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Highly excited negative parity baryons in the $1/N_c$ expansion

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The masses of experimentally known highly excited baryons of negative parity supposed to belong to the $[\mathbf{70}, \ell^-]$ multiplets ($\ell = 1, 2, 3$) of the $N = 3$ band are calculated in the $1/N_c$ expansion method to order $1/N_c$ by using a procedure which allows to considerably reduce the number of linearly independent operators entering the mass formula. The numerical fits to present data show that the coefficients encoding the QCD dynamics have large, comparable values, for the flavor and spin operators. It implies that these operators contribute dominantly to the flavor-spin SU(6) symmetry breaking, like for the $[\mathbf{70}, 1^-]$ multiplet of the $N = 1$ band.

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I. INTRODUCTION

The $1/N_c$ expansion method, where N_c is the number of colors, [1–4] is a powerful and systematic tool to study ground state baryons [5–8]. The method is based on the observation that, for N_f flavors, the ground state baryons display an exact contracted SU($2N_f$) symmetry when $N_c \to \infty$. At large, but finite N_c, this symmetry is broken by contributions of order of $1/N_c$, leading to mass splittings.

Subsequently, efforts have been made to extend this method to excited states. These states can be grouped into the so-called excitation bands $N = 1, 2, 3$, etc. following a harmonic oscillator notation. In this way, one can organize the states into SU(6) × O(3) multiplets. So far the resonances corresponding to the $N = 1$ band have drawn a particular attention, being well known experimentally. It turned out that the problem is more complicated technically than for the ground states, because these states belong to the SU(6) × O(3) $[\mathbf{70}, 1^-]$ multiplet, thus have mixed symmetric orbital and flavor-spin parts of the total wave function. In such a case the SU($2N_f$) symmetry is broken at order $\mathcal{O}(N_c^0)$. The standard analysis is based on the separation of the system into a ground state core + an excited quark, either for $N_f = 2$ [9–16] or for $N_f = 3$ [17]. A simpler method, avoiding this separation has been proposed in Ref. [18] for $N_f = 2$ and extended to $N_f = 3$ in Refs. [19,20].

The $N = 2$ band contains five SU(6) × O(3) multiplets from which four have a physical relevance. The $[\mathbf{56}', 0^+]$ and $[\mathbf{56}, 2^+]$, having symmetric orbital and spin-flavor states, have been analyzed in Refs. [21,22] respectively, in close analogy to ground states. The masses of the multiplet of mixed orbital symmetry $[\mathbf{70}, \ell^+]$, with $\ell = 0$ and 2, have been calculated by extending the ground state + excited quark method to an excited symmetric core + excited quark [23] for $N_f = 2$. The method has been extended to $N_f = 3$ in Ref. [24].

The $N = 3$ band contains eight SU(6) × O(3) multiplets. In the notation of Ref. [25] these are $[\mathbf{56}, 1^-]$, $[\mathbf{56}, 3^-]$, $[\mathbf{70}', 1^-]$, $[\mathbf{70}'', 1^-]$, $[\mathbf{70}, 2^-]$, $[\mathbf{70}, 3^-]$, $[\mathbf{20}, 1^-]$, and $[\mathbf{20}, 3^-]$, where $[\mathbf{70}', 1^-]$ and $[\mathbf{70}'', 1^-]$ correspond to radial excitations. This classification provides 45 nonstrange states (1 state $N9/2^-$, 1 state $\Delta 9/2^-$, 5 states $N7/2^-$, 2 states $\Delta 7/2^-$, 8 states $N5/2^-$, 4 states $\Delta 5/2^-$, 9 states $N3/2^-$, 5 states $\Delta 3/2^-$, 7 states $N1/2^-$, and 3 states $\Delta 1/2^-$). On the other hand in the 1900 –2400 MeV region only about ten nonstrange resonances have been observed so far. The interest in the $N = 3$ band has been largely hindered from a theoretical analysis in the $1/N_c$ expansion method, because of the scarcity of experimental data on the one hand, and because of its complex multiplet structure on the other hand. To our knowledge the only $1/N_c$ expansion study existing so far is that of Ref. [26] in conjunction with Regge trajectories. The analysis included a series of multiplets belonging to the $N = 1, 2, 3, 4, 5$, and 6 bands, in particular, the $[\mathbf{70}, 3^-]$ multiplet of $N = 3$. The mass operator was reduced to a few terms containing simplified operators, considered to capture the main features of the spectrum, but the only term of interest was the leading spin-flavor singlet term proportional to N_c.

The $N = 4$ band has 17 SU(6) × O(3) multiplets [27] from which only the $[\mathbf{56}, 4^+]$ has been analyzed in the $1/N_c$ expansion method, being the lowest one and also technically simple, due to its symmetric orbital and spin-flavor parts [28].

The above studies revealed a systematic dependence of the contribution of the dominant terms in the mass formula, with the excitation energy, or alternatively with the band number, as presented in Ref. [23]. It turns out that the coefficient c_1 of the leading spin-flavor singlet term,

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proportional to N_c, is raising linearly with N. It was also found that the coefficient of the spin-orbit operator having matrix elements of order $\mathcal{O}(N_c^0)$, decreases with N and tends to vanish at large excitation energy. The coefficient of the spin-spin term, with matrix elements of order $\mathcal{O}(N_c^{-1})$, which brings the largest contribution to the splitting, decreases with the excitation energy. Results for the $N = 3$ band were however absent in that analysis. Note that the energy dependence of the mass formula obtained in the $1/N_c$ expansion method is remarkably compatible with the energy dependence obtained within the framework of quark models with a chromomagnetic hyperfine interaction [29].

Here we present the first systematic attempt towards studying the $N = 3$ band in the $1/N_c$ expansion method. We include all mixed symmetric multiplets $[\mathbf{70}, \ell^-]$ ($\ell = 1$, 2, and 3) of the band, therefore more experimental data to analyze. An ultimate aim is to see whether or not the results are compatible with the systematic analysis of Ref. [23] and to clarify the role of the isospin operator, found so important in the $N = 1$ band.

At this stage it is useful to mention that both the symmetric core + excited quark procedure and our way of handling the problem of mixed symmetric states are algebraic methods in the spirit of the Gell-Mann-Okubo mass formula. There is no radial dependence in the picture. The symmetric core + excited quark was originally proposed [10] as an extension of the ground state treatment to excited states and was inspired by the Hartree picture. In this way, in the flavor-spin space, the problem was reduced to the knowledge of matrix elements of the SU($2N_f$) generators between symmetric states, already known from the ground state studies. Accordingly, the wave function was approximately given by the coupling of an excited quark to a ground state core of $N_c - 1$ quarks, without performing antisymmetrization. In our approach, all identical quarks are treated on the same footing and we have an exact wave function in the orbital-flavor-spin space. As a result, the knowledge of the matrix elements of the SU($2N_f$) generators between mixed symmetric states is required. We have calculated and provided all matrix elements for SU(4) in Ref. [18] and for SU(6) in Refs. [19,20], by considering an extension of the Wigner-Eckart theorem. While in the symmetric core + excited quark procedure the number of terms entering the mass formula is excessively large our mass formula has fewer terms and is physically more transparent. The calculated spectrum pointed out the important role of the isospin operator (indirectly of the flavor-spin operator, as a part of the SU($2N_f$) Casimir operator), systematically neglected in the symmetric core + an excited quark procedure. We consider that a basic aim of the $1/N_c$ expansion is to find the most dominant terms with a physical meaning.

Later on, the symmetric core + an excited quark approach was strongly supported by the authors of Ref. [30]. Starting from a general large N_c constituent quark model Hamiltonian and an exact wave function, they used transformation properties of states and interactions under the permutation group S_{N_c} and arrived at an expectation value of the Hamiltonian in terms of matrix elements of the approximate wave function of Ref. [13] and operators acting either on the symmetric core or on the excited quark. The difference with the algebraic methods is that the space degree of freedom enters the discussion. As a consequence extra terms appear in the mass formula. They were gathered together to match various quark models Hamiltonians. This matching implies constraints on the ratios of different coefficients (expressed in terms of radial integrals). The numerical application to the Isgur-Karl model was partially successful [31] by reproducing the spin-spin term but the tensor term could not be reproduced. More work and a deeper understanding is therefore desirable.

Before describing the $1/N_c$ expansion method let us recall some elements of the history of the $N = 3$ band within the framework of the constituent quark model. An important wave of interest has been triggered by the need of finding an assignment of the $D_{35}(1930)$ resonance announced by Cutkosky *et al.* [32]. The quark model calculations of Cutkosky and Hendrick [33] and later on of Capstick and Isgur [34], incorporating a linear confinement and relativistic effects, predicted a mass of about 200 MeV above the experimental value. An earlier analysis based on sum rules derived in a harmonic oscillator basis by Dalitz *et al.* [35] provided a mass of 2088 ± 25 MeV describing this resonance as a pure $[\mathbf{56}, 1^-]$, $J = 5/2$ state of the $N = 3$ band, following the suggestion of Cutkosky *et al.* [32]. A similar mass range has been obtained in Ref. [25] in a semirelativistic constituent quark model with a linear confinement and a chromomagnetic interaction. The spin-independent part of the model used in Ref. [25], which has a linear confinement, makes the $[\mathbf{56}, 1^-]$ multiplet the lowest one among those compatible with the quantum numbers of the $D_{35}(1930)$ resonance. However this resonance remains an open problem in quark models, inasmuch as its mass is about 200 MeV too high above the experimental value.

As already mentioned, here we are concerned with resonances which can be interpreted as members of the mixed symmetric multiplets of the $N = 3$ band. An important incentive to this work was that there is new experimental interest, for example, in the photo-production of η mesons off protons which suggest a new resonance $N(2070)D_{15}$ which can belong to the $N = 3$ band [36]. Moreover a recent multichannel partial wave analysis including high-lying resonances, in the so-called fourth resonance region [37,38] suggests the existence of a high-lying spin quartet

$$N(2150)_{3/2^-}, \quad N(2060)_{5/2^-}, \quad N(2190)_{7/2^-}, \quad N(2250)_{9/2^-}, \tag{1}$$

with $\ell = 3$. In the following we shall compare this suggestion with our predictions.

The paper is organized as follows. In the next section we introduce the mass operator defined within the $1/N_c$ expansion method. In Sec. III we present the results of four distinct fits of the dynamical coefficients in the mass formula and calculate the mass of the fitted resonances obtained from one of these numerical fits. We discuss our results in a general context by analogy to results obtained for the $N = 1$ band and compare our interpretation of resonances with that of Refs. [36–38]. Some conclusions are drawn in the last section.

II. THE MASS OPERATOR

When hyperons are included in the analysis, the SU(3) symmetry must be broken and the mass operator takes the following general form [39]:

$$M = \sum_i c_i O_i + \sum_i d_i B_i. \tag{2}$$

The formula contains two types of operators. The first type are the operators O_i, which are invariant under SU(N_f) and are defined as

$$O_i = \frac{1}{N_c^{n-1}} O_\ell^{(k)} \cdot O_{SF}^{(k)}, \tag{3}$$

where $O_\ell^{(k)}$ is a k-rank tensor in SO(3) and $O_{SF}^{(k)}$ a k-rank tensor in SU(2)-spin. Thus O_i are rotational invariant. For the ground state one has $k = 0$. The excited states also require $k = 1$ and $k = 2$ terms. The rank $k = 2$ tensor operator of SO(3) is

$$L^{(2)ij} = \frac{1}{2}\{L^i, L^j\} - \frac{1}{3}\delta_{i,-j}\vec{L}\cdot\vec{L}, \tag{4}$$

which we choose to act on the orbital wave function $|\ell m_\ell\rangle$ of the whole system of N_c quarks (see Ref. [23] for the normalization of $L^{(2)ij}$). The second type are the operators B_i which are SU(3) breaking and are defined to have zero expectation values for nonstrange baryons. Because of the scarcity of data on the hyperons here we consider only one hyperon and accordingly include only one of these operators, namely, $B_1 = -\mathcal{S}$, where $\mathcal{S}$ is the strangeness.

The values of the coefficients c_i and d_i which encode the QCD dynamics are determined from numerical fits to data. Table I gives the list of O_i and B_i operators together with their coefficients, which we believe to be the most relevant for the present study. The choice is based on our previous experience with the $N = 1$ band [20]. In this table the first nontrivial operator is the spin-orbit operator O_2. In the spirit of the Hartree picture [2], generally adopted for the description of baryons, we identify the spin-orbit operator with the single-particle operator

$$\ell \cdot s = \sum_{i=1}^{N_c} \ell(i) \cdot s(i), \tag{5}$$

the matrix elements of which are of order N_c^0. For simplicity we ignore the two-body part of the spin-orbit operator, denoted by $1/N_c(\ell \cdot S_c)$ in Ref. [13], as being of a lower order (there the lower-case operators $\ell(i)$ act on the excited quark and S_c is the core-spin operator). The analytic expression of the matrix elements of O_2 is given in the Appendix.

The spin operator O_3 and the flavor operator O_4 are two-body and linearly independent. The expectation values of O_3 are simply equal to $\frac{1}{N_c}S(S+1)$ where S is the spin of the whole system. They are given in Table II.

Note that the definition of the operator O_4, indicated in Table I, is such as to recover the matrix elements of the usual $1/N_c(T^aT^a)$ in SU(4) by subtracting the quantity $(N_c + 6)/12$. This is understood by using Eq. (30) of Ref. [19] for the matrix elements of $1/N_c(T^aT^a)$ extended to SU(6). Then, as one can see from Table II it turns out that the expectation values of O_4 are positive for octets and decuplets and of order N_c^{-1}, as in SU(4), and negative and of order N_c^0 for flavor singlets (see the Appendix for details).

The operators O_5 and O_6 are also two-body, which means that they carry a factor $1/N_c$ in the definition. However, as G^{ia} sums coherently, it introduces an extra factor N_c and makes the matrix elements of O_5 and O_6 of order N_c^0, as seen from Table II. These matrix elements are

TABLE I. Operators and their coefficients in the mass formula obtained from numerical fits. The values of c_i and d_i are indicated under the heading Fit n ($n = 1, 2, 3, 4$).

Operator	Fit 1 (MeV)	Fit 2 (MeV)	Fit 3 (MeV)	Fit 4 (MeV)
$O_1 = N_c \mathbb{1}$	$c_1 = 672 \pm 8$	$c_1 = 673 \pm 7$	$c_1 = 672 \pm 8$	$c_1 = 673 \pm 7$
$O_2 = \ell^i s^i$	$c_2 = 18 \pm 19$	$c_2 = 17 \pm 18$	$c_2 = 19 \pm 9$	$c_2 = 20 \pm 9$
$O_3 = \frac{1}{N_c} S^i S^i$	$c_3 = 121 \pm 59$	$c_3 = 115 \pm 46$	$c_3 = 120 \pm 58$	$c_3 = 112 \pm 42$
$O_4 = \frac{1}{N_c}[T^aT^a - \frac{1}{12}N_c(N_c+6)]$	$c_4 = 202 \pm 41$	$c_4 = 200 \pm 40$	$c_4 = 205 \pm 27$	$c_4 = 205 \pm 27$
$O_5 = \frac{3}{N_c} L^i T^a G^{ia}$	$c_5 = 1 \pm 13$	$c_5 = 2 \pm 12$		
$O_6 = \frac{15}{N_c} L^{(2)ij} G^{ia} G^{ja}$	$c_6 = 1 \pm 6$		$c_6 = 1 \pm 5$	
$B_1 = -\mathcal{S}$	$d_1 = 108 \pm 93$	$d_1 = 108 \pm 92$	$d_1 = 109 \pm 93$	$d_1 = 108 \pm 92$
$\chi^2_{\rm dof}$	1.23	0.93	0.93	0.75

TABLE II. Diagonal matrix elements of the operators O_i for the $[\mathbf{70}, \ell^-]$ multiplets ($\ell = 1, 2, 3$) of the $N = 3$ band.

	O_1	O_2	O_3	O_4	O_5	O_6
$^4N[\mathbf{70}, 3^-]_{9/2}$	N_c	$\frac{3}{2}$	$\frac{15}{4N_c}$	$\frac{3}{4N_c}$	$-\frac{9(N_c+3)}{4N_c}$	$-\frac{75(N_c-1)}{8N_c}$
$^2N[\mathbf{70}, 3^-]_{7/2}$	N_c	$\frac{2N_c-3}{2N_c}$	$\frac{3}{4N_c}$	$\frac{3}{4N_c}$	$\frac{9}{2N_c}$	0
$^4N[\mathbf{70}, 3^-]_{5/2}$	N_c	$-\frac{7}{6}$	$\frac{15}{4N_c}$	$\frac{3}{4N_c}$	$-\frac{7(N_c+3)}{4N_c}$	$\frac{45(N_c-1)}{8N_c}$
$^2N[\mathbf{70}, 3^-]_{5/2}$	N_c	$-\frac{2(2N_c+3)}{3N_c}$	$\frac{3}{4N_c}$	$\frac{3}{4N_c}$	$-\frac{6}{N_c}$	0
$^4N[\mathbf{70}, 3^-]_{3/2}$	N_c	-2	$\frac{15}{4N_c}$	$\frac{3}{4N_c}$	$-\frac{3(N_c+3)}{N_c}$	$-\frac{45(N_c-1)}{2N_c}$
$^2N[\mathbf{70}', 1^-]_{3/2}$	N_c	$\frac{2N_c-3}{6N_c}$	$\frac{3}{4N_c}$	$\frac{3}{4N_c}$	$\frac{3}{2N_c}$	0
$^2N[\mathbf{70}', 1^-]_{1/2}$	N_c	$-\frac{2N_c-3}{3N_c}$	$\frac{3}{4N_c}$	$\frac{3}{4N_c}$	$-\frac{3}{N_c}$	0
$^2\Delta[\mathbf{70}, 3^-]_{7/2}$	N_c	$-\frac{1}{2}$	$\frac{3}{4N_c}$	$\frac{15}{4N_c}$	$\frac{9(N_c+1)}{4N_c}$	0
$^2\Delta[\mathbf{70}, 2^-]_{5/2}$	N_c	$-\frac{1}{3}$	$\frac{3}{4N_c}$	$\frac{15}{4N_c}$	$\frac{3(N_c+1)}{2N_c}$	0
$^2\Lambda[\mathbf{70}, 3^-]_{7/2}$	N_c	$\frac{3}{2}$	$\frac{3}{4N_c}$	$-\frac{2N_c+3}{4N_c}$	$-\frac{3(N_c-3)}{4N_c}$	0

obtained from the formulas (B2) and (B4) of Ref. [20] where the multiplet $[70, 1^-]$ has been discussed. Interestingly, when $N_c = 3$, the contribution of O_5 cancels out for flavor singlets, like for $\ell = 1$ [20]. This property follows from the analytic form of the isoscalar factors given in Ref. [20].

We remind that the SU(6) generators S^i, T^a, and G^{ia} and the SO(3) generators L^i of Eq. (4) act on the total wave function of the N_c system of quarks as proposed in Refs. [18–20]. The advantage of this procedure over the standard one, where the system is separated into a ground state core + an excited quark [13], is that the number of relevant operators needed in the fit is usually smaller than the number of data and it allows a better understanding of their role in the mass formula, in particular, the role of the isospin operator O_4 which has always been omitted in the symmetric core + excited quark procedure. We should also mention that in our approach the permutation symmetry is exact [18].

Among the operators containing angular momentum components, besides the spin-orbit, we have included the operators O_5 and O_6, to check whether or not they bring insignificant contributions, as it was in the $N = 1$ band. From Table I one can see that their coefficients are indeed negligible either included together as in Fit 1 or separately as in Fit 2 and 3.

III. RESULTS AND DISCUSSION

We have performed four distinct numerical fits of the mass formula (2) to the experimental data. The corresponding dynamical coefficients c_i and d_i together with the values of $\chi^2_{\rm dof}$ are listed in Table I. Fit 1 is made with all operators. Fit 2 and Fit 3 are made by removing one operator and Fit 4 is made only with the first four operators. It turns out that the contributions of angular-dependent operators O_5 and O_6 are negligible but that of the spin-orbit operator, which is quite small, remains important. The values of $\chi^2_{\rm dof}$ are good and the error bars of the coefficients suggest that the choice of the operators we have made provides a reliable fit.

As already mentioned, in the 1.9–2.4 GeV region the experimental data are rather scarce. For this reason, besides the eight resonances provided by the Particle Data Group (PDG) [40] we have also included two more, proposed in Refs. [37,38]. They do not have a status yet. Therefore, in all, we have included ten resonances in the fit from which one is a hyperon.

The experimental masses of four star resonances are from the Summary Table of PDG. For the two-star resonance $D_{15}(2200)$ we took the mass indicated in the Baryon Particle Listings of PDG as due to Cutkosky *et al.* [41] and for the one-star resonance $D_{13}(2080)$ the mass due to Hoehler *et al.* [42]. The experimental mass of $S_{11}(2090)$ was taken as the average of the masses obtained by Hoehler *et al.* and Cutkosky *et al.* in the partial-wave analysis of the πN scattering.

The baryon masses obtained from the mass formula (2) with the coefficients from Fit 4 and matrix elements from Table II are presented in Table III. Partial contributions of different operators to the total mass are also indicated. One can see that the spin operator O_3 brings a dominant contribution to the splitting in $^4N[\mathbf{70}, 3^-]$ states and the isospin operator is as important, or even more so, in the Δ and the Λ resonances. In the latter, the negative sign of O_4 matrix elements helps in lowering the calculated mass close down to the experimental value.

The first observation regarding the multiplet structure is related to the $G_{17}(2190)$ four-star resonance. To obtain a good fit we had to interpret it as a $^2N[\mathbf{70}, 3^-]$ state. If so, Table III implies that it forms a doublet with the newly suggested $D_{15}(2060)$ resonance of Refs. [37,38]. If this resonance is the same as the newly suggested $N(2070)D_{15}$ resonance of Ref. [36] we should be in

TABLE III. Partial contributions and the total mass (MeV) predicted by the $1/N_c$ expansion method obtained from Fit 4. The last two columns indicate the empirically known masses and the resonance name and status (whenever known).

	Part. contrib. (MeV)							
	c_1O_1	c_2O_2	c_3O_3	c_4O_4	d_1B_1	Total (MeV)	Exp. (MeV)	Name, status
$^4N[\mathbf{70}, 3^-]_{9/2}$	2018	29	140	51	0	2238 ± 46	2275 ± 75	$G_{19}(2250)$****
$^2N[\mathbf{70}, 3^-]_{7/2}$	2018	10	28	51	0	2107 ± 17	2150 ± 50	$G_{17}(2190)$****
$^4N[\mathbf{70}, 3^-]_{5/2}$	2018	−23	140	51	0	2186 ± 41	2180 ± 80	$D_{15}(2200)$**
$^2N[\mathbf{70}, 3^-]_{5/2}$	2018	−39	28	51	0	2058 ± 14	2060 ± 15	$D_{15}(2060)$
$^4N[\mathbf{70}, 3^-]_{3/2}$	2018	−39	140	51	0	2170 ± 42	2150 ± 60	$D_{13}(2150)$
$^2N[\mathbf{70}', 1^-]_{3/2}$	2018	3	28	51	0	2101 ± 14	2081 ± 20	$D_{13}(2080)$*
$^2N[\mathbf{70}', 1^-]_{1/2}$	2018	−7	28	51	0	2091 ± 12	2100 ± 20	$S_{11}(2090)$*
$^2\Delta[\mathbf{70}, 3^-]_{7/2}$	2018	−10	28	256	0	2292 ± 25	2200 ± 80	$G_{37}(2220)$*
$^2\Delta[\mathbf{70}, 2^-]_{5/2}$	2018	−7	28	256	0	2295 ± 25	2305 ± 26	$D_{35}(2350)$*
$^2\Lambda[\mathbf{70}, 3^-]_{7/2}$	2018	29	28	−153	108	2030 ± 82	2030 ± 82	$G_{07}(2100)$****

agreement with the latter authors, who proposed a doublet.

As a matter of fact we have found out that the inclusion of the resonance $S_{11}(2090)$, interpreted as a radial excitation belonging to the $[\mathbf{70}', 1^-]$ multiplet, improves the fit, which gives confidence in this interpretation. This resonance appears therefore as the spin-orbit partner of $D_{13}(2080)$.

Regarding the Δ resonances, our analysis shows that we have interpreted $D_{35}(2350)$ as a member of a $[\mathbf{70}, 2^-]$ multiplet. This is inspired by the quark model results of Ref. [25] where this multiplet, having $\ell = 2$, is the highest in the spectrum of the spin-independent Hamiltonian with a relativistic kinetic energy and a linear confinement. Such a high value is expected to lead to a mass as large as that of the mentioned resonance. This choice suggests a kind of agreement between quark models and the present fit, well in the spirit of Ref. [29].

The value of the coefficient c_1 found in our best fit $c_1 = 673 \pm 7$ MeV is smaller than $c_1 = 731 \pm 17$ MeV of Ref. [26] but not far from the estimate $c_1 \approx 640$ MeV which can be extracted from Fig. 1 of Ref. [23] where a linear dependence of c_1 on the excitation energy, or alternatively on the band number N, was found. Note that such an energy dependence is reproduced by the formula (29) of Ref. [29] where the compatibility between the $1/N_c$ expansion method and semirelativistic quark models with a linear confinement was discussed. This compatibility is confirmed by the present results. The value of $c_2 = 20 \pm 9$ MeV is practically identical with that obtainable from Fig. 1 of Ref. [23].

From the comparison of our results with the "new" resonances reported in Refs. [37,38] we can make the following comments. We do not support the interpretation of the $D_{15}(2060)$ resonance as a member of the quartet (1), inasmuch as we interpret this resonance as a member of the doublet $^2N[\mathbf{70}, 3^-]$. But $D_{13}(2150)$ is a member of the quartet $^4N[\mathbf{70}, 3^-]$ together with $G_{19}(2250)$ and $D_{15}(2200)$. If this interpretation is valid it remains to find the $J = 7/2$ member, not observed yet.

IV. CONCLUSIONS

Using the $1/N_c$ expansion method we have analyzed the multiplet structure of high-lying negative parity resonances, located in the 1900 –2400 MeV region, supposed to belong to the $N = 3$ band in the SU(6) × O(3) classification. Our results are largely consistent with the recent experimental analysis of Refs. [36–38]. A possible future observation of a $7/2^-$ resonance would be of great help in understanding the $^4N[\mathbf{70}, 3^-]$ multiplet and the structure of the $N = 3$ band in general.

The simplified method we have used allows us to include a small number of terms in the mass formula and easily identify the most dominant operators to order $\mathcal{O}(N_c^{-1})$. As a common feature with the SU(4) and SU(6) analysis of the $N = 1$ band, we found that the isospin operator O_4, neglected in the standard core + excited quark approach, contributes to the mass of Δ's with a coefficient c_4 with a magnitude comparable to that of the coefficient c_3 of the spin operator O_3 in N^* resonances. In addition the role of the operator O_4 is crucial in describing the flavor singlet four-star resonance $\Lambda(2100)G_{07}$ included in the fit.

Future discoveries will help to improve our study and confirm or infirm the present multiplet interpretation.

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APPENDIX A

We remind that the matrix elements of the spin-orbit operator O_2 between states with spins S and S' are given by

$$\langle \ell' S' J' J_3'; I' I_3' | \ell \cdot s | \ell S J J_3; I I_3 \rangle_{p=2} = (-1)^{J+\ell+1/2} \delta_{J'J} \delta_{J_3' J_3} \delta_{\ell'\ell} \delta_{I'I} \delta_{I_3' I_3} \sqrt{\frac{3}{2}(2S+1)(2S'+1)\ell(\ell+1)(2\ell+1)} \begin{Bmatrix} \ell & \ell & 1 \\ S & S' & J \end{Bmatrix} \times \sum_{p_1, p_2} (-1)^{-S_c} c_{p_1 p_2}^{[N_c-1,1]}(S') c_{p_1 p_2}^{[N_c-1,1]}(S) \begin{Bmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ S_c & S & S' \end{Bmatrix}. \quad \text{(A1)}$$

The quantities $c_{p_1 p_2}^{[N_c-1,1]}(S)$, are a shorthand notation for the isoscalar factors of the permutation group of N_c quarks, denoted by $K([f']p'[f'']p''|[f]p)$ in Ref. [43] (see also Ref. [44]). In that notation p, p_1 and p_2 represent the position of the N_c-th quark in the spin-flavor, spin, and flavor parts of the wave function, of partitions $[f]$, $[f']$, and $[f'']$, respectively. Actually, $c_{p_1 p_2}^{[N_c-1,1]}(S)$ are functions of the spin S and the number N_c of quarks. By definition, the core + excited quark wave function [13] has $p = 2$. Equation (A1) is equivalent to Eq. (A7) of Ref. [13]. The correspondence in the isoscalar factors denoted there by $c_{\rho\eta}$ is

$$c_{11}^{[N_c-1,1]}(S) \to c_{0-}; \qquad c_{22}^{[N_c-1,1]}(S) \to c_{0+}; \\ c_{12}^{[N_c-1,1]}(S) \to c_{++}; \qquad c_{21}^{[N_c-1,1]}(S) \to c_{--}. \quad \text{(A2)}$$

We also remind that the matrix elements of the isospin operator O_4 as defined in Table I requires the knowledge of the expectation value of the SU(3) Casimir operator $T^a T^a$. Labelling the flavor states by (λ, μ) this is

$$\langle T^a T^a \rangle = \tfrac{1}{3}(\lambda^2 + \mu^2 + \lambda\mu + 3\lambda + 3\mu). \quad \text{(A3)}$$

For the flavor states of N_c quarks we have

(i)

$$^2 8 \text{ or } ^4 8\text{: } \lambda = 1, \qquad \mu = \frac{N_c - 1}{2}, \\ \langle T^a T^a \rangle = \frac{(N_c+3)^2}{12},$$

(ii)

$$^2 10\text{: } \lambda = 3, \qquad \mu = \frac{N_c - 3}{2}, \\ \langle T^a T^a \rangle = \frac{N_c^2 + 6N_c + 45}{12},$$

(iii)

$$^2 1\text{: } \lambda = 0, \qquad \mu = \frac{N_c - 3}{2}, \\ \langle T^a T^a \rangle = \frac{N_c(N_c+6) - 3(2N_c+3)}{12}.$$

The last case has been discussed in Ref. [19]. Accordingly the expectation value of O_4 is negative for flavor singlets.

The analytic expressions of the matrix elements of O_5 and O_6 can be found in Ref. [20] together with the corresponding isoscalar factors of the SU(3) generators which we do not reproduce here.

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