Delta^{(TT;M^2)}_{aba'b'}(x,x')$ to be twice the two-point function for the transverse-traceless symmetric tensor field with mass $M \neq 0$, satisfying

$$[\Box_x - (2+M^2)]\Delta^{(TT;M^2)}_{aba'b'}(x,x') = 0. \tag{5.54}$$

It can be given in the mode-sum form as

$$\begin{aligned}\Delta^{(TT;M^2)}_{aba'b'}(x,x') &= 2\sum_{m=0}^{2}\sum_{\ell=2}^{\infty}\sum_{\sigma}\langle H^{(M^2;m\ell\sigma)}, H^{(M^2;m\ell\sigma)}\rangle^{-1}_{\mathrm{KG}} \times H^{(M^2;m\ell\sigma)}_{ab}(x)\overline{H^{(M^2;m\ell\sigma)}_{a'b'}(x')}\\ &= \sum_{\ell=2}^{\infty}\sum_{\sigma}H^{(M^2;2\ell\sigma)}_{ab}(x)\overline{H^{(M^2;2\ell\sigma)}_{a'b'}(x')} + \frac{1}{M^2}\sum_{m=0}^{1}\sum_{\ell=2}^{\infty}\sum_{\sigma}(-1)^{m+1} \times H^{(M^2;m\ell\sigma)}_{ab}(x)\overline{H^{(M^2;m\ell\sigma)}_{a'b'}(x')}.\end{aligned} \tag{5.55}$$

(See Ref. [31] for an explicit form of $\Delta^{(TT,M^2)}_{aba'b'}$.) Then, we find from Eq. (5.35) and the definition $E^{(1;m\ell\sigma)}_{ab} = H^{(0;m\ell\sigma)}_{ab}$ for $m = 0, 1$ [see Eq. (4.28)]

$$\Delta^{(\mathrm{phys})}_{aba'b'}(x,x') + \Delta^{(G)}_{aba'b'}(x,x') = \Delta^{(TT)}_{aba'b'}(x,x') + \Delta^{(V)}_{aba'b'}(x,x'), \tag{5.56}$$

where

$$\Delta^{(TT)}_{aba'b'}(x,x') = \lim_{M^2\to 0}\frac{\partial}{\partial M^2}[M^2\Delta^{(TT;M^2)}_{aba'b'}(x,x')], \tag{5.57}$$

$$\Delta^{(V)}_{aba'b'}(x,x') = -2\alpha\lim_{\mu^2\to -6}[\nabla_{(a}\nabla_{|a'|}\Delta^{(V;(1)\mu^2)}_{b)b'}(x,x') + \nabla_{(a}\nabla_{|b'|}\Delta^{(V;(1)\mu^2)}_{b)a'}(x,x')]. \tag{5.58}$$

These expressions will be used in Appendix B to compare the two-point function found here and the corresponding result in the Euclidean approach [11].

VI. SUMMARY

In this paper, we investigated the relationship between the covariant graviton Wightman two-point function and the physical transverse-traceless and synchronous one in global coordinates. We defined two Wightman graviton two-point functions, $\Delta^{(1)}_{aba'b'}(x,x')$ and $\Delta^{(2)}_{aba'b'}(x,x')$, to be physically equivalent in linearized gravity if they differ by a bitensor of the form $\nabla_{(a}Q_{b)a'b'}(x,x') + \nabla_{(a'}Q_{|ab|b')}(x,x')$ and showed that the covariant two-point function is physically equivalent to the physical two-point function in global coordinates. Our result is perhaps not surprising, but since there has been much controversy over infrared properties of graviton two-point functions, we believe that it is a worthwhile addition to the body of knowledge about gravitational perturbation in de Sitter spacetime.

Although our result holds for all α and β except $\beta = -L_0(L_0+3)$, L_0 non-negative integer, in global de Sitter spacetime, one encounters some difficulties if $\beta < 0$ in the Poincaré patch because the scalar sector $\Delta^{(S)}_{aba'b'}(x,x')$ becomes tachyonic. This is also the case for the vector sector

$\Delta^{(V)}_{aba'b'}(x,x')$ if $\alpha \neq 0$. (See also Ref. [32] for related difficulties with $\alpha \neq 0$.) However, none of the objections raised in Refs. [10,32] are relevant with the choices of gauge parameters $\alpha = 0$ and $\beta > 0$ and the de Sitter-covariant two-point function can be constructed without any ambiguities even in the Poincaré patch. It will be interesting to construct $\Delta^{(TT)}_{aba'b'}(x,x')$ in Eq. (5.57) in the Poincaré patch by the mode-sum method because this is the only place where nontrivial IR issues arise with $\beta > 0$ and $\alpha = 0$.

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APPENDIX A: EXPLICIT SOLUTIONS TO FREE-FIELD EQUATIONS

In this appendix, we give the solutions to free-field equations discussed in Sec. II explicitly, following Ref. [16]. The scalar solutions are

$$\phi^{(\mu^2;\ell\sigma)} = N_{L_0\ell}(\cosh t)^{-1}\mathrm{P}^{-(\ell+1)}_{L_0+1}(i\sinh t)Y^{(0\ell\sigma)}, \quad \text{(A1)}$$

where $N_{L_0\ell}$ is defined by

$$N_{L_0\ell} = \frac{1}{\sqrt{2}}[\Gamma(\ell - L_0)\Gamma(\ell + L_0 + 3)]^{1/2}. \quad \text{(A2)}$$

The transverse vector solutions $A_a^{(\mu^2;m\ell\sigma)}$ are

$$A_0^{(\mu^2;1\ell\sigma)} = 0, \quad \text{(A3)}$$

$$A_i^{(\mu^2;1\ell\sigma)} = \tilde{N}_{L_1\ell}\mathrm{P}^{-(\ell+1)}_{L_1+1}(i\sinh t)Y_i^{(1\ell\sigma)}, \quad \text{(A4)}$$

where

$$\tilde{N}_{L_1\ell} = \begin{cases} N_{L_1\ell} & \text{if } \ell \geq 2, \\ \sqrt{\mu^2+6}N_{L_1\ell} & \text{if } \ell = 1, \end{cases} \quad \text{(A5)}$$

and

$$A_0^{(\mu^2;0\ell\sigma)} = \sqrt{\frac{\ell(\ell+2)}{(L_1+1)(L_1+2)}} \times \tilde{N}_{L_1\ell}(\cosh t)^{-2}\mathrm{P}^{-(\ell+1)}_{L_1+1}(i\sinh t)Y^{(0\ell\sigma)}, \quad \text{(A6)}$$

$$A_i^{(\mu^2;0\ell\sigma)} = -\frac{\tilde{N}_{L_1\ell}}{\sqrt{(L_1+1)(L_1+2)\ell(\ell+2)}}\left(\frac{\partial}{\partial t} + \tanh t\right) \times \mathrm{P}^{-(\ell+1)}_{L_1+1}(i\sinh t)\tilde{\nabla}_i Y^{(0\ell\sigma)}. \quad \text{(A7)}$$

Finally, the transverse-traceless symmetric tensor solutions are

$$H_{0a}^{(M^2;2\ell\sigma)} = 0, \quad \text{(A8)}$$

$$H_{ij}^{(M^2;2\ell\sigma)} = \sqrt{2}N_{L_2\ell}\cosh t\,\mathrm{P}^{-(\ell+1)}_{L_2+1}(i\sinh t)Y_{ij}^{(2\ell\sigma)}, \quad \text{(A9)}$$

$$H_{00}^{(M^2;1\ell\sigma)} = 0, \quad \text{(A10)}$$

$$H_{0i}^{(M^2;1\ell\sigma)} = -i\sqrt{(\ell-1)(\ell+3)} \times N_{L_2\ell}(\cosh t)^{-1}\mathrm{P}^{-(\ell+1)}_{L_2+1}(i\sinh t)Y_i^{(1\ell\sigma)}, \quad \text{(A11)}$$

$$H_{ij}^{(M^2;1\ell\sigma)} = i\frac{N_{L_2\ell}}{\sqrt{(\ell-1)(\ell+3)}}\cosh t\left(\frac{\partial}{\partial t} + 2\tanh t\right) \times \mathrm{P}^{-(\ell+1)}_{L_2+1}(i\sinh t)(\tilde{\nabla}_i Y_j^{(1\ell\sigma)} + \tilde{\nabla}_j Y_i^{(1\ell\sigma)}), \quad \text{(A12)}$$

and

$$H_{00}^{(M^2;0\ell\sigma)} = -iN'_{L_2\ell}(\cosh t)^{-3}\mathrm{P}^{-(\ell+1)}_{L_2+1}(i\sinh t)Y^{(0\ell\sigma)}, \quad \text{(A13)}$$

$$H_{0i}^{(M^2;0\ell\sigma)} = iN'_{L_2\ell}\frac{(\cosh t)^{-1}}{\ell(\ell+2)}\left(\frac{\partial}{\partial t} + \tanh t\right) \times \mathrm{P}^{-(\ell+1)}_{L_2+1}(i\sinh t)\tilde{\nabla}_i Y^{(0\ell\sigma)}, \quad \text{(A14)}$$

$$H_{ij}^{(M^2;0\ell\sigma)} = -iN'_{L_2\ell}\left\{\frac{3}{2(\ell-1)(\ell+3)}\left[\frac{\cosh t}{\ell(\ell+2)}\left(\frac{\partial}{\partial t} + 2\tanh t\right)\left(\frac{\partial}{\partial t} + \tanh t\right) + \frac{1}{3\cosh t}\right]\mathrm{P}^{-(\ell+1)}_{L_2+1}(i\sinh t)\left[\tilde{\nabla}_i\tilde{\nabla}_j + \frac{\ell(\ell+2)}{3}\tilde{\eta}_{ij}\right]Y^{(0\ell\sigma)} + \frac{1}{3\cosh t}\tilde{\eta}_{ij}\mathrm{P}^{-(\ell+1)}_{L_2+1}(i\sinh t)Y^{(0\ell\sigma)}\right\}, \quad \text{(A15)}$$

where

$$N'_{L_2\ell} = \sqrt{\frac{4(\ell-1)\ell(\ell+2)(\ell+3)}{3(L_2+1)(L_2+2)}}N_{L_2\ell}. \quad \text{(A16)}$$

To show Eq. (2.25), we used the associated Legendre equation,

$$\left[\frac{d^2}{dt^2}+\tanh t\frac{d}{dt}+\frac{(\ell+1)^2}{\cosh^2 t}-(L+1)(L+2)\right]\mathrm{P}_{L+1}^{-(\ell+1)}(i\sinh t)=0, \tag{A17}$$

and the lowering and raising differential operators for the associated Legendre functions,

$$\cosh t\left[\frac{d}{dt}-(L+1)\tanh t\right]\mathrm{P}_{L+1}^{-(\ell+1)}(i\sinh t)=i(L-\ell)\mathrm{P}_{L}^{-(\ell+1)}(i\sinh t), \tag{A18}$$

$$\cosh t\left[\frac{d}{dt}+(L+1)\tanh t\right]\mathrm{P}_{L}^{-(\ell+1)}(i\sinh t)=-i(L+\ell+2)\mathrm{P}_{L+1}^{-(\ell+1)}(i\sinh t). \tag{A19}$$

APPENDIX B: ORTHOGONALITY OF SCALAR-TYPE AND TENSOR-VECTOR-TYPE SOLUTIONS

In this appendix, we show that the symplectic product vanishes between a scalar-type solution $h_{ab}^{(S)}=S_{ab}^{(A;\ell\sigma)}$ given by Eqs. (4.16) and (4.17) and any vector-tensor-type solution h_{ab} satisfying $\nabla_a\nabla_b h^{ab}=0$ and $h=h^c{}_c=0$. This result implies that the scalar and tensor-vector sectors can be treated separately as we did.

We consider the symplectic product between a scalar-type solution $h_{ab}^{(S)}=\nabla_a\nabla_b B+g_{ab}\Psi$, with B and Ψ satisfying Eqs. (4.9) and (4.10), and the complex conjugate of a vector-tensor-type solution h_{ab}:

$$(\bar{h},h^{(S)})_{\mathrm{symp}}=-i\int_\Sigma d\Sigma_a X^a(h,h^{(S)}), \tag{B1}$$

where

$$X^a(h,h^{(S)})\equiv h_{bc}p^{(S)abc}-p^{abc}h_{bc}^{(S)}, \tag{B2}$$

and where Σ is a Cauchy surface, e.g. a $t=$ constant hypersurface. The conjugate momentum current p^{abc} here is given by Eq. (5.1) with the conditions $\nabla_a\nabla_b h^{ab}=0$ and $h=0$ imposed. The contribution to p_{inv}^{abc} defined by Eq. (5.2) from the part $\nabla_a\nabla_b B$ in $h_{ab}^{(S)}$ can be found as

$$p_{\mathrm{inv}}^{(B)abc}=-\tfrac{1}{2}\nabla^a\nabla^b\nabla^c B-\tfrac{3}{2}g^{bc}\nabla^a B+\tfrac{1}{4}[g^{ab}\nabla^c(\Box+6)B+g^{ac}\nabla^b(\Box+6)B]. \tag{B3}$$

The conjugate momentum current for the scalar-type solution $h_{ab}^{(S)}$ is

$$p^{(S)abc}=p_{\mathrm{inv}}^{(B)abc}+p_{\mathrm{inv}}^{(\Psi)abc}+p_{\mathrm{gf}}^{(S)abc}, \tag{B4}$$

where $p_{\mathrm{inv}}^{(\Psi)abc}$ is the contribution to $p_{\mathrm{inv}}^{(S)abc}$ from $\nabla_a\nabla_b\Psi$. We have

$$p_{\mathrm{inv}}^{(\Psi)abc}+p_{\mathrm{gf}}^{(S)abc}=-\frac{1}{\beta}\nabla^a\Psi \tag{B5}$$

[see Eq. (5.45)]. Then, we find after a tedious but straightforward calculation

$$\begin{aligned}X^a(h,h^{(S)})=&-\frac{1}{2}h_{bc}\nabla^a\nabla^b\nabla^c B+\frac{1}{2}h^{ab}\nabla_b(\Box+6)B\\&+\frac{1}{2}\nabla^a h^{bc}\nabla_b\nabla_c B+\left(\frac{1}{\alpha}-1\right)\nabla_c h^{bc}\nabla^a\nabla_b B\\&+\left(\frac{1}{2}-\frac{1+\beta}{\alpha\beta}\right)\nabla_b h^{ab}\Box B\\&+\frac{(\alpha-3)\beta-4}{\alpha\beta}\nabla_b h^{ab}\Psi.\end{aligned} \tag{B6}$$

To show that $\int_\Sigma d\Sigma_a X^a(h,h^{(S)})=0$, we first note that

$$X^a(h,h^{(S)})=Y^a(h,h^{(S)})+\nabla_b F^{(1)ab}, \tag{B7}$$

where

$$F^{(1)ab}=-\tfrac{1}{2}(h^{bc}\nabla^a\nabla_c B-h^{ac}\nabla^b\nabla_c B)+\tfrac{1}{2}(\nabla^a h^{bc}\nabla_c B-\nabla^b h^{ac}\nabla_c B), \tag{B8}$$

and, with the definition $C^a=\nabla_b h^{ab}$,

$$\begin{aligned}Y^a(h,h^{(S)})=&\left(\frac{1}{\alpha}-\frac{1}{2}\right)C^b\nabla^a\nabla_b B+\left(\frac{1}{2}-\frac{1+\beta}{\alpha\beta}\right)C^a\Box B\\&+\left[-\frac{1}{2\alpha}\nabla^a C^b+\left(\frac{1}{2}-\frac{1}{2\alpha}\right)\nabla^b C^a\right]\nabla_b B\\&-\frac{4-(\alpha-3)\beta}{\alpha\beta}C^a\Psi.\end{aligned} \tag{B9}$$

We have used the field equation (4.3) to solve for $\Box h_{ab}$. Since $F^{(1)ab}$ is an antisymmetric tensor, we have

$$\int_\Sigma d\Sigma_a\nabla_b F^{(1)ab}=0 \tag{B10}$$

by the generalized Stokes theorem. Hence,

$$(\bar{h},h^{(S)})_{\mathrm{symp}}=-i\int_\Sigma d\Sigma_a Y^a(h,h^{(S)}). \tag{B11}$$

Next, we find

$$Y^a(h,h^{(S)})=\nabla_b F^{(2)ab}-\frac{1}{\alpha\beta}C^a\{(\Box-3\beta)B+[4-(\alpha-3)\beta]\Psi\}, \tag{B12}$$

where

$$F^{(2)ab}=\left(\frac{1}{\alpha}-\frac{1}{2}\right)(C^b\nabla^a B-C^a\nabla^b B)+\frac{1}{2\alpha}B(\nabla^b C^a-\nabla^a C^b) \tag{B13}$$

by using the equation

$$\nabla_b(\nabla^b C^a-\nabla^a C^b)=-6C^a \tag{B14}$$

[see Eq. (4.32)]. Finally, by Eq. (4.9) and antisymmetry of $F^{(2)ab}$, we find $(\bar{h}, h^{(S)})_{\rm symp} = 0$ from Eqs. (B11) and (B12).

APPENDIX C: TWO-POINT FUNCTION FOR TACHYONIC SCALAR FIELD

It has been pointed out in Ref. [10] that the two-point function for the scalar field with negative mass squared is IR-divergent if it is expanded in terms of momentum eigenfunctions in the Poincaré patch and that as a result the de Sitter-invariant graviton two-point function is IR-divergent for $\beta < 0$. This is true even if β is not one of the discrete values for which it is IR-divergent in the Euclidean approach [8,11].

In this appendix, we verify that *in global coordinates*, the de Sitter-invariant two-point function is IR-finite even if the field is tachyonic unless the mass squared μ^2 is of the form $-L_0(L_0+3)$, $L_0 = 0, 1, 2, \ldots$ by explicitly constructing it. We also point out that this two-point function is recovered also in the Poincaré patch if an appropriate IR subtraction is made.

1. Construction of the scalar two-point function in global coordinates

We first show that the scalar two-point function can be constructed by the mode-sum method in global coordinates without any IR divergences even with tachyonic mass unless the mass squared satisfies $\mu^2 = -L_0(L_0+3)$, $L_0 = 0, 1, 2, \ldots$.

We write the metric on the unit S^3 as

$$d\Omega^2 = d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\varphi^2), \qquad \text{(C1)}$$

where $0 \le \chi \le \pi$ and where θ and φ are the usual spherical polar coordinates on S^2. The positive-frequency mode functions corresponding to the Bunch-Davies vacuum are given by Eq. (2.5):

$$\Phi^{(\ell\ell_2 m)}(t,\chi,\theta,\varphi) = \frac{1}{\cosh t} \mathrm{P}^{-(\ell+1)}_{L_0+1}(i\sinh t) Y^{(0\ell\ell_2 m)}(\chi,\theta,\varphi), \qquad \text{(C2)}$$

where

$$Y^{(0\ell\ell_2 m)}(\chi,\theta,\chi) = \frac{\ell+1}{\sqrt{\sin\chi}} \mathrm{P}^{-(\ell_2+(1/2))}_{\ell+(1/2)}(\cos\chi) Y_{\ell_2 m}(\theta,\varphi). \qquad \text{(C3)}$$

The $Y_{\ell_2 m}(\theta,\varphi)$ are the standard spherical harmonics on S^2.

The Wightman two-point function with one point at $\chi = 0$ is given as [16]

$$G(t_1,t_2,\chi) \equiv \sum_{\ell=0}^{\infty} \frac{\Gamma(\ell - L_0)\Gamma(\ell+L_0+3)}{2} \times \Phi^{(\ell 00)}(t_1,\chi,\theta_1,\varphi_1)\overline{\Phi^{(\ell 00)}(t_2,0,\theta_2,\varphi_2)}. \qquad \text{(C4)}$$

If $L_0 > 0$, i.e. if $\mu^2 < 0$, then some modes have negative coefficients, i.e. have negative norm.

We assume that L_0 is not an integer. If L_0 is an integer, then this two-point function is indeed IR-divergent. We note in passing that the modes $\Phi^{(\ell\ell_2 m)}$ with positive norm form a unitary representation of the de Sitter group if L_0 *is* an integer, whereas for a positive noninteger value of L_0, no unitary representation exists because of the negative norm modes [16,33].

Since only the $\ell_2 = 0$ modes contribute in Eq. (C4), the function $G(t_1,t_2,\chi)$ is independent of θ_1, θ_2, φ_1 and φ_2. By noting that

$$\mathrm{P}^{-(1/2)}_{\ell+(1/2)}(\cos\chi) = \sqrt{\frac{2}{\pi}} \frac{\sin(\ell+1)\chi}{(\ell+1)\sin\chi}, \qquad \text{(C5)}$$

we obtain

$$G(t_1,t_2,\chi) = \frac{1}{4\pi^2}\sum_{\ell=0}^{\infty}(\ell+1)\Gamma(\ell-L_0)\Gamma(\ell+L_0+3) \times \frac{1}{\cosh t_1}\mathrm{P}^{-(\ell+1)}_{L_0+1}(i\sinh t_1+\epsilon) \times \frac{1}{\cosh t_2}\mathrm{P}^{-(\ell+1)}_{L_0+1}(-i\sinh t_2+\epsilon)\frac{\sin(\ell+1)\chi}{\sin\chi}, \qquad \text{(C6)}$$

where we inserted the "infinitesimal" positive number ϵ for UV regularization. This series can be shown to be convergent by using

$$\mathrm{P}^{-(\ell+1)}_{L_0+1}(z) = \frac{1}{(\ell+1)!}\left(\frac{1-z}{1+z}\right)^{\ell+1} \times F(-L_0-1, L_0+2; \ell+2; (1-z)/2) \approx \frac{1}{(\ell+1)!}\left(\frac{1-z}{1+z}\right)^{\ell+1} \text{if } \ell \gg 1. \qquad \text{(C7)}$$

By using the identity $\Gamma(u)\Gamma(1-u) = \pi/\sin\pi u$, we find that Eq. (C6) can be written

$$G(t_1,t_2,\chi) = -\frac{\Gamma(-L_0-1)\Gamma(L_0+2)}{4\pi^2\cosh t_1\cosh t_2\sin\chi}\sum_{\ell=0}^{\infty}(\ell+1) \times \frac{\Gamma(L_0+\ell+3)}{\Gamma(L_0-\ell+1)}\mathrm{P}^{-(\ell+1)}_{L_0+1}(i\sinh t_1+\epsilon) \times \mathrm{P}^{-(\ell+1)}_{L_0+1}(-i\sinh t_2+\epsilon) \times \sin[(\ell+1)(\pi-\chi)]. \qquad \text{(C8)}$$

Now, an addition theorem for the associated Legendre functions (8.794.1 of Ref. [34]) can be adapted to the series here as

$$\begin{aligned} &\mathrm{P}_{L_0+1}(\sinh t_1 \sinh t_2 - \cosh t_1 \cosh t_2 \cos\chi + i\epsilon(t_1 - t_2)) \\ &\quad = \mathrm{P}_{L_0+1}(i\sinh t_1 + \epsilon)\mathrm{P}_{L_0+1}(-i\sinh t_2 + \epsilon) \\ &\qquad + 2\sum_{\ell=0}^{\infty}\frac{\Gamma(L_0+\ell+3)}{\Gamma(L_0-\ell+1)}\mathrm{P}_{L_0+1}^{-(\ell+1)}(i\sinh t_1 + \epsilon) \\ &\qquad \times \mathrm{P}_{L_0+1}^{-(\ell+1)}(-i\sinh t_2 + \epsilon)\cos[(\ell+1)(\pi-\chi)]. \end{aligned} \tag{C9}$$

By differentiating both sides with respect to χ and substituting the result into Eq. (C8), we obtain

$$\begin{aligned} G(t_1, t_2, \chi) = -&\frac{\Gamma(-L_0-1)\Gamma(L_0+2)}{8\pi^2\cosh t_1\cosh t_2\sin\chi} \\ &\times \frac{d}{d\chi}\mathrm{P}_{L_0+1}(\sinh t_1\sinh t_2 \\ &- \cosh t_1\cosh t_2\cos\chi + i\epsilon(t_1-t_2)). \end{aligned} \tag{C10}$$

Finally, by using Eq. (C7) with $\mathrm{P}_{L_0+1}(z) = \mathrm{P}^0_{L_0+1}(z)$ in Eq. (C8) and

$$\frac{d}{du}F(a,b;c;u) = \frac{ab}{c}F(a+1,b+1;c+1;u), \tag{C11}$$

we find

$$\begin{aligned} G(t_1,t_2,\chi) = &\frac{\Gamma(-L_0)\Gamma(L_0+3)}{16\pi^2} \\ &\times F(-L_0, L_0+3; 2; (1+Z-i\epsilon(t_1-t_2))/2), \end{aligned} \tag{C12}$$

$$Z \equiv -\sinh t_1\sinh t_2 + \cosh t_1\cosh t_2\cos\chi, \tag{C13}$$

which is the standard result [15,20]. Note that our result is valid also for $L_0 > 0$, i.e. for tachyonic scalar fields, as long as L_0 is not an integer.

2. Two-point function for tachyonic scalar field in the Poincaré patch

In this subsection, we show that, even though the two-point function for tachyonic scalar field is IR-divergent in the momentum expansion in the Poincaré patch, one can still recover the two-point function found in the previous subsection by subtracting the IR-divergent terms.

In the spatially flat coordinate system, the metric of de Sitter spacetime can be given as

$$ds^2 = \frac{1}{\eta^2}(-d\eta^2 + d\mathbf{x}^2), \qquad \eta \in (-\infty, 0). \tag{C14}$$

The Wightman two-point function between points $(\eta_1, \mathbf{x}_1)$ and $(\eta_2, \mathbf{x}_2)$ with $\| \mathbf{x}_1 - \mathbf{x}_2 \| = r$ is found as

$$\begin{aligned} G_{\text{flat}}(\eta_1, \eta_2, r) = &\frac{(\eta_1\eta_2)^{3/2}}{8\pi r} \\ &\times \int_0^\infty dk k \sin kr H_\nu^{(1)}(-k\eta_1)\overline{H_\nu^{(1)}(-k\eta_2)}, \end{aligned} \tag{C15}$$

where

$$\nu \equiv L_0 + \tfrac{3}{2} = \sqrt{\tfrac{9}{4} - \mu^2}. \tag{C16}$$

The Hankel function is given in terms of the Bessel function as

$$H_\nu^{(1)}(u) = \frac{i}{\sin\pi\nu}[e^{-i\pi\nu}J_\nu(u) - J_{-\nu}(u)]. \tag{C17}$$

The integral (C15) converges if $\mu^2 > 0$, and the result of the integral is known to agree with $G(t_1, t_2, \chi)$ in Eq. (C12) [20] with

$$Z = \frac{\eta_1^2 + \eta_2^2 - r^2}{2\eta_1\eta_2} \tag{C18}$$

in this case. We have

$$J_{-\nu}(u) \approx \frac{1}{\Gamma(1-\nu)}\left(\frac{2}{u}\right)^\nu, \qquad |u| \ll 1. \tag{C19}$$

Hence, the integral (C15) diverges in the infrared if $\nu \geq \frac{3}{2}$, i.e. if $\mu^2 \leq 0$.

Let us first separate out the term causing the IR divergences as

$$G_{\text{flat}}(\eta_1, \eta_2, r) = G_{\text{flat}}^{(\text{reg})}(\eta_1, \eta_2, r) + G_{\text{flat}}^{(\infty)}(\eta_1, \eta_2, r), \tag{C20}$$

where the IR-divergent contribution for $\nu \geq \frac{3}{2}$ is given by

$$\begin{aligned} G_{\text{flat}}^{(\infty)}(\eta_1, \eta_2, r) = &\frac{(\eta_1\eta_2)^{3/2}}{8\pi r\sin\pi\nu}\int_0^\lambda dk k\sin kr J_{-\nu}(-k\eta_1) \\ &\times J_{-\nu}(-k\eta_2), \quad \lambda > 0. \end{aligned} \tag{C21}$$

(The case with integer ν needs to be treated as a limit of cases with noninteger ν.) The function $G_{\text{flat}}^{(\text{reg})}(\eta_1, \eta_2, r)$ is the IR-regularized two-point function with the IR cutoff λ. If $\mathrm{Re}\,\nu < \frac{3}{2}$, then the integral in Eq. (C21) is convergent and tends to zero as $\lambda \to 0$. Now, this can be analytically continued to $\mathrm{Re}\,\nu > \frac{3}{2}$ as

$$\begin{aligned} G_{\text{flat}}^{(\text{sub})}(\eta_1, \eta_2, r) \equiv &\frac{(\eta_1\eta_2)^{3/2}}{8\pi r\sin\pi\nu(1+e^{-2\pi i\nu})} \\ &\times \int_C dk k \sin kr J_{-\nu}(-k\eta_1)J_{-\nu}(-k\eta_2), \end{aligned} \tag{C22}$$

where C is a path on the complex k plane from $-\lambda$ to λ which avoids the origin in the upper half-plane. This means that the two-point function defined by

$$G^{(\mathrm{inv})}_{\mathrm{flat}}(\eta_1,\eta_2,r) \equiv G^{(\mathrm{reg})}_{\mathrm{flat}}(\eta_1,\eta_2,r) + G^{(\mathrm{sub})}_{\mathrm{flat}}(\eta_1,\eta_2,r) \tag{C23}$$

is the two-point function $G(t_1, t_2, \chi)$ given by Eq. (C12) expressed in spatially flat coordinates even for $\mathrm{Re}\,\nu > \frac{3}{2}$. Thus, $G^{(\mathrm{reg})}_{\mathrm{flat}}$ is the IR-regularized two-point function as mentioned before, and $G^{(\mathrm{sub})}_{\mathrm{flat}}$ is the IR-subtraction term needed to recover the de Sitter-invariant two-point function. Note that this scheme does not work if ν is a half-odd integer because $G^{(\mathrm{sub})}_{\mathrm{flat}}(\eta_1,\eta_2,r)$ is infinite in this case.

Let us examine the IR-subtraction term $G^{(\mathrm{sub})}_{\mathrm{flat}}$ more closely for $\frac{3}{2} < \nu < \frac{5}{2}$ in the limit $\lambda \to 0$. Choosing C to be the upper semicircle from $-\lambda$ to λ, we find

$$G^{(\mathrm{reg})}_{\mathrm{flat}}(\eta_1,\eta_2,r) = -\frac{[\Gamma(\nu)]^2\lambda^{3-2\nu}}{\pi^2(2\nu-3)}\left(\frac{4}{\eta_1\eta_2}\right)^{\nu} + O(\lambda^{5-2\nu}). \tag{C24}$$

Note that $\lambda^{5-2\nu} \to 0$ as $\lambda \to 0$ by our assumption $\nu < \frac{5}{2}$. Hence, we have

$$G^{(\mathrm{inv})}_{\mathrm{flat}}(\eta_1,\eta_2,r) = \lim_{\lambda\to 0}\left[\frac{(\eta_1\eta_2)^{3/2}}{8\pi r}\times\int_\lambda^\infty dk\,\sin kr\,H^{(1)}_\nu(-k\eta_1)\overline{H^{(1)}_\nu(-k\eta_2)} - \frac{[\Gamma(\nu)]^2}{\pi^3(2\nu-3)}\left(\frac{\lambda^2\eta_1\eta_2}{4}\right)^{(3/2)-\nu}\right]. \tag{C25}$$

Thus, to recover the de Sitter-covariant two-point function for $\frac{3}{2} < \nu < \frac{5}{2}$, we need to remove the IR divergences by subtracting some zero-mode contribution.

Finally, we verify that the large r behavior of $G^{(\mathrm{inv})}_{\mathrm{flat}}(\eta_1,\eta_2,r)$ is correctly reproduced by Eq. (C25). From Eq. (C12), we find, using a transformation formula for hypergeometric functions and the doubling formula for the Gamma function,

$$G^{(\mathrm{inv})}_{\mathrm{flat}}(\eta_1,\eta_2,r) \approx \frac{1}{4\pi^{5/2}}\Gamma\left(\frac{3}{2}-\nu\right)\Gamma(\nu)\left(\frac{r^2}{\eta_1\eta_2}\right)^{\nu-(3/2)}. \tag{C26}$$

By examining the $\eta_1\eta_2$ dependence of this term, we find that this term comes entirely from the leading term in the k expansion of $H^{(1)}_\nu(-k\eta_1)\overline{H^{(1)}_\nu(-k\eta_2)}$ in Eq. (C25). Thus, we find

$$G^{(\mathrm{inv})}_{\mathrm{flat}}(\eta_1,\eta_2,r) \approx \lim_{\lambda\to 0}\left\{\frac{[\Gamma(\nu)]^2}{\pi^3}\left(\frac{\eta_1\eta_2}{4}\right)^{(3/2)-\nu}\times\int_\lambda^\infty dk\,k^{2-2\nu}\frac{\sin kr}{kr} - \frac{[\Gamma(\nu)]^2}{\pi^3(2\nu-3)}\left(\frac{\lambda^2\eta_1\eta_2}{4}\right)^{(3/2)-\nu}\right\}. \tag{C27}$$

Upon integration by parts, the second term cancels out the boundary term, and we obtain

$$G^{(\mathrm{inv})}_{\mathrm{flat}}(\eta_1,\eta_2,r) \approx \frac{[\Gamma(\nu)]^2}{\pi^3(2\nu-3)}\left(\frac{\eta_1\eta_2}{4r^2}\right)^{(3/2)-\nu}\times\int_0^\infty du\,u^{2-2\nu}\left(\cos u - \frac{\sin u}{u}\right), \tag{C28}$$

where we have let $u \equiv kr$. We find Eq. (C26) by evaluating this integral.

APPENDIX D: COMPARISON WITH THE EUCLIDEAN APPROACH

In this paper, we found the covariant graviton two-point function using the mode-sum method. It can be written as

$$\Delta_{aba'b'}(x,x') = \Delta^{(TT)}_{aba'b'}(x,x') + \Delta^{(V)}_{aba'b'}(x,x') + \Delta^{(S)}_{aba'b'}(x,x'), \tag{D1}$$

where $\Delta^{(TT)}_{aba'b'}$, $\Delta^{(V)}_{aba'b'}$ and $\Delta^{(S)}_{aba'b'}$ are given by Eqs. (5.57), (5.58), and (5.53), respectively. Now, this two-point function can also be found in the Euclidean approach. In this approach, $\Delta_{aba'b'}(x,x')$ can also be given as a sum of three parts:

$$\Delta_{aba'b'}(x,x') = G^{(TT)}_{aba'b'}(x,x') + G^{(V)}_{aba'b'}(x,x') + G^{(S)}_{aba'b'}(x,x'). \tag{D2}$$

(See, e.g. Refs. [11,24]. Our graviton two-point functions are twice that of Ref. [24].) The function $G^{(TT)}_{aba'b'}(x,x')$ is transverse-traceless and $G^{(V)}_{aba'b'}(x,x')$ is a symmetric derivative in each of the sets of indices (ab) and $(a'b')$ of a vector two-point function like $\Delta^{(V)}_{aba'b'}(x,x')$ in the mode-sum case. However, these functions are not equal to $\Delta^{(TT)}_{aba'b'}(x,x')$ and $\Delta^{(V)}_{aba'b'}(x,x')$, respectively. We also find that the scalar part in the Euclidean approach, $G^{(S)}_{aba'b'}(x,x')$, given in Ref. [11] is different from $\Delta^{(S)}_{aba'b'}(x,x')$. In this appendix, we verify that Eqs. (D1) and (D2) give the same two-point function for spacelike-separated points x and x' in spite of these differences.

Let us describe the difference between $\Delta^{(S)}_{aba'b'}$ given by Eq. (5.53) and the scalar part $G^{(S)}_{aba'b'}$ in the Euclidean approach. For spacelike-separated points x and x', the two-point function $\Delta_{\mu^2}(x,x')$ for a scalar field in de Sitter spacetime is identical to the corresponding Green's function on S^4 as a function of the geodesic distance between x and x'. If we let $\psi^{(n\nu)}(x)$, $n = 0, 1, 2, \ldots$, be a complete set of orthonormal scalar modes on S^4 satisfying

$$[\Box + n(n+3)]\psi^{(n\nu)}(x) = 0, \qquad n = 0, 1, 2, \ldots, \tag{D3}$$

where ν represents all labels other than n, and

$$\int_{S^4} dS \overline{\psi^{(n\nu)}(x)} \psi^{(n'\nu')}(x) = \delta^{nn'}\delta^{\nu\nu'}, \tag{D4}$$

then one can readily see that the equation for the Green's function

$$(-\Box_x + \mu^2)\Delta_{\mu^2}(x, x') = \delta(x, x'), \tag{D5}$$

where

$$\delta(x, x') = \sum_{n=0}^{\infty}\sum_{\nu} \psi^{(n\nu)}(x)\overline{\psi^{(n\nu)}(x')}, \tag{D6}$$

is uniquely solved by

$$\Delta_{\mu^2}(x, x') = \sum_{n=0}^{\infty}\sum_{\nu} \frac{\psi^{(n\nu)}(x)\overline{\psi^{(n\nu)}(x')}}{n(n+3)+\mu^2}. \tag{D7}$$

We define

$$\Delta^{-}_{\mu^2}(x, x') \equiv \sum_{n=1}^{\infty}\sum_{\nu} \frac{\psi^{(n\nu)}(x)\overline{\psi^{(n\nu)}(x')}}{n(n+3)+\mu^2}, \tag{D8}$$

$$\Delta^{--}_{\mu^2}(x, x') \equiv \sum_{n=2}^{\infty}\sum_{\nu} \frac{\psi^{(n\nu)}(x)\overline{\psi^{(n\nu)}(x')}}{n(n+3)+\mu^2}. \tag{D9}$$

Then the scalar part in the Euclidean approach, $G^{(S)}_{aba'b'}(x, x')$, is given in Ref. [11] as

$$\begin{aligned} G^{(S)}_{aba'b'}(x, x') = {} & \Delta^{(S)}_{aba'b'}(x, x') + \frac{\alpha}{9}\nabla_a\nabla_b\nabla_{a'}\nabla_{b'}\Delta^{-}_{0}(x, x') \\ & - \frac{1}{3}\left(\nabla_a\nabla_b - \frac{1}{4}g_{ab}\Box\right) \\ & \times \left(\nabla_{a'}\nabla_{b'} - \frac{1}{4}g_{a'b'}\Box'\right)\Delta^{--}_{-4}(x, x'). \end{aligned} \tag{D10}$$

Hence, the Euclidean and mode-sum approaches will be consistent with each other if

$$G^{(V)}_{aba'b'}(x, x') = \Delta^{(V)}_{aba'b'}(x, x') - \frac{\alpha}{9}\nabla_a\nabla_b\nabla_{a'}\nabla_{b'}\Delta^{-}_{0}(x, x'), \tag{D11}$$

$$\begin{aligned} G^{(TT)}_{aba'b'}(x, x') = {} & \Delta^{(TT)}_{aba'b'}(x, x') + \frac{1}{3}\left(\nabla_a\nabla_b - \frac{1}{4}g_{ab}\Box\right) \\ & \times \left(\nabla_{a'}\nabla_{b'} - \frac{1}{4}g_{a'b'}\Box'\right)\Delta^{--}_{-4}(x, x'). \end{aligned} \tag{D12}$$

We will verify these relations in the rest of this appendix.

To show Eq. (D11), we first need to define the Green's function $G^{(V;\mu^2)}_{aa'}(x, x')$ for the transverse vector field with mass μ^2 in the Euclidean approach. Let $V^{(n\nu)}_a(x)$, $n = 1, 2, \ldots$, form a complete orthonormal set of transverse solutions to the eigenvalue equation on S^4,

$$\begin{aligned} & \nabla^b(\nabla_a V^{(n\nu)}_b - \nabla_b V^{(n\nu)}_a) \\ & \quad = (n+1)(n+2)V^{(n\nu)}_a, \qquad n = 1, 2, \ldots, \end{aligned} \tag{D13}$$

satisfying $\nabla^a V^{(n\nu)}_a = 0$ and

$$\int_{S^4} dS \overline{V^{(n\nu)}_a(x)} V^{(n'\nu')a}(x) = \delta^{nn'}\delta^{\nu\nu'}. \tag{D14}$$

Then, we define the transverse Green's function for the operator

$$L^{(V)b}_a V_b \equiv \nabla^b(\nabla_a V_b - \nabla_b V_a) + \mu^2 V_a \tag{D15}$$

by

$$G^{(V;\mu^2)}_{aa'}(x, x') \equiv \sum_{n=1}^{\infty}\sum_{\nu} \frac{V^{(n\nu)}_a(x)\overline{V^{(n\nu)}_{a'}(x')}}{(n+1)(n+2)+\mu^2}. \tag{D16}$$

This Green's function satisfies

$$L^{(V)b}_a G^{(V;\mu^2)}_{ba'}(x, x') = \delta^{(V)}_{aa'}(x, x'), \tag{D17}$$

where

$$\delta^{(V)}_{aa'}(x, x') = \sum_{n=1}^{\infty}\sum_{\nu} V^{(n\nu)}_a(x)\overline{V^{(n\nu)}_{a'}(x')}. \tag{D18}$$

On the other hand, the Euclidean Green's function $\Delta^{(V;\mu^2)}_{aa'}(x, x')$ which becomes the Feynman propagator and hence the Wightman two-point function for spacelike-separate points after appropriate analytic continuation satisfies [24]

$$L^{(V)b}_a \Delta^{(V;\mu^2)}_{ba'}(x, x') = \delta_{aa'}(x, x'), \tag{D19}$$

where

$$\begin{aligned} \delta_{aa'}(x, x') & = \delta^{(V)}_{aa'}(x, x') + \sum_{n=1}^{\infty}\sum_{\nu} \frac{\nabla_a\psi^{(n\nu)}(x)\nabla_{a'}\overline{\psi^{(n\nu)}(x')}}{n(n+3)} \\ & = \delta^{(V)}_{aa'}(x, x') + \nabla_a\nabla_{a'}\Delta^{-}_{0}(x, x'). \end{aligned} \tag{D20}$$

The two-point function $\Delta^{-}_{0}(x, x')$ is defined by Eq. (D8). By noting that

$$L^{(V)b}_a \nabla_b\nabla_{a'}\Delta^{-}_{0}(x, x') = \mu^2\nabla_a\nabla_{a'}\Delta^{-}_{0}(x, x'), \tag{D21}$$

we readily find [24]

$$G^{(V;\mu^2)}_{aa'}(x, x') = \Delta^{(V;\mu^2)}_{aa'}(x, x') - \frac{1}{\mu^2}\nabla_a\nabla_{a'}\Delta^{-}_{0}(x, x'). \tag{D22}$$

The vector part of the propagator in the Euclidean approach is [11,24]

$$G^{(V)}_{aba'b'}(x,x') = 4\alpha \sum_{n=2}^{\infty}\sum_{\nu} \frac{\nabla_{(a}V^{(n\nu)}_{b)}(x)\nabla_{(a'}\overline{V^{(n\nu)}_{b')}(x')}}{[(n+1)(n+2)-6]^2}. \tag{D23}$$

Note that there is no contribution from the vectors $V_a^{(n=1,\nu)}$ because they are Killing vectors on S^4. Using the definition (D16), we find

$$G^{(V)}_{aba'b'}(x,x') = -2\alpha \lim_{\mu^2\to -6}\frac{\partial}{\partial\mu^2}[\nabla_{(a}\nabla_{|a'|}G^{(V;\mu^2)}_{b)b'}(x,x') + \nabla_{(a}\nabla_{|b'|}G^{(V;\mu^2)}_{b)a'}(x,x')]. \tag{D24}$$

[Notice the similarity of this equation with Eq. (5.58).] From Eq. (D22), we readily find Eq. (D11).

Next, we show Eq. (D12). The transverse-traceless part of the two-point function in the Euclidean approach is [24]

$$G^{(TT)}_{aba'b'}(x,x') = 2\sum_{n=2}^{\infty}\sum_{\nu}\frac{K^{(n\nu)}_{ab}(x)\overline{K^{(n\nu)}_{a'b'}(x')}}{n(n+3)}, \tag{D25}$$

where $K^{(n\nu)}_{ab}(x)$ form a complete orthonormal set of transverse-traceless eigenfunctions satisfying

$$L^{(\mathrm{inv})cd}_{ab}K^{(n\nu)}_{cd} = (-\Box + 2)K^{(n\nu)}_{ab} = n(n+3)K^{(n\nu)}_{ab}, \tag{D26}$$

and

$$\int_{S^4} dS\overline{K^{(n\nu)}_{ab}(x)}K^{(n'\nu')ab}(x) = \delta^{nn'}\delta^{\nu\nu'}. \tag{D27}$$

It is convenient to define the massive Green's function $G^{(TT;M^2)}_{aba'b'}(x,x')$ by

$$\begin{aligned} L^{(M^2)cd}_{ab}G^{(TT;M^2)}_{cda'b'}(x,x') &\equiv L^{(\mathrm{inv})cd}_{ab}G^{(TT;M^2)}_{cda'b'}(x,x') + \tfrac{1}{2}M^2G^{(TT)}_{aba'b'}(x,x') \\ &\quad - \tfrac{1}{2}M^2 g_{ab}g^{cd}G^{(TT;M^2)}_{cda'b'}(x,x') \\ &= \tfrac{1}{2}(-\Box + 2 + M^2)G^{(TT;M^2)}_{aba'b'}(x,x') \\ &= \delta^{(TT)}_{aba'b'}(x,x'), \end{aligned} \tag{D28}$$

where $L^{(\mathrm{inv})cd}_{ab}$ is defined by Eq. (3.2). The transverse-traceless delta function is

$$\delta^{(TT)}_{aba'b'}(x,x') = \sum_{n=2}^{\infty}\sum_{\nu}K^{(n\nu)}_{ab}(x)\overline{K^{(n\nu)}_{a'b'}(x')}. \tag{D29}$$

We clearly have

$$G^{(TT;M^2)}_{aba'b'}(x,x') = 2\sum_{n=2}^{\infty}\sum_{\nu}\frac{K^{(n\nu)}_{ab}(x)\overline{K^{(n\nu)}_{a'b'}(x')}}{n(n+3)+M^2}, \tag{D30}$$

and

$$G^{(TT)}_{aba'b'}(x,x') = \lim_{M\to 0}G^{(TT;M^2)}_{aba'b'}(x,x'). \tag{D31}$$

For spacelike-separated points x and x', the Lorentzian tensor two-point function $\Delta^{(TT;M^2)}_{aba'b'}(x,x')$ equals the Green's function on S^4 satisfying the same equation as $G^{(TT;M^2)}_{aba'b'}(x,x')$, i.e. the first line of Eq. (D28), but with the transverse-traceless delta function $\delta^{(TT)}_{aba'b'}(x,x')$ replaced by the full delta function given by [24]

$$\delta_{aba'b'}(x,x') = \delta^{(TT)}_{aba'b'}(x,x') + \delta^{(TV)}_{aba'b'}(x,x') + \delta^{(TS)}_{aba'b'}(x,x'), \tag{D32}$$

where

$$\delta^{(TV)}_{aba'b'}(x,x') = \sum_{n=2}^{\infty}\sum_{\nu}\frac{2\nabla_{(a}V^{(n\sigma)}_{b)}(x)\overline{\nabla_{(a'}V^{(n\sigma)}_{b')}(x')}}{(n+1)(n+2)-6}, \tag{D33}$$

and, with the definition $\lambda_n = n(n+3)$,

$$\begin{aligned} \delta^{(TS)}_{aba'b'}(x,x') &= \sum_{n=2}^{\infty}\sum_{\nu}\frac{4}{3\lambda_n(\lambda_n-4)}\left(\nabla_a\nabla_b + \frac{\lambda_n}{4}g_{ab}\right)\psi^{(n\nu)}(x) \\ &\quad\times\left(\nabla_{a'}\nabla_{b'} + \frac{\lambda_n}{4}g_{a'b'}\right)\overline{\psi^{(n\nu)}(x')} \\ &\quad + \frac{1}{4}g_{ab}g_{a'b'}\sum_{n=0}^{\infty}\sum_{\nu}\psi^{(n\nu)}(x)\overline{\psi^{(n\nu)}(x')}. \end{aligned} \tag{D34}$$

One can find $\Delta^{(TT;M^2)}_{aba'b'}$ in the form

$$\Delta^{(TT;M^2)}_{aba'b'}(x,x') = G^{(TT;M^2)}_{aba'b'}(x,x') + G^{(TV;M^2)}_{aba'b'}(x,x') + G^{(TS;M^2)}_{aba'b'}(x,x'), \tag{D35}$$

where

$$L^{(M^2)cd}_{ab}G^{(TV)}_{cda'b'}(x,x') = \delta^{(TV)}_{aba'b'}(x,x'), \tag{D36}$$

$$L^{(M^2)cd}_{ab}G^{(TS)}_{cda'b'}(x,x') = \delta^{(TS)}_{aba'b'}(x,x'). \tag{D37}$$

By noting that

$$L_{ab}{}^{(M^2)cd}(\nabla_c V_d + \nabla_d V_c) = \frac{M^2}{2}(\nabla_a V_b + \nabla_b V_a), \tag{D38}$$

one can readily solve Eq. (D36) as

$$G^{(TV;M^2)}_{aba'b'}(x,x') = \frac{2}{M^2}\delta^{(TV)}_{aba'b'}(x,x') = \frac{2}{M^2}\lim_{\mu^2\to -6}[\nabla_{(a}\nabla_{|a'|}G^{(V;\mu^2)}_{b)b'}(x,x') + \nabla_{(a}\nabla_{|b'|}G^{(V;\mu^2)}_{b)a'}(x,x')]$$
$$= \frac{2}{M^2}\lim_{\mu^2\to -6}[\nabla_{(a}\nabla_{|a'|}\Delta^{(V;\mu^2)}_{b)b'}(x,x') + \nabla_{(a}\nabla_{|b'|}\Delta^{(V;\mu^2)}_{b)a'}(x,x')] + \frac{2}{3M^2}\nabla_a\nabla_b\nabla_{a'}\nabla_{b'}\Delta_0^-(x,x'). \quad \text{(D39)}$$

To find $G^{(TS;M^2)}_{aba'b'}(x,x')$, we first observe

$$L_{ab}{}^{(M^2)cd}\nabla_c\nabla_d\psi^{(n\nu)} = \frac{M^2}{2}\nabla_a\nabla_b\psi^{(n\nu)} + \frac{M^2}{2}\lambda_n g_{ab}\psi^{(n\nu)}, \quad \text{(D40)}$$

$$L_{ab}{}^{(M^2)cd}g_{cd}\psi^{(n\nu)} = -\nabla_a\nabla_b\psi^{(n\nu)} - \left(\lambda_n - 3 + \frac{3}{2}M^2\right)g_{ab}\psi^{(n\nu)}. \quad \text{(D41)}$$

The function $G^{(TS;M^2)}_{aba'b'}(x,x')$ can be found as the inverse of the operator $L^{(M^2)cd}_{ab}$ for the modes $g_{ab}\psi^{(n\nu)}$ and $(\nabla_a\nabla_b + \frac{\lambda_n}{4}g_{ab})\psi^{(n\nu)}$ as

$$G^{(TS;M^2)}_{aba'b'}(x,x') = -\frac{2}{3}\sum_{n=2}^{\infty}\sum_{\nu}\left(\frac{1}{M^2\lambda_n} + \frac{1}{(2-M^2)(\lambda_n-4)}\right)\left(\nabla_a\nabla_b + \frac{\lambda_n}{4}g_{ab}\right)\left(\nabla_{a'}\nabla_{b'} + \frac{\lambda_n}{4}g_{a'b'}\right)\psi^{(n\nu)}(x)\overline{\psi^{(n\nu)}(x')}$$
$$+ \frac{1}{3M^2(2-M^2)}\sum_{n=2}^{\infty}\sum_{\nu}\Bigg[g_{ab}\psi^{(n\nu)}(x)\left(\nabla_{a'}\nabla_{b'} + \frac{\lambda_n}{4}g_{a'b'}\right)\overline{\psi^{(n\nu)}(x')}$$
$$+ g_{a'b'}\overline{\psi^{(n\nu)}(x')}\left(\nabla_a\nabla_b + \frac{\lambda_n}{4}g_{ab}\right)\psi^{(n\nu)}(x)\Bigg]$$
$$+ \sum_{n=0}^{\infty}\sum_{\nu}\frac{-\lambda_n + 2M^2}{12M^2(2-M^2)}g_{ab}g_{a'b'}\psi^{(n\nu)}(x)\overline{\psi^{(n\nu)}(x')}. \quad \text{(D42)}$$

Some terms on the right-hand side have support only for $x = x'$ on S^4. For example,

$$\sum_{n=0}^{\infty}\sum_{\nu}\frac{-\lambda_n + 2M^2}{M^2(2-M^2)}\psi^{(n\nu)}(x)\psi^{(n\nu)}(x') = \frac{\Box + 2M^2}{M^2(2-M^2)}\delta(x,x'). \quad \text{(D43)}$$

Thus, for $x \neq x'$ on S^4, or for spacelike-separated points x and x' in de Sitter spacetime, we have

$$G^{(TS;M^2)}_{aba'b'}(x,x') = -\frac{2}{3M^2}\nabla_a\nabla_b\nabla_{a'}\nabla_{b'}\Delta_0^-(x,x') - \frac{2}{3(2-M^2)}\left(\nabla_a\nabla_b - \frac{1}{4}g_{ab}\Box\right)\left(\nabla_{a'}\nabla_{b'} - \frac{1}{4}g_{a'b'}\Box'\right)\Delta^{--}_{-4}(x,x'), \quad \text{(D44)}$$

where we have used the fact that $\Box\Delta_0^-(x,x')$ is a constant [11]. By substituting this equation and Eq. (D39) into Eq. (D35), we find

$$\Delta^{(TT;M^2)}_{aba'b'}(x,x') - G^{(TT;M^2)}_{aba'b'}(x,x') = \frac{2}{M^2}\lim_{\mu^2\to -6}[\nabla_{(a}\nabla_{|a'|}\Delta^{(V;\mu^2)}_{b)b'}(x,x') + \nabla_{(a}\nabla_{|b'|}\Delta^{(V;\mu^2)}_{b)a'}(x,x')]$$
$$-\frac{2}{3(2-M^2)}\left(\nabla_a\nabla_b - \frac{1}{4}g_{ab}\Box\right)\left(\nabla_{a'}\nabla_{b'} - \frac{1}{4}g_{a'b'}\Box'\right)\Delta^{--}_{-4}(x,x'), \quad \text{(D45)}$$

where $G^{(V;\mu^2)}_{aa'}(x,x')$ is defined by Eq. (D16). Then, noting that

$$\lim_{M\to 0}\left\{\Delta^{(TT;M^2)}_{aba'b'}(x,x') - \frac{2}{M^2}\lim_{\mu^2\to -6}[\nabla_{(a}\nabla_{|a'|}\Delta^{(V;\mu^2)}_{b)b'}(x,x') + \nabla_{(a}\nabla_{|b'|}\Delta^{(V;\mu^2)}_{b)a'}(x,x')]\right\}$$
$$= \sum_{\ell=2}^{\infty}\sum_{\sigma}H^{(0;2\ell\sigma)}_{ab}(x)\overline{H^{(0;2\ell\sigma)}_{a'b'}(x')} + \lim_{M\to 0}\frac{1}{M^2}\sum_{m=0}^{1}\sum_{\ell=2}^{\infty}\sum_{\sigma}(-1)^{m+1}[H^{(M^2;m\ell\sigma)}_{ab}(x)\overline{H^{(M^2;m\ell\sigma)}_{a'b'}(x')} - H^{(0;m\ell\sigma)}_{ab}(x)\overline{H^{(0;m\ell\sigma)}_{a'b'}(x')}]$$
$$= \Delta^{(TT)}_{aba'b'}(x,x') \quad \text{(D46)}$$

and using Eq. (5.57), we indeed find Eq. (D12).

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