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Electromagnetic multipole moments of elementary spin-1/2, 1, and 3/2 particles

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We study multipole decompositions of the electromagnetic currents of spin-1/2, 1, and 3/2 particles described in terms of representation-specific wave equations which are second order in the momenta and which emerge within the recently elaborated Poincaré covariant-projector method, where the respective Lagrangians explicitly depend on the Lorentz group generators of the representations of interest. The currents are then the ordinary linear Noether currents related to phase invariance, and present themselves always as two-terms motion–plus spin-magnetization currents. The spin-magnetization currents appear weighted by the gyromagnetic ratio g, a free parameter in the method which we fix either by unitarity of forward Compton scattering amplitudes in the ultraviolet for spin-1 and spin-3/2, or in the spin-1/2 case, by their asymptotic vanishing, thus ending up in all three cases with the universal g value of $g = 2$. Within the method under discussion, we calculate the electric multipoles of the above spins for the spinor, the four-vector, and the four-vector–spinor representations, and find it favorable in some aspects, specifically in comparison with the conventional Proca and Rarita-Schwinger frameworks. We furthermore attend to the most general non-Lagrangian spin-3/2 currents, which are allowed by Lorentz invariance to be up to third order in the momenta and construct the linear-current equivalent of identical multipole moments of one of them. We conclude that nonlinear non-Lagrangian spin-3/2 currents are not necessarily more general and more advantageous than the linear spin-3/2 Lagrangian current emerging within the covariant-projector formalism. Finally, we test the representation dependence of the multipoles by placing spin-1 and spin-3/2 in the respective $(1,0)\oplus(0,1)$ and $(3/2,0)\oplus(0,3/2)$ single-spin representations. We observe representation independence of the charge monopoles and the magnetic dipoles, in contrast to the higher multipoles, which turn out to be representation-dependent. In particular, we find the bi-vector $(1,0)\oplus(0,1)$ to be characterized by an electric quadrupole moment of opposite sign to the one found in $(1/2,1/2)$, and consequently to the W boson. This observation allows us to explain the positive electric quadrupole moment of the ρ meson extracted from recent analyses of the ρ meson electric form factor. Our finding points toward the possibility that the ρ-meson could transform as part of an antisymmetric tensor with an a_1 mesonlike state as its representation companion, a possibility consistent with the empirically established ρ and a_1 vector meson dominance of the hadronic vector and axial-vector currents.

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I. INTRODUCTION

The electromagnetic characteristics of elementary particles continue being one of the key issues in contemporary physics research, both experimental and theoretical. The reason is that a great deal of our knowledge of the fundamental properties of matter is in the first instance obtained from theoretical analyses of measured cross-sections of electromagnetic processes, such as nuclear reactions induced by real photons or elastic and inelastic electron scattering off proton or nuclear targets. In these types of processes, particles with spins higher than 1/2 can arise both as intermediate virtual or real outgoing states. As an example, we wish to mention the reaction of a quasifree knockout of a $\Delta(1232)$ particle in the inelastic electron scattering on ^{3}He [1]. The evaluation of this process relies upon the electromagnetic moments of the $\Delta(1232)$ particle, a spin-3/2 state. The electromagnetic spin-3/2 current has been widely analyzed, predominantly within the Rarita-Schwinger framework [2] where a spin-J fermion is considered as the highest spin, $J = K + \frac{1}{2}$, in the totally

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symmetric rank-K tensor-spinor $\psi_{\mu_1 \ldots \mu_K}$. As is well-known, the Rarita-Schwinger Lagrangian is linear in the momenta and takes into account certain auxiliary conditions, supposed to restrict the tensor-spinor space to the desired $2(2J+1)$ fermion–antifermion degrees of freedom. Although widely used, the Rarita-Schwinger framework is known to suffer several pathologies, among them, the acausal propagation of the classical wave fronts of the particle within an electromagnetic environment. Recently, a second-order formalism for the description of the electromagnetic interactions of particles with spins in terms of $\psi_{\mu_1 \ldots \mu_K}$ has been developed in Ref. [3], which was shown to be free from the problem of acausal propagation for $K = 1$, provided the spin-3/2 gyromagnetic ratio, $g_{(3/2)}$, were to take the universal value of $g_{(3/2)} = 2$. Within the latter formalism, which naturally incorporates the necessary auxiliary conditions, the $J = K + \frac{1}{2}$ sector of $\psi_{\mu_1 \ldots \mu_K}$ is pinned down by a covariant projector constructed from the two Casimir operators of the Poincaré group, the squared linear momentum P^2 and the squared Pauli-Lubanski vector W^2. The formalism is applicable to boson fields as well and has been employed in the calculation of the Compton scattering cross-section off vector particles [4]. In the latter work, the case has been made that while Poincaré invariance prescribes the cross-section under investigation to depend on only two parameters, in turn identified as the spin-1 gyromagnetic factor g_1 and the coupling ξ associated to parity-violating Lorentz structures, it is insufficient to fix their values. To fix those values, additional dynamical requirements need to be imposed on the cross-sections. In particular, in imposing the unitarity condition on the Compton scattering amplitudes in the ultraviolet, the $\xi = 0$ value, and again the universal $g_1 = 2$ value have been encountered in Ref. [4].

The same path has been pursued in Ref. [5] with the aim to evaluate the Compton scattering process off a spin-3/2 target. Confining to parity conservation, the amplitudes have been found again to depend on g alone. Moreover, for $g_{(3/2)} = 2$, the forward differential cross-section was shown to become finite in accord with unitarity. In this way, an independent confirmation of the causality argument for $g = 2$ universality has been obtained. Knowing the value of the gyromagnetic ratio provides a reliable basis for the calculation of the multipole moments of particles with spins ranging from 1/2 to 3/2.

The goal of the present work is to find within the Poincaré covariant second-order projector formalism expressions for the electromagnetic multipole moments of particles with spins-1/2, 1, and 3/2, exploring both single-spin and multiple-spin representations of the Lorentz group.

In the literature, the electromagnetic multipole moments of a fundamental Dirac particle have been well understood, [6,7] and reproducing their values is mandatory for establishing credibility of any new high spin method, including the present one. In the following, we show that the Poincaré covariant second-order formalism yields the same multipoles for a spin-1/2 particle as the Dirac theory. As to the higher spins-1 and 3/2, and in comparison to the Proca and Rarita-Schwinger frameworks, the formalism under investigation will be shown to bring some notable improvements in our understanding of the electromagnetic properties of the abovementioned elementary particles.

To be specific, the electromagnetic current of a spin-3/2 particle is often designed in the literature by decomposing it into the basis of the most general tensors compatible with Lorentz invariance, not necessarily restricted to first order in the momenta [2,8–11]. The Poincaré covariant-projector method instead yields a current expressed in terms of the generators of the Poincaré group for the representation of interest, which is linear in the momenta and generates a full-flashed spin-3/2 contribution to the electric quadrupole moment of the particle under discussion, in contrast to the Rarita-Schwinger formalism which, as we will show, incorporates only the contribution of the corresponding spin-1/2 sector.

The paper is structured as follows. In the next section, we review the Poincaré covariant second-order projector formalism for the spinor and vector-spinor representations and elaborate the formalism for the $(1,0) \oplus (0,1)$ (bi-vector) and $(3/2,0) \oplus (0,3/2)$ representations with the emphasis on the emerging electromagnetic currents. Special attention is paid to the development of the formalism for the bi-vector representation in the form of an antisymmetric second-rank tensor. Sec. III summarizes the definitions of the electromagnetic multipole moments to be used in the paper. In Sec. IV, we present a side-by-side comparison of the electromagnetic multipole moments of spin-1/2, 1, and 3/2 particles following from the respective Dirac, Proca, and Rarita-Schwinger frameworks on the one side with the same observables following from the Poincaré covariant second-order formalism on the other. In Sec. V, we discuss and summarize our results. The paper closes with concise conclusions and has one Appendix.

II. COVARIANT-PROJECTOR FORMALISM AND ELECTROMAGNETIC CURRENTS

The idea underlying the Poincaré covariant second-order projector formalism is that a state, $\psi^{(m,s)}$, of mass m, and spin-s at rest, residing within a given representation of the Poincaré group, can be pinned down unambiguously in any inertial frame by means of an appropriately designed covariant projector, $\mathcal{P}^{(m,s)}$. The latter expresses in terms of the two Casimir operators of the group: the squared momentum operator P^2 and the squared Pauli-Lubanski operator W^2. For illustrative purposes, we here recall the form of such a projector for the $(1/2,1/2) \otimes [(1/2,0) \oplus (0,1/2)]$ representation (four-vector–spinor representation in the following) whose wave function, ψ_μ, has 16 degrees of freedom distributed over one spin-3/2 and two spin-1/2 fermions of opposite parities. In this case, one finds [3]

$$\mathcal{P}^{(m,(3/2))}\psi^{(m,(3/2))} = \psi^{(m,(3/2))},$$
$$\mathcal{P}^{(m,(3/2))} = -\frac{1}{3}\left(\frac{W^2}{m^2} + \frac{3}{2}\left(\frac{3}{2} - 1\right)\frac{P^2}{m^2}\mathbf{1}_{16\times 16}\right), \tag{2.1}$$

where $\mathbf{1}_{16\times 16}$ stands for the unit matrix in the 16-dimensional vector-spinor representation. Notice that $\mathcal{P}^{(m,(3/2))}$ is a 16×16-dimensional four-vector–spinor object which carries two Lorentz indices, as usually denoted by lowercase Greek letters, and two Dirac-spinor labels, here denoted by capital Latin letters. Subsequently, as explained in Ref. [3], the wave equation for ψ_μ can be cast in the most general covariant form according to

$$(-\Gamma_{AB\alpha\eta;\mu\nu}\partial^\mu\partial^\nu - m^2\delta_{AB}g_{\alpha\eta})\psi_B^{(m,(3/2))\eta} = 0, \tag{2.2}$$

and in terms of the Lorentz tensor $\Gamma_{AB\alpha\eta;\mu\nu}$, carrying four covariant and two spinor indices. In the labeling of the Γ tensor, we placed a semicolon as a demarcation sign between the Lorentz indices of the tensor which contract with the Lorentz indices of the four-vector representation and those which contract with the two derivatives. The explicit form of $\Gamma_{AB\alpha\eta;\mu\nu}$ has been worked out in Ref. [3] and will be presented again in due place below. For the time being, suffice it to recall that $\Gamma_{AB\alpha\eta;\mu\nu}$ can be split into a symmetric (SYM) and an antisymmetric (AS) tensor, according to

$$\begin{aligned}\Gamma_{AB\alpha\eta;\mu\nu} &= \Gamma^{\rm SYM}_{AB\alpha\eta;\mu\nu} + \Gamma^{\rm AS}_{AB\alpha\eta;\mu\nu},\\ \Gamma^{\rm SYM}_{AB\alpha\eta;\mu\nu} &= \tfrac{1}{2}(\Gamma_{AB\alpha\eta;\mu\nu} + \Gamma_{AB\alpha\eta;\nu\mu}),\\ \Gamma^{\rm AS}_{AB\alpha\eta;\mu\nu} &= \tfrac{1}{2}(\Gamma_{AB\alpha\eta;\mu\nu} - \Gamma_{AB\alpha\eta;\nu\mu}),\end{aligned} \tag{2.3}$$

and that in the absence of interactions, the $\Gamma^{\rm AS}_{AB\alpha\eta;\mu\nu}$ term does not provide any contribution. In order to verify this, we drop the Dirac spinor indices and the mass and spin labels as well, in which case Eq. (2.2) becomes more transparent:

$$(-\Gamma_{\alpha\eta;\mu\nu}\partial^\mu\partial^\nu - m^2\delta_{\alpha\eta})\psi^\eta = 0. \tag{2.4}$$

Because of the commutativity of ∂^μ and ∂^ν, it is obvious that for free particles, contributions of the type $\Gamma^{\rm AS}_{\alpha\eta;\mu\nu}\partial^\mu\partial^\nu$ nullify. This contrasts with the situation in which the particles are interacting via a gauge field, $\partial^\mu \to D^\mu = \partial^\mu + ieA^\mu$, in which case the covariant derivatives no longer commute:

$$[D^\mu, D^\nu] = ieF^{\mu\nu}. \tag{2.5}$$

One of the essential advantages of the Poincaré covariant-projector formalism over Rarita-Schwinger's framework is that within the former, the antisymmetric part of $\Gamma_{\alpha\eta;\mu\nu}$ can be designed in the most general way compatible with Poincaré invariance, thus guaranteeing completeness of the equation. Subsequently, we will occasionally refer to the Poincaré covariant second-order projector formalism developed in Ref. [3] as NKR formalism, for brevity. Towards our goal, the description of the electromagnetic multipole moments of particles with spin-1/2, spin-1, and spin-3/2, we present below expressions for the respective electromagnetic currents as they emerge within the NKR formalism.

A. The spin-1/2 current in the fundamental $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$ spinor representation

The NKR formalism for the fundamental spinor (S) representation, $(1/2, 0) \oplus (0, 1/2)$, was addressed in Ref. [12] and the one-loop structure was studied in Ref. [13]. The Γ tensor corresponding to the one defined in Eq. (2.2) carries only two Lorentz indices and the related equation of motion reads

$$(-\Gamma^S_{\mu\nu}\partial^\mu\partial^\nu - m^2)\psi^{(m,(1/2))} = 0, \tag{2.6}$$

where we suppressed the mass and spin labels of the field for the sake of simplifying notations. In the present work we restrict to parity conserving processes, in which case the $\Gamma^S_{\mu\nu}$ decomposition simplifies to

$$\Gamma^S_{\mu\nu} = g_{\mu\nu} - ig_S M^S_{\mu\nu}. \tag{2.7}$$

Here, $M^S_{\mu\nu}$ are the Lorentz group generators in the $(1/2, 0) \oplus (0, 1/2)$ representation, and are given by

$$M^S_{\mu\nu} = \tfrac{1}{2}\sigma_{\mu\nu}, \qquad \sigma_{\mu\nu} = \tfrac{i}{2}[\gamma_\mu, \gamma_\nu], \tag{2.8}$$

where γ_μ stand for the conventional Dirac matrices. Upon introducing small Latin letters, $i, j = 1, 2, 3$, to denote as usual the spacelike Lorentz indices, the M^S_{ij} and M^S_{0i} generators become in their turn the well-known pseudovectorial and vectorial generators of rotations $\mathbf{J}$ and boosts, $\mathbf{K}$, according to

$$(M^S)^{0i} = K_i = -\frac{\sigma_i}{2}, \qquad (M^S)^{ij} = \epsilon_{ijk}J_k = \epsilon_{ijk}\frac{i\sigma_k}{2}, \tag{2.9}$$

where σ_k denote the standard Pauli matrices. The associated free Lagrangian is then obtained as

$$\mathcal{L}^S_{\rm free} = (\partial^\mu\bar\psi)\Gamma^S_{\mu\nu}\partial^\nu\psi - m^2\bar\psi\psi. \tag{2.10}$$

The introduction of the electromagnetic interaction is standard and brought about by the gauge principle leading to the covariant derivative

$$\partial_\mu \to D_\mu = \partial_\mu + ieA_\mu, \tag{2.11}$$

where e is the charge of the particle, the resulting gauged Lagrangian being

$$\mathcal{L}^S_{\rm int} = \mathcal{L}^S_{\rm free} - j_\mu A^\mu + e^2\bar\psi(\Gamma^S_{\mu\nu} + \Gamma^S_{\nu\mu})\psi A^\mu A^\nu. \tag{2.12}$$

Switching to momentum space, the electromagnetic current emerges as

$$j^S_\mu(\mathbf{p}', \lambda'; \mathbf{p}, \lambda) = e\bar{u}(\mathbf{p}', \lambda')(\Gamma^S_{\nu\mu} p'^\nu + \Gamma^S_{\mu\nu} p^\nu) u(\mathbf{p}, \lambda). \tag{2.13}$$

We here are interested in parity-conserving interactions and employ the $u(\mathbf{p}, \lambda)$ and $v(\mathbf{p}, \lambda)$ Dirac spinors, the amplitudes of the quantum parity states transforming in the $(1/2, 0) \oplus (0, 1/2)$ representation of the homogeneous Lorentz group (HLG).

Substituting for the explicit form of $\Gamma^S_{\mu\nu}$ in Eq. (2.13), we arrive at

$$j^S_\mu(\mathbf{p}', \lambda'; \mathbf{p}, \lambda) = e\bar{u}(\mathbf{p}', \lambda')[(p' + p)_\mu + i g_S M^S_{\mu\nu}(p' - p)^\nu] u(\mathbf{p}, \lambda). \tag{2.14}$$

Here, in a natural way, the spinless part of the current, proportional to $(p' + p)^\mu$, appears separated from the spin contribution to the interaction, i.e., from the $i g_S M^S_{\mu\nu}(p' - p)^\nu$ term. The expression in Eq. (2.13) is the counterpart in the NKR formalism to the Gordon-decomposed Dirac (D) current [14],

$$j^D_\mu(\mathbf{p}', \lambda'; \mathbf{p}, \lambda) = e\overline{u_D}(\mathbf{p}', \lambda')[(p' + p)_\mu + i2 M^S_{\mu\nu}(p' - p)^\nu] u_D(\mathbf{p}, \lambda), \tag{2.15}$$

associated with the phase invariance of the Dirac Lagrangian,

$$\mathcal{L}_D = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi. \tag{2.16}$$

Notice that the Gordon-decomposed Dirac current emerges out of the Clifford algebra of the γ matrices in combination with the explicit use of the on shell Dirac equation, and cannot be obtained directly as a Noether current of a Lagrangian linear in the momenta. In contrast to this, within the second-order formalism discussed here, the same expression Eq. (2.14) appears directly as a Noether current due to phase invariance of the Lagrangian. However, unlike the Dirac current, where the gyromagnetic ratio is fixed by the algebra of the γ matrices to $g_{(1/2)} = 2$, in Eq. (2.14) the value of its counterpart g_S in the second-order formalism at first remains unspecified because, as usual, Poincaré invariance alone correctly identifies only the mass and the spin of the particle associated with a given representation and is insufficient to fix the values of the Lagrangian parameters prior to interactions. The above parameter has been fixed to $g_S = 2$ from the requirement on the asymptotic vanishing of the Compton scattering cross-section [12] with energy increase.

B. The spin-1 current in the $(\frac{1}{2}, \frac{1}{2})$ four-vector representation

In this case, the equation of motion takes the form [4]

$$(\Gamma^V_{\alpha\beta\mu\nu} p^\mu p^\nu - m^2 g_{\alpha\beta}) V^\beta = 0, \tag{2.17}$$

where V^β denotes the wave function of an ordinary vectorial, i.e., $J^P = 1^-$, particle. The $\Gamma^V_{\alpha\beta\mu\nu}$ tensor relevant for parity-conserving processes is obtained as

$$\Gamma^V_{\alpha\beta\mu\nu} = g_{\alpha\beta} g_{\mu\nu} - \tfrac{1}{2}(g_{\alpha\nu} g_{\beta\mu} + g_{\alpha\mu} g_{\beta\nu}) - i\left(g_V - \tfrac{1}{2}\right)[M^V_{\mu\nu}]_{\alpha\beta}, \tag{2.18}$$

where g_V stands for the gyromagnetic ratio of a spin-1 particle as it appears within the covariant-projector formalism. As usual, $M^V_{\mu\nu}$ are the Lorentz group generators within the $(1/2, 1/2)$ representation, which are known to be

$$[M^V_{\mu\nu}]_{\alpha\beta} = i(g_{\alpha\mu} g_{\beta\nu} - g_{\alpha\nu} g_{\beta\mu}). \tag{2.19}$$

The associated free Lagrangian reads

$$\mathcal{L}^V_{\text{free}} = -(\partial^\mu V^\alpha)^\dagger \Gamma^V_{\alpha\beta\mu\nu} \partial^\nu V^\beta + m^2 V^{\alpha\dagger} V_\alpha. \tag{2.20}$$

Correspondingly, the gauged Lagrangian emerges as

$$\mathcal{L}^V_{\text{int}} = \mathcal{L}^V_{\text{free}} - j_\mu A^\mu - e^2 V^{\alpha\dagger}(\Gamma^V_{\alpha\beta\mu\nu} + \Gamma^V_{\alpha\beta\nu\mu}) V^\beta A^\mu A^\nu. \tag{2.21}$$

The related electromagnetic current in momentum space is then obtained as

$$j^V_\mu(\mathbf{p}', \lambda'; \mathbf{p}, \lambda) = -e\eta^{\alpha *}(\mathbf{p}', \lambda')(\Gamma^V_{\alpha\beta\nu\mu} p'^\nu + \Gamma^V_{\alpha\beta\mu\nu} p^\nu)\eta^\beta(\mathbf{p}, \lambda). \tag{2.22}$$

Above, $V^\alpha(x) = \int d^4p (\eta^\alpha(\mathbf{p}, \lambda) e^{-ix\cdot p} a_\lambda(\mathbf{p}) + \eta^{\alpha *}(\mathbf{p}, \lambda) \times a^\dagger_\lambda(\mathbf{p}) e^{ix\cdot p})$ and $\eta(\mathbf{p}, \lambda)$ are the amplitudes of the spin-1 quantum states that transform in the $(1/2, 1/2)$ representation of the homogeneous Lorentz group [14]. As long as $p^\alpha \eta_\alpha(\mathbf{p}, \lambda) = 0$ holds valid on mass shell, and using the explicit form of $\Gamma^V_{\alpha\beta\mu\nu}$ in Eq. (2.18), the expression for the current in momentum space simplifies to

$$j^V_\mu(\mathbf{p}', \lambda'; \mathbf{p}, \lambda) = -e\eta^{\alpha *}(\mathbf{p}', \lambda')[(p' + p)_\mu g_{\alpha\beta} + i g_V [M^V_{\mu\nu}]_{\alpha\beta}(p' - p)^\nu - p'_\beta g_{\beta\mu} - p_\beta g_{\alpha\mu}]\eta^\beta(\mathbf{p}, \lambda). \tag{2.23}$$

In effect, on mass shell one finds

$$j^V_\mu(\mathbf{p}', \lambda'; \mathbf{p}, \lambda) = -e\eta^{\alpha *}(\mathbf{p}', \lambda')[(p' + p)_\mu g_{\alpha\beta} + i g_V [M^V_{\mu\nu}]_{\alpha\beta}(p' - p)^\nu]\eta^\beta(\mathbf{p}, \lambda). \tag{2.24}$$

This current again parallels the Gordon decomposition of the Dirac current, and, as we shall see below, coincides with the one used in the standard model Lagrangian for the W boson. The current in Eq. (2.24) is the counterpart in the NKR formalism to the Proca (P) current,

$$j^P_\mu(\mathbf{p}', \lambda'; \mathbf{p}, \lambda) = -e\eta^{\alpha *}(\mathbf{p}', \lambda')[(p' + p)_\mu g_{\alpha\beta} - p'_\beta g_{\alpha\mu} - p_\alpha g_{\beta\mu}]\eta^\beta(\mathbf{p}, \lambda), \tag{2.25}$$

corresponding to the Lagrangian,

$$\mathcal{L}_P = -\frac{1}{2}[U^\dagger]^{\alpha\beta} U_{\alpha\beta} + m^2 [V^\dagger]^\alpha V_\alpha, \tag{2.26}$$

with $U_{\alpha\beta} = D_\alpha V_\beta - D_\beta V_\alpha$. Again because of $p^\beta \eta_\beta(\mathbf{p}, \lambda) = 0$, this current equivalently rewrites as

$$j^P_\mu(\mathbf{p}', \lambda'; \mathbf{p}, \lambda) = -e\eta^{*\alpha}(\mathbf{p}', \lambda')[(p'+p)_\mu g_{\alpha\beta} + i[M^V_{\mu\nu}]_{\alpha\beta}(p'-p)^\nu]\eta^\beta(\mathbf{p}, \lambda). \tag{2.27}$$

Comparing with Eq. (2.24), we see that Proca's theory comes with a previously built-in gyromagnetic factor of $g_P = 1$, a circumstance that will seriously spoil the prediction of the quadrupole moment.

C. The spin-1 current in the antisymmetric tensor (1, 0) ⊕ (0, 1) representation

The $(1,0)\oplus(0,1)$ single-spin representation has six components and it is a well-known fact [15] that they can be viewed as the components of a totally antisymmetric Lorentz tensor of second rank, $F^{\mu\nu}$. This tensor transforms according to the antisymmetric part of the direct product of two four-vectors,

$$(F^{\mu\nu})' = \tfrac{1}{2}(\Lambda^\mu{}_\alpha \Lambda^\nu{}_\beta - \Lambda^\mu{}_\beta \Lambda^\nu{}_\alpha)F^{\alpha\beta} \equiv \Lambda^{\mu\nu}{}_{\alpha\beta}F^{\alpha\beta}, \tag{2.28}$$

where

$$\Lambda^\mu{}_\alpha = g^\mu{}_\alpha - i\left(M^V_{\rho\sigma}\frac{\theta^{\rho\sigma}}{2}\right)^\mu{}_\alpha + \mathcal{O}(\theta^2). \tag{2.29}$$

Here, $\theta^{\rho\sigma}$ stand for the parameters of the Lorentz transformation, with the $M^V_{\rho\sigma}$ generators from Eq. (2.19). Infinitesimally, one finds

$$\Lambda^{\mu\nu}{}_{\alpha\beta} = \frac{1}{2}(g^\mu_\alpha g^\nu_\beta - g^\nu_\beta g^\mu_\alpha) - i[(\mathcal{M}^V_{\rho\sigma})^{\mu\nu}{}_{\alpha\beta}]\frac{\theta^{\rho\sigma}}{2} + O(\theta^2), \tag{2.30}$$

which allows us to identify the $(1,0)\oplus(0,1)$ generators as

$$(\mathcal{M}_{\rho\sigma})^{\mu\nu}{}_{\alpha\beta} = \tfrac{1}{2}[(M^V_{\rho\sigma})^\mu{}_\alpha g_{\nu\beta} + g_{\mu\alpha}(M^V_{\rho\sigma})^\nu{}_\beta - (M^V_{\rho\sigma})^\nu{}_\beta g_{\nu\alpha} - g_{\mu\beta}(M^V_{\rho\sigma})^\mu{}_\alpha]. \tag{2.31}$$

The transformation properties of the components of opposite parities $V_i = F^{0i}$ and $A_i = \frac{1}{2}\varepsilon_{ijk}F^{jk}$, spanning this space, are now obtained as

$$\mathbf{A}' = \mathbf{A} + \theta \times \mathbf{A} + \phi \times \mathbf{V}, \qquad \mathbf{V}' = \mathbf{V} - \phi \times \mathbf{A} + \theta \times \mathbf{V}, \tag{2.32}$$

where we introduced special notations for the vectorial boost parameters, $\phi_i \equiv \theta_{0i}$, and the pseudovectorial rotation parameters, $\theta^{ij} \equiv \varepsilon_{ijk}\theta_k$. The unit operator in this space is

$$\mathbf{1}^{ab}{}_{cd} = \tfrac{1}{2}(g^a{}_c g^b{}_d - g^a{}_d g^b{}_c), \tag{2.33}$$

while the parity operator reads

$$(\Pi)^{ab}{}_{cd} = \tfrac{1}{2}(g_{ac}g_{bd} - g_{ad}g_{bc}), \qquad a, b, c, \ldots = 0, 1, 2, 3. \tag{2.34}$$

Notice that in this case we have Lorentz indices associated with the $(1,0)\oplus(0,1)$ representation in its antisymmetric tensor form. In order to keep track of the products of operators in this space, we use small Latin letters for the Lorentz indices of the components of the states and operators in the $(1,0)\oplus(0,1)$, and hope that such will not lead to confusion.

The explicit connection with the conventional $(1,0)\oplus(0,1)$ states is established upon casting Eqs. (2.32) in matrix form as

$$\begin{pmatrix} i\mathbf{A}' \\ \mathbf{V}' \end{pmatrix} = \begin{pmatrix} 1 - i\mathbf{L}\cdot\theta & \mathbf{L}\cdot\phi \\ \mathbf{L}\cdot\phi & 1 - i\mathbf{L}\cdot\theta \end{pmatrix}\begin{pmatrix} i\mathbf{A} \\ \mathbf{V} \end{pmatrix}, \tag{2.35}$$

with $(L_i)_{jk} = -i\epsilon_{ijk}$, meaning that we can associate to the antisymmetric tensor the following bi-vector field

$$F^{\mu\nu} \to \begin{pmatrix} i\mathbf{A} \\ \mathbf{V} \end{pmatrix}, \tag{2.36}$$

which is Lorentz transformed by the generators

$$\mathbf{J}_{VA} = \begin{pmatrix} \mathbf{L} & 0 \\ 0 & \mathbf{L} \end{pmatrix}, \qquad \mathbf{K}_{VA} = \begin{pmatrix} 0 & i\mathbf{L} \\ i\mathbf{L} & 0 \end{pmatrix}. \tag{2.37}$$

The parity operator within this bi-vector space in the above basis is

$$\Pi_{VA} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2.38}$$

Now we turn to the construction of the Poincaré projector. A straightforward calculation using Eq. (2.31) yields

$$\frac{1}{4}(\mathcal{M}^{\alpha\beta}\mathcal{M}_{\alpha\beta})^{ab}{}_{cd} = s(s+1)\mathbf{1}^{ab}{}_{cd},$$
$$(\mathcal{M}_{\mu\alpha}\mathcal{M}^\alpha{}_\nu)^{ab}{}_{cd} = s(s+1)g_{\mu\nu}\mathbf{1}^{ab}{}_{cd} - i(\mathcal{M}_{\mu\nu})^{ab}{}_{cd},$$

with $s = 1$. As a result, the squared Pauli-Lubanski operator is obtained as

$$(W^2)^{ab}{}_{cd} = \left(-\tfrac{1}{2}(\mathcal{M}^{\alpha\beta}\mathcal{M}_{\alpha\beta})^{ab}{}_{cd}g^{\mu\nu} + (\mathcal{M}_\alpha{}^\mu\mathcal{M}^{\alpha\nu})^{ab}{}_{cd}\right)p_\mu p_\nu. \tag{2.39}$$

Correspondingly, the Poincaré projector for this representation reads

$$-\frac{W^2}{s(s+1)m^2} = \left(g^{\mu\nu}\mathbf{1} + \frac{1}{s(s+1)}i\mathcal{M}^{\mu\nu}\right)\frac{p_\mu p_\nu}{m^2}, \tag{2.40}$$

where we now drop the small Latin letter indices. The antisymmetric part of the corresponding tensor remains undetermined by the Poincaré projector, and as usual it will be set as the most general antisymmetric tensor (preserving parity for the purposes of this work). In so doing, we find the following equation of motion:

$$[(T_{\mu\nu})^{ab}{}_{cd}p^\mu p^\nu - m^2 \mathbf{1}^{ab}{}_{cd}]F^{cd} = 0. \tag{2.41}$$

Here,

$$(T_{\mu\nu})^{ab}{}_{cd} = g_{\mu\nu}\mathbf{1}^{ab}{}_{cd} - ig_{VA}(\mathcal{M}_{\mu\nu})^{ab}{}_{cd}, \tag{2.42}$$

with g_{VA} being the free parameter, to be associated in the following with the gyromagnetic ratio in $(1,0)\oplus(0,1)$. The gauged Lagrangian is then

$$\mathcal{L}_{\text{int}} = (D^\mu F_{ab})^\dagger (T_{\mu\nu})^{ab}{}_{cd} D^\nu F^{cd} - m^2 (F^{ab})^\dagger F_{ab}, \tag{2.43}$$

which yields the following interactions:

$$\begin{aligned}\mathcal{L}_{\text{int}} = & -ie[(F^{ab})^\dagger T_{\mu\nu abcd}\partial^\nu F^{cd} \\ & - (\partial^\nu F^{ab})^\dagger T_{\nu\mu abcd}F^{cd}]A^\mu \\ & + e^2 (F^{ab})^\dagger T_{\mu\nu abcd}F^{cd}A^\mu A^\nu.\end{aligned} \tag{2.44}$$

The free particle solutions can be written as usual as

$$F^{ab}(x) = \mathcal{F}^{ab}(\mathbf{p},\lambda)e^{-ip.x}, \tag{2.45}$$

yielding the electromagnetic current in momentum space as

$$\begin{aligned}J_\mu(\mathbf{p},\lambda;\mathbf{p}',\lambda') = & e\mathcal{F}^\dagger_{ab}(\mathbf{p}',\lambda')[(p'+p)_\mu \mathbf{1}^{ab}{}_{cd} \\ & + ig_{VA}(\mathcal{M}_{\mu\nu})^{ab}{}_{cd}(p'-p)^\nu]\mathcal{F}^{cd}(\mathbf{p},\lambda), \\ & \lambda = \pm 1, 0.\end{aligned} \tag{2.46}$$

In general, there are six independent degrees of freedom in $(1,0)\oplus(0,1)$ and we are free to choose the corresponding basis according to the physics we aim to describe. Concerning electromagnetic interactions, it is appropriate to work within a basis of well-defined parity. Below, in Subsec. C of Sec. IV, we shall present in detail the explicit construction of antisymmetric tensors describing the negative parity states of interest. Charge conjugation is especially simple in the tensor basis. Indeed, in position space, the field F satisfies the gauged equation of motion

$$[(T_{\mu\nu})^{ab}{}_{cd}(\partial^\mu + ieA^\mu)(\partial^\nu + ieA^\nu) - m^2\mathbf{1}^{ab}{}_{cd}]F^{cd} = 0. \tag{2.47}$$

Taking the complex conjugate of Eq. (2.47), one arrives at

$$[T^*_{\mu\nu}(\partial^\mu - ieA^\mu)(\partial^\nu - ieA^\nu) - m^2]F^* = 0, \tag{2.48}$$

where we skipped out the representation indices for simplicity. On the other side, Eqs. (2.31), (2.19), and (2.42) imply

$$T^*_{\mu\nu} = T_{\mu\nu}, \tag{2.49}$$

and the complex conjugate field satisfies same gauged Eq. (2.47) but with an inverse sign of the charge,

$$[T_{\mu\nu}(\partial^\mu - ieA^\mu)(\partial^\nu - ieA^\nu) - m^2]F^* = 0. \tag{2.50}$$

Hence, the charge conjugated field for this basis is obtained by simple complex conjugation

$$F^c = F^*. \tag{2.51}$$

It is obvious that the parity operator in Eq. (2.34) commutes with charge conjugation in Eq. (2.51); thus the charge conjugate states carry equal spatial parities.

D. The spin-3/2 current in the $(\frac{1}{2},\frac{1}{2})\otimes[(\frac{1}{2},\mathbf{0})\oplus(\mathbf{0},\frac{1}{2})]$ four-vector–spinor representation

1. Covariant-projector formalism

In this case, the equation of motion takes the form [3]

$$(\Gamma_{\alpha\beta\mu\nu}p^\mu p^\nu - m^2 g_{\alpha\beta})\psi^\beta = 0. \tag{2.52}$$

Here, $\Gamma_{\alpha\beta\mu\nu}$ contains five independent parity-conserving antisymmetric Lorentz tensors, weighted by the five free parameters, c, d, f, g_V, and g_S, and expresses as [3]

$$\begin{aligned}\Gamma_{\alpha\beta\mu\nu} = & -\tfrac{1}{3}g_{\alpha\nu}g_{\beta\mu} - \tfrac{1}{6}i\sigma_{\alpha\nu}g_{\beta\mu} - \tfrac{1}{3}g_{\alpha\mu}g_{\beta\nu} + \tfrac{2}{3}g_{\alpha\beta}g_{\mu\nu} \\ & + \tfrac{i}{3}g_{\mu\nu}\sigma_{\alpha\beta} - \tfrac{i}{6}g_{\beta\nu}\sigma_{\alpha\mu} + \tfrac{i}{6}g_{\alpha\nu}\sigma_{\beta\mu} + \tfrac{i}{6}g_{\alpha\mu}\sigma_{\beta\nu} \\ & - g_S\tfrac{i}{2}g_{\alpha\beta}\sigma_{\mu\nu} + g_V(g_{\alpha\mu}g_{\beta\nu} - g_{\beta\mu}g_{\alpha\nu}) \\ & + id(g_{\beta\nu}\sigma_{\alpha\mu} - g_{\beta\mu}\sigma_{\alpha\nu}) + ic(g_{\alpha\mu}\sigma_{\beta\nu} \\ & - g_{\alpha\nu}\sigma_{\beta\mu}) + if\gamma^5\epsilon_{\alpha\beta\mu\nu}.\end{aligned} \tag{2.53}$$

Contracting the wave equation in Eq. (2.53) first by p^α and then by γ^α amounts to $-m^2 p_\beta\psi^\beta = 0$ and $-m^2\gamma_\beta\psi^\beta = 0$, respectively, meaning that the restrictions,

$$p^\beta\psi_\beta = 0, \tag{2.54a}$$

$$\gamma^\beta\psi_\beta = 0, \tag{2.54b}$$

needed to eliminate the undesired spin-1/2 sectors, are inherent to Eq. (2.53). The associated Lagrangian describing the free negative parity states is

$$\begin{aligned}\mathcal{L}_{\text{free}} &= -(\partial^\mu\psi^\alpha)^\dagger\Gamma_{\alpha\beta\mu\nu}\partial^\nu\psi^\beta + m^2\psi^{\alpha\dagger}\psi_\alpha, \\ \psi_\alpha &= u_\alpha(\mathbf{p},\lambda)e^{-ip\cdot x}.\end{aligned} \tag{2.55}$$

Upon gauging one finds

$$\mathcal{L}_{\text{int}} = \mathcal{L}_{\text{free}} - j_\mu A^\mu - e^2\bar{\psi}^\alpha(\Gamma_{\alpha\beta\mu\nu} + \Gamma_{\alpha\beta\nu\mu})\psi^\beta A^\mu A^\nu. \tag{2.56}$$

Then the electromagnetic transition current in momentum space emerges as

$$\begin{aligned}j_\mu(\mathbf{p}',\lambda';\mathbf{p},\lambda) = & -e\bar{u}^\alpha(\mathbf{p}',\lambda')(\Gamma_{\alpha\beta\nu\mu}p'^\nu \\ & + \Gamma_{\alpha\beta\mu\nu}p^\nu)u^\beta(\mathbf{p},\lambda), \\ & \lambda,\lambda' = \pm\tfrac{1}{2},\pm\tfrac{3}{2},\end{aligned} \tag{2.57}$$

where $\psi^\alpha = u^\alpha(\mathbf{p},\lambda)e^{-ip\cdot x}$ are the amplitudes of the quantum spin-3/2 states of negative parity transforming in the $(1/2,1/2)\otimes[(1/2,0)\oplus(0,1/2)]$ representation of the HLG.

Some of the five undetermined parameters in Eq. (2.58) can be fixed by imposing physical requirements. In Ref. [3], the simplest case $f=0$ was studied in detail. Here instead we consider the most general case of a nonvanishing f. We begin with the gauged equation of motion

$$\begin{aligned}[\Gamma_{\alpha\beta\mu\nu}D^\mu D^\nu + m^2 g_{\alpha\beta}]\psi^\beta &= \mathcal{D}_{\alpha\beta}\psi^\beta + f(-\gamma_\alpha\gamma_\mu\gamma_\nu\gamma_\beta + \gamma_\alpha\gamma_\beta g_{\mu\nu})D^\mu D^\nu\psi^\beta + \tfrac{e}{2}(2f+g_S)M^S_{\mu\nu}g_{\alpha\beta}F^{\mu\nu}\psi^\beta \\ &\quad + \tfrac{e}{2}\Big(g_V - c + d - f + \tfrac{2}{3}\Big)[M^V_{\mu\nu}]_{\alpha\beta}F^{\mu\nu}\psi^\beta + ie\Big(c+f-\tfrac{1}{6}\Big)F_{\alpha\mu}\gamma^\mu\gamma_\beta\psi^\beta \\ &\quad - ie\Big(d-f+\tfrac{1}{6}\Big)\gamma_\alpha\gamma^\mu F_{\mu\beta}\psi^\beta, \end{aligned} \tag{2.58}$$

with

$$\mathcal{D}_{\alpha\beta} = (D^2+m^2)g_{\alpha\beta} + \tfrac{1}{3}(\gamma_\alpha\gamma^\mu D_\mu - 4D_\alpha)D_\beta + \tfrac{1}{3}(D_\alpha\gamma^\mu D_\mu - \gamma_\alpha D^2)\gamma_\beta. \tag{2.59}$$

As long as the last two lines of Eq. (2.58) invoke the undesirable spin-1/2 structure $(\gamma\cdot\psi)$ into the coupling with the electromagnetic field, we shall eliminate these terms by nullifying their coefficients, yielding

$$c = -d = \tfrac{1}{6} - f. \tag{2.60}$$

In effect, we are left with

$$\begin{aligned}[\Gamma_{\alpha\beta\mu\nu}D^\mu D^\nu + m^2 g_{\alpha\beta}]\psi^\beta &= \mathcal{D}_{\alpha\beta}\psi^\beta + f(-\gamma_\alpha\gamma_\mu\gamma_\nu\gamma_\beta + \gamma_\alpha\gamma_\beta g_{\mu\nu})D^\mu D^\nu\psi^\beta + \tfrac{e}{2}(2f+g_S)M^S_{\mu\nu}g_{\alpha\beta}F^{\mu\nu}\psi^\beta \\ &\quad + \tfrac{e}{2}\Big(g_V + f + \tfrac{1}{3}\Big)[M^V_{\mu\nu}]_{\alpha\beta}F^{\mu\nu}\psi^\beta. \end{aligned} \tag{2.61}$$

As a next step, we aim to identify the spin-3/2 gyromagnetic factor. In order to ensure proportionality to $g_{3/2}\mathbf{S}\cdot\mathbf{B}$ of the coupling energy of the particle's spin to the external field, where $\mathbf{S}$ stands for the spin operator within this representation, we demand the following form of corresponding interaction Lagrangian:

$$\mathcal{L}_M = g_{3/2}\frac{e}{2}\bar{\psi}^\alpha[M^{3/2}_{\mu\nu}]_{\alpha\beta}\psi^\beta F^{\mu\nu}, \tag{2.62}$$

with $g_{3/2}$ denoting the gyromagnetic factor. Given the form of the generators,

$$[M^{3/2}_{\mu\nu}]_{\alpha\beta} = M^{1/2}_{\mu\nu}g_{\alpha\beta} + [M^V_{\mu\nu}]_{\alpha\beta}, \tag{2.63}$$

and taking into account that $\gamma_\beta\psi^\beta = 0$ holds valid to leading order in a perturbative expansion, one observes that the second term on the right-hand side of Eq. (2.61) identically vanishes, allowing us to impose the restrictions

$$g_S = g_{3/2} - 2f, \qquad g_V = g_{3/2} - f - \frac{1}{3}. \tag{2.64}$$

With that, one is finally able to write down the tree-level electromagnetic current in terms of a single free parameter, $g_{3/2}$, as

$$\begin{aligned} j_\mu(\mathbf{p}',\lambda';\mathbf{p},\lambda) &= -e\bar{u}^\alpha(\mathbf{p}',\lambda')[g_{\alpha\beta}(p'+p)_\mu \\ &\quad + ig_{3/2}[M^{3/2}_{\mu\nu}]_{\alpha\beta}(p'-p)^\nu]u^\beta(\mathbf{p},\lambda). \end{aligned} \tag{2.65}$$

The conclusion is that in the NKR formalism, the spin-3/2 current in the four-vector–spinor associated with local phase invariance naturally decomposes into a motion (convection) part and a spin-magnetization part, which, as we will see below, in reality provides contributions to all the allowed higher multipoles of the particle. As a reminder, in Ref. [3], it was shown that the causal propagation of spin-3/2 waves in an electromagnetic background requires $g_{3/2}=2$, a conclusion also independently verified by the unitarity of the differential cross-section in the forward direction for Compton scattering [5].

The expression in Eq. (2.65) is the counterpart in the NKR formalism of the Gordon decomposition of the Rarita-Schwinger current

$$\begin{aligned} j^{\rm RS}_\mu(\mathbf{p}',\lambda';\mathbf{p},\lambda) &= -2me\bar{u}^\alpha(\mathbf{p}',\lambda')\gamma_\mu u_\alpha(\mathbf{p},\lambda), \\ &= -e\bar{u}^\alpha(\mathbf{p}',\lambda')[(p'+p)^\mu \\ &\quad + ig_{(1/2)}M^{1/2}_{\mu\nu}(p'-p)^\nu]u_\alpha(\mathbf{p},\lambda). \end{aligned} \tag{2.66}$$

Notice that the principal difference between Eqs. (2.65) and (2.66) is due to the appearance of the purely Dirac spin-magnetization tensor $g_{(1/2)}M^{1/2}_{\mu\nu}$ in (2.66) in place of the genuine four-vector–spinor spin-magnetization tensor $g_{(3/2)}[M^{3/2}_{\mu\nu}]_{\alpha\beta}$ in Eq. (2.65). In order to illuminate the origin of this crucial difference, we take in the next section a closer look on the eigenvalue problem of the squared Pauli-Lubanski vector in ψ_μ and its relationship to the Rarita-Schwinger formalism for spin-3/2.

2. *Shortcomings of the Rarita-Schwinger framework from the perspective of the covariant projector*

We begin by recalling that in the $(1/2,1/2)\otimes[(1/2,0)\oplus(0,1/2)]$ direct product space, the principal ingredient of the respective covariant projector, the Pauli-Lubanski vector $\mathcal{W}_\mu$, is obtained as the direct sum of the Pauli-Lubanski vectors W_μ and w_μ, in the respective $(1/2, 1/2)$- and Dirac-building blocks according to

$$[[\mathcal{W}_\mu]_\alpha{}^\beta]_{AB} = [w_\mu]_{AB} g_\alpha{}^\beta + [W_\mu]_\alpha{}^\beta \delta_{AB},$$
$$w_\mu = \tfrac{1}{2}\gamma_5(p_\mu - \gamma_\mu \not{p}), \qquad [W_\mu]_\alpha{}^\beta = i\epsilon_{\mu\alpha}{}^\beta{}_\sigma p^\sigma. \quad (2.67)$$

The squared Pauli-Lubanski vector in ψ_μ is then calculated as

$$[\mathcal{W}^2]_\alpha{}^\beta = w^2 g_\alpha{}^\beta + [W^2]_\alpha{}^\beta + 2(W^\mu)_\alpha{}^\beta w_\mu, \quad (2.68a)$$
$$w^2 = -\tfrac{1}{4}\sigma_{\lambda\mu}\sigma^\lambda{}_\nu p^\mu p^\nu, \quad (2.68b)$$
$$[W^2]_\alpha{}^\beta = -2(g_\alpha{}^\beta g_{\mu\nu} - g_{\alpha\nu} g^\beta{}_\mu) p^\mu p^\nu, \quad (2.68c)$$
$$2(W^\mu)_\alpha{}^\beta w_\mu = i\epsilon^\mu{}_\alpha{}^\beta{}_\sigma p^\sigma \gamma_5 (p_\mu - \gamma_\mu \not{p}) = -i\epsilon^\mu{}_\alpha{}^\beta{}_\sigma p^\sigma \gamma_5 \gamma_\mu \not{p}, \quad (2.68d)$$

where for the sake of simplicity, we suppressed the spinorial indices. Then the wave equation takes the form in Eq. (2.52) with

$$\Gamma_{\alpha\beta\mu\nu} = \tfrac{2}{3}(g_{\alpha\beta} g_{\mu\nu} - g_{\alpha\nu} g_{\beta\mu}) + \tfrac{1}{6}(\epsilon^\lambda{}_{\alpha\beta\mu}\gamma^5 \sigma_{\lambda\nu} + \epsilon^\lambda{}_{\alpha\beta\nu}\gamma^5\sigma_{\lambda\mu}) + \tfrac{1}{12}\sigma_{\lambda\mu}\sigma^\lambda{}_\nu g_{\alpha\beta} - \tfrac{1}{4} g_{\mu\nu} g_{\alpha\beta}. \quad (2.69)$$

The first and third terms in Eq. (2.69) take their origins in turn from the contributions to the projector of W^2 and w^2 according to Eq. (2.68), while the second term arises due to the contribution of the interference term $2W \cdot w$ in Eq. (2.68d). On mass shell, Eq. (2.69) leads to

$$(i\epsilon_{\alpha\beta\mu\sigma}\gamma^5\gamma^\mu p^\sigma \not{p} - m^2 g_{\alpha\beta} + 2p_\beta p_\alpha)\psi^{(m,3/2)\beta} = 0. \quad (2.70)$$

The latter equation bears strong resemblance to a version of the Rarita-Schwinger equation of frequent use [16],

$$(i\epsilon_{\alpha\beta\mu\sigma}\gamma^5\gamma^\mu p^\sigma - m g_{\alpha\beta} + \gamma_\alpha\gamma_\beta)\psi^{(m,3/2)\beta} = 0, \quad (2.71)$$

as visible upon substituting $\gamma \cdot \psi = 0$ in Eq. (2.71) by the more fundamental $p \cdot \psi = 0$ and $\not{p}\psi^{(m,3/2)} = m\psi^{(m,3/2)}$. The above considerations show that the Rarita-Schwinger framework solely captures the piece of the covariant projector given by the interference term in Eq. (2.69) while ignoring the rest. The omission of the W^2 and w^2 contributions to Eq. (2.69) and the subsequent linearization of the covariant projector by the Rarita-Schwinger framework seriously prejudice the coupling of spin-3/2 to the electromagnetic field and are at the root of the inconsistencies of the interacting Rarita-Schwinger theory.

E. Spin-3/2 current in the single-spin $(3/2, 0) \oplus (0, 3/2)$ representation

Even though the most interesting cases of spin 3/2 particles belong to the $(1/2, 1/2) \otimes [(1/2, 0) \oplus (0, 1/2)]$ representation, we include for completeness the multipole moments of the $(3/2, 0) \oplus (0, 3/2)$ representation as well. To obtain the current for this representation, we need to work out the corresponding Poincaré projector whose calculation can be done for an arbitrary half-integer spin j residing in the $(j, 0) \oplus (0, j)$ representation.

In choosing a rotational $\{|j, \lambda\rangle\}$ basis within any one of the $(j, 0)$ and $(0, j)$ subspaces, the generators for the $(j, 0) \oplus (0, j)$ representation can be written as

$$\mathbf{J} = \begin{pmatrix} \mathbf{J}^{(j)} & 0 \\ 0 & \mathbf{J}^{(j)} \end{pmatrix}, \qquad \mathbf{K} = \begin{pmatrix} i\mathbf{J}^{(j)} & 0 \\ 0 & -i\mathbf{J}^{(j)} \end{pmatrix}. \quad (2.72)$$

Here, $\mathbf{J}^{(j)}$ stand for the conventional $(2j+1) \times (2j+1)$ rotation matrices for spin-j in the $\{|j, \lambda\rangle\}$ basis. The $(j, 0)$, and $(0, j)$ states are in turn associated with right- and left-chiralities, respectively. In momentum space one encounters

$$\psi(\mathbf{p}, \lambda) = \begin{pmatrix} \phi_R(\mathbf{p}, \lambda) \\ \phi_L(\mathbf{p}, \lambda) \end{pmatrix}, \qquad \lambda = \pm\frac{1}{2}, \ldots, \pm j, \quad (2.73)$$

a spinor which transforms according to

$$\Lambda = \begin{pmatrix} \Lambda_R & 0 \\ 0 & \Lambda_L \end{pmatrix}, \quad (2.74)$$

where the Λ_R and Λ_L matrices transform the respective right- ($(j, 0)$) and left-handed ($(0, j)$) subspaces, whose generators were given in the diagonals of the matrices in Eq. (2.72). The generators of the combined $(j, 0) \oplus (0, j)$ representation satisfy

$$\mathbf{K}^2 = -\mathbf{J}^2. \quad (2.75)$$

The conventional Lorentz generators $M^{\mu\nu}$ are now identified as $M^{0i} = K_i$, $M^{ij} = \varepsilon_{ijk} J_k$, and it is straightforward to prove that they satisfy the standard Lorentz algebra

$$[M^{\mu\nu}, M^{\alpha\beta}] = -i(g^{\mu\alpha}M^{\nu\beta} - g^{\mu\beta}M^{\nu\alpha} - g^{\nu\alpha}M^{\mu\beta} + g^{\nu\beta}M^{\mu\alpha}). \quad (2.76)$$

Squaring the Pauli-Lubanski vector,

$$W_\alpha = \tfrac{1}{2}\varepsilon_{\alpha\rho\sigma\mu} M^{\rho\sigma} p^\mu, \quad (2.77)$$

amounts to

$$W^2 = \left(-\tfrac{1}{2}M^{\alpha\beta}M_{\alpha\beta}g^{\mu\nu} + M_\alpha{}^\mu M^{\alpha\nu}\right)p_\mu p_\nu. \quad (2.78)$$

A straightforward calculation with the generators in Eq. (2.72) yields for any j,

$$\tfrac{1}{2}M^{\alpha\beta}M_{\alpha\beta} = (\mathbf{J}^2 - \mathbf{K}^2) = 2\mathbf{J}^2 = 2j(j+1),$$
$$M_\alpha{}^\mu M^{\alpha\nu} = j(j+1)g^{\mu\nu} - iM^{\mu\nu}.$$

Correspondingly, the Poincaré projector emerges as

$$\mathcal{P}^{(m,s)} = \frac{p^2}{m^2}\left[-\frac{W^2}{j(j+1)p^2}\right] = -\frac{W^2}{j(j+1)m^2}. \quad (2.79)$$

As long as the antisymmetric part of this operator remains unspecified by Poincaré invariance, we as usual shall take advantage of this freedom and extend the Lagrangian by the most general covariant antisymmetric tensor. In so doing, the following equation of motion in the representation under consideration is found:

$$(T_{\mu\nu}p^\mu p^\nu - m^2)\psi = 0, \tag{2.80}$$

with

$$T_{\mu\nu} = g_{\mu\nu} - igM_{\mu\nu}. \tag{2.81}$$

This tensor depends on one sole free parameter, g, provided we have restricted the formalism to parity-conserving processes.

The Lagrangian for a particle in the $(j, 0) \oplus (0, j)$ representation interacting with an electromagnetic field is

$$\mathcal{L}_{\rm int} = \overline{D^\mu \psi} T_{\mu\nu} D^\nu \psi - m^2 \bar{\psi}\psi, \tag{2.82}$$

where $D^\mu = \partial^\mu + ieA^\mu$ and e is the charge of the particle. Here, one defines the adjoint spinor as

$$\bar{\psi} = \psi^\dagger \Pi, \tag{2.83}$$

with Π being the parity operator, which in the chiral basis coincides with the off-diagonal 6×6 unit matrix

$$\Pi = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{2.84}$$

Explicitly, the electromagnetic interactions described by $\mathcal{L}_{\rm int}$ in Eq. (2.82) are

$$\mathcal{L}_{\rm int} = -ie[\bar{\psi} T_{\mu\nu}\partial^\nu \psi - (\partial^\nu \bar{\psi})^\dagger T_{\nu\mu}\psi]A^\mu - e^2 \bar{\psi} T_{\mu\nu}\psi A^\mu A^\nu. \tag{2.85}$$

Using the momentum space functions $w(\mathbf{p}, \lambda)e^{-ip\cdot x}$ with $\lambda = \pm\frac{1}{2}, \ldots, \pm j$ yields the following electromagnetic current:

$$j^{(j)}_\mu(\mathbf{p}', \lambda'; \mathbf{p}, \lambda) = e\bar{w}(\mathbf{p}', \lambda')[(p' + p)_\mu + ig^{(j)} M^{(j)}_{\mu\nu}(p' - p)^\nu]w(\mathbf{p}, \lambda). \tag{2.86}$$

Here, we attached the representation index j to the generators, the free parameter, and the current. The states $w(\mathbf{p}, \lambda)$ of well-defined parity are obtained from the chiral states in Eq. (2.73) by multiplication by the matrix

$$M = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}. \tag{2.87}$$

Consequently, the Lorentz transformations of the parity basis express in terms of the Lorentz transformations of the chiral states as

$$\Lambda = \frac{1}{2}\begin{pmatrix} \Lambda_R + \Lambda_L & \Lambda_R - \Lambda_L \\ \Lambda_R - \Lambda_L & \Lambda_R + \Lambda_L \end{pmatrix}. \tag{2.88}$$

Finally, before closing this section and for the needs of what follows, we wish to bring the boost operator in the $(3/2, 0) \oplus (0, 3/2)$ in this basis,

$$B(\mathbf{p}) = \begin{pmatrix} \cosh(\mathbf{J}\cdot\mathbf{n}\varphi) & \sinh(\mathbf{J}\cdot\mathbf{n}\varphi) \\ \sinh(\mathbf{J}\cdot\mathbf{n}\varphi) & \cosh(\mathbf{J}\cdot\mathbf{n}\varphi) \end{pmatrix}. \tag{2.89}$$

III. MULTIPOLE EXPANSIONS

The electromagnetic moments of a particle are defined by means of a multipole expansion of a corresponding current density. The current densities used here are obtained in transforming the electromagnetic currents from above to the Breit (B) frame:

$$J^B_\mu(\mathbf{q}, s, \lambda) = \frac{1}{\omega} j^{(s)}_\mu(\mathbf{p}', \lambda; \mathbf{p}, \lambda), \qquad p' = (\omega/2, \mathbf{q}/2), \quad p = (\omega/2, -\mathbf{q}/2), \tag{3.1}$$

with $\omega = \sqrt{4m^2 + \mathbf{q}^2}$. Expressions for the Cartesian electromagnetic moments for a particle of spin s and polarization λ can be found in Ref. [17] and read

$$Q^l_E(\mathbf{q}, s, \lambda) = b^{l0}(-i\partial_\mathbf{q})\varrho_E(\mathbf{q}, s, \lambda)|_{\mathbf{q}=0}, \tag{3.2}$$

$$Q^l_M(\mathbf{q}, s, \lambda) = \frac{1}{l+1} b^{l0}(-i\partial_\mathbf{q})\varrho_M(\mathbf{q}, s, \lambda)|_{\mathbf{q}=0}, \tag{3.3}$$

where the electric density $\varrho_E(\mathbf{q}, s, \lambda)$ and the magnetic density $\varrho_M(\mathbf{q}, s, \lambda)$ are

$$\varrho_E(\mathbf{q}, s, \lambda) = j^0_B(\mathbf{q}, s, \lambda), \quad \varrho_M(\mathbf{q}, s, \lambda) = \partial_\mathbf{q} \cdot [\mathbf{j}_B(\mathbf{q}, s, \lambda) \times \mathbf{q}]. \tag{3.4}$$

The $b^{l0}(-i\partial_\mathbf{q})$ operators can be obtained from their definitions in position space in terms of the spherical harmonics,

$$b^{l0}(\mathbf{r}) = l!\sqrt{4\pi/(2l+1)}\, r^l Y_{l0}(\Omega), \tag{3.5}$$

upon Fourier transformations toward momentum space,

$$b^{00}(-i\partial_\mathbf{q}) = 1, \tag{3.6}$$

$$b^{10}(-i\partial_\mathbf{q}) = -i\frac{\partial}{\partial q_z}, \tag{3.7}$$

$$b^{20}(-i\partial_\mathbf{q}) = \frac{\partial^2}{\partial q_x^2} + \frac{\partial^2}{q_y^2} - 2\frac{\partial^2}{\partial q_z^2}, \tag{3.8}$$

$$b^{30}(-i\partial_\mathbf{q}) = -3i\frac{\partial}{\partial q_z}\left(3\frac{\partial^2}{\partial q_x^2} + 3\frac{\partial^2}{\partial q_y^2} - 2\frac{\partial^2}{\partial q_z^2}\right), \quad \text{etc.} \tag{3.9}$$

IV. ELECTROMAGNETIC MULTIPOLES FROM THE SECOND-ORDER FORMALISM

In this section, we perform side-by-side calculations of the electromagnetic multipole moments of elementary particles with spins-1/2, 1, and 3/2 within the NKR formalism and the respective Dirac, Proca, and Rarita-Schwinger frameworks.

A. Spin-1/2 in $(1/2, 0) \oplus (0, 1/2)$

We first examine the multipole decompositions of the currents in Eqs. (2.14) and (2.15) under employment of the well-known spinors

$$u\left(\mathbf{p},+\frac{1}{2}\right)=N\begin{pmatrix} m+p_0\\ 0\\ p_z\\ p_x+ip_y\end{pmatrix},$$

$$u\left(\mathbf{p},-\frac{1}{2}\right)=N\begin{pmatrix} 0\\ m+p_0\\ p_x-ip_y\\ -p_z\end{pmatrix}, \tag{4.1}$$

with $N=[2m(m+p_0)]^{-1/2}$. The Breit frame densities of Eq. (3.4) corresponding to the current in Eq. (2.14) are found as

$$\varrho_E^S(\mathbf{q},1/2,\pm 1/2)=\frac{e(8m^2-(g_S-2)\mathbf{q}^2)}{4m\omega}, \tag{4.2a}$$

$$\varrho_M^S(\mathbf{q},1/2,\pm 1/2)=\pm\frac{ieg_S q_z}{\omega}. \tag{4.2b}$$

The associated multipoles are then

$$(Q_E^0)^{\mathrm{NKR}}=e, \tag{4.3a}$$

$$(Q_M^1)^{\mathrm{NKR}}=\frac{eg_S}{2m}\langle S_z\rangle, \tag{4.3b}$$

where we have used the following compact notations:

$$Q_E^0\equiv Q_E^0(s,\lambda), \tag{4.4a}$$

$$\langle\mathcal{O}\rangle\equiv\langle s,\lambda|\mathcal{O}|s,\lambda\rangle, \tag{4.4b}$$

with $s=\frac{1}{2}$, $\lambda=\pm\frac{1}{2}$. In Eqs. (4.3), Q_E^0 and Q_M^1 denote in turn the electric monopole and magnetic dipole moments; all other moments vanish. The above expressions reproduce the electric monopole and the magnetic dipole moments of the Dirac electron for $g_S=2$. Within the NKR method, the g_S value has been fixed to $g_S=2$ in Ref. [12] from the requirement of reproducing the correct asymptotic behavior of the Compton scattering cross-sections with energy increase. Therefore, the results on the multipole moments for a fundamental spin-1/2 particle predicted by the covariant-projector formalism are the same as those following from the Dirac Lagrangian. However, this is not to be so for the higher spins $s=1$ and $s=3/2$, where one detects problems in the Proca and Rarita-Schwinger approaches, which need special attention.

B. Spin-1 in (1/2, 1/2)

Next we turn to the vector case. We shall be calculating the multipole expansions of the currents in Eqs. (2.24) and (2.25). The basis vectors, $\eta(\mathbf{p},\lambda)$, with $\lambda=\pm1,0$, used by us are

$$\eta(\mathbf{p},1)=\frac{\mathcal{N}}{\sqrt{2}}\begin{pmatrix} -(m+p_0)(p_x+ip_y)\\ -m^2-p_0m-p_x^2-ip_xp_y\\ -i\left(p_y^2-ip_xp_y+m(m+p_0)\right)\\ -(p_x+ip_y)p_z\end{pmatrix}, \tag{4.5a}$$

$$\eta(\mathbf{p},0)=\mathcal{N}\begin{pmatrix} (m+p_0)p_z\\ p_xp_z\\ p_yp_z\\ p_z^2+m(m+p_0)\end{pmatrix}, \tag{4.5b}$$

$$\eta(\mathbf{p},-1)=\frac{\mathcal{N}}{\sqrt{2}}\begin{pmatrix} (m+p_0)(p_x-ip_y)\\ m^2+p_0m+p_x^2-ip_xp_y\\ -i\left(p_y^2+ip_xp_y+m(m+p_0)\right)\\ (p_x-ip_y)p_z\end{pmatrix}, \tag{4.5c}$$

with $\mathcal{N}=[m(m+p_0)]^{-1}$. The transverse densities are now calculated as

$$\varrho_E^V(\mathbf{q},1,\pm1)=\frac{e(4m^2+q_x^2+q_y^2)(8m^2+4\omega m+\mathbf{q}^2)}{4m^2(2m+\omega)^2}-\frac{eg_V(q_x^2+q_y^2)(4m^2+2\omega m+\mathbf{q}^2)}{4m^2(2m+\omega)^2} \tag{4.6a}$$

$$=\frac{e[4m^2-(g_V-1)(q_x^2+q_y^2)]}{4m^2}, \tag{4.6b}$$

$$\varrho_M^V(\mathbf{q},1,\pm1)=\pm\frac{ig_Ve_z(4m^2+2\omega m+\mathbf{q}^2)}{m\omega(2m+\omega)}=\pm\frac{ig_Ve_z}{m}. \tag{4.6c}$$

For the longitudinal densities ($\lambda = 0$), we find

$$\varrho_E^V(\mathbf{q}, 1, 0) = \frac{e(2m^2 + q_z^2)(8m^2 + 4\omega m + \mathbf{q}^2)}{2m^2(2m+\omega)^2} - \frac{e g_V q_z^2(4m^2 + 2\omega m + \mathbf{q}^2)}{2m^2\omega(2m+\omega)} \tag{4.7a}$$

$$= \frac{e[2m^2 - (g_V - 1)q_z^2]}{2m^2}, \tag{4.7b}$$

$$\varrho_M^V(\mathbf{q}, 1, 0) = 0. \tag{4.7c}$$

Correspondingly, in terms of the expectation values of the $\mathbf{S}^2$ and S_z^2 operators, one encounters

$$(Q_E^0)^V = e, \tag{4.8a}$$

$$(Q_M^1)^V = \frac{e g_V}{2m}\langle S_z\rangle, \tag{4.8b}$$

$$(Q_E^2)^V = \frac{e(1 - g_V)}{m^2}\langle 3S_z^2 - \mathbf{S}^2\rangle, \tag{4.8c}$$

where we have adopted the notation Eq. (4.4) with $s = 1$ in this case. The g_V value has been fixed in Ref. [4] to $g_V = 2$ from the requirement on unitarity of the Compton scattering amplitudes in the ultraviolet. All multipoles are nonvanishing and interrelated by the g_V value, as it should be, due to the impossibility of separating the electric and magnetic fields covariantly. Thus the NKR method predicts a negative electric quadrupole moment of the W boson in agreement with the Standard Model (see, for example, Refs. [18–20]) and in line with the empirical observations [21]. In contrast to this, the Proca Lagrangian predicts a vanishing electric quadrupole, as visible upon substituting $g_V = 1$ in Eq. (4.6). This shortcoming of Proca's framework is removed by the standard model on the cost of introducing a non-Abelian current [20],

$$J_\mu^{\mathrm{NA}}(\mathbf{p}', \lambda'; \mathbf{p}, \lambda) = -ie\eta^{*\alpha}(\mathbf{p}', \lambda')[M_{\mu\nu}^V]_{\alpha\beta} \times (p' - p)^\nu \eta^\beta(\mathbf{p}, \lambda), \tag{4.9}$$

which leads to a gyromagnetic W-boson ratio of $g_W = 2$. The róle of this current is the same as in the NKR method, namely, removing the incompleteness of Proca's Lagrangian brought about by the omission of the most general antisymmetric tensor allowed by Poincaré invariance that respects the exclusion of the redundant spin-0 (timelike) component of the four-vector. In effect, tree-level Compton scattering within the NKR method results are the same as within the Standard Model, though the former is valid for any spin-1 in $(1/2, 1/2)$, no matter whether it is Abelian or non-Abelian (c.f. Ref. [4] for details).

C. Spin-1 in $(1, 0) \oplus (0, 1)$

In the current section, we explore the representation dependence of the multipole moments of a vector particle. For this purpose, we study the electromagnetic properties of a fundamental single spin-1 residing in the $(1, 0) \oplus (0, 1)$ representation space. In the following, this representation will be termed either as antisymmetric second-rank tensor or, equivalently, as bi-vector. In order to calculate the multipole decomposition of the current given in Eq. (2.46) above, we need to explicitly construct the $\mathcal{F}^{\mu\nu}$ tensor components; equivalently, the $\mathbf{A}$ and $\mathbf{V}$ vectors in Eq. (2.32), describing spin-1 particles. This can be done easily with the aid of the boost generators in Eq. (2.37). In so doing, one finds the boost operator as

$$B(\mathbf{p}) = \begin{pmatrix} \cosh(\mathbf{L}\cdot\mathbf{n}\varphi) & \sinh(\mathbf{L}\cdot\mathbf{n}\varphi) \\ \sinh(\mathbf{L}\cdot\mathbf{n}\varphi) & \cosh(\mathbf{L}\cdot\mathbf{n}\varphi) \end{pmatrix}. \tag{4.10}$$

Using $(\mathbf{L}\cdot\mathbf{n})^3 = \mathbf{L}\cdot\mathbf{n}$, we find

$$\cosh(\mathbf{L}\cdot\mathbf{n}\varphi) = 1 + (\mathbf{L}\cdot\mathbf{n})^2(\cosh\varphi - 1), \tag{4.11}$$

$$\sinh(\mathbf{L}\cdot\mathbf{n}\varphi) = \mathbf{L}\cdot\mathbf{n}(\sinh\varphi). \tag{4.12}$$

A straightforward calculation yields

$$(\mathbf{L}\cdot\mathbf{n})_{ij} = -i\epsilon_{ijm}n_m \qquad (\mathbf{L}\cdot\mathbf{n})_{ij}^2 = (\mathbf{n}^2\delta_{ij} - n_i n_j), \tag{4.13}$$

which allows us to explicitly construct the boost for the bi-vector state and therefore to produce explicit expressions for the corresponding $\mathbf{A}$, $\mathbf{V}$ components. The antisymmetric tensor describing negative parity spin-1 particles obtained in this fashion has the following components:

$$\left(i\mathbf{A}_{(\zeta)}(\mathbf{p})\right)_i = -i\epsilon_{i\zeta m}\frac{p_m}{m},$$

$$\left(\mathbf{V}_{(\zeta)}(\mathbf{p})\right)_i = \frac{p_0}{m}\delta_{i\zeta} - \frac{p_i p_\zeta}{m(p_0 + m)}, \tag{4.14}$$

with $\zeta = 1, 2, 3$ labeling the three independent bi-vectors. The set of three $\mathcal{F}_{(\zeta)}^{\mu\nu}$ tensors calculated in this way is equivalent to the following construct (see Ref. [22] and references therein for more details) in terms of the $(1/2, 1/2)$ spinors $\eta(\mathbf{p}, \lambda)$ in Eqs. (4.5):

$$\mathcal{F}_{(\zeta)}^{\mu\nu}(\mathbf{p}) = e_0^\mu(\mathbf{p})e_{(\zeta)}^\nu(\mathbf{p}) - e_{(\zeta)}^\mu(\mathbf{p})e_0^\nu(\mathbf{p}), \tag{4.15}$$

with

$$e_0^\alpha(\mathbf{p}) = \frac{1}{m}p^\alpha, \tag{4.16a}$$

$$e_{(x)}^\alpha(\mathbf{p}) = \frac{1}{\sqrt{2}}(\eta^\alpha(\mathbf{p}, -1) - \eta^\alpha(\mathbf{p}, +1)), \tag{4.16b}$$

$$e_{(y)}^\alpha(\mathbf{p}) = \frac{i}{\sqrt{2}}(\eta^\alpha(\mathbf{p}, -1) + \eta^\alpha(\mathbf{p}, +1)), \tag{4.16c}$$

$$e_{(z)}^\alpha(\mathbf{p}) = \eta^\alpha(\mathbf{p}, 0). \tag{4.16d}$$

Notice that at rest, $\mathbf{V}_{(\zeta)}$ reduce to the three Cartesian unit vectors, $\hat{\mathbf{e}}_{(\zeta)}$, while $\mathbf{A}_{(\zeta)}$ vanish. We will not discuss here in detail the positive parity solutions, but they can be obtained as the dual tensors to the ones in Eq. (4.17).

It has to be noticed that a set of six bi-vectors spanning the $(1,0)\oplus(0,1)$ representation space has already been constructed earlier in Ref. [23], though on the grounds of a different representation of the J_i generators. The main purpose in Ref. [23] was the construction of Feynman propagators from the explicit construction of the states and without reference to a Lagrangian formalism. The problem of the identification of the degrees of freedom of the corresponding antisymmetric tensor has not been addressed there. We here instead are interested in the electromagnetic couplings of general spin-1 particles, including composite ones, and entirely focus on the $(1,0)\oplus(0,1)\sim\mathcal{F}^{\mu\nu}_{(\zeta)}$ map. We construct an electromagnetic current that follows from the Lagrangian underlying the covariant projector wave equation for $\mathcal{F}^{\mu\nu}_{(\zeta)}$. An additional motivation to work with this field is the straightforward generalization of the dimensional regularization method in the calculation of quantum corrections (work in progress).

In the calculation of the multipole moments, we will need the explicit form of the states $\mathbf{A}(\mathbf{p},\lambda)/\mathbf{V}(\mathbf{p},\lambda)$ of well-defined parity and angular momentum, which relate to the Cartesian states $\mathbf{A}_{(\zeta)}(\mathbf{p})/\mathbf{V}_{(\zeta)}(\mathbf{p})$ from Eq. (4.14) by combinations of the type given in Eq. (4.16). Then the three corresponding antisymmetric tensors are

$$\mathcal{F}^{\mu\nu}(\mathbf{p},\lambda)=\begin{pmatrix} 0 & V^1(\mathbf{p},\lambda) & V^2(\mathbf{p},\lambda) & V^3(\mathbf{p},\lambda)\\ -V^1(\mathbf{p},\lambda) & 0 & A^3(\mathbf{p},\lambda) & -A^2(\mathbf{p},\lambda)\\ -V^2(\mathbf{p},\lambda) & -A^3(\mathbf{p},\lambda) & 0 & A^1(\mathbf{p},\lambda)\\ -V^3(\mathbf{p},\lambda) & A^2(\mathbf{p},\lambda) & -A^1(\mathbf{p},\lambda) & 0\end{pmatrix},\qquad \lambda=+1,0,-1. \tag{4.17}$$

The explicit form of the polar and axial three-vectors are calculated as

$$\mathbf{V}(\mathbf{p},+1)=\frac{\mathcal{N}}{\sqrt{2}}\begin{pmatrix} p_0^2+mp_0-p_xp_+\\ i(p_0^2+mp_0+ip_yp_+)\\ -p_zp_+\end{pmatrix},\qquad \mathbf{A}(\mathbf{p},+1)=\frac{\mathcal{N}}{\sqrt{2}}\begin{pmatrix}-i(m+p_0)p_z\\ (m+p_0)p_z\\ i(m+p_0)p_+\end{pmatrix},$$

$$\mathbf{V}(\mathbf{p},0)=\mathcal{N}\begin{pmatrix} p_xp_z\\ p_yp_z\\ -(p_0^2+mp_0-p_z^2)\end{pmatrix},\qquad \mathbf{A}(\mathbf{p},0)=\mathcal{N}\begin{pmatrix}-(m+p_0)p_y\\ (m+p_0)p_x\\ 0\end{pmatrix},$$

$$\mathbf{V}(\mathbf{p},-1)=\frac{\mathcal{N}}{\sqrt{2}}\begin{pmatrix} -p_0^2-mp_0+p_xp_-\\ i(p_0^2+mp_0-ip_yp_+)\\ p_zp_-\end{pmatrix},\qquad \mathbf{A}(\mathbf{p},-1)=\frac{\mathcal{N}}{\sqrt{2}}\begin{pmatrix}-i(m+p_0)p_z\\ -(m+p_0)p_z\\ i(m+p_0)p_-\end{pmatrix}, \tag{4.18}$$

where $p_\pm=p_x\pm ip_y$, $\mathcal{N}=[m(m+p_0)]^{-1}$. The above states satisfy

$$\frac{\mathbf{p}}{p_0}\times\mathbf{V}(\mathbf{p},\lambda)=\mathbf{A}(\mathbf{p},\lambda),\qquad \mathbf{p}\cdot\mathbf{A}(\mathbf{p},\lambda)=0. \tag{4.19}$$

Using these states, we calculate the multipole moments for the current in Eq. (2.46). The gyromagnetic ratio, here denoted by g_{VA}, remains, as usual, unspecified so far. The electric and magnetic densities corresponding to the current in Eq. (2.46) are labeled by (VA). The transverse densities are found as

$$\varrho_E^{VA}(\mathbf{q},1,\pm1)=\frac{e((\mathbf{q}^2+\omega^2)(q_x^2+q_y^2-2\mathbf{q}^2)-4m^2(q_x^2+q_y^2))}{8m^2(4m^2-\omega^2)}-\frac{eg_{VA}\mathbf{q}^2(q_x^2+q_y^2-2\mathbf{q}^2)}{4m^2(4m^2-\omega^2)} \tag{4.20a}$$

$$=\frac{e(4m^2-(g_{VA}-1)(q_x^2+q_y^2+2q_z^2))}{4m^2}, \tag{4.20b}$$

$$\varrho_M^{VA}(\mathbf{q},1,\pm1)=\pm\frac{ig_{VA}e_z}{m}. \tag{4.20c}$$

For the longitudinal polarization $\lambda=0$, we encounter

$$\varrho_E^{VA}(\mathbf{q},1,0)=\frac{e(4m^2(q_x^2+q_y^2-\mathbf{q}^2)-(\mathbf{q}^2+\omega^2)(q_x^2+q_y^2))}{4m^2(4m^2-\omega^2)}+\frac{eg_{VA}\mathbf{q}^2(q_x^2+q_y^2)}{2m^2(4m^2-\omega^2)} \tag{4.21a}$$

$$=\frac{e(2m^2-(g_{VA}-1)(q_x^2+q_y^2))}{2m^2}, \tag{4.21b}$$

$$\varrho_M^{VA}(\mathbf{q},1,0)=0. \tag{4.21c}$$

The three resulting multipole moments are then calculated as

$$(Q_E^0)^{VA} = e, \qquad (Q_M^1)^{VA} = \frac{eg_{VA}}{2m}\langle S_z\rangle,$$
$$(Q_E^2)^{VA} = -\frac{e(1-g_{VA})}{m^2}\langle 3S_z^2 - \mathbf{S}^2\rangle. \tag{4.22}$$

The spin-1 multipole moments in Eqs. (4.22) are same as those of spin-1 residing in the four-vector and given in Eqs. (4.8) for $g_{VA} = 2$, *except for a relative sign between the electric quadrupole moments*. This sign of $(Q_E^2)^{VA}$ is inverse relative to that of $(Q_E^2)^V$ and technically reflects the opposite signs of q_z^2 in the combinations $(\mathbf{q}^2 \mp q_z^2)$ determining $\varrho_E^V(1,1)$ in Eq. (4.6), and $\varrho_E^{VA}(1,1)$ in Eq. (4.20), respectively. The different dependencies of the electric densities in the $(1,0)\oplus(0,1)$ and $(1/2,1/2)$ representations on q_i^2 is of pure kinematic origin, and can be thought of as a difference in the orientations of the four-vector and bi-vector quadrupoles in the Breit frame. It definitively appears in consequence of the different forms taken by the boost in the respective four-vector, and bi-vector representations. Setting $g_V = g_{VA} = 2$, and for the maximal polarization, our formalism predicts a negative quadrupole moment of $Q_E^2 = -e/m^2$ for the four-vector (same as the W in the Standard Model) and a positive one, $Q_E^2 = +e/m^2$, for the bi-vector. It is interesting to notice that a positive electric quadrupole moment, $Q_E^2 = 0.33e/m^2$, has been extracted from studying the ρ meson in the light-front quark model [24,25]. This result and our findings are indicative of the possibility that the ρ meson could transform in the bi-vector representation. An interesting dynamical aspect of the map between the antisymmetric Lorentz tensor in Eq. (4.17) and the Lorentz bi-vector in Eq. (2.36) is revealed upon associating the $\mathcal{F}^{\mu\nu}$ tensor with the ρ meson and its dual (opposite parity) with the a_1 meson. The bi-vector space is interesting as a carrier space for the ρ meson in so far as it easily allows for a tensor coupling, and is at variance to the electroweak gauge bosons, whose interactions with the matter fields are restricted by the principle of minimal couplings. The interactions of the nongauge ρ and a_1 mesons with baryons can be of any type compatible with Lorentz covariance and the symmetries of the light sector of QCD. The ρ and a_1 mesons are well-known to dominate the respective vector and axial vector hadron currents; they have tensor couplings and are essential, among others, in the design of chiral Lagrangians. Within this context, the tensor electromagnetic currents of these mesons acquire importance. Such hadronic versions of $\mathcal{F}^{\mu\nu}$ and its dual hint on the importance of the ρ and a_1 meson clouds surrounding the nucleon (N) for the description of multipole moments in the transition of the nucleon to spin-3/2 resonances (N^*) such as the $\Delta(1232)$ and $D(1520)$ resonances, where the $NN^*\gamma$ vertex (to be considered in a forthcoming work) is designed by a contraction of the electromagnetic field-strength tensor with a third-rank Lorentz tensor sandwiched between the nucleon and the N^* resonance state [26].

D. Spin-3/2 multipole moments

The present section is devoted to the multipole decompositions of the currents in Eqs. (2.65) and (2.66). Again, if one wishes to calculate the spin-3/2 multipole moment, knowledge on the four-vector–spinors is required. We use the spin-3/2 basis previously employed in Ref. [5]:

$$u^\alpha(\mathbf{p},3/2) = \eta^\alpha(\mathbf{p},1)u(\mathbf{p},1/2), \tag{4.23a}$$
$$u^\alpha(\mathbf{p},1/2) = \frac{1}{\sqrt{3}}\eta^\alpha(\mathbf{p},1)u(\mathbf{p},-1/2) + \sqrt{\frac{2}{3}}\eta^\alpha(\mathbf{p},0)u(\mathbf{p},1/2), \tag{4.23b}$$
$$u^\alpha(\mathbf{p},-1/2) = \frac{1}{\sqrt{3}}\eta^\alpha(\mathbf{p},-1)u(\mathbf{p},1/2) + \sqrt{\frac{2}{3}}\eta^\alpha(\mathbf{p},0)u(\mathbf{p},-1/2), \tag{4.23c}$$
$$u^\alpha(\mathbf{p},-3/2) = \eta^\alpha(\mathbf{p},-1)u(\mathbf{p},-1/2), \tag{4.23d}$$

where $\bar{u}^\alpha(\mathbf{p},\lambda)\cdot u_\alpha(\mathbf{p},\lambda) = -1$ with $u(\mathbf{p},\lambda)$ and $\eta^\alpha(\mathbf{p},\lambda)$ being defined in Eqs. (4.1) and (4.5) respectively.

1. Rarita-Schwinger multipoles

The calculation of the spin-3/2 multipole moments within the Rarita-Schwinger framework in Eq. (2.66) amounts to the following densities:

$$\varrho_E^{\mathrm{RS}}(\mathbf{q},3/2,\pm3/2) = \frac{e(4m^2 - q_z^2 + \mathbf{q}^2)(8m^2 - (g_S-2)\mathbf{q}^2)}{16m^3\omega}, \tag{4.24}$$

$$\varrho_M^{\mathrm{RS}}(\mathbf{q},3/2,\pm3/2) = \mp\frac{ieg_Sq_z(2m^2 + q_z^2 - \omega^2)}{2m^2\omega}. \tag{4.25}$$

For $\lambda = \pm1/2$ one has

$$\varrho_E^{\mathrm{RS}}(\mathbf{q},3/2,\pm1/2) = \frac{e(12m^2 + 3q_z^2 + \mathbf{q}^2)(8m^2 - (g_S-2)\mathbf{q}^2)}{48m^3\omega}, \tag{4.26}$$

$$\varrho_M^{\mathrm{RS}}(\mathbf{q},3/2,\pm1/2) = \pm\frac{ieg_Sq_z(22m^2 + 9q_z^2 - 5\omega^2)}{6m^2\omega}. \tag{4.27}$$

The general expressions for any polarization in the notations of Eq. (4.4) and for the maximal polarization are then

$$(Q_E^0)^{\mathrm{RS}} = e, \tag{4.28a}$$
$$(Q_M^1)^{\mathrm{RS}} = \frac{eg_S}{2m}\frac{1}{3}\langle S_z\rangle, \tag{4.28b}$$
$$(Q_E^2)^{\mathrm{RS}} = \frac{e}{m^2}\frac{1}{3}\langle\mathcal{A}\rangle, \tag{4.28c}$$
$$(Q_M^3)^{\mathrm{RS}} = \frac{eg_S}{2m^3}\langle\mathcal{B}\rangle. \tag{4.28d}$$

The operators $\mathcal{A}$ and $\mathcal{B}$ in the four-vector–spinor representation read

$$\mathcal{A} = 3S_z^2 - \mathbf{S}^2, \qquad \mathcal{B} = S_z\left(15S_z^2 - \frac{41}{5}\mathbf{S}^2\right). \quad (4.29a)$$

One sees that it is exclusively the Dirac spin-tensor which determines the dipole and octupole magnetic moments without contributing to the electric quadrupole moment. The spin-tensor of the underlying spin-1 sector does not show up anywhere.

2. Non-Lagrangian currents and multipoles for the four-vector spinor

In order to gain a deeper insight into the problematics of Eq. (4.28), it is quite instructive to compare Eq. (2.66) to the most general form of a covariant current density, satisfying time-reversal, parity, and gauge invariance for the $(1/2, 1/2) \otimes [(1/2, 0) \oplus (0, 1/2)]$ representation, and given by Ref. [11],

$$j_\mu^{a_1,a_2,b_1,c_1,c_2}(\mathbf{p}', \lambda'; \mathbf{p}, \lambda) = -e\bar{u}^\alpha(\mathbf{p}', \lambda')V_{\alpha\beta\mu}(p', p; a_1, a_2, b_1, c_1, c_2)u^\beta(\mathbf{p}, \lambda). \quad (4.30)$$

Here,

$$V_{\alpha\beta\mu}(p', p; a_1, a_2, b_1, c_1, c_2) = a_1 2m\gamma_\mu g_{\alpha\beta} + a_2(p' + p)_\mu g_{\alpha\beta} + b_1((p' - p)_\alpha g_{\mu\beta} - (p' - p)_\beta g_{\mu\alpha}) + \frac{1}{(2m)^2} \times (p' - p)_\alpha (p' - p)_\beta (2mc_1\gamma_\mu + c_2(p' + p)_\mu). \quad (4.31)$$

The multipole moments resulting from the latter non-Lagrangian current density are calculated as

$$Q_E^0 = e\,(a_1 + a_2), \quad (4.32a)$$

$$Q_M^1 = \frac{e}{2m}\frac{2}{3}(a_1 + b_1)\langle S_z\rangle, \quad (4.32b)$$

$$Q_E^2 = \frac{e}{m^2}\frac{1}{6}(2a_1 + 2a_2 - 2b_1 - c_1 - c_2)\langle\mathcal{A}\rangle, \quad (4.32c)$$

$$Q_M^3 = \frac{e}{2m^3}(2a_1 - c_1)\langle\mathcal{B}\rangle. \quad (4.32d)$$

The Rarita-Schwinger Lagrangian which is linear in the momentum corresponds to setting all parameters equal to zero except a_1. This inevitably restricts the parameters in $j_\mu^{a_1,a_2,b_1,c_1,c_2}(\mathbf{p}', \lambda'; \mathbf{p}, \lambda)$ to

$$a_1 = \frac{g_S}{2}, \qquad a_2 = -\frac{g_S}{2} + 1, \qquad b_1 = c_1 = c_2 = 0, \qquad g_S = 2. \quad (4.33)$$

Instead, the NKR current is recovered for the following parameter set:

$$a_1 = \frac{g}{2}, \qquad a_2 = -\frac{g}{2} + 1, \qquad b_1 = g, \qquad c_1 = c_2 = 0, \quad (4.34)$$

to be inserted in Eqs. (4.32). In effect, the essential difference between the NKR and the RS spin-3/2 multipole moments in the respective Eqs. (4.34) and (4.33), is brought about by the nonvanishing value of the b_1 parameter. The b_1 dependent term in Eq. (4.30) takes its origin from the vector part of the generators of the $(1/2, 1/2) \otimes [(1/2, 0) \oplus (0, 1/2)]$ representation and is completely missing from the Rarita-Schwinger formalism. A $b_1 = 0$ value is responsible for the absence of a contribution to the magnetic dipole moment by the vector part of ψ_μ and for the $g \neq 2$ value in the Rarita-Schwinger framework. This shortcoming has been corrected in Ref. [27], where a non-Lagrangian current has been placed on the light cone (LC). The LC moments reported in Ref. [27] would correspond to the following $j_\mu^{a_1,a_2,b_1,c_1,c_2}(\mathbf{p}', \lambda'; \mathbf{p}, \lambda)$ parametrization:

$$a_1 = 3, \qquad a_2 = -2, \qquad b_1 = 0, \qquad c_1 = 8, \qquad c_2 = 0, \quad (4.35)$$

and are listed in Table I. We see that $c_1 \neq 0$, in consequence of which the current becomes second order in the momenta.

Our point here is that the multipole moments of this quadratic in the momenta current can be equally well obtained also from a current linear in the momentum. It is straightforward to verify that such a current would correspond to the following parametrization:

$$a_1 = -1, \qquad a_2 = 2, \qquad b_1 = 4, \qquad c_1 = c_2 = 0. \quad (4.36)$$

It translates into the following three-term linear current:

$$j_\mu(\mathbf{p}', \lambda'; \mathbf{p}, \lambda)|_{\mathrm{LC}} = -e\bar{u}^\alpha(\mathbf{p}', \lambda')[g_{\alpha\beta}(p' + p)_\mu + i(-2M^S_{\mu\nu}g_{\alpha\beta} + 4[M^V_{\mu\nu}]_{\alpha\beta}) \times (p' - p)^\nu]u^\beta(\mathbf{p}, \lambda), \quad (4.37)$$

which mimics the second-order Light Cone current constructed by Lorcé in Ref. [27]. Subsequently, $j_\mu(\mathbf{p}', \lambda'; \mathbf{p}, \lambda)|_{\mathrm{LC}}$ will be termed as Lorcé-like linear current. The above considerations show that the non-Lagrangian spin-3/2 currents of second order are not really inevitable for the explanation of the multipole moments. The current in Eq. (4.37) excludes equally well the coupling of the spin-1/2 sector from ψ_μ to the electromagnetic field, though it ceases to be a genuine Poincaré covariant projector current due to the inadequate combination between the spin-magnetization tensors of the spinor- and four-vector sectors of ψ_μ. We here tested the Lorcé-like current in Eq. (4.37) in the calculation of Compton scattering off a spin-3/2 target. The essentials of the calculation are given in the Appendix. We observe that the tree-level forward Compton scattering cross corresponding to the current in Eq. (4.37) grows infinitely with the energy according to

$$\left[\frac{d\sigma}{d\Omega}\right]_{\theta=0} = \frac{1}{81}(4712\eta^4 - 16\eta^2 + 81)r_0^2. \quad (4.38)$$

Here, $r_0 = e^2/(4\pi m) = \alpha/m$ and $\eta = \omega/m$, where ω and m stand in turn for the energy of the incident photon and the mass of the target. This contrasts the high-energy forward scattering cross-section concluded from the genuinely NKR

TABLE I. Summary of the multipole moments. The abbreviations SM, RS, LC, and NKR stand in their turn for Standard Model, Rarita-Schwinger, Light Cone [27], and the second-order formalism of Ref. [3]. For other notation, see the main body of the text. Notice that both the LC and NKR predictions on the spin-1 multipoles are in accord with the Standard Model.

Formalism	Representation	Spin	g-factor	Q_E^0	Q_M^1	Q_E^2	Q_M^3
Dirac	$(\frac{1}{2},0)\oplus(0,\frac{1}{2})$	$\frac{1}{2}$	$g_D=2$	e	$\frac{eg_D}{2m}\langle S_z\rangle$	0	0
NKR	$(\frac{1}{2},0)\oplus(0,\frac{1}{2})$	$\frac{1}{2}$	$g_S=2$	e	$\frac{eg_S}{2m}\langle S_z\rangle$	0	0
Proca	$(\frac{1}{2},\frac{1}{2})$	1	$g_P=1$	e	$\frac{eg_P}{2m}\langle S_z\rangle$	0	0
SM	$(\frac{1}{2},\frac{1}{2})$	1	$g_W=2$	e	$\frac{eg_W}{2m}\langle S_z\rangle$	$-\frac{e(g_W-1)}{m^2}\langle\mathcal{A}\rangle$	0
LC	$(\frac{1}{2},\frac{1}{2})$	1	$g_{LC}=2$	e	$\frac{eg_{LC}}{2m}\langle S_z\rangle$	$-\frac{e(g_{LC}-1)}{m^2}\langle\mathcal{A}\rangle$	0
NKR	$(\frac{1}{2},\frac{1}{2})$	1	$g_V=2$	e	$\frac{eg_V}{2m}\langle S_z\rangle$	$-\frac{e(g_V-1)}{m^2}\langle\mathcal{A}\rangle$	0
NKR	$(1,0)\oplus(0,1)$	1	g_{VA}	e	$\frac{eg_{VA}}{2m}\langle S_z\rangle$	$\frac{e(g_{VA}-1)}{m^2}\langle\mathcal{A}\rangle$	0
RS	$(\frac{1}{2},\frac{1}{2})\otimes(\frac{1}{2},0)\oplus(0,\frac{1}{2})$	$\frac{3}{2}$	$\frac{g_D}{3}=\frac{2}{3}$	e	$\frac{g_D}{3}\frac{e}{2m}\langle S_z\rangle$	$\frac{e}{m^2}\frac{1}{3}\langle\mathcal{A}\rangle$	$\frac{eg_D}{2m^3}\frac{1}{3}\langle\mathcal{B}\rangle$
LC	$(\frac{1}{2},\frac{1}{2})\otimes(\frac{1}{2},0)\oplus(0,\frac{1}{2})$	$\frac{3}{2}$	$g_{LC}=2$	e	$\frac{eg_{LC}}{2m}\langle S_z\rangle$	$-\frac{e}{m^2}\langle\mathcal{A}\rangle$	$-\frac{e}{2m^3}\langle 2\mathcal{B}\rangle$
NKR	$(\frac{1}{2},\frac{1}{2})\otimes(\frac{1}{2},0)\oplus(0,\frac{1}{2})$	$\frac{3}{2}$	$g_{3/2}=2$	e	$\frac{eg_{3/2}}{2m}\langle S_z\rangle$	$-\frac{e(g_{3/2}-1)}{m^2}\frac{1}{3}\langle\mathcal{A}\rangle$	$\frac{eg_{3/2}}{2m^3}\langle\mathcal{B}\rangle$
NKR	$(\frac{3}{2},0)\oplus(0,\frac{3}{2})$	$\frac{3}{2}$	g	e	$\frac{eg}{2m}\langle S_z\rangle$	$\frac{e(g-1)}{m^2}\langle\mathcal{A}\rangle$	$-\frac{eg}{2m^3}3\langle\mathcal{B}\rangle$

spin-3/2 current in Eq. (2.65), in which case one finds finite values according to Ref. [5],

$$\left[\frac{d\sigma}{d\Omega}\right]_{\theta=0}=\frac{1}{24}\Big((g-2)^4\eta^2+24\Big)r_0^2. \tag{4.39}$$

We conclude that nonlinear non-Lagrangian spin-3/2 currents are not necessarily more general and more advantageous than the linear spin-3/2 Lagrangian current emerging within the covariant-projector formalism. Furthermore, a Lagrangian method is always more advantageous for field-theoretical considerations. The above observation does not exclude by any means the possibility that Compton scattering within Lorcé 's Light Cone framework [27], and under employment of the genuine second-order Lorcé current, may throughout create a result consistent with unitarity.

3. Covariant-projector multipoles

The transverse spin-3/2 NKR densities of the current in Eq. (2.65) for the maximal polarizations are calculated as

$$\varrho_E(\mathbf{q},3/2,\pm 3/2)=\frac{e\omega(\omega^2-q_z^2)}{8m^3}+\frac{g_{(3/2)}e(-3\omega^4+3(4m^2+q_z^2)\omega^2-4m^2q_z^2)}{16m^3\omega}, \tag{4.40}$$

$$\varrho_M(\mathbf{q},3/2,\pm 3/2)=\mp\frac{ig_{(3/2)}q_ze(2m^2+q_z^2-2\omega^2)}{2m^2\omega}. \tag{4.41}$$

For $\lambda=\pm 1/2$ one finds,

$$\varrho_E(\mathbf{q},3/2,\pm 1/2)=\frac{e\omega(8m^2+3q_z^2+\omega^2)}{24m^3}+\frac{g_{(3/2)}e(32m^4+4(3q_z^2+\omega^2)m^2-3(\omega^4+3q_z^2\omega^2))}{48m^3\omega}, \tag{4.42}$$

$$\varrho_M(\mathbf{q},3/2,\pm 1/2)=\pm\frac{ig_{(3/2)}q_ze(22m^2+9q_z^2-4\omega^2)}{6m^2\omega}. \tag{4.43}$$

The electromagnetic moments for the above densities are then found as

$$(Q_E^0)^{\rm NKR}=e, \tag{4.44a}$$

$$(Q_M^1)^{\rm NKR}=\frac{eg_{(3/2)}}{2m}\langle S_z\rangle, \tag{4.44b}$$

$$(Q_E^2)^{\rm NKR}=\frac{e(1-g_{(3/2)})}{m^2}\frac{1}{3}\langle\mathcal{A}\rangle, \tag{4.44c}$$

$$(Q_M^3)^{\rm NKR}=\frac{eg_{(3/2)}}{2m^3}\langle\mathcal{B}\rangle, \tag{4.44d}$$

again with the use of Eq. (4.4) for $s=3/2$ and $\lambda=\pm 1/2$, $\pm 3/2$.

E. Spin-3/2 multipole moments in the $(3/2,0)\oplus(0,3/2)$ representation

The current for representations of this type is given in Eq. (2.86) and for the $j=3/2$ value of interest reads

$$j_\mu^{(3/2)}(\mathbf{p}',\lambda';\mathbf{p},\lambda)=e\bar{w}(\mathbf{p}',\lambda')[(p'+p)_\mu+igM_{\mu\nu}^{(3/2)}(p'-p)^\nu]w(\mathbf{p},\lambda), \tag{4.45}$$

where we have simply denoted by g the corresponding gyromagnetic factor. The rotational generators in this single spin-3/2 representation satisfy $(\mathbf{J}\cdot\mathbf{n})^4=\frac{5}{2}\times(\mathbf{J}\cdot\mathbf{n})^2-(\frac{3}{4})^2$, a relationship that we employ in the calculation of the boost elements,

$$\exp(\pm\mathbf{J}\cdot\mathbf{n})\varphi = \cosh\frac{\varphi}{2}\left(1-\frac{1}{2}\sinh^2\frac{\varphi}{2}\right) \pm (\mathbf{J}\cdot\mathbf{n})\sinh\frac{\varphi}{2} \times\left(2-\frac{1}{3}\sinh^2\frac{\varphi}{2}\right) + 2(\mathbf{J}\cdot\mathbf{n})^2\cosh\frac{\varphi}{2} \times\sinh^2\frac{\varphi}{2} \pm \frac{4}{3}(\mathbf{J}\cdot\mathbf{n})^3\sinh^3\frac{\varphi}{2}. \tag{4.46}$$

This relation can be used to explicitly calculate the boost generator in Eq. (2.89). The states of well-defined parity in any frame are constructed by boosting the corresponding rest frame states. We aim to compare the spin-$3/2^-$ multipole moments of the four-vector–spinor and focus on the four negative parity states,

$$v(\mathbf{p},+3/2) = \frac{1}{4(m(m+p_0))^{3/2}}\begin{pmatrix} (m+p_0+p_z)^3 \\ \sqrt{3}p_+(m+p_0+p_z)^2 \\ \sqrt{3}p_+^2(m+p_0+p_z) \\ p_+^3 \\ -(m+p_0-p_z)^3 \\ \sqrt{3}p_+(m+p_0-p_z)^2 \\ -\sqrt{3}p_+^2(m+p_0-p_z) \\ p_+^3 \end{pmatrix}, \tag{4.47a}$$

$$v(\mathbf{p},+1/2) = \frac{1}{4(m(m+p_0))^{3/2}}\begin{pmatrix} \sqrt{3}p_-(m+p_0+p_z)^2 \\ -(m+p_0+p_z)(3p_z^2+(m-3p_0)(m+p_0)) \\ p_+((m+p_0)(m+3p_0)-3p_z^2) \\ \sqrt{3}p_+^2(m+p_0-p_z) \\ \sqrt{3}p_-(m+p_0-p_z)^2 \\ (m+p_0-p_z)(3p_z^2+(m-3p_0)(m+p_0)) \\ p_+((m+p_0)(m+3p_0)-3p_z^2) \\ -\sqrt{3}p_+^2(m+p_0+p_z) \end{pmatrix}, \tag{4.47b}$$

$$v(\mathbf{p},-1/2) = \frac{1}{4(m(m+p_0))^{3/2}}\begin{pmatrix} \sqrt{3}p_-^2(m+p_0+p_z) \\ p_-((m+p_0)(m+3p_0)-3p_z^2) \\ -(m+p_0-p_z)(3p_z^2+(m-3p_0)(m+p_0)) \\ \sqrt{3}p_+(m+p_0-p_z)^2 \\ -\sqrt{3}p_-^2(m+p_0-p_z) \\ p_-((m+p_0)(m+3p_0)-3p_z^2) \\ (m+p_0+p_z)(3p_z^2+(m-3p_0)(m+p_0)) \\ \sqrt{3}p_+(m+p_0+p_z)^2 \end{pmatrix}, \tag{4.47c}$$

$$v(\mathbf{p},-3/2) = \frac{1}{4(m(m+p_0))^{3/2}}\begin{pmatrix} p_-^3 \\ \sqrt{3}p_-^2(m+p_0-p_z) \\ \sqrt{3}p_-(m+p_0-p_z)^2 \\ (m+p_0-p_z)^3 \\ p_-^3 \\ -\sqrt{3}p_-^2(m+p_0+p_z) \\ \sqrt{3}p_-(m+p_0+p_z)^2 \\ -(m+p_0+p_z)^3 \end{pmatrix}, \tag{4.47d}$$

where $p_\pm = p_x \pm ip_y$. The transverse electric densities for the maximal polarizations obtained with these states are:

$$\varrho_E\left(\frac{3}{2},\pm\frac{3}{2}\right) = -\frac{e\omega(12m^2+3q_x^2+3q_y^2-4\omega^2)}{8m^3} + \frac{3e(-16m^4-4(q_x^2+q_y^2-5\omega^2)m^2-4\omega^4+3(q_x^2+q_y^2)\omega^2)}{16m^3\omega}, \tag{4.48}$$

$$\varrho_E\left(\frac{3}{2}, \pm\frac{1}{2}\right) = \frac{e(4m^2 + 3(q_x^2 + q_y^2))\omega}{8m^3} + \frac{e(16m^4 + 4(3q_x^2 + 3q_y^2 - \omega^2)m^2 - 9(q_x^2 + q_y^2)\omega^2)}{16m^3\omega}, \tag{4.49}$$

where $\omega^2 = \mathbf{q}^2 - 4m^2$. The magnetic densities are calculated as

$$\varrho_M\left(\pm\frac{3}{2}, \pm\frac{3}{2}\right) = \pm\frac{3iegq_z(2m^2 + q_z^2)}{2m^2\omega} \tag{4.50}$$

$$\varrho_M\left(\pm\frac{3}{2}, \pm\frac{1}{2}\right) = \mp\frac{iegq_z(-2m^2 - 6q_x^2 - 6q_y^2 + 3q_z^2)}{2m^2\omega}. \tag{4.51}$$

From these densities, we find the following multipole moments for a particle transforming in the $(3/2, 0) \oplus (0, 3/2)$ representation

$$Q_E^0 = e, \tag{4.52a}$$

$$Q_M^1 = \frac{e}{2m}\langle S_z\rangle, \tag{4.52b}$$

$$Q_E^2 = -\frac{e(1-g)}{m^2}\langle\mathcal{A}\rangle, \tag{4.52c}$$

$$Q_M^3 = -\frac{e}{2m^3}3\langle\mathcal{B}\rangle, \tag{4.52d}$$

with the operators $\mathcal{A}$ and $\mathcal{B}$ defined in Eq. (4.29). We observe that according to the covariant-projector formalism, all multipoles of a state transforming in the $(3/2, 0) \oplus (0, 3/2)$ are dictated by the value of a single parameter, the gyromagnetic factor g. Furthermore, comparing with the expressions in Eq. (4.44) for the four-vector–spinor, we see that only the charge and magnetic moment coincide. Higher multipole moments turn out to be representation specific. This is so because magnetic dipole moments, in describing the rest-frame coupling to the magnetic field, are exclusively determined by the generators of rotations in the representation of interest, which are necessarily the same for equal spins. This is no longer valid for higher multipoles, which are sensitive to the dependence of the boost operator on the momentum, a dependence which varies with the representation.

V. SUMMARY AND DISCUSSION

In Table I, all the results obtained so far have been summarized.

To recapitulate, we wish to emphasize the following two main points reflected by the above results. The first one concerns the incompleteness of the Proca and Rarita-Schwinger formalisms, which becomes detectable in the former case through a vanishing electric quadrupole moment, a result which is a consequence of the inbuilt deficient $g_P = 1$ value instead of the universal $g = 2$ value. In the latter case, the deficient $g_{\mathrm{RS}} = \frac{2}{3}$ reflects the omission of the coupling to the magnetic field of the spin-magnetization current of the vector sector of the four-vector–spinor, an issue explained in Subsec. D of Sec. II above. This very omission is furthermore at the root of the violation of unitarity in Compton scattering in the forward direction [5]. The second point concerns the representation dependence of the multipole moments higher than the charge monopole and the magnetic dipole of particles of equal spins transforming in different Lorentz representations.

This result might seem surprising, but it is quite natural indeed once we recall that

(i) the magnetic dipole moment, in describing the rest frame coupling to the magnetic field, exclusively invokes the generators of rotations. In the multipole expansion, this moment is dictated by the linear terms in the momentum expansion of the magnetic current density. The linear terms are already contained in the operators of the electromagnetic currents; hence this moment is insensitive to the momentum dependence of the states which compose the electromagnetic current.

(ii) the momentum dependence of the boost operator varies with the representation. This is reflected in a different momentum dependence of the corresponding magnetic-, and electric-current densities. Higher multipoles are dictated by the quadratic and higher terms in the momentum expansion of these current densities and are sensitive to the momentum dependence of the states, which is different for different representations.

VI. CONCLUSIONS

In the present investigation we studied the electromagnetic multipole moments of spin-1/2, spin-1, and spin-3/2 particles within the Poincaré covariant second-order formalism of Ref. [3], and we compared our results with those of the Dirac, Proca, Standard Model, and Rarita-Schwinger theories. Introducing the electromagnetic interactions via the gauge principle, we were able to show that in the scheme under discussion, we reproduce for spin-1/2 the couplings and multipole moments known from the Dirac theory. Concerning spin-1, we studied two distinct Lorentz representations: the four-vector and the bi-vector. As for the former, we explicitly showed that Proca's theory is incomplete in the sense that it yields vanishing electric quadrupole as a result of the insufficient prediction for the gyromagnetic factor as $g_P = 1$. Then, predictions of the Poincaré covariant formalism on the electromagnetic multipole moments of the four-vector representation coincided with the tree-level predictions of the Standard Model. This achievement is not exclusive to the Poincaré covariant-projector formalism. More recently, the Standard Model properties of the W boson have also been successfully reproduced by Lorcé [27] within a Light Cone framework. We furthermore found that the electric-charge and magnetic dipole moments of the bi-vector representation coincide with those of the four-vector, while the electric quadrupole moment came out of equal absolute value but of opposite sign.

Regarding spin-3/2, we showed that the linear Rarita-Schwinger framework can be obtained from the second-order Poincaré projector only on shell. This linearization is the main culprit for the decoupling of the four-vector building block from the electromagnetic current, which happens to capture only contributions from the underlying spinor sector. The principal advantage of the second-order Poincaré covariant-projector formalism over the linear Rarita-Schwinger framework consists of the incorporation on equal footing of the vectorial and spinorial building blocks of ψ_μ into the expression for the current. In this fashion, the spin-3/2 multipole moments are no longer determined by an incomplete $g = \frac{2}{3}$ but appear interrelated by a full-flashed spin-3/2 gyromagnetic factor that takes the universal value of $g_{(3/2)} = 2$ and is in accord with unitarity of forward Compton scattering in the ultraviolet.

We compared in the same representation our results with those following from a particular non-Lagrangian current, allowed to be second-order in the momenta, and constructed its counterpart within the covariant-projector formalism, i.e., we found its linear current equivalent of identical multipole moments. We concluded that, at least as it concerns the electromagnetic multipoles, the non-Lagrangian spin-3/2 currents of higher order in the momenta are not necessarily more general and more advantageous than the two-term spin-3/2 current emerging within the covariant-projector method.

We also worked out the predictions of the Poincaré projector formalism for the multipoles of a particle transforming in the single-spin $(3/2, 0) \oplus (0, 3/2)$ representation, finding similar results as in the spin-1 case, i.e., charge and magnetic moments coincide with those of the spinor-vector representation, but the higher multipole moments (electric quadrupole and magnetic octupole) differ, this time both in sign and in magnitude.

We furthermore observed that if the electric quadrupole moments of the ρ meson and the W boson were to be of opposite signs, as suggested by light-front quark-model calculations [24,25], then the relativistic wave function of a fundamental ρ meson polarized in ζ direction has necessarily to transform as the vector field of a totally antisymmetric tensor, $\mathcal{F}^{\mu\nu}_{(\zeta)}$. The axial-vector companion to the ρ meson in $\mathcal{F}^{\mu\nu}_{(\zeta)}$ is described by the dual tensor and carries the quantum numbers of the a_1 meson. This observation is compatible with the ρ- and a_1-vector meson dominance hypothesis of the respective strong vector and axial-vector currents.

Finally, we like to emphasize that the spin-3/2 description within the Poincaré covariant-projector method is based on minimal gauging of a second-order Lagrangian, thus avoiding the commonly used *ad hoc* extension of the linear Rarita-Schwinger Lagrangian by terms describing nonminimal couplings [28]. The distinction between minimal and nonminimal couplings is best visualized within the geometric interpretation of gauge theories, where the former stand for parallel transport while the latter would invoke rare torsion couplings to the electromagnetic field.

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APPENDIX: FORWARD COMPTON SCATTERING FOR NON-LAGRANGIAN LIGHT CONE ELECTROMAGNETIC MOMENTS

Here we highlight the calculation of Compton scattering off a spin-3/2 target using the linear Lorcé-like current in Eq. (4.37), constructed within the NKR framework as a current having the same multipole moments as the genuine second-order Lorcé current in Ref. [27]. The linear Lorcé-like and NKR currents are characterized by equal charge and magnetic dipole moments and are distinct through the higher multipoles. The aim is to figure out whether the above difference will show up in the evaluation of the process under investigation.

The calculation is executed along the line of Ref. [5]; however, we are using a differently parametrized $\Gamma_{\alpha\beta\mu\nu}$ tensor in Eq. (2.53). Namely, we fixed those parameters in such a way that the electromagnetic multipole moments associated with the Lorcé-like current, Eq. (4.37), are reproduced. This is achieved by the following replacements:

$$\tfrac{1}{2}(2f + g_S) \rightarrow -1, \qquad -\tfrac{1}{2}(2f + g_S - 2) \rightarrow +2,$$
$$-\tfrac{1}{3}(3f + 3g_V + 1) \rightarrow -4, \tag{A1}$$

in combination with Eq. (2.60), with the purpose of eliminating spin-$3/2 \leftrightarrow$ spin-$1/2$ transitions.

From the corresponding second-order interaction Lagrangian, one can extract the Feynman rules needed for constructing the tree-level Compton scattering amplitudes and use as a propagator the inverse of the equation of motion:

$$\Pi_{\alpha\beta}(p) = (\Gamma_{\alpha\beta\mu\nu}p^\mu p^\nu - m^2 g_{\alpha\beta})^{-1}$$
$$= \frac{1}{p^2 - m^2 + i\epsilon}\left[P^{(3/2)}_{\alpha\beta}(p) - \frac{p^2 - m^2}{m^2}P^{(1/2)}_{\alpha\beta}(p)\right]. \tag{A2}$$

Here $P^{(1/2)}_{\alpha\beta}(p)$ and $P^{(3/2)}_{\alpha\beta}(p)$ are the respective spin-1/2 and spin-3/2 projectors,

$$P^{(1/2)}_{\alpha\beta}(p) = \frac{1}{3}\gamma_\alpha\gamma_\beta + \frac{1}{3p^2}(\not{p}\gamma_\alpha p_\beta + p_\alpha\gamma_\beta\not{p}), \tag{A3}$$

$$P^{(3/2)}_{\alpha\beta}(p) = g_{\alpha\beta} - \frac{1}{3}\gamma_\alpha\gamma_\beta - \frac{1}{3p^2}(\not{p}\gamma_\alpha p_\beta + p_\alpha\gamma_\beta\not{p}). \quad (A4)$$

The averaged squared amplitude is then found as

$$\overline{|\mathcal{M}|^2} = \frac{1}{8}\sum_{\text{pol}}|\mathcal{M}|^2 = \frac{e^4}{8}\,\text{Tr}[P^{\eta\alpha}(p')U_{\alpha\beta\mu\nu}P^{\beta\zeta}(p)\bar{U}^{\zeta\eta\mu\nu}], \quad (A5)$$

with

$$U_{\mu\nu} = V(p', Q)_{\alpha\gamma\mu}\Pi^{\gamma\delta}(Q)V(Q,p)_{\delta\beta\nu} + V(p',R)_{\alpha\gamma\nu}\Pi^{\gamma\delta}(R)V(R,p)_{\delta\beta\mu} + V_{\alpha\beta\mu\nu}, \quad (A6)$$

$$\bar{U}_{\mu\nu} = V(p,Q)_{\zeta\phi\nu}\Pi^{\phi\theta}(Q)V(Q,p')_{\theta\eta\mu} + V(p,R)_{\zeta\phi\mu}\Pi^{\phi\theta}(R)V(R,p')_{\theta\eta\nu} + V_{\zeta\eta\mu\nu}, \quad (A7)$$

where $Q = p + q = p' + q'$ and $R = p' - q = p - q'$ are in turn the momenta of the intermediate states in the s and u channels, $p(p')$ stands for the momentum of the incident (scattered) fermion, and $q(q')$ is the momentum of the incident (scattered) photon. The first- and second-order vertices are given by

$$V(p',p)_{\alpha\beta\mu} = \Gamma_{\alpha\beta\nu\mu}p'^{\nu} + \Gamma_{\alpha\beta\mu\nu}p^{\nu}, \quad (A8)$$

$$V_{\alpha\beta\mu\nu} = -(\Gamma_{\alpha\beta\mu\nu} + \Gamma_{\alpha\beta\nu\mu}), \quad (A9)$$

and satisfy the Ward-Takahashi identity:

$$(k'-k)^{\mu}V(k',k)_{\alpha\beta\mu} = \Pi^{-1}_{\alpha\beta}(k') - \Pi^{-1}_{\alpha\beta}(k). \quad (A10)$$

The latter relation ensures gauge invariance of the scattering amplitude and, as we can see, the right-hand side of Eq. (A10) is independent of the undetermined parameters, meaning the Ward-Takahashi identity holds for any parametrization in the second-order formalism.

The negative parity projector is found as

$$P_{\alpha\beta}(p) = \sum_{\lambda}u_\alpha(p,\lambda)\bar{u}_\beta(p,\lambda) = \frac{\not{p}-m}{2m}P^{(3/2)}_{\alpha\beta}(p). \quad (A11)$$

The corresponding differential cross-section of the process is then calculated as

$$\frac{d\sigma}{d\Omega} = \frac{1}{4}\frac{1}{(4\pi)^2}\frac{\overline{|\mathcal{M}|^2}}{m^2}\left(\frac{\omega'}{\omega}\right)^2, \quad (A12)$$

with $\omega(\omega')$ being the energy of the incident (scattered) photon. Performing all the calculations with the aid of the FEYNCALC package, we express the final result in terms of the energy variable $\eta = \omega/m$ and the scattering angle in the laboratory frame θ according to

$$\frac{d\sigma}{d\Omega} = \frac{r_0^2}{(1-(x-1)\eta)^7}\sum_{l=0}^{10}h_l\eta^l, \qquad x = \cos\theta. \quad (A13)$$

Here,

$$h_0 = +\tfrac{1}{2}(x^2+1), \quad (A14a)$$
$$h_1 = -\tfrac{5}{2}(x-1)(x^2+1), \quad (A14b)$$
$$h_2 = +\tfrac{1}{162}(x(5x(18x(9x-19)+581)-3852)+1815), \quad (A14c)$$
$$h_3 = -\tfrac{1}{81}(x-1)(x(5x(9x(9x-22)+676)-5274)+2415), \quad (A14d)$$
$$h_4 = +\tfrac{1}{162}(x(x(x(15x(9x(3x-16)+809)-35632)+51459)-33248)+16465), \quad (A14e)$$
$$h_5 = -\tfrac{1}{162}(x-1)(x(x(x(x(9x(9x-76)+7957)-29336)+51883)-35820)+34191), \quad (A14f)$$
$$h_6 = -\tfrac{1}{162}(x-1)(x(x(x(x(5x(18x-617)+17331)-46468)+57194)-61287)+39361), \quad (A14g)$$
$$h_7 = -\tfrac{1}{81}(x-1)^2(x(x(x(x(255x-2099)+6972)-10508)+15525)-13281), \quad (A14h)$$
$$h_8 = -\tfrac{1}{81}(x-1)^3(x(x(x(162x-701)+1685)-4215)+6205), \quad (A14i)$$
$$h_9 = -\tfrac{56}{81}(x-1)^4(9x-37), \quad (A14j)$$
$$h_{10} = -\tfrac{392}{81}(x-1)^5. \quad (A14k)$$

We notice that the linear Lorcé-like current constructed within the NKR formalism has the correct low-energy limit $(x^2+1)(r_0^2/2)$. The result for the forward direction is then deduced from the last equation as

$$\left[\frac{d\sigma}{d\Omega}\right]_{\theta=0} = \frac{1}{81}(4712\eta^4 - 16\eta^4 - 16^2 + 81)r_0^2. \quad (A15)$$

In Fig. 1 we compare the predictions of the Rarita-Schwinger, the NKR, and the Lorcé-like currents for the low-energy differential cross-section Eq. (A13). We observe that all three formulations amount to equally good and realistic predictions in the low-energy regime and posses the correct classical limit prescribed by the Thompson cross-section (dotted line in Fig. 1). This is

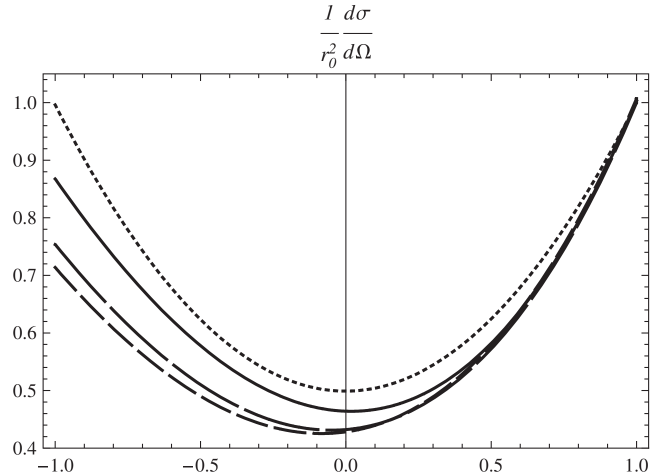

FIG. 1. Differential cross-section for Compton scattering off spin-3/2 particles at $\eta = 0.1$. The dotted line corresponds to the Thompson classical limit, the short-dashed line represents the Rarita-Schwinger result, and the long-dashed line indicates the prediction based on the Lorcé-like first order current. The continuous line refers to the prediction by the NKR method.

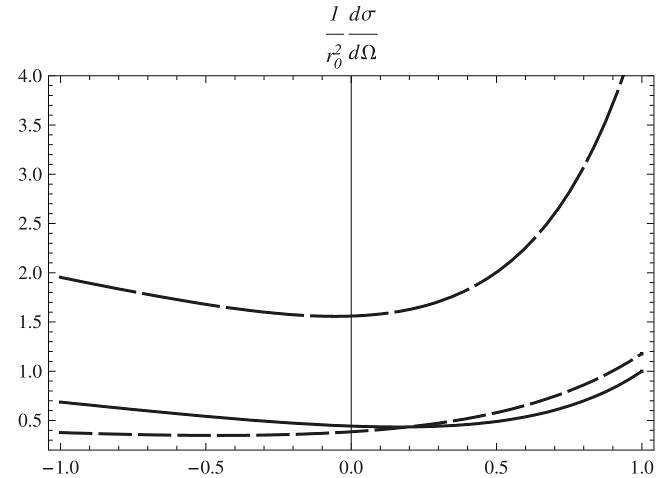

FIG. 2. Differential cross-section for Compton scattering off spin-3/2 particles as a function of $x = \cos\theta$ at $\eta = 0.5$, for the Rarita-Schwinger theory (short-dashed line), the Lorcé-like first order current (long-dashed line), and the NKR formalism (continuous line).

due to the circumstance that the Thomson limit is entirely governed by the lowest multipole, which is the electric monopole, and is insensitive to both the spin degrees of freedom and the boost. Beyond this limit, we observe that the current in Eq. (4.37) gives a differential cross-section which grows rapidly with the energy in the forward direction, as shown in Fig. 2 for $\eta = 0.5$. Only the line corresponding to the parametrization of the NKR current leads to a differential cross-section independent of the energy in the forward direction. On these grounds, we conclude that linear Lorentz invariant currents of equal electric-monopole and magnetic-dipole moments do not necessarily lead to equivalent forward Compton scattering cross-sections. A realistic description is provided only by the current consistent with the NKR spin-3/2 Lagrangian. This finding is restricted to the NKR method alone and does not rule out the possibility of obtaining a correct high-energy limit of the Compton scattering cross-section within a consistent Light Cone approach based on the genuine second-order Lorcé current.

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