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Note on the scale evolution of the Efremov-Teryaev-Qiu-Sterman function $T_F(x, x)$

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We reexamine the scale dependence of the Efremov-Teryaev-Qiu-Sterman twist-3 matrix element that has been studied already by the four different groups with conflicting results [Z.-B. Kang and J.-W. Qiu, Phys. Rev. D **79**, 016003 (2009); J. Zhou, F. Yuan, and Z.-T. Liang, Phys. Rev. D **79**, 114022 (2009); W. Vogelsang and F. Yuan, Phys. Rev. D **79**, 094010 (2009); and V. M. Braun, A. N. Manashov, and B. Pirnay, Phys. Rev. D **80**, 114002 (2009)]. We find that we can in fact reproduce the results of V. M. Braun, A. N. Manashov, and B. Pirnay, Phys. Rev. D **80**, 114002 (2009) with the methods of J. Zhou, F. Yuan, and Z.-T. Liang, Phys. Rev. D **79**, 114022 (2009) when we treat some subtleties with greater care, thus easing the mentioned conflict.

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The Efremov-Teryaev-Qiu-Sterman (ETQS) matrix element plays an important role in the theoretical description of transverse single spin asymmetries in the framework of collinear factorization. The control of Q evolution is not only necessary to describe QCD dynamics correctly and to reduce the dependence of theory predictions on the factorization scale adopted but also such evolution equations give insight into the functional form of higher-twist distribution functions. The idea there is to start evolution at a low scale and use the fact that the resulting form at a high scale is only little dependent on the low-scale input [1]. The latter is especially important in view of the limited experimental input one has to determine these functions.

The corresponding calculation was done in Refs. [2–5]. However, the result obtained in Ref. [5] differs from that derived in Refs. [2–4] by two extra terms. It was settled rather easily that one of these terms is due to a Feynman diagram that was missed in Refs. [2–4]. The second additional term in [5] that is proportional to $\delta(1-z)$ could not be reproduced by the other calculations so far. We show in this short contribution, how this term arises within the formalism of Ref. [3] due to a rather subtle fact related to the noncommutativity of a certain limit and a certain integration. We now hope to be able to perform this calculation consistently in the light cone gauge.

The ETQS function T_F is defined through the following matrix element:

$$\int \frac{dy^-}{2\pi}\frac{dy_1^-}{2\pi} e^{ixP^+y^-}\langle PS|\bar\psi_\beta(0) gF^{+\mu}(y_1^-)\psi_\alpha(y^-)|PS\rangle = \frac{M}{2}T_F(x,x)\epsilon_\perp^{\nu\mu}S_{\perp\nu}\not p. \tag{1}$$

In Ref [3], the light cone gauge ($A^+=0$) with the retarded boundary condition, i.e., $A_\perp(-\infty^-)=0$ was chosen such that $T_F(x,x)$ can be rewritten as

$$T_F(x) = \int \frac{dy^-}{8\pi^2 M} e^{ixP^+y^-} \times \langle PS|\bar\psi(0)\not n\epsilon_\perp^{\nu\mu}S_{\perp\nu} i\partial_{\perp\mu}\psi_\alpha(y^-)|PS\rangle. \tag{2}$$

To calculate the splitting function, one has to take into account the contributions from the operators $(\bar\psi\partial_\perp\psi)$ and $(\bar\psi A_\perp\psi)$, because they are of the same twist. We plot the Feynman diagrams contributions for the real gluon radiation in Fig. 1, where (a) is the contribution from the partial derivative acting on the quark field, and $(b-d)$ are those from $A_\perp$ contributions. Virtual corrections only contribute to the contribution proportional to $(\bar\psi\partial_\perp\psi)$. Their contribution is the same as for the quark self-energy correction.

Following the procedure presented in Ref. [3], we perform a collinear expansion for the hard scattering part to calculate the contribution from Fig. 1(a). The linear $k_\perp$ expansion term combining with the quark field will lead to the quark-gluon correlation function $T_F(x,x)$. In the

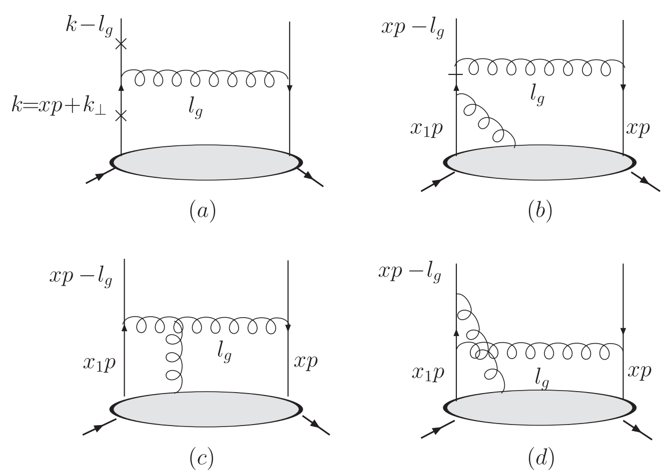

FIG. 1. Real gluon radiation contribution to the evolution equation for the ETQS function $T_F(x,x)$. Crosses in (a) and horizontal bar in (b) indicate $k_\perp$ flow and special propagator, respectively.

collinear expansion in terms of $k_\perp$, we can fix the transverse momentum of the probing quark (l_q) or the radiated gluon (l_g), because of momentum conservation and we are integrating over them to obtain $T_F(x,x)$. We have also checked that they will generate the same result. Following Ref. [3], we fix l_g in the collinear expansion to simplify the calculation.

For the $A_\perp$ contribution, we notice that $F^{+\mu} = \partial^+ A_\perp^\mu$ in the light cone gauge. Therefore, one can relate the corresponding soft matrix to the correlation function $T_F(x, x_1)$ in the following way:

$$\frac{i}{x - x_1 + i\epsilon} \int \frac{dy^- dy_1^-}{4\pi} e^{ix_1 P^+ y^-} e^{i(x-x_1)P^+ y_1^-} \langle PS|\bar{\psi}_\beta(0^-) \not{n} \epsilon_\perp^{\nu\mu} S_{\perp\nu} g F^+{}_\mu(y_1^-)\psi_\alpha(y^-)|PS\rangle$$
$$= \int \frac{dy^- dy_1^-}{4\pi} P^+ e^{ix_1 P^+ y^-} e^{i(x-x_1)P^+ y_1^-} \langle PS|\bar{\psi}_\beta(0^-) \not{n} \epsilon_\perp^{\nu\mu} S_{\perp\nu} g A_{\perp\mu}(y_1^-)\psi_\alpha(y^-)|PS\rangle. \quad (3)$$

In above formula, the soft gluon pole appearing in the first line is generated by partial integration. The pole prescription has been determined by our choice of a retarded boundary condition. For the same reason, we have to regulate the light cone propagator in a consistent manner. The gluon propagator appearing in Fig. 1(c) in the light cone gauge with the retarded boundary condition is given by

$$D^{\alpha\beta}(l) = \frac{-i}{l^2 + i\epsilon}\left(g^{\alpha\beta} - \frac{l^\alpha n^\beta + n^\alpha l^\beta}{l \cdot n + i\epsilon}\right), \quad (4)$$

where l is the gluon propagator momentum flowing out from the quark-gluon vertex in Fig. 1(c).

We now deviate from the original calculation [3] in two ways:

(a) in [3] the integral $\int_{x_g'} dx_g \frac{x_g' \delta(x_g')}{x_g^2}$ was simply neglected;

(b) one of the two absorptive parts of the free propagator was not taken into account.

We will discuss next these two points in more details, arguing that the neglected contributions have to be taken into account. When computing the hard pole contribution from Fig. 1(c), for the left cut diagram, one has

$$T_F^{(1)}|^{\text{hp-left}}_{\text{Fig. 1(c)}}(x_B) = \frac{\alpha_s}{4\pi} \int_{xB} \frac{dx}{x} \int_{x_g'} dx_g \frac{dl_{g\perp}^2}{l_{g\perp}^2} \frac{C_A}{2} \times \left[\frac{(x + x_B)(2x_g - x_g')}{2x_g^2}\right] \times \delta(x_g') T_F(x - x_g, x), \quad (5)$$

where $x_g' = l \cdot n/p^+$ with $x_g' = x_B - x + x_g$. By noticing that $\int_{x_g'} dx_g \frac{x_g' \delta(x_g')}{x_g^2} = \delta(x_B - x)$, rather than zero, and summing left and right cut diagrams, one obtains

$$T_F^{(1)}|^{\text{hp}}_{\text{Fig. 1(c)}}(x_B) = \frac{\alpha_s}{2\pi} \int_{xB} \frac{dx}{x} \frac{dl_{g\perp}^2}{l_{g\perp}^2} \frac{C_A}{2} \left[\frac{1+z}{1-z} T_F(x_B, x) - \delta(1-z) T_F(x_B, x_B)\right], \quad (6)$$

where $z = x_B/x$. The second term is missing in Ref. [3].

Next we discuss the second contribution that was overlooked in Ref. [3]. Since we work in the light cone gauge with a retarded boundary condition, the free propagator possesses two absorptive parts [6],

$$\text{disc}\, D^{\alpha\beta}(l_g) = 2\pi\theta(l_g^0)\delta(l_g^2)\left[-g^{\alpha\beta} + \frac{2l_g^-(l_g^\alpha n^\beta + n^\alpha l_g^\beta)}{l_{g\perp}^2}\right] - 2\pi\theta(l_g^0)\delta(l_g^+)\frac{(l_g^\alpha n^\beta + n^\alpha l_g^\beta)}{l_{g\perp}^2}. \quad (7)$$

In [3] only the first absorptive part was taken into account. In order to carry out the calculation in a consistent manner, one must include the contribution from the second part. However, if one still picks up the same imaginary part as we did above, this contribution will cancel between the different cut diagrams as it happens when both gluon lines go on shell. On the other side, the additional imaginary part may come from the artificial pole that appears in Eq. (3). Such pole-absorptive part combination gives the contribution

$$T_F^{(1)}|^{\text{LC-left}}_{\text{Fig. 1(c)}}(x_B) = \frac{\alpha_s}{4\pi} \int \frac{dl_{g\perp}^2}{l_{g\perp}^2} \int_{xB} dx \int_0^\infty dl_g^- \int_{x_g'} dx_g \delta(x_g - x_g')\delta(x_g) \frac{C_A}{2} \left[\frac{2(2x_B - x_g')(x_g + x_g')}{2(2l_g^- x_B + l_{g\perp}^2)x_g'} - \frac{2x_B l_{g\perp}^2}{(2l_g^- x_B + l_{g\perp}^2)^2}\right] T_F(x - x_g, x). \quad (8)$$

Integrating over x_g, $l_g^- = l_g \cdot p$ and summing the two cut diagrams, we obtain

$$T_F^{(1)}|_{\text{Fig. 1(c)}}^{\text{LC-left}}(x_B) = \frac{\alpha_s}{2\pi}\int\frac{dl_{g\perp}^2}{l_{g\perp}^2}\int_{xB}\frac{dx}{x}\frac{C_A}{2}\left[\int_0^1 dy\frac{2}{1-y} - 1\right]\delta(1-z)T_F(x_B, x_B). \tag{9}$$

Taking into account the contribution from the second part of the absorptive part, Eq. (20) in Ref. [3] should be modified as follows:

$$-\frac{\alpha_s}{2\pi}\frac{C_A}{2}\int_{x_B}\frac{dx}{x}T_F(x,x)d^2l_{g\perp}\left[\frac{\partial}{\partial l_{g\perp}^\mu}\hat{H}_0(xP, l_{g\perp})\right]\times(-l_{g\perp}^\mu) = -\frac{\alpha_s}{2\pi}\frac{C_A}{2}\int_{x_B}\frac{dx}{x}T_F(x,x)d^2l_{g\perp}\hat{H}_0(xP, l_{g\perp})$$
$$= -\frac{\alpha_s}{2\pi}\frac{C_A}{2}\int_{x_B}\frac{dx}{x}\frac{dl_{g\perp}^2}{l_{g\perp}^2}\left[\frac{1+z^2}{1-z} - \delta(1-z)\int_0^1 dy\frac{2}{1-y}\right]T_F(x,x). \tag{10}$$

Collecting all pieces, we eventually arrive at the following scale evolution equation for $T_F(x, x)$:

$$\frac{\partial T_F(x_B, x_B, \mu^2)}{\partial\ln\mu^2} = \frac{\alpha_s}{2\pi}\int_{x_B}\frac{dx}{x}\left[C_F\left\{\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right\}T_F(x,x) + \frac{C_A}{2}\left\{\frac{1+z}{1-z}T_F(xz,x)\right.\right.$$
$$\left.\left. - \frac{1+z^2}{1-z}T_F(x,x) - 2\delta(1-z)T_F(x,x) + \tilde{T}_F(xz,x)\right\}\right], \tag{11}$$

which coincides with the result given in Ref. [5].

Following a similar procedure, one can also recover an extra term $-N_c\delta(1-z)T_F^{(\sigma)}(x,x)$ at the scale evolution equation for $T_F^{(\sigma)}(x,x)$ [3]. It would be interesting to investigate if such an extra term also shows up in the spin dependent cross section, and to check if the observed matching between the TMD factorization and collinear factorization at intermediate transverse momentum will be affected or not by this extra term. We leave this part for future study.

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Note added.—When finishing this paper, we learned that the extra boundary term also can be recovered in both Kang-Qiu's approach [7] and Vogelsang-Yuan's approach [8] and also in [9] from a different approach.

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