

Background field method as a canonical transformationD. Binosi¹ and A. Quadri²¹*European Centre for Theoretical Studies in Nuclear Physics and Related Areas (ECT*) and Fondazione Bruno Kessler, Villa Tambosi, Strada delle Tabarelle 286, I-38123 Villazzano (Trento) Italy*²*Dipartimento di Fisica, Università degli Studi di Milano, via Celoria 16, I-20133 Milano, Italy and INFN, Sezione di Milano, via Celoria 16, I-20133 Milano, Italy*

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We construct explicitly the canonical transformation which controls the full dependence (local and nonlocal) of the vertex functional of a Yang-Mills theory on a background field. After showing that the canonical transformation found is nothing but a direct field-theoretic generalization of the Lie transform of classical analytical mechanics, we comment on a number of possible applications, and, in particular, the nonperturbative implementation of the background field method on the lattice, the background field formulation of the two-particle irreducible formalism, and, finally, the formulation of the Schwinger-Dyson series in the presence of topologically nontrivial configurations.

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I. INTRODUCTION

Quantization of non-Abelian gauge theories around background field configurations is a subject of considerable interest. Since the pioneering work of 't Hooft [1], the path-integral around topologically nontrivial field configurations has been used in many different applications, ranging from chiral soliton models in effective approaches to low-energy QCD [2] to the study of critical solitons in supersymmetric models [3].

Quantum fluctuations around the classical background can be treated perturbatively by fixing a gauge while retaining explicit (background) gauge invariance. This is the so-called background field method (BFM) [4], which has been widely used to simplify, technically as well as conceptually, calculations in gauge theories.

While within perturbation theory the BFM has been extended to all orders both in the continuum [5] and on the lattice [6], there is yet no clear-cut prescription on how to handle the BFM quantization in nonperturbative approaches to non-Abelian gauge theories, like, e.g., their *nonperturbative* formulation on the lattice [7] or their treatment in the two-particle irreducible (2-PI) formalism of Cornwall, Jackiw, and Toumbulis [8]. One of the open issues in realizing this program is the existence of Gribov copies [9] which prevents a direct generalization of the perturbative treatment.

Clearly, if one were able to implement the BFM for *both* nonperturbative lattice gauge theory and the 2-PI formalism, one could make significant progress in the computation of nonperturbative lattice quantities, as well as in understanding the matching with their continuum counterparts. For instance, the one-loop correspondence [10] between the pinch technique [11] and the BFM Green's functions has been shown to hold true to all orders for the background-dependent amplitudes [12,13]. Then, the simulation of the background gluon two-point function on a lattice gauge fixed in the BFM (Feynman) gauge would

allow one to construct a renormalization group invariant propagator (which is independent of the renormalization point μ chosen, or, conversely, of the lattice space chosen) by simply multiplying the unrenormalized propagator by the square of the unrenormalized charge, as in QED. Moreover, the ability to extend the BFM to the aforementioned contexts would open up a wide range of gauge-invariant simulations and variational estimates. This would translate into very useful phenomenology for addressing the properties of the infrared sector of Yang-Mills theories and, in particular, phenomena like confinement, chiral symmetry breaking, and/or dynamical gluon mass generation.

Surprisingly enough, it turns out that one can give a very simple characterization of the dependence of the effective action on the background which holds in a very general setting. Specifically, as we will show in this paper, it turns out that, whenever the extended Slavnov Taylor (ST) identity in the presence of the background is fulfilled, the background dependence of the effective action is governed by a canonical transformation with respect to (w.r.t.) the fundamental Batalin-Vilkovisky (BV) bracket of the underlying gauge theory. Consequently, one can draw a fruitful analogy with the theory of Lie transforms in classical analytical mechanics and obtain simple and powerful formulas for the finite canonical transformation which fix (uniquely) the background-dependent amplitudes in terms of those at zero background.

In a purely nonperturbative setting, the definition of the ST identity is a delicate issue requiring a careful analysis, which exceeds the purpose and scope of this paper. Nevertheless, we would like to point out that the approach proposed in the present paper does not require the presence of dynamical ghosts. This represents an advantage w.r.t. the conventional techniques for implementing the BFM on the lattice, since, as we will explicitly see, it allows in principle to evade the Neuberger's 0/0 problem [14].

II. BACKGROUND FIELDS AND CANONICAL TRANSFORMATIONS

Within the BV framework, the *complete* vertex functional Γ of a SU(N) Yang-Mills theory, quantized in a linear background gauge, can be written in terms of the fields $\phi = (A_\mu^a, c^a, \bar{c}^a, b^a)$, the antifields $\phi^* = (A_\mu^{*a}, c^{*a})$, the background field \hat{A}_μ^a , and its associated source Ω_μ^a [15]. The antifields (\bar{c}^{*a}, b^{*a}) are not needed, since the fields b^a and \bar{c}^a form a Becchi-Rouet-Stora-Tyutin (BRST) doublet [16,17], i.e., a set of variables u, v transforming under the BRST differential s according to $su = v, sv = 0$. This, together with the fact that the b dependence is confined at tree level, allows one to eliminate the doublet (\bar{c}^a, b^a) by means of a canonical transformation yielding the so-called *reduced* functional.

For example, if one considers the background Lorentz-covariant gauge-fixing function $\hat{\mathcal{F}}^a = [\hat{\mathcal{D}}^\mu (A - \hat{A})_\mu]^a$ with $\hat{\mathcal{D}}_\mu^{ab} = \delta^{ab} \partial_\mu + f^{acb} \hat{A}_\mu^c$ the background covariant derivative, the complete tree-level vertex functional reads

$$\begin{aligned} \Gamma^{(0)} = \int d^4x & \left[-\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} - \bar{c}^a (\hat{\mathcal{D}}_\mu \mathcal{D}^\mu c)^a \right. \\ & - (\mathcal{D}^\mu \bar{c})^a \Omega_\mu^a - \frac{\xi}{2} (b^a)^2 + b^a [\hat{\mathcal{D}}^\mu (A - \hat{A})_\mu]^a \\ & \left. + A_\mu^{*a} (\mathcal{D}^\mu c)^a + \frac{1}{2} f^{abc} c^{*a} c^b c^c \right]. \end{aligned} \quad (1)$$

The reduced functional is then obtained by first defining

$$\tilde{\Gamma} = \Gamma - \int d^4x b^a [\hat{\mathcal{D}}^\mu (A - \hat{A})_\mu]^a + \frac{\xi}{2} \int d^4x (b^a)^2$$

and then eliminating \bar{c}^a through the antifield redefinition $\tilde{A}_\mu^{*a} = A_\mu^{*a} + (\hat{\mathcal{D}}_\mu \bar{c})^a$, which, due to the antighost equation $\frac{\delta \tilde{\Gamma}}{\delta \bar{c}^a} = -\hat{\mathcal{D}}_\mu^{ab} \frac{\delta \tilde{\Gamma}}{\delta A_\mu^{*b}} + (\mathcal{D}^\mu \Omega_\mu)^a$, represents the only combination through which the vertex functional could possibly depend on \bar{c}^a . In what follows, we will always use the reduced vertex functional and thus drop the tilde symbols on all quantities.

As shown in Ref. [18], the *extended* ST identity in the presence of a background field can be written as

$$\int d^4x \Omega_\mu^a(x) \frac{\delta \Gamma}{\delta \hat{A}_\mu^a(x)} = -\frac{1}{2} \{\Gamma, \Gamma\}, \quad (2)$$

where $\{X, Y\}$ represents the BV bracket defined as (only left derivative assumed in what follows) [19]

$$\begin{aligned} \{X, Y\} = \int d^4x \sum_\phi & \left[(-1)^{\epsilon_\phi(\epsilon_X+1)} \frac{\delta X}{\delta \phi} \frac{\delta Y}{\delta \phi^*} \right. \\ & \left. - (-1)^{\epsilon_{\phi^*}(\epsilon_X+1)} \frac{\delta X}{\delta \phi^*} \frac{\delta Y}{\delta \phi} \right]. \end{aligned} \quad (3)$$

The sum runs over the fields $\phi = (A_\mu^a, c^a)$ and the corresponding antifields $\phi^* = (A_\mu^{*a}, c^{*a})$, with $\epsilon_\phi, \epsilon_{\phi^*}$ and ϵ_X

representing the statistics of the field ϕ , the antifield ϕ^* and the functional X , respectively. For the graded properties of the BV bracket, the reader is referred to Ref. [19].

If one now takes the derivative of Eq. (2) w.r.t. Ω_μ^a and sets the latter source equal to zero afterward, the resulting equation [18]

$$\frac{\delta \Gamma}{\delta \hat{A}_\mu^a(x)} \Big|_{\Omega=0} = - \left[\frac{\delta \Gamma}{\delta \Omega_\mu^a(x)}, \Gamma \right] \Big|_{\Omega=0} \quad (4)$$

shows that the derivative of the vertex functional w.r.t. the background field equals the effect of an infinitesimal canonical transformation (w.r.t. the BV bracket) on the vertex functional itself. Then, since the BV bracket does not depend on either \hat{A}_μ^a or Ω_μ^a , if one were able to write the finite canonical transformation generated by the fermion $\Psi_\mu^a(x) = \frac{\delta \Gamma}{\delta \Omega_\mu^a(x)}$, one would control the full dependence of Γ on the background fields; and this would happen not only at the level of the counterterms of Γ , but rather for the full 1-PI Green's functions, thus giving control even over the nonlocal dependence on the background.

The problem can be thus stated as follows: given the field and antifield variables ϕ, ϕ^* , which are canonical w.r.t. the BV bracket (3), i.e.,

$$\begin{aligned} \{\phi_i(x), \phi_j(y)\} &= \{\phi_i^*(x), \phi_j^*(y)\} = 0 \\ \{\phi_i(x), \phi_j^*(y)\} &= \delta_{ij} \delta^4(y-x), \end{aligned}$$

and the background field \hat{A}_μ^a , find the canonical mapping

$$(\phi(x), \phi^*(x); \hat{A}_\mu^a(x)) \mapsto (\Phi(x), \Phi^*(x)),$$

to the new field and antifield variables Φ and Φ^* such that the ST identity (4) written in these new variables is automatically satisfied. This last condition translates in a relatively straightforward fashion, in determining the canonical variables Φ and Φ^* which are also solutions of the two equations:

$$\begin{aligned} \frac{\delta \Phi(y)}{\delta \hat{A}_\mu^a(x)} &= \frac{\delta \Psi_\mu^a(x)}{\delta \Phi^*(y)} = \{\Phi(y), \Psi_\mu^a(x)\}, \\ \frac{\delta \Phi^*(y)}{\delta \hat{A}_\mu^a(x)} &= -\frac{\delta \Psi_\mu^a(x)}{\delta \Phi(y)} = \{\Phi^*(y), \Psi_\mu^a(x)\}. \end{aligned} \quad (5)$$

Before proceeding to construct explicitly the canonical mapping, let us notice that a (recursive) solution of the *finite* canonical transformation has been already derived by means of homotopy techniques in Ref. [18], where it was found that this solution fails to respect the (naively expected) exponentiation pattern, due to the dependence of the generating functional Ψ_μ^a on the background field \hat{A}_μ^a .

In order to find the explicit canonical transformation, let us then introduce the operator

$$\Delta_{\Psi_\mu^a(x)} = \{\cdot, \Psi_\mu^a(x)\} + \frac{\delta}{\delta \hat{A}_\mu^a(x)}.$$

The first term above represents a (graded) generalization (to the BV bracket and a fermionic generator) of the classical Lie derivative w.r.t a (bosonic) generator (in which case the bracket would be the usual Poisson bracket); the second term takes into account the above observation on the exponentiation failure.

Using then the properties of the BV bracket, it is not particularly difficult to establish the following relations:

$$\begin{aligned}\Delta_{\Psi_\mu^a(x)}(\alpha X + \beta Y) &= \alpha \Delta_{\Psi_\mu^a(x)} X + \beta \Delta_{\Psi_\mu^a(x)} Y, \\ \Delta_{\Psi_\mu^a(x)}(XY) &= X \Delta_{\Psi_\mu^a(x)} Y + (-1)^{\epsilon_X \epsilon_Y} Y \Delta_{\Psi_\mu^a(x)} X, \\ \Delta_{\Psi_\mu^a(x)}\{X, Y\} &= \{\Delta_{\Psi_\mu^a(x)} X, Y\} + \{X, \Delta_{\Psi_\mu^a(x)} Y\}.\end{aligned}$$

The first two equations above establish that $\Delta_{\Psi_\mu^a(x)}$ gives rise to a graded derivation with the usual statistics, while the last formula allows us to determine the important result

$$\begin{aligned}\int_1 \dots \int_n \hat{A}_1 \dots \hat{A}_n \Delta_{\Psi_n} \dots \Delta_{\Psi_1} \{X, Y\} \\ = \sum_{0 \leq m \leq n} \binom{n}{m} \{\Delta_{\Psi_1} \dots \Delta_{\Psi_m} X, \Delta_{\Psi_{m+1}} \dots \Delta_{\Psi_n} Y\},\end{aligned}\quad (6)$$

where we have introduced the shorthand notation $\int_i = \int d^4 y_i$, $\hat{A}_i = \hat{A}_{\mu_i}^{a_i}(y_i)$ and $\Psi_i = \Psi_{\mu_i}^{a_i}(y_i)$.

From the operator Δ_Ψ , one can then define a mapping E_Ψ given in terms of a formal power series in the background field \hat{A} as follows:

$$\begin{aligned}\Phi(x) &= E_\Psi(\phi(x)) \\ &\equiv \sum_{n \geq 0} \frac{1}{n!} \int_1 \dots \int_n \hat{A}_1 \dots \hat{A}_n [\Delta_{\Psi_n} \dots \Delta_{\Psi_1} \phi(x)]_{\hat{A}=0},\end{aligned}\quad (7)$$

with an identical expansion holding for the antifields variables. Then Eq. (7) constitutes the sought-for canonical mapping between the old and the new variables.

Indeed, on the one hand, the canonicity property is a direct consequence of Eq. (6) above, since the latter directly implies the identity $E_\Psi\{X, Y\} = \{E_\Psi X, E_\Psi Y\}$. On the other hand, to see that the new variables are indeed solutions of Eqs. (5), let us concentrate on the case of a bosonic field Φ and expand both the latter and the fermionic generator Ψ_μ^a in power series w.r.t. the background field \hat{A} . Schematically, one has

$$\begin{aligned}\Phi &= \phi + \sum_{n \geq 0} \frac{1}{n!} \int_1 \dots \int_n \hat{A}_1 \dots \hat{A}_n \Phi_{1\dots n}, \\ \Psi_0 &= \psi_0 + \sum_{n \geq 0} \frac{1}{n!} \int_1 \dots \int_n \hat{A}_1 \dots \hat{A}_n \Psi_{01\dots n},\end{aligned}$$

and finds up to third order in \hat{A}

$$\begin{aligned}\Delta_{\Psi_1} \phi|_{\hat{A}=0} &= \{\phi, \psi_1\}, \\ \Delta_{\Psi_2} \Delta_{\Psi_1} \phi|_{\hat{A}=0} &= \{\{\phi, \psi_1\}, \psi_2\} + \{\phi, \Psi_{12}\}, \\ \Delta_{\Psi_3} \Delta_{\Psi_2} \Delta_{\Psi_1} \phi|_{\hat{A}=0} &= \{\{\{\phi, \psi_1\}, \psi_2\}, \psi_3\} + \{\phi, \Psi_{123}\} \\ &\quad + \{\{\phi, \Psi_{12}\}, \psi_3\} + \{\{\phi, \Psi_{23}\}, \psi_1\} \\ &\quad + \{\{\phi, \Psi_{31}\}, \psi_2\},\end{aligned}$$

where in the last equation, we have symmetrized all indices and used the (graded) Jacobi identity together with the result $\int_1 \int_2 \hat{A}_1 \hat{A}_2 \frac{\delta}{\delta \hat{A}_3} \{\Psi_1, \Psi_2\} = 0$. It can then be checked that the above terms are indeed the solutions (up to third order in \hat{A}) of the first of Eqs. (5). The fermionic case, e.g., a fermionic antifield Φ^* , can be treated in exactly the same way.

III. DISCUSSION

There are several comments which can be made w.r.t. the canonical transformation (7).

To begin with, it should be noticed that such transformation, together with the method used for constructing it, is nothing but a direct generalization of the procedure developed long ago by Deprit [20] to construct canonical mappings in the form of (formal) power series in a small parameter ϵ , in cases where the generating function itself explicitly depends on ϵ . In this case, the problem one tries to address is the following: given a function V which depends on the (canonical) variables q, p and a parameter ϵ , find a canonical mapping $(q, p; \epsilon) \mapsto (Q, P)$, such that the new variables Q and P satisfy the equations

$$\frac{dQ}{d\epsilon} = \frac{\partial}{\partial P} V(p, q; \epsilon); \quad \frac{dP}{d\epsilon} = -\frac{\partial}{\partial Q} V(p, q; \epsilon).$$

The solution to this problem is found [20] by enlarging the concept of a Lie series through the introduction of the operator $\Delta_V = \{\cdot, V\} + \frac{\partial}{\partial \epsilon}$ (the bracket being now the usual Poisson bracket) and next defining, for any given function f of the variables p, q and ϵ , the formal power series (Lie transform generated by V)

$$E_V(f) = \sum_n \frac{1}{n!} \epsilon^n \Delta_{V(q,p;\epsilon)}^n f(q, p; \epsilon)|_{\epsilon=0}.$$

Then, the new canonical variables are given by the Lie transforms $Q = E_V(q)$ and $P = E_V(p)$. We thus see that the canonical transformation found has a sound geometric interpretation, being a direct field-theoretic generalization of the Lie transform in classical analytical mechanics. Given this connection, one might conversely wonder what advantages could bring the use of the homotopy techniques of Ref. [18] in a completely classical context.

Second, the mapping (7) provides a new set of field variables such that when the conventional Green's functions are written in terms of these new variables, they would coincide with those calculated in the BFM, thus

explaining the aforementioned correspondence between them. It would be then very interesting to supplement the current formulation of the canonical transformation with Nielsen identities [21] and study the flow of Eq. (7) as ξ moves towards the critical value $\xi = 1$, where it is known that the BFM Green's functions acquire additional physical properties [22].

Third, it should be noticed that at no point in this analysis, we have relied on the Ward identity usually associated with background linear gauge fixings such as the one used for illustrative purposes in Eq. (1). Indeed, the only requirement we have on the gauge-fixing fermion is that it is possible to construct the canonical mapping which eliminates the BRST doublet (\bar{c}^a, b^a) , thus allowing for the writing of the reduced vertex functional and ultimately of the extended ST identity (2) (which happens in the vast majority of cases). This shows that it is the (extended) ST identity and not the Ward identity which forces the strongest constraints on the theory, in agreement with the findings of Ref. [23], where it was shown that the background Ward identity alone is not able to guarantee physical unitarity (i.e., the cancellation of the intermediate ghost states in physical amplitudes) in the absence of the ST identity.

As for applications, we can think of at least three. The first one is in relation to the nonperturbative formulation of the BFM on the lattice. At first sight, this claim looks surprising, since such a formulation would require a BRST-invariant integration over link variables, and it is well known that the so-called Neuberger 0/0 problem [14] forbids a direct nonperturbative generalization of the BRST symmetry. Thus, the extended ST identity, which clearly constitutes the central pillar of our construction, would not be present either. However, notice that the canonical transformation (7) can also be written in a (gauge-invariant) model where the ghosts are replaced by external classical anticommuting sources, i.e., the BV bracket spans the gauge field A_μ^a and its antifield A_μ^{*a} and, for a gauge fixing obtained by minimizing some functional $F[g]$ over the gauge group, the parameters of the group element g and their antifields. Thus, the fact that *dynamical* ghosts need not be present in the formulation overcomes the absence at the nonperturbative level of the BRST symmetry. Assume then that one is able to fix a background gauge, e.g., through the minimization of a suitable functional $F[g]$ (recall that our derivation does not rely on the particular gauge fixing chosen); assume also that at a *fixed* background \hat{A} , such functional depends on the gauge field only through the combination $A_\mu^g - \hat{A}_\mu$ (leading to the most economical generalization of the ordinary Landau gauge functional) with $A_\mu^g = g^\dagger A_\mu g - i\partial_\mu g^\dagger g$ and g a gauge group element. The simplest generalization of the ordinary Landau gauge functional is

$$F[g] = - \int d^4x \text{Tr}(A_\mu^g - \hat{A}_\mu)^2. \quad (8)$$

When minimized, it gives the background Landau gauge condition $\hat{\mathcal{D}}_\mu(A_\mu^g - \hat{A}_\mu) = 0$.¹ Then, *on the minimum* of the functional (8), the mapping

$$A \rightarrow A^g(A, \hat{A}) - \hat{A} \quad (9)$$

defines the action of the canonical transformation on the gauge field, thus generalizing nonperturbatively the background quantum splitting. In the presence of Gribov copies, multiple minima exist which are parametrized by different functions $g_i(A, \hat{A})$; however, the canonical mapping (9) allows for reconstructing the full dependence on the background also in such case, provided that we restrict ourselves to the region of validity of each g_i .

At this point, the strategy would then be to reconstruct the dependence on the background of the various quantities calculated through this canonical transformation (with a suitable extension to the gauge antifield). Notice that this canonical mapping provides highly nontrivial constraints, relating quantum and background Green's functions, which can therefore be tested, at least in principle, on the lattice.

A second application is the BFM formulation of the 2-PI formalism [8]. To get an idea of how this can be accomplished, observe that the extended ST identity (2) can be rewritten in terms of the generator of the connected Green's functions $W[J] = \Gamma[\Phi] + \int J\Phi$ as

$$\int d^4x \Omega_\mu^a(x) \frac{\delta W}{\delta \hat{A}_\mu^a(x)} = - \int d^4x J(x) \frac{\delta W}{\delta \Phi^*(x)}. \quad (10)$$

Starting from the connected diagrams, which are assumed to satisfy Eq. (10), one can perform a double Legendre transform

$$W[J, K] = \Gamma[\Phi, G] + \int J\Phi + 1/2 \iint \Phi K \Phi + 1/2\hbar \iint GK$$

and derive the corresponding extended ST identity for the 2-PI effective action $\Gamma[\Phi, G]$. This, however, implies the introduction of the BRST doublet $s\chi = \pm K$ and $sK = 0$ (the \pm sign corresponding to bosonic/fermionic fields) and the addition to the tree-level action of the composite operator term

$$s \frac{1}{2} \iint \Phi \chi \Phi = \frac{1}{2} \iint \Phi K \Phi + \iint \chi \frac{\delta \Gamma}{\delta \Phi^*} \Phi.$$

Because of the nilpotency of the BRST operator, this term does not violate the ST identity; then, one can study the

¹Notice that the background Landau gauge condition for the quantum field Q_μ can be obtained by finding the extrema of the functional $\int d^4x \text{Tr} Q_\mu \hat{A}_\mu^g$, where \hat{A}_μ^g is the gauge-transformed background field, i.e., one gauge-rotates the background field by keeping Q_μ fixed. We remark, however, that this procedure does not select in general a unique representative along the gauge orbit of Q_μ .

extra terms which are bound to appear in Eq. (10) and at the same time keep under control the renormalization of the operators added.

The third application is the study of the conditions under which the (nonperturbative) Schwinger-Dyson (SD) equations can be reliably trusted when expanding around non-trivial vacua. As shown in Ref. [24] for some toy models, a naive SD expansion is poor when the potential admits more than one minimum; on the other hand, a modified SD formulation is required in order to improve on the saddle point approximation in such cases. The formalism developed here and in Ref. [18] can indeed help in formulating the SD expansion in the presence of topologically non-trivial vacuum configurations (instantons, center vortices, monopoles, etc.); indeed, the framework of Ref. [25] and its related truncation scheme, which in the Landau gauge

compares favorably with large-volume lattice simulations, could be generalized to study the effects due to the presence of such solitons in the theory vacuum.

Concluding, in this paper, we have explicitly constructed the canonical transformation which controls the full dependence of the vertex functional Γ on the background field \hat{A}_μ^a . Though, being an interesting result in its own right, especially given its connection with classical analytical mechanics, its strongest appeal resides in the many interesting directions it opens up.

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