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KATRIN sensitivity to sterile neutrino mass in the shadow of lightest neutrino mass

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The presence of light sterile neutrinos would strongly modify the energy spectrum of the tritium β electrons. We perform an analysis of the KArlsruhe TRItium Neutrino (KATRIN) experiment's sensitivity by scanning almost all the allowed region of neutrino mass-squared difference and mixing angles of the $3 + 1$ scenario. We consider the effect of the unknown absolute mass scale of active neutrinos on the sensitivity of KATRIN to the sterile neutrino mass. We show that after 3 years of data-taking, the KATRIN experiment can be sensitive to mixing angles as small as $\sin^2 2\theta_s \sim 10^{-2}$. Particularly we show that for small mixing angles, $\sin^2 2\theta_s \lesssim 0.1$, the KATRIN experiment can give the strongest limit on active-sterile mass-squared difference.

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I. INTRODUCTION

There is a consensus that nonzero masses of neutrinos and the nontrivial mixing between them is the plausible framework to explain the outstanding results of plenty of neutrino oscillation experiments. The standard approach is to have a three-active-neutrino scenario, with at least two nonzero masses and two reasonably large mixing angles [1].

An interesting extension to this standard scenario is the existence of extra light sterile neutrino states which arose for the first time in light of the LSND experiment [2], showing evidence for $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillation (see also the recent MiniBooNE data [3] which seems to corroborate this). Another evidence of the presence of light sterile neutrinos is the so-called reactor neutrino anomaly [4]. This anomaly is the departure from unity of the ratio of the observed rate of events to the predicted rate in very short-baseline reactor neutrino experiments. This departure prompted by the reevaluation of $\bar{\nu}_e$ reactor flux which revealed a 2.5% increase in the flux [4]. Also, the Gallium anomaly [5] shows a deficit of ν_e produced by intense radioactive sources, such that the ratio of the observed rate to the predicted rate is 0.86 ± 0.05 [5]. All these anomalies can be understood by adding one (or more) light sterile neutrino state(s) to the scenario with three active neutrinos, with active-sterile mass-squared difference $\Delta m^2_{\rm SBL} \gtrsim 0.1$ eV^2. The $3 + 1$ scenario [6,7] is the simplest scenario to accommodate the presence of light sterile neutrinos. The model is composed of 4 flavor states ν_α, $\alpha = e, \mu, \tau$, and s, which are the mixture of mass eigenstates ν_i with masses m_i, $i = 1, 2, 3, 4$. We associate the mass scale which induces the very short-baseline oscillations, $\Delta m^2_{\rm SBL}$, with the mass difference $\Delta m^2_{41} \equiv m_4^2 - m_1^2$. It should be noticed that in order to explain the reactor anomaly, a nonzero component of ν_4 in the state ν_e is necessary.

A new generation of tritium beta decay experiments, like the KArlsruhe TRItium Neutrino (KATRIN) experiment [8], was proposed to search kinematically for the neutrino mass by measuring the energy spectrum of the electrons from the beta decay of tritium ${}^3\mathrm{H} \to {}^3\mathrm{He}^+ + e^- + \bar{\nu}_e$ (see also other proposals [9]). A nonzero neutrino mass results in displacement of the endpoint energy in the electron spectrum which is the focus region to probe in KATRIN. In this paper, we analyze the capability of the KATRIN experiment in the search for endpoint displacement in the energy spectrum of β electrons due to the lightest neutrino mass m_1 and the irregularities in the shape of energy spectrum due to the heavier (mostly sterile) neutrino mass m_4 and its nonzero mixing with electron neutrino. We discuss for the first time the role of the lightest neutrino mass m_1 in the determination of the sensitivity of the KATRIN experiment to the oscillation parameter Δm^2_{41}. We show that with the present KATRIN design, this experiment is the only one sensitive to the part of parameter space corresponding to very small active-sterile mixing angle and large active-sterile mass-squared difference.

II. TRITIUM BETA DECAY AT KATRIN

The KATRIN experiment [8] has the following setup. Injected molecular tritium gas at Tritium Laboratory Karlsruhe provides high luminosity β electrons emitting isotropically. The electrons will be guided by the gradient of a magnetic field to the so-called MAC-E-Filter

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(Magnetic Adiabatic Collimation combined with an Electrostatic Filter) spectrometer. The ratio of the minimum magnetic field at the central plane of the spectrometer ($B_A = 3 \times 10^{-4}$ T) to the maximum magnetic field near the tritium source ($B_{\max} = 6$ T) determines the relative sharpness of the energy filtering of MAC-E-Filter. Also, applying a magnetic field $B_S = 3.6$ T at the tritium source suppresses the entrance of electrons with large initial emission angle. In the spectrometer, with the help of an electric field parallel to the electron's propagation path, it is possible to make an electrostatic barrier qU which can be passed just by electrons with energy higher than the height of the barrier. Taking all together, the transmission function of the KATRIN spectrometer as a function of electron kinetic energy K_e and retarding potential qU is

$$T(K_e, qU) = \begin{cases} 0 & \text{if } K_e - qU < 0 \\ \dfrac{1-\sqrt{1-\frac{K_e - qU}{K_e}\frac{B_S}{B_A}}}{1-\sqrt{1-\frac{\Delta K_e}{K_e}\frac{B_S}{B_A}}} & \text{if } 0 \le K_e - qU \le \Delta K_e\,, \\ 1 & \text{if } K_e - qU > \Delta K_e \end{cases}$$

where $\Delta K_e / K_e = B_A / B_{\max}$ is the relative sharpness of the filter. However, electrons can undergo inelastic scattering with tritium molecules in the source which can change the spectrum. Taking into account the probability of multiple inelastic scattering, the transmission function modifies to the following convoluted form:

$$T'(K_e, qU) = \int_0^{K_e - qU} T(K_e - \epsilon, qU)[P_0 \delta(\epsilon) + P_1 f(\epsilon) + P_2 (f \otimes f)(\epsilon) + \ldots] d\epsilon, \quad (1)$$

where P_n is the probability that the electron scatters n times off the tritium molecules before leaving the source and $f(\epsilon)$ is the energy-loss function at each scattering [10]. The symbol $\otimes$ defines the following convolution:

$$(f \otimes f)(\epsilon) = \int_0^{K_e - qU} f(\epsilon') f(\epsilon - \epsilon') d\epsilon'.$$

The rate of the electrons passing the potential barrier qU and arriving at the detector is

$$S(Q, qU, [U_{ei}], [m_\nu]) = \int_0^\infty \beta(K_e, Q, [U_{ei}], [m_\nu]) \times T'(K_e, qU) dK_e, \quad (2)$$

where the symbols $[m_\nu] = \{m_1, \ldots, m_n\}$ and $[U_{ei}] = \{U_{e1}, \ldots, U_{en}\}$ denote, respectively, the set of the masses of neutrino mass eigenstates ν_i and the elements of the first row of Pontecorvo-Maki-Nakagawa-Sakata (PMNS) U_{PMNS} [11] mixing matrix. The function β gives the spectrum of electrons in beta decay

$$\beta(K_e, Q, [U_{ei}], [m_\nu]) = N_s F(Z, K_e) E_e p_e \sum_{i,j} [p_i \mathcal{E}_i |U_{ej}|^2 \sqrt{\mathcal{E}_i^2 - m_j^2}\, \Theta(\mathcal{E}_i - m_j)], \quad (3)$$

where $\mathcal{E}_i = Q - W_i - K_e$. In the above equation, E_e and p_e are, respectively, the electron's energy and momentum; $F(Z, K_e)$ is the Fermi function which takes into account the electrostatic interaction of the emitted electron with the daughter nucleus with $Z = 2$ [12]; W_i and p_i are, respectively, the excitation energy and transition probability for the excited state i of the daughter nucleus [13]; and the Heaviside step function Θ guarantees the conservation of energy. The index i runs over the excited states, and index j runs over the neutrino mass eigenstates. The factor N_s determines the total number of emitted electrons which for the KATRIN design parameters is $1.47 \times 10^{-13}\ \mathrm{s}^{-1}\,\mathrm{eV}^{-5}$ [8].

III. SENSITIVITY OF KATRIN TO STERILE NEUTRINO

The functional form of the β-electron spectrum in Eq. (3) depends on the set of masses $[m_\nu] = \{m_1, \ldots, m_n\}$. As discussed in Ref. [14], for the case that the energy resolution of the experiment near the endpoint and the energy interval which is probed by the experiment is much larger than the mass splittings, it is possible to replace the set of masses $[m_\nu]$ with an effective mass $m_\beta \equiv \sqrt{\sum_i m_i^2 |U_{ei}|^2}$. However, in the case of sterile neutrino with a mass-squared difference $\sim 1\ \mathrm{eV}^2$, this approximation fails, and the error of using effective mass in the fit of spectrum becomes large. Here, we use the exact form of Eq. (3) with four mass parameters $\{m_1, m_2, m_3, m_4\}$ in the case of the $3+1$ scheme. However, from these four masses, just m_1 and m_4 enter the analysis, and the other two are fixed by $m_2 = \sqrt{m_1^2 + \Delta m_{21}^2}$ and $m_3 = \sqrt{m_1^2 + \Delta m_{31}^2}$, where the values of Δm_{21}^2 and Δm_{31}^2 are fixed by oscillation phenomenology [1]. For the elements of the PMNS mixing matrix, we use such parametrization of the $U_{4\times4}$ that its 3×3 submatrix for the light active masses reduces to the Particle Data Group parametrization [15]. Thus, the element U_{e4} just depends on one mixing angle θ_s, through $|U_{e4}| = \sin\theta_s$. With the above-mentioned considerations, the rate of the events in Eq. (2) is a function of parameters $(Q, qU, U_{e4}, m_1, m_4)$. The total rate is the sum of signal rate S in Eq. (2) and the expected rate of the background events N_b which for the KATRIN is 10 mHz [8]. For the Q value of the tritium beta decay, we use the central value of a recent measurement

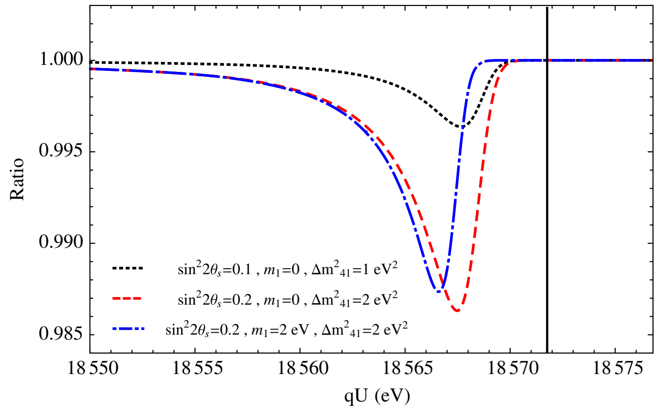

FIG. 1 (color online). The ratios of the total rate in Eq. (4) for various mixing and mass parameters. The vertical line shows the $Q = 18571.8$ eV.

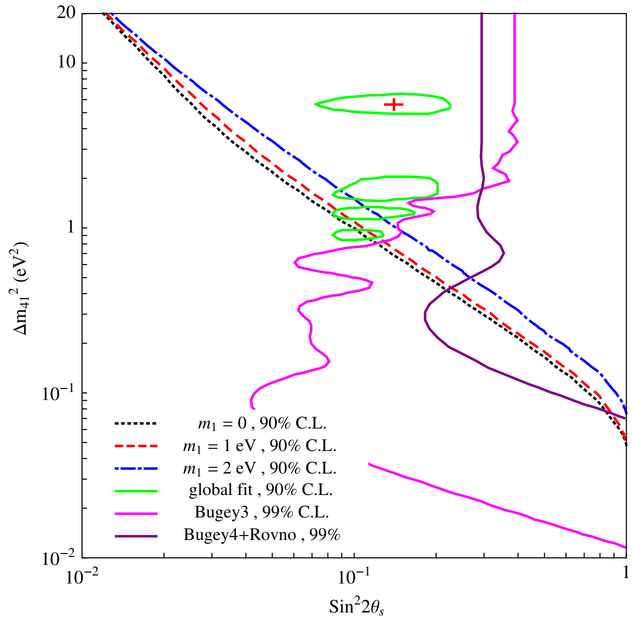

FIG. 2 (color online). The 90% C.L. contours of the KATRIN experiment in the $(\sin^2 2\theta_s, \Delta m^2_{41})$ plane. The black (dotted), red (dashed), and blue (dotted-dashed) curves correspond, respectively, to $m_1 = 0$, 1, 2 eV. The green (solid) curves show the 90% C.L. allowed region and the red cross the best-fit point for the global fit of the data for the 3 + 1 scheme [7]. The magenta and purple curves show, respectively, the Bugey3 and Bugey4 + Rovno exclusion curves [5].

$Q = 18571.8 \pm 1.2$ eV [16]. To illustrate the behavior of sterile admixture, we define the following ratio:

$$\frac{S(\sin^2 2\theta_s, m_1, \Delta m^2_{41}) + N_b}{S(\sin^2 2\theta_s = 0, m_1, \Delta m^2_{41} = 0) + N_b}, \tag{4}$$

where $\sin^2 2\theta_s = 4|U_{e4}|^2(1 - |U_{e4}|^2)$. We plotted this ratio in Fig. 1 for a different set of the parameters $\sin^2 2\theta_s$, m_1, and Δm^2_{41}. Comparing the black (dotted) curve with the red (dashed) curve in Fig. 1, it is easy to see the change in β spectrum for different values of the mass m_4 for a vanishing light mass $m_1 = 0$. The height of minimum depends on the values of $\sin^2 2\theta_s$ and Δm^2_{41}, and the position of the minimum depends only on m_4. However, in the comparison between the red (dashed) and blue (dotted-dashed) curves in Fig. 1, we see that by the inclusion of a nonzero value for m_1, the two curves with the same Δm^2_{41} cross each other, implying that the lack of knowledge about the value of m_1 can lay a shadow on the determination of Δm^2_{41}. To quantify the sensitivity of the KATRIN experiment to the sterile neutrino mass, we define the following χ^2 function:

$$\chi^2(Q, U_{e4}, m_1, m_4, R_s, R_b) = \sum_i \frac{(N_{\rm exp}([qU]_i) - N_{\rm th}(Q, [qU]_i, U_{e4}, m_1, m_4, R_s, R_b))^2}{\sigma^2}, \tag{5}$$

where $\sigma = \sqrt{N_{\rm exp}}$ is the statistical standard deviation. In Eq. (5), the $N_{\rm th}$ corresponds to the number of detected β electrons when the retarding potential has the value $[qU]_i$ which is calculated by multiplying the rate in Eq. (2) by the time spent for the measurement,

$$N_{\rm th}(\ldots, [qU]_i, \ldots) = t[i] \cdot (R_s S(\ldots, [qU]_i, \ldots) + R_b N_b),$$

where R_s and R_b are, respectively, the normalization factors of signal and background events, and $N_b = 10$ mHz. The $N_{\rm exp}$ denotes the experimental number of events assuming m_4 and U_{e4} equal zero. The index i in Eq. (5) runs in 31 steps such that the retarding potential covers the range $qU \in [Q - 20\text{ eV}, Q + 5\text{ eV}]$. For the time $t[i]$ spent at each step of the retarding potential, we use the optimized measurement time proposed by the KATRIN collaboration (see Figure 131 in Ref. [8]). We minimize the χ^2 function with respect to the normalization factors R_s and R_b analytically and with respect to Q numerically in its uncertainty range. Figure 2 shows the 90% C.L. sensitivity contours of the KATRIN experiment in the $(\sin^2 2\theta_s, \Delta m^2_{41})$ plane for the total measurement time $\sum_i t[i] = 3$ years. The black (dotted), red (dashed), and blue (dotted-dashed) curves correspond, respectively, to $m_1 = 0$, 1, 2 eV. The green (solid) curves show the 90% C.L. allowed region from the global fit of the short-baseline oscillation data [7], and the red cross shows the best-fit value.

We can conclude from Fig. 2 that after three years of data-taking, the KATRIN experiment can exclude the main part of the current allowed region of the 3 + 1 scenario. Beside that, the KATRIN is the most sensitive experiment in the upper-left part of the $(\sin^2 2\theta_s, \Delta m^2_{41})$ plane. As can be seen, the experiment is sensitive to mixing angles as small as $\sin^2 2\theta_s \sim 10^{-2}$. For comparison, we have shown in Fig. 2 the present exclusion curves of Bugey3 (magenta solid curve) and Bugey4 + Rovno (purple solid curve) [5].

We have also found that, for the first time, varying the value of lightest neutrino mass m_1 can affect the sensitivity to the large mass m_4. For example, for a fixed value of mixing angle $\sin^2 2\theta_s = 0.1$, the sensitivity of the experiment is $\Delta m_{41}^2 = 0.98$, 1.1 and 1.5 eV^2 for, respectively, $m_1 = 0$, 1 and 2 eV, as can be seen from Fig. 2. This implies a correlation between the discovery potential of Δm_{41}^2 and the value of m_1. For smaller (larger) values of m_1, the correlation is weaker (stronger), and also the correlation depends on the mixing angle. For very small mixing angles, the correlation disappears because of the weak m_4 contribution, and for the $\sin^2 2\theta_s = 1$ case, which corresponds to equal admixture of ν_1 and ν_4 in ν_e, the correlation still exists. Also, it should be noted that the large values of m_1 are in potential conflict with the standard ΛCDM model of cosmology, although extensions to nonminimal cosmological models can reduce the conflict [17].

IV. COMPARISON WITH EARLIER WORKS

The effect of light sterile neutrinos in beta-decay experiments was formerly discussed in Refs. [14,18–20]. Our results are in agreement with the estimations of Refs. [14,18]. In Ref. [19], a general analysis was not performed, but we can conclude that we have similar results for small values of m_1. In Ref. [20], the authors assume null value for the lightest mass m_1 which can be compared with the black (dotted) curve in Fig. 2. For mixing angles $\sin^2 2\theta_s \sim 0.1$, our exclusion is ~ 0.6 stronger in $\log_{10}\Delta m_{41}^2$. This stronger exclusion can be the result of two issues: i) we use the optimized running time in the measurement of the spectrum; ii) a different χ^2 function can be used in Ref. [20]. For the very small mixing angles $\sin^2 2\theta_s \lesssim 0.05$, our analysis does not show the wiggling behavior in Ref. [20] which is not expected in kinematical mass measurement experiments. Generally, we have an agreement in the limiting case of very small light mass m_1 with the previous results. However, for nonsmall m_1 masses, we found an interplay between the mixing parameter U_{e4} and the two free mass scales m_1 and m_4, which was not noticed in all previous analyses.

V. CONCLUSIONS

In this work, we analyzed the sensitivity of beta-decay experiment KATRIN in determining the mass scale associated with the presence of a light sterile neutrino state. Motivation comes from the $\bar{\nu}_e$ reactor anomaly, the Gallium anomaly, and the LSND and MiniBooNE experiments which favor the presence of light sterile neutrinos which mix with electron neutrinos, compatible with $\Delta m_{\mathrm{SBL}}^2 \gtrsim 0.1$ eV^2 and small mixing angles $\sin^2 2\theta_s$.

For the first time, we considered the effect of light mass scale m_1 in the determination of oscillation parameter Δm_{41}^2 in KATRIN. We exploited a general treatment of nonzero values for the lightest mass scale m_1 and the heavier mass scale m_4. We have shown that varying the unknown mass scale $m_1 \in [0, 2]$ eV induces 0.2 uncertainty in the sensitivity of KATRIN to $\log_{10}\Delta m_{41}^2$. However, we have shown that despite this uncertainty, with 3 years of data-taking, KATRIN can exclude the main part of the current allowed region in the $(\sin^2 2\theta_s, \Delta m_{41}^2)$ plane indicated by the global fit of short-baseline oscillation experiments. Also, we have shown that for very small mixing angles $\sin^2 2\theta_s \lesssim 10^{-1}$, the KATRIN experiment gives the strongest bound on the oscillation parameter Δm_{41}^2.

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