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Reappraisal of two-loop contributions to the fermion electric dipole moments in *R*-parity violating supersymmetric models

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We reexamine the *R*-parity violating contribution to the fermion electric and chromo-electric dipole moments in the two-loop diagrams. It is found that the leading Barr-Zee-type two-loop contribution is smaller than the result found in previous works, and that electric dipole moment experimental data provide looser limits on *R*-parity violating couplings.

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The supersymmetry is known to resolve many theoretical problems which have been encountered in the standard model (SM), such as the cancellation of the power divergences in radiative corrections, and its supersymmetric extension is therefore one of the promising candidates of new physics. The supersymmetric SM can be extended to allow baryon number or lepton number violating interactions, known as the *R*-parity violating (RPV) interactions, and they have been constrained from the analysis of various phenomena [1].

The electric dipole moment (EDM) is an excellent observable to investigate the underlying mechanisms of the *P* and *CP* violations and can be measured in a variety of systems [2]. Since the contribution of the SM to the EDM is, in general, small [3], it is a very good experimental observable to examine the supersymmetric models and other candidates of new physics. In the past three decades, many analyses of the supersymmetric models with [4–7] and without [8–13] the conservation of *R* parity have been done using the EDMs.

In the RPV supersymmetric model with trilinear RPV interactions, it has been found that the fermion (quark or lepton) EDM does not receive any one-loop contribution [9], and the two-loop contribution has been analyzed in detail to give the Barr-Zee-type diagram as the leading contribution [11]. In this paper, we reexamine the RPV Barr-Zee-type contribution which turns out to be in disagreement with previous works [10,11,13]. We will show that the RPV Barr-Zee-type diagram actually has a smaller contribution than that given in previous works.

The RPV interactions are generated by the following superpotential:

$$W_{\not R} = \frac{1}{2}\lambda_{ijk}\epsilon_{ab}L_i^a L_j^b (E^c)_k + \lambda'_{ijk}\epsilon_{ab}L_i^a Q_j^b (D^c)_k, \quad (1)$$

with i, j, $k = 1, 2, 3$ indicating the generation, and a, $b = 1, 2$ the $SU(2)_L$ indices. L and E^c denote the lepton doublet and singlet left-chiral superfields. Q, U^c, and D^c denote, respectively, the quark doublet, up quark singlet, and down quark singlet left-chiral superfields. The RPV baryon number violating interactions are irrelevant in this analysis since they do not contribute to the Barr-Zee-type diagrams, and are not included in our current analysis. The bilinear RPV interactions were not considered either. The RPV Lagrangian of interest is then given as

$$\begin{aligned}\mathcal{L}_{\not R} = &-\tfrac{1}{2}\lambda_{ijk}[\tilde{\nu}_i \bar{e}_k P_L e_j + \tilde{e}_{Lj}\bar{e}_k P_L \nu_i + \tilde{e}^\dagger_{Rk}\bar{\nu}^c_i P_L e_j \\ &- (i\leftrightarrow j)] + (\text{H.c.}) - \lambda'_{ijk}[\tilde{\nu}_i \bar{d}_k P_L d_j + \tilde{d}_{Lj}\bar{d}_k P_L \nu_i \\ &+ \tilde{d}^\dagger_{Rk}\bar{\nu}^c_i P_L d_j - \tilde{e}_{Li}\bar{d}_k P_L u_j - \tilde{u}_{Lj}\bar{d}_k P_L e_i \\ &- \tilde{d}^\dagger_{Rk}\bar{e}^c_i P_L u_j] + (\text{H.c}), \end{aligned} \quad (2)$$

where $P_L \equiv \frac{1}{2}(1-\gamma_5)$, and we also define $P_R \equiv \frac{1}{2}(1+\gamma_5)$ for later use. These RPV interactions are lepton number violating Yukawa interactions.

The EDM d_F of the fermion is defined as follows:

$$\mathcal{L}_{\text{EDM}} = -i\frac{d_F}{2}\bar{\psi}\gamma_5\sigma^{\mu\nu}\psi F_{\mu\nu}, \quad (3)$$

where $F_{\mu\nu}$ is the electromagnetic field strength. With the RPV Lagrangian (2), the sneutrino exchange Barr-Zee-type diagrams shown in Fig. 1 contribute to the EDM. Here the emission (absorption) of the sneutrino from the fermion is accompanied by a P_R (P_L) projection operator, as is apparent from Eq. (2).

At first, we give the expression of the two-photon decay amplitude of annihilation and production of the sneutrino with an internal fermion loop shown in Fig. 2, given as

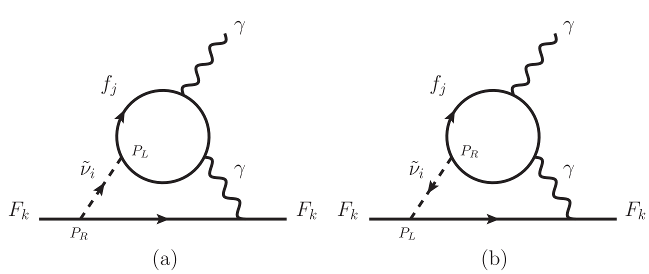

FIG. 1. Examples of Barr-Zee-type two-loop contributions to the fermion EDM within RPV interactions. The projections of the chirality (P_L and P_R) were explicitly given for the RPV vertex.

$$\epsilon_\mu^*(q_1)\epsilon_\nu^*(q_2)i\mathcal{M}_L^{\mu\nu}(q_1,q_2) = -\hat{\lambda}_{ijj}n_c(Q_f e)^2\epsilon_\mu^*(q_1)\epsilon_\nu^*(q_2)\int\frac{d^4k}{(2\pi)^4}\frac{\mathrm{Tr}[(\not{k}+\not{q}_1+m_{f_j})\gamma^\mu(\not{k}+m_{f_j})\gamma^\nu(\not{k}-\not{q}_2+m_{f_j})P_L]}{[(k+q_1)^2-m_{f_j}^2][k^2-m_{f_j}^2][(k-q_2)^2-m_{f_j}^2]}$$
$$\approx\frac{im_{f_j}\hat{\lambda}_{ijj}n_c(Q_f e)^2}{(4\pi)^2}\epsilon_\mu^*(q_1)\epsilon_\nu^*(q_2)\int_0^1 dx\frac{(1-2x(1-x))(q_2^\mu q_1^\nu-(q_1\cdot q_2)g^{\mu\nu})-i\epsilon^{\mu\nu}{}_{\alpha\beta}q_1^\alpha q_2^\beta}{m_{f_j}^2-x(1-x)q_1^2}, \quad (4)$$

$$\epsilon_\mu^*(q_1)\epsilon_\nu^*(q_2)i\mathcal{M}_R^{\mu\nu}(q_1,q_2) = -\hat{\lambda}^*_{ijj}n_c(Q_f e)^2\epsilon_\mu^*(q_1)\epsilon_\nu^*(q_2)\int\frac{d^4k}{(2\pi)^4}\frac{\mathrm{Tr}[(\not{k}+\not{q}_1+m_{f_j})\gamma^\mu(\not{k}+m_{f_j})\gamma^\nu(\not{k}-\not{q}_2+m_{f_j})P_R]}{[(k+q_1)^2-m_{f_j}^2][k^2-m_{f_j}^2][(k-q_2)^2-m_{f_j}^2]}$$
$$\approx\frac{im_{f_j}\hat{\lambda}^*_{ijj}n_c(Q_f e)^2}{(4\pi)^2}\epsilon_\mu^*(q_1)\epsilon_\nu^*(q_2)\int_0^1 dx\frac{(1-2x(1-x))(q_2^\mu q_1^\nu-(q_1\cdot q_2)g^{\mu\nu})+i\epsilon^{\mu\nu}{}_{\alpha\beta}q_1^\alpha q_2^\beta}{m_{f_j}^2-x(1-x)q_1^2}, \quad (5)$$

where i and j denote the flavor indices of $\tilde{\nu}$ and the loop fermion, respectively. $\hat{\lambda}$ is the R-parity violating coupling: $\hat{\lambda}=\lambda$ when the charged lepton runs in the loop, and $\hat{\lambda}=\lambda'$ in the case of a down-type quark. $n_c=1$ ($n_c=3$) if f_j is a lepton (quark). m_{f_j} and Q_f are the mass and the charge in units of e of the loop fermion, respectively. The second line is the approximated expression when q_2 is small. To be precise, the Levi-Civita tensor is defined by $\epsilon^{0123}\equiv+1$ and $\gamma_5\equiv i\gamma^0\gamma^1\gamma^2\gamma^3$.

We now insert the effective $\tilde{\nu}\gamma\gamma$ vertices (4) and (5) into the whole Barr-Zee-type diagram. Then we end up with

$$i\mathcal{M}_{\mathrm{BZ}} = -\tilde{\lambda}^*_{ikk}Q_F e\epsilon_\nu^*(q)\int\frac{d^4k}{(2\pi)^4}\frac{\bar{u}(p-q)\gamma_\mu(\not{p}-\not{q}-\not{k}+m_{F_k})P_R u(p)\cdot\mathcal{M}_L^{\mu\nu}(k,q)}{k^2[(q+k)^2-m_{\tilde{\nu}_i}^2][(p-q-k)^2-m_{F_k}^2]}$$
$$-\tilde{\lambda}_{ikk}Q_F e\epsilon_\nu^*(q)\int\frac{d^4k}{(2\pi)^4}\frac{\bar{u}(p-q)\gamma_\mu(\not{p}-\not{q}-\not{k}+m_{F_k})P_L u(p)\cdot\mathcal{M}_R^{\mu\nu}(k,q)}{k^2[(q+k)^2-m_{\tilde{\nu}_i}^2][(p-q-k)^2-m_{F_k}^2]}$$
$$\approx i\,\mathrm{Im}(\hat{\lambda}_{ijj}\tilde{\lambda}^*_{ikk})\frac{\alpha_{\mathrm{em}}}{(4\pi)^3}n_c Q_f^2 Q_F e\frac{1}{m_{f_j}}\left\{f\left(\frac{m_{f_j}^2}{m_{\tilde{\nu}_i}^2}\right)-g\left(\frac{m_{f_j}^2}{m_{\tilde{\nu}_i}^2}\right)\right\}\epsilon_\nu^*(q)\bar{u}\sigma^{\mu\nu}q_\mu\gamma_5 u, \quad (6)$$

where $\tilde{\lambda}=\lambda$ for the lepton EDM contribution and $\tilde{\lambda}=\lambda'$ for the quark EDM contribution, and f and g are defined as

$$f(z)=\frac{z}{2}\int_0^1 dx\frac{1-2x(1-x)}{x(1-x)-z}\ln\left(\frac{x(1-x)}{z}\right), \quad (7)$$

$$g(z)=\frac{z}{2}\int_0^1 dx\frac{1}{x(1-x)-z}\ln\left(\frac{x(1-x)}{z}\right), \quad (8)$$

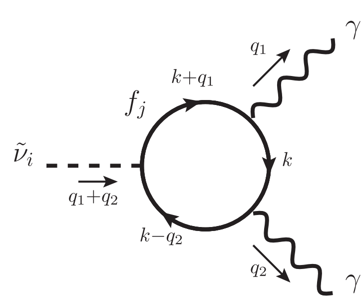

FIG. 2. One-loop $\tilde{\nu}\gamma\gamma$ vertex generated with RPV interactions.

in the original notation of Barr and Zee [14]. For small z, we have $g(z)\approx\frac{z}{2}(\frac{\pi^2}{3}+(\ln z)^2)$ and $f(z)\approx\frac{z}{2}(\frac{\pi^2}{3}+4+2\ln z+(\ln z)^2)$. In the last line of Eq. (6), we have taken only the part of $i\mathcal{M}_{\mathrm{BZ}}$ which contributes to the EDM, disregarding $\mathrm{Re}(\hat{\lambda}_{ijj}\tilde{\lambda}^*_{ikk})$. For each diagram of Fig. 1, there are also diagrams with the internal fermion loop reversed and those with internal photon and sneutrino lines interchanged. They all give the same amplitude, and Eq. (6) should be multiplied by 4. The total EDM of the fermion F from the Barr-Zee-type diagrams with an R-parity violating interaction is then

$$d_{F_k}=\mathrm{Im}(\hat{\lambda}_{ijj}\tilde{\lambda}^*_{ikk})\frac{\alpha_{\mathrm{em}}n_c Q_f^2 Q_F e}{16\pi^3 m_{f_j}}\cdot\{f(\tau)-g(\tau)\}$$
$$\approx\mathrm{Im}(\hat{\lambda}_{ijj}\tilde{\lambda}^*_{ikk})\frac{\alpha_{\mathrm{em}}n_c Q_f^2 Q_F e}{16\pi^3 m_{f_j}}\cdot\tau(2+\ln\tau+\cdots), \quad (9)$$

where $\tau=m_{f_j}^2/m_{\tilde{\nu}_i}^2$. The flavor index, electric charge, and number of color of the fermion F are denoted, respectively, by k, $Q_F e$, and n_c ($n_c=3$ if the inner loop fermion is a quark, otherwise $n_c=1$), and j and $Q_f e$ are, respectively,

TABLE I. The electron EDM d_e [10^{-27} e cm]. The coupling constants of RPV interactions are set to $\mathrm{Im}(\lambda_{233}\lambda^*_{211}) = \mathrm{Im}(\sum_i \lambda'_{i33}\lambda^*_{i11}) = 10^{-5}$ and the masses of the b-quark and tau lepton are set to $m_b = 4.2$ GeV and $m_\tau = 1.78$ GeV. For comparison, we have shown the EDM calculated by replacing $f - g$ by $f + g$.

$m_{\tilde{\nu}}$	Tau lepton		Bottom quark	
(TeV)	$f - g$ (ours)	$f + g$	$f - g$ (ours)	$f + g$
0.1	3.14	−32.1	1.80	−15.9
1	5.50×10^{-2}	-7.89×10^{-1}	3.64×10^{-2}	-4.64×10^{-1}
5	2.87×10^{-3}	-4.99×10^{-2}	1.98×10^{-3}	-3.12×10^{-2}

the flavor index and electric charge of the inner loop fermion f. The second line of the above equation is the approximated expression for small τ. Note also that the Barr-Zee-type diagram gives an EDM contribution only to down-type quarks, and the same property holds also for the chromo-EDM (cEDM) seen below.

We see from Eq. (6) (see also Fig. 1) that the chirality structure of the scalar exchange between the internal loop and external line (RPV vertices with sneutrino exchange) has the form $P_L \otimes P_R$ and $P_R \otimes P_L$, which is a consequence of the lepton number conservation of the whole EDM process. This gives, as a result, the structure

$$f(\tau) - g(\tau) \approx \tau(2 + \ln\tau), \tag{10}$$

in the final formula (9). This is consistent with the result obtained in the analysis of the Barr-Zee-type diagram analogues with the exchange of Higgs bosons in the two Higgs doublet model, done originally by Barr and Zee [14] (see also [15]). In the two Higgs doublet model, there are also additional contributions with the structures $P_L \otimes P_L$ and $P_R \otimes P_R$ that yield a contribution proportional to

$$f(\tau) + g(\tau) \approx \tau\left(\frac{\pi^2}{3} + 2 + \ln\tau + (\ln\tau)^2\right), \tag{11}$$

which is absent in the RPV supersymmetric models.

The small τ behavior in Eq. (9) is in contradiction with the result presented in Refs. [10,11,13], where the RPV Barr-Zee-type diagrams receive the leading contribution proportional to $\tau(\ln\tau)^2$. If one would replace $f(\tau) - g(\tau)$ in Eq. (9) by $f(\tau) + g(\tau)$, the formula given in Refs. [10,11,13] would be obtained. The difference between these two results is large. For example, if we consider the Barr-Zee-type diagram with the bottom quark and tau lepton in the loop, the electron EDM evaluated from our formula in Eq. (9) is 1 order of magnitude smaller than those of Refs. [10,11,13], and even the sign of the electron EDM is different, as shown in Table I. By using our correct formula, the experimental upper bounds on RPV interactions given from the RPV Barr-Zee-type contribution are loosened by 1 order of magnitude.

We can also evaluate Barr-Zee-type diagrams which contribute to the quark cEDM. The Lagrangian of the cEDM interaction is given by

$$\mathcal{L}_{\mathrm{cEDM}} = -i\frac{d_q^c}{2}\bar{\psi}\gamma_5\sigma^{\mu\nu}T_a\psi F^a_{\mu\nu}, \tag{12}$$

where $F^a_{\mu\nu}$ is the gluon field strength. The Barr-Zee-type contribution of the down-type quark q_k is then

$$d^c_{q_k} = \mathrm{Im}(\lambda'_{ijj}\lambda'^*_{ikk})\frac{\alpha_s g_s}{32\pi^3 m_{q_j}}\cdot\{f(\tau) - g(\tau)\}, \tag{13}$$

where $\tau = m^2_{q_j}/m^2_{\tilde{\nu}_i}$. The flavor indices of the quark q_k and the quark of the inner loop are denoted, respectively, by k and j.

In conclusion, we have reanalyzed the RPV supersymmetric contribution to the Barr-Zee-type two-loop diagrams, and have found that the result gives a fermion EDM that is 1 order of magnitude smaller than the previous analyses [10,11,13] (for sneutrino mass = 1 TeV). This difference is significant, as it can alter the relative size of other contributing processes such as the four-fermion interactions [10,13]. Nevertheless, our finding does not alter the dominance of the Barr-Zee-type diagrams over the other two-loop diagrams, as shown in the analysis of Chang *et al.* [11]. The conclusion of Ref. [11] is still very important.

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