Lightest *CP*-even Higgs boson mass in the testable flipped $SU(5) \times U(1)_X$ models from *F* theory

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We study the lightest *CP*-even Higgs boson mass in five kinds of testable flipped $SU(5) \times U(1)_X$ models from *F*-theory. Two kinds of models have vectorlike particles around the TeV scale, while the other three kinds also have the vectorlike particles at the intermediate scale that can be considered as messenger fields in gauge-mediated supersymmetry breaking. We require that the Yukawa couplings for the TeV-scale vectorlike particles and the third family of the Standard Model (SM) fermions are smaller than three from the electroweak scale to the $SU(3)_C \times SU(2)_L$ unification scale. With the two-loop renormalization group equation running for the gauge couplings and Yukawa couplings, we obtain the maximal Yukawa couplings between the TeV-scale vectorlike particles and Higgs fields. To calculate the lightest *CP*-even Higgs boson mass upper bounds, we employ the renormalization group improved effective Higgs potential approach, and consider the two-loop leading contributions in the supersymmetric SM and one-loop contributions from the TeV-scale vectorlike particles. We assume maximal mixings between the stops and between the TeV-scale vectorlike scalars. The numerical results for these five kinds of models are roughly the same. In particular, we show that the lightest *CP*-even Higgs boson can have mass up to 146 GeV naturally, which is the current upper bound from the CMS and ATLAS collaborations.

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I. INTRODUCTION

The Higgs boson mass in the SM is not stable against quantum corrections and has quadratic divergences. Because the reduced Planck scale is about 2.4×10^{18} GeV while the electroweak (EW) scale is around 100 GeV, there exists huge fine-tuning to have the EW-scale Higgs boson mass, which is called the gauge hierarchy problem. Supersymmetry is a symmetry between the bosonic and fermionic states, and it naturally solves this problem due to the cancellations between the bosonic and fermionic quantum corrections.

In the Minimal Supersymmetric Standard Model (MSSM) with *R* parity, under which the SM particles are even while the supersymmetric particles (sparticles) are odd, the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge couplings can be unified around 2×10^{16} GeV [1]; the lightest supersymmetric particle such as the neutralino can be a cold dark matter candidate [2,3]; and the EW precision constraints can be evaded, etc. Especially, the gauge coupling unification strongly suggests Grand Unified Theories (GUTs), which can explain the SM fermion quantum numbers. However, in the supersymmetric $SU(5)$ models, there exist the doublet-triplet splitting problem and dimension-five proton decay problem. Interestingly, these problems can be solved elegantly in the flipped $SU(5) \times U(1)_X$ models via missing partner mechanism [4–6]. Previously, the flipped $SU(5) \times U(1)_X$ models were constructed systematically in the free fermionic string constructions at the Kac-Moody level one [7,8]. To solve the little hierarchy problem between the traditional unification scale and the string scale, two of us (T. L. and D. V. N.) with Jiang have proposed the testable flipped $SU(5) \times U(1)_X$ models, where the TeV-scale vectorlike particles are introduced [9]. There is a two-step unifcation: the $SU(3)_C \times SU(2)_L$ gauge couplings are unified at the scale M_{32} around the usual GUT scale, and the $SU(5) \times U(1)_X$ gauge couplings are unified at the final unification scale $M_{\mathcal{F}}$ around 5×10^{17} GeV [9]. Moreover, such kinds of models have been constructed locally from the *F*-theory model building [10,11] and are dubbed as $\mathcal{F}$-$SU(5)$ [11]. In particular, these models are very interesting from the phenomenological point of view [11]: The vectorlike particles can be observed at the Large Hadron Collider (LHC); proton decay is within the reach of the future Hyper-Kamiokande [12] and Deep Underground Science and Engineering Laboratory [13] experiments [14,15]; the hybrid inflation can be naturally realized; and the correct cosmic primodial density fluctuations can be generated [16].

With no-scale boundary conditions at $M_{\mathcal{F}}$ [17], two of us (T. L. and D. V. N.), with Maxin and Walker, have described an extraordinarily constrained "golden point" [18] and "golden strip" [19] that satisfied all the latest experimental constraints and have an imminently observable proton decay rate [14]. Especially, the UV boundary condition $B_\mu = 0$ gives very strong constraint on the viable parameter space, where B_μ is the soft bilinear Higgs mass term in the MSSM. In addition, exploiting a "super-no-scale" condition, we dynamically determined the universal gaugino mass $M_{1/2}$ and the ratio of the Higgs vacuum expectation values (VEVs) $\tan\beta$. Since $M_{1/2}$ is related to the modulus field of the internal space in string models, we stabilized the modulus dynamically [20,21]. Interestingly, the sparticle spectra generially have a light stop and gluino, which are lighter than all the other squarks. Thus, we can test such kinds of models at the LHC by looking for the ultra-high jet signals [22,23]. Moreover, the complete viable parameter space in no-scale $\mathcal{F}$-$SU(5)$ has been studied by considering a set of "bare minimal" experimental constaints [24]. For the other LHC and dark matter phenomenological studies, see Refs. [25–27].

It is well-known that one of main LHC goals is to detect the SM or SM-like Higgs boson. Recently, both the CMS [28] and ATLAS [29] collaborations have presented their combined searches for the SM Higgs boson based on the integrated luminosities between 1 fb^{-1} and 2.3 fb^{-1}, depending on the search channels. For the light SM Higgs boson mass region preferred by the EW precision data, they have excluded the SM Higgs boson with mass larger than 145 GeV and 146 GeV, respectively. In the no-scale $\mathcal{F}$-$SU(5)$, the lightest CP-even Higgs boson mass is generically about 120 GeV if the contributions from the vectorlike particles are neglected [30]. Thus, the interesting question is whether the lightest CP-even Higgs boson can have mass up to 146 GeV naturally if we include the contributions from the additional vectorlike particles.

In this paper, we consider five kinds of testable flipped $SU(5) \times U(1)_X$ models from F-theory. Two kinds of models have only vectorlike particles around the TeV scale. Because the gauge-mediated supersymmetry breaking can be realized naturally in the F-theory GUTs [31], we also introduce vectorlike particles with mass around 10^{11} GeV [31], which can be considered as messenger fields, in the other three kinds of models. We require that the Yukawa couplings for the TeV-scale vectorlike particles and the third family of the SM fermions are smaller than three from the EW scale to the scale M_{32} from the perturbative bound; i.e., the Yukawa coupling squares are less than 4π. With the two-loop Renormalization Group Equation (RGE) running for the gauge couplings and Yukawa couplings, we obtain the maximal Yukawa couplings for the TeV-scale vectorlike particles. To calculate the lightest CP-even Higgs boson mass upper bounds, we employ the Renormalization Group (RG) improved effective Higgs potential approach and consider the two-loop leading contributions in the MSSM and one-loop contributions from the TeV-scale vectorlike particles. For simplicity, we assume that the mixings between both the stops and the TeV-scale vectorlike scalars are maximal. In general, we shall increase the lightest CP-even Higgs boson mass upper bounds if we increase the supersymmetry breaking scale or decrease the TeV-scale vectorlike particle masses. The numerical results for our five kinds of models are roughly the same. For the TeV-scale vectorlike particles and sparticles with masses around 1 TeV, we show that the lightest CP-even Higgs boson can have mass up to 146 GeV naturally.

This paper is organized as follows. In Sec. II, we briefly review the testable flipped $SU(5) \times U(1)_X$ models from F-theory and present five kinds of models. We calculate the lightest CP-even Higgs boson mass upper bounds in Sec. III. Sec. IV is our Conclusion. In Appendices, we present all the RGEs in five kinds of models.

II. TESTABLE FLIPPED $SU(5) \times U(1)_X$ MODELS FROM F-THEORY

We first briefly review the minimal flipped $SU(5)$ model [4–6]. The gauge group for flipped $SU(5)$ model is $SU(5) \times U(1)_X$, which can be embedded into $SO(10)$ model. We define the generator $U(1)_{Y'}$ in $SU(5)$ as

$$T_{U(1)_{Y'}} = \text{diag}\left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2}\right). \quad (1)$$

The hypercharge is given by

$$Q_Y = \frac{1}{5}(Q_X - Q_{Y'}). \quad (2)$$

There are three families of the SM fermions whose quantum numbers under $SU(5) \times U(1)_X$ are

$$F_i = (\mathbf{10}, \mathbf{1}), \qquad \bar{f}_i = (\bar{\mathbf{5}}, -\mathbf{3}), \qquad \bar{l}_i = (\mathbf{1}, \mathbf{5}), \quad (3)$$

where $i = 1, 2, 3$. The SM particle assignments in F_i, $\bar{f}_i$ and $\bar{l}_i$ are

$$F_i = (Q_i, D_i^c, N_i^c), \qquad \bar{f}_i = (U_i^c, L_i), \qquad \bar{l}_i = E_i^c, \quad (4)$$

where Q_i and L_i are, respectively, the superfields of the left-handed quark and lepton doublets and U_i^c, D_i^c, E_i^c and N_i^c are the CP conjugated superfields for the right-handed up-type quarks, down-type quarks, leptons, and neutrinos, respectively. To generate the heavy right-handed neutrino masses, we need to introduce three SM singlets ϕ_i [32].

To break the GUT and electroweak gauge symmetries, we introduce two pairs of Higgs fields

$$H = (\mathbf{10}, \mathbf{1}), \qquad \bar{H} = (\overline{\mathbf{10}}, -\mathbf{1}),$$
$$h = (\mathbf{5}, -\mathbf{2}), \qquad \bar{h} = (\bar{\mathbf{5}}, 2). \quad (5)$$

Interestingly, we can solve the doublet-triplet splitting problem via the missing partner mechanism [6]. Also, the Higgsino-exchange–mediated proton decays are negligible, i.e., we do not have the dimension-5 proton decay problem.

To achieve the string-scale gauge coupling unification [9,33], we introduce the vectorlike particles which form complete flipped $SU(5) \times U(1)_X$ multiplets. The quantum numbers for these additional vectorlike particles under the $SU(5) \times U(1)_X$ gauge symmetry are [9]

$$XF = (\mathbf{10}, \mathbf{1}), \qquad \overline{XF} = (\overline{\mathbf{10}}, -1), \quad (6)$$

$$Xf = (\mathbf{5}, \mathbf{3}), \qquad \overline{Xf} = (\bar{\mathbf{5}}, -\mathbf{3}), \tag{7}$$

$$Xl = (\mathbf{1}, -\mathbf{5}), \qquad \overline{Xl} = (\mathbf{1}, \mathbf{5}), \tag{8}$$

$$Xh = (\mathbf{5}, -\mathbf{2}), \qquad \overline{Xh} = (\bar{\mathbf{5}}, \mathbf{2}). \tag{9}$$

Moreover, the particle contents from the decompositions of XF, $\overline{XF}$, Xf, $\overline{Xf}$, Xl, $\overline{Xl}$, Xh, and $\overline{Xh}$ under the SM gauge symmetry are

$$XF = (XQ, XD^c, XN^c), \qquad \overline{XF} = (XQ^c, XD, XN), \tag{10}$$

$$Xf = (XU, XL^c), \qquad \overline{Xf} = (XU^c, XL), \tag{11}$$

$$Xl = XE, \qquad \overline{Xl} = XE^c, \tag{12}$$

$$Xh = (XD, XL), \qquad \overline{Xh} = (XD^c, XL^c), \tag{13}$$

where XQ, XU^c, XD^c, XL, XE^c, and XN^c have the same SM quantum numbers as Q_i, U_i^c, D_i^c, L_i, E_i^c, and N_i^c, respectively.

To separate the mass scales M_{32} and $M_{\mathcal{F}}$ in our F-theory flipped $SU(5) \times U(1)_X$ models, we need to introduce sets of vectorlike particles around the TeV scale or intermediate scale whose contributions to the one-loop beta functions satisfy $\Delta b_1 < \Delta b_2 = \Delta b_3$. To avoid the Landau pole problem, we have shown that there are only five possible such sets of vectorlike particles as follows, due to the quantizations of the one-loop beta functions [9]

$$Z0\text{: } XF + \overline{XF}; \tag{14}$$

$$Z1\text{: } XF + \overline{XF} + Xl + \overline{Xl}; \tag{15}$$

$$Z2\text{: } XF + \overline{XF} + Xf + \overline{Xf}; \tag{16}$$

$$Z3\text{: } XF + \overline{XF} + Xl + \overline{Xl} + Xh + \overline{Xh}; \tag{17}$$

$$Z4\text{: } XF + \overline{XF} + Xh + \overline{Xh}. \tag{18}$$

We have systematically constructed flipped $SU(5) \times U(1)_X$ models with generic sets of vectorlike particles around the TeV scale and/or around the intermediate scale from the F-theory. In addition, gauge mediated supersymmetry breaking can be realized naturally in the F-theory GUTs [31], and there may exist vectorlike particles as the messenger fields at the intermediate scale around 10^{11} GeV [31]. Therefore, in this paper, we shall calculate the lightest CP-even Higgs boson mass in five kinds of the flipped $SU(5) \times U(1)_X$ models from F-theory: (i) In Model I, we introduce the $Z0$ set of vectorlike particles $(XF, \overline{XF})$ at the TeV scale, and we shall add superheavy vectorlike particles around M_{32} so that the $SU(5) \times U(1)_X$ unification scale is smaller than the reduced Planck scale; (ii) In Model II, we introduce the vectorlike particles $(XF, \overline{XF})$ at the TeV scale and the vectorlike particles $(Xf, \overline{Xf})$ at the intermediate scale, which can be considered as the messenger fields; (iii) In Model III, we introduce the vectorlike particles $(XF, \overline{XF})$ at the TeV scale and the vectorlike particles $(Xf, \overline{Xf})$ and $(Xl, \overline{Xl})$ at the intermediate scale; (iv) In Model IV, we introduce the $Z1$ set of vectorlike particles $(XF, \overline{XF})$ and $(Xl, \overline{Xl})$ at the TeV scale; (v) In Model V, we introduce the vectorlike particles $(XF, \overline{XF})$ and $(Xl, \overline{Xl})$ at the TeV scale, and the vectorlike particles $(Xf, \overline{Xf})$ at the intermediate scale. In particular, we emphasize that the vectorlike particles at the intermediate scale in Models II, III, and V will give us the generalized gauge-mediated supersymmetry breaking if they are the messenger fields [34]. By the way, if we introduce the vectorlike particles $(Xh, \overline{Xh})$ at the intermediate scale, which are the traditional messenger fields in gauge mediation, the discussions are similar and the numerical results are almost the same. Thus, we will not study such kinds of models here.

TABLE I. The vectorlike particle contents in Model I, Model II, Model III, Model IV, and Model V.

Models	Vectorlike Particles at M_V	Vectorlike Particles at M_I
Model I	$(XF, \overline{XF})$	...
Model II	$(XF, \overline{XF})$	$(Xf, \overline{Xf})$
Model III	$(XF, \overline{XF})$	$(Xf, \overline{Xf})$, $(Xl, \overline{Xl})$
Model IV	$(XF, \overline{XF})$, $(Xl, \overline{Xl})$	...
Model V	$(XF, \overline{XF})$, $(Xl, \overline{Xl})$	$(Xf, \overline{Xf})$

For simplicity, we assume that the masses for the vectorlike particles around the TeV scale or the intermediate scale are universal. Also, we denote the universal mass for the vectorlike particles at the TeV scale as M_V and the universal mass for the vectorlike particles at the intermediate scale as M_I. With this convention, we present the vectorlike particle contents of our five kinds of models in Table I. In the following discussions, we shall choose $M_I = 1.0 \times 10^{11}$ GeV. Moreover, we will assume universal supersymmetry breaking at low energy and denote the universal supersymmetry breaking scale as M_S.

It is well-known that there exists a few percent fine-tuning for the lightest CP-even Higgs boson mass in the MSSM to be larger than 114.4 GeV. In all the previously mentioned five kinds of models, we have the vectorlike particles XF and $\overline{XF}$ at the TeV scale. Then we can introduce the following Yukawa interaction terms between the MSSM Higgs fields and these vectorlike particles in the superpotential in the flipped $SU(5) \times U(1)_X$ models:

$$W = \tfrac{1}{2} Y_{xd} XF XF h + \tfrac{1}{2} Y_{xu} \overline{XF}\,\overline{XF}\,\overline{h}, \tag{19}$$

where Y_{xd} and Y_{xu} are Yukawa couplings. After the gauge symmetry $SU(5) \times U(1)_X$ is broken down to the SM gauge symmetry, we have the following relevant Yukawa coupling terms in the superpotential

$$W = Y_{xd} XQ XD^c H_d + Y_{xu} XQ^c XD H_u. \tag{20}$$

To have the upper bounds on the lightest CP-even Higgs boson mass, we first need to calculate the upper bounds on the Yukawa couplings Y_{xu} and Y_{xd}. In this paper, employing the two-loop RGE running, we will require that all the Yukawa couplings, including Y_{xu} and Y_{xd}, are smaller than three (perturbative bound) below the $SU(3)_C \times SU(2)_L$ unification scale M_{32} for simplicity, since M_{32} is close to the $SU(5) \times U(1)_X$ unification scale $M_{\mathcal{F}}$. The other point is that above the scale M_{32}, there might exist other superheavy threshold corrections, and then the RGE running for the gauge couplings and Yukawa couplings might be very complicated. Moreover, we will not give the two-loop RGEs in the SM and the MSSM, which can be easily found in the literature; for example, in Refs. [35,36]. We shall present the RGEs in the SM with vectorlike particles and Models I to V in Appendices A, B, C, D, E, and F.

III. THE LIGHTEST *CP*-EVEN HIGGS BOSON MASS

In our calculations, we employ the RG-improved effective Higgs potential approach. The two-loop leading contributions to the lightest *CP*-even Higgs boson mass m_h in the MSSM are [37,38]

$$[m_h^2]_{\rm MSSM} = M_Z^2\cos^2 2\beta\left(1 - \frac{3}{8\pi^2}\frac{m_t^2}{v^2}t\right) + \frac{3}{4\pi^2}\frac{m_t^4}{v^2}\left[t + \frac{1}{2}X_t + \frac{1}{(4\pi)^2}\left(\frac{3}{2}\frac{m_t^2}{v^2} - 32\pi\alpha_s\right)(X_t t + t^2)\right], \quad (21)$$

where M_Z is the Z boson mass, m_t is the $\overline{MS}$ top quark mass, v is the SM Higgs vacuum expectation values, and α_S is the strong coupling constant. Also, t and X_t are given as follows

$$t = \log\frac{M_S^2}{M_t^2}, \qquad X_t = \frac{2\tilde{A}_t^2}{M_S^2}\left(1 - \frac{\tilde{A}_t^2}{12M_S^2}\right), \qquad \tilde{A}_t = A_t - \mu\cot\beta, \quad (22)$$

where M_t is the top quark pole mass and A_t denotes the trilinear soft term for the top quark Yukawa coupling term.

Moreover, we use the RG-improved one-loop effective Higgs potential approach to calculate the contributions to the lightest *CP*-even Higgs boson mass from the vectorlike particles [39,40]. Such contributions in our models are

$$\begin{aligned}\Delta m_h^2 = & -\frac{N_c}{8\pi^2}M_Z^2\cos^2 2\beta(\hat{Y}_{xu}^2 + \hat{Y}_{xd}^2)t_V + \frac{N_c v^2}{4\pi^2}\Bigg\{\hat{Y}_{xu}^4\left[t_V + \frac{1}{2}X_{xu}\right] + \hat{Y}_{xu}^3\hat{Y}_{xd}\left[-\frac{2M_S^2(2M_S^2+M_V^2)}{3(M_S^2+M_V^2)^2} - \frac{\tilde{A}_{xu}(2\tilde{A}_{xu}+\tilde{A}_{xd})}{3(M_S^2+M_V^2)}\right] \\ & + \hat{Y}_{xu}^2\hat{Y}_{xd}^2\left[-\frac{M_S^4}{(M_S^2+M_V^2)^2} - \frac{(\tilde{A}_{xu}+\tilde{A}_{xd})^2}{3(M_S^2+M_V^2)}\right] + \hat{Y}_{xu}\hat{Y}_{xd}^3\left[-\frac{2M_S^2(2M_S^2+M_V^2)}{3(M_S^2+M_V^2)^2} - \frac{\tilde{A}_{xd}(2\tilde{A}_{xd}+\tilde{A}_{xu})}{3(M_S^2+M_V^2)}\right] + \hat{Y}_{xd}^4\left[t_V + \frac{1}{2}X_{xd}\right]\Bigg\},\end{aligned} \quad (23)$$

where

$$\begin{aligned}\hat{Y}_{xu} &= Y_{xu}\sin\beta, \qquad \hat{Y}_{xd} = Y_{xd}\cos\beta, \\ t_V &= \log\frac{M_S^2 + M_V^2}{M_V^2}, \\ X_{xu} &= -\frac{2M_S^2(5M_S^2 + 4M_V^2) - 4(3M_S^2 + 2M_V^2)\tilde{A}_{xu}^2 + \tilde{A}_{xu}^4}{6(M_V^2 + M_S^2)^2}, \\ X_{xd} &= -\frac{2M_S^2(5M_S^2 + 4M_V^2) - 4(3M_S^2 + 2M_V^2)\tilde{A}_{xd}^2 + \tilde{A}_{xd}^4}{6(M_V^2 + M_S^2)^2}, \\ \tilde{A}_{xu} &= A_{xu} - \mu\cot\beta, \qquad \tilde{A}_{xd} = A_{xd} - \mu\tan\beta, \end{aligned} \quad (24)$$

where A_{xu} and A_{xd} denote the supersymmetry breaking trilinear soft terms for the superpotential Yukawa terms $Y_{xu}XQ^cXDH_u$ and $Y_{xd}XQXD^cH_d$, respectively.

The third, fourth, fifth, and sixth terms in Eq. (23) are suppressed by the inverses of $\tan\beta$, $\tan^2\beta$, $\tan^3\beta$, and $\tan^4\beta$, respectively. To have the lightest *CP*-even Higgs boson mass upper bounds, we usually need $\tan\beta \sim 22$ from the numerical calculations. Especially, in order to increase the lightest *CP*-even Higgs boson mass, we should choose relatively large Y_{xu} and small Y_{xd} [39,40]. Thus, for simplicity, we only employ the first and second terms in our calculations, i.e., the first line of Eq. (23). In order to have larger corrections to the lightest *CP*-even Higgs boson mass, we consider the maximal mixings X_t and X_{xu}, respectively, for both the stops and the TeV-scale vectorlike scalars, i.e., $X_t = 6$ with $\tilde{A}_t^2 = 6M_S^2$, and $X_{xu} = \frac{8}{3} + \frac{M_S^2(5M_S^2+4M_V^2)}{3(M_S^2+M_V^2)}$ with $\tilde{A}_{xu}^2 = 6M_S^2 + 4M_V^2$.

In this section, we shall calculate the lightest *CP*-even Higgs boson mass in our five kinds of models. The relevant parameters are the universal supersymmetry breaking scale M_S, the light vectorlike particle mass M_V, the intermediate scale M_I, the mixing terms X_t and X_V, respectively, for the stops and TeV-scale vectorlike scalars, and the two new Yukawa couplings for TeV-scale vectorlike particles Y_{xu} and Y_{xd}. Because we consider low-energy supersymmetry, we choose M_S from 360 GeV to 2 TeV. In order to increase the lightest *CP*-even Higgs boson mass, we need to choose small M_V as well. The experimental lower bound on M_V is about 325 GeV [41], so we will choose M_V from 360 GeV to 2 TeV. In our numerical calculations, we will use the SM input parameters at scale M_Z from the Particle Data Group [42]. In particular, we use the updated top quark pole mass $M_t = 172.9$ GeV, and the corresponding $\overline{MS}$ top quark mass $m_t = 163.645$ GeV [42].

In this paper, we require that all the Yukawa couplings for both the TeV-scale vectorlike particles and the third family of SM fermions are less than three (perturbative bound) from the EW scale to the scale M_{32}. To obtain the upper bounds on the Yukawa couplings Y_{xu} and Y_{xd} at low energy, we consider the two-loop RGE running for both the SM gauge couplings and all the Yukawa couplings. The only exception is that when $M_V < M_S$, we use the two-loop RGE running for the SM gauge couplings and one-loop RGE running for all the Yukawa couplings from M_V to M_S (see Appendix A for details). Because in this case M_V is still close to M_S, such small effects are negligible. After we obtain the upper bounds on Y_{xu} and Y_{xd}, we use the maximal Y_{xu} to calculate the upper bounds on the lightest CP-even Higgs boson mass with the maximal mixings for stops and TeV-scale vectorlike scalars.

First, we consider $Y_{xd} = 0$, $M_S = 800$ GeV, and $M_I = 1.0 \times 10^{11}$ GeV. We choose three values for M_V: $M_V =$ 400 GeV, 1000 GeV, and 2000 GeV. In Fig. 1, we present the upper bounds on the lightest CP-even Higgs boson mass by varying $\tan\beta$ from 2 to 50. We find that for the same M_V, the upper bounds on the lightest CP-even Higgs boson mass are almost the same for five kinds of models. In particular, the small differences are less than 0.4 GeV. Because the gauge couplings will give negative contributions to the Yukawa coupling RGEs, we will have a little bit larger maximal Yukawa couplings Y_{xu} if the vectorlike particles contribute more to the gauge coupling RGE running. Thus, the model order for the lightest CP-even Higgs boson mass upper bounds from small to large is Model I, Model IV, Model II, Model III, Model V. Also, the upper bounds on the lightest CP-even Higgs boson mass will decrease when we increase M_V, which is easy to understand from a physics point of view. Moreover, the maximal Yukawa couplings Y_{xu} are about 0.96, 1.03, and 1.0 for $\tan\beta = 2$, $\tan\beta \sim 23$, and $\tan\beta = 50$, respectively. In addition, for $M_V = 400$ GeV and $\tan\beta \simeq 21$; $M_V =$ 1000 GeV and $\tan\beta \simeq 23.5$; and $M_V = 2000$ GeV and $\tan\beta \simeq 24.5$; we obtain the lightest CP-even Higgs boson mass upper bounds around 153.5 GeV, 141.6 GeV, and 136.8 GeV, respectively.

Second, we consider $Y_{xd} = Y_{xu}$ at the scale M_V, $M_S =$ 800 GeV, and $M_I = 1.0 \times 10^{11}$ GeV. We choose three values for M_V: $M_V = 400$ GeV, 1000 GeV, and 2000 GeV. In Fig. 2, we present the upper bounds on the lightest CP-even Higgs boson mass by varying $\tan\beta$ from 2 to 50. For $\tan\beta < 40$, we find that the lightest CP-even Higgs boson mass upper bounds are almost the same as those in Fig. 1. However, for $\tan\beta > 40$, we find that the lightest CP-even Higgs boson mass upper bounds decrease quickly when $\tan\beta$ increases. At $\tan\beta = 50$, the upper bounds on the lightest CP-even Higgs boson mass are smaller than 130 GeV for all our scenarios. The reasons are the following: For $\tan\beta < 40$, the Yukawa couplings Y_{xu} and Y_t are easy to run out of the perturbative bound, while for $\tan\beta > 40$, the Yukawa couplings Y_{xd}, Y_b, and especially Y_τ are easy to run out, where Y_t, Y_b, and Y_τ are Yukawa couplings for the top quark, bottom quark, and tau lepton, respectively. In particular, for $\tan\beta = 50$, the maximal Yukawa couplings $Y_{xd} = Y_{xu}$ are as small as 0.67, while they are about 1.025 for $\tan\beta < 40$.

Third, we consider $Y_{xd} = 0$, $\tan\beta = 20$, $M_S =$ 800 GeV, and $M_I = 1.0 \times 10^{11}$ GeV. In Fig. 3, we present the upper bounds on the lightest CP-even Higgs boson mass by varying M_V from 360 GeV to 2 TeV. We can see that as the value of M_V increases from 360 GeV to 2 TeV, the upper bounds on the lightest CP-even Higgs boson mass decrease from 155 GeV to 137 GeV. In particular, to have the lightest CP-even Higgs boson mass upper bounds larger than 146 GeV, we obtain that M_V is smaller than

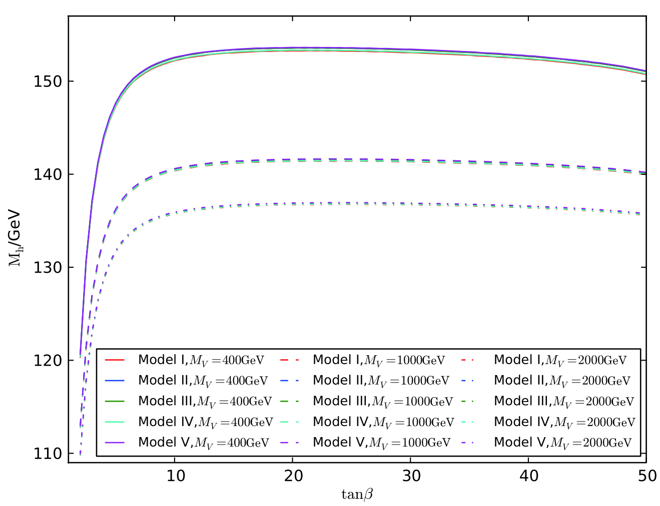

FIG. 1 (color online). . The upper bounds on the lightest CP-even Higgs boson mass versus $\tan\beta$ for our five kinds of models with $Y_{xd} = 0$, $M_S = 800$ GeV, and $M_I = 1.0 \times 10^{11}$ GeV. The upper lines, middle lines, and lower lines are for $M_V = 400$ GeV, 1000 GeV, and 2000 GeV, respectively.

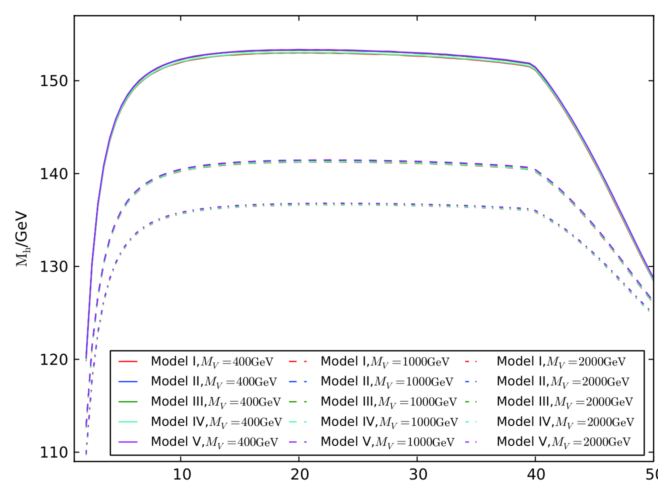

FIG. 2 (color online). . The upper bounds on the lightest CP-even Higgs boson mass versus $\tan\beta$ for our five kinds of models with $Y_{xd}(M_V) = Y_{xu}(M_V)$, $M_S = 800$ GeV, and $M_I =$ 1.0×10^{11} GeV. The upper lines, middle lines, and lower lines are for $M_V = 400$ GeV, 1000 GeV, and 2000 GeV, respectively.

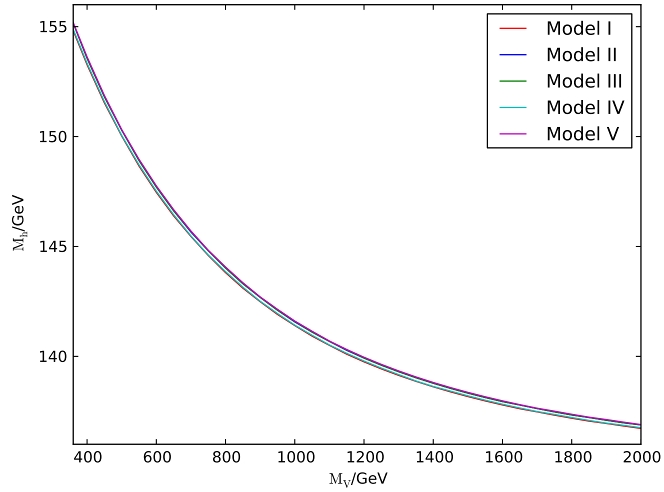

FIG. 3 (color online). . The upper bounds on the lightest CP-even Higgs boson mass versus M_V for our five kinds of models with $Y_{xd} = 0$, $\tan\beta = 20$, $M_S = 800$ GeV, and $M_I = 1.0 \times 10^{11}$ GeV.

about 700 GeV. Moreover, the maximal Yukawa couplings Y_{xu} vary only a little bit, decreasing from about 1.029 to 1.016 for M_V from 360 GeV to 2 TeV.

Fourth, we consider $Y_{xd} = 0$, $\tan\beta = 20$, and $M_I = 1.0 \times 10^{11}$ GeV. We choose three values for M_V: $M_V = 400$ GeV, 1000 GeV, and 2000 GeV. In Fig. 4, we present the upper bounds on the lightest CP-even Higgs boson mass by varying M_S from 360 GeV to 2 TeV. As the value of M_S increases, the upper bounds on the lightest CP-even Higgs boson mass increase from about 143 GeV to 162 GeV, from about 136 GeV to 150 GeV, and from about 134 GeV to 141 GeV, for $M_V = 400$ GeV, 1000 GeV, and 2000 GeV, repectively. Especially, to have the lightest CP-even Higgs boson mass upper bounds larger than 146 GeV, we obtain that M_S is larger than about 430 GeV and 1260 GeV for $M_V = 400$ GeV and 1000 GeV, respectively. Moreover, the maximal Yukawa couplings Y_{xu} decrease from about 1.049 to 1.007 for M_S from 360 GeV to 2 TeV.

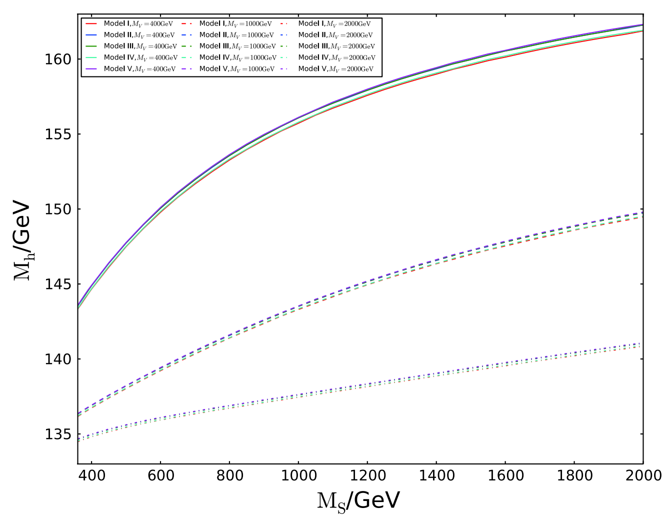

FIG. 4 (color online). . The upper bounds on the lightest CP-even Higgs boson mass versus M_S for our five kinds of models with $Y_{xd} = 0$, $\tan\beta = 20$, and $M_I = 1.0 \times 10^{11}$ GeV. The upper lines, middle lines, and lower lines are for $M_V = 400$ GeV, 1000 GeV, and 2000 GeV, respectively.

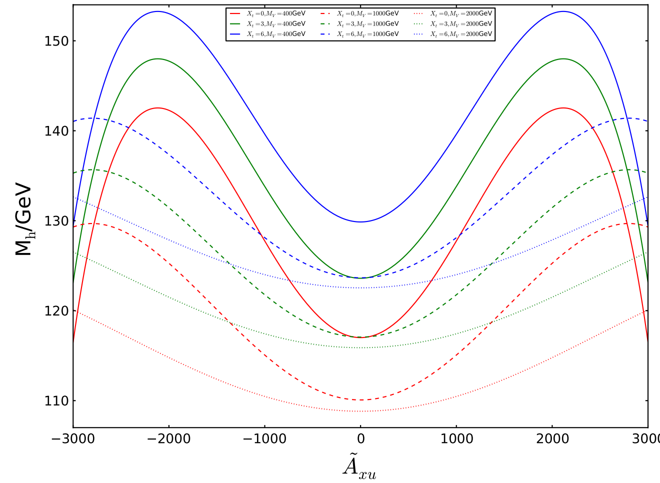

FIG. 5 (color online). . The upper bounds on the lightest CP-even Higgs boson mass versus X_{xu} in Model I with $Y_{xd} = 0$, $\tan\beta = 20$, $M_S = 800$ GeV; $M_V = 400$ GeV, 1000 GeV, and 2000 GeV; and $X_t = 0$, 3, and 6.

Fifth, we consider $Y_{xd} = 0$, $\tan\beta = 20$, and $M_S = 800$ GeV. Also, we choose three values for M_V: $M_V = 400$ GeV, 1000 GeV, and 2000 GeV, and three values for X_t: $X_t = 0$, 3, and 6. For simplicity, we only consider Model I here. In Fig. 5, we present the upper bounds on the lightest CP-even Higgs boson mass by varying $\tilde{A}_{xu}$. As we expected, they behave just like the variations of the lightest CP-even Higgs boson mass upper bounds with varying stop mixing X_t, which have been studied extensively in Refs. [43–46].

IV. CONCLUSION

We calculated the lightest CP-even Higgs boson mass in five kinds of testable flipped $SU(5) \times U(1)_X$ models from F-theory. Two kinds of models have vectorlike particles around the TeV scale, while the other three kinds also have the vectorlike particles at the intermediate scale as the messenger fields in gauge mediation. The Yukawa couplings for the TeV-scale vectorlike particles and the third family of the SM fermions are required to be smaller than three from the EW scale to the scale M_{32}. With the two-loop RGE running for both the gauge couplings and Yukawa couplings, we obtained the maximal Yukawa couplings between the TeV-scale vectorlike particles and Higgs fields. To calculate the lightest CP-even Higgs boson mass upper bounds, we used the RG-improved effective Higgs potential approach, and considered the two-loop leading contributions in the MSSM and one-loop contributions from the TeV-scale vectorlike particles. For simplicity, we assumed that the mixings both between the stops and between the TeV-scale vectorlike scalars are maximal. The numerical results for these five kinds of models are roughly the same. With M_V and M_S around 1 TeV, we showed that the lightest *CP*-even Higgs boson mass can be close to 146 GeV naturally, which is the upper bound from the current CMS and ATLAS collaborations.

ACKNOWLEDGMENTS

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Note Added.—In February 2012, the ATLAS and CMS Collaborations reported an excess of events for the SM-like Higgs boson with mass about 126 GeV and 124 GeV, respectively [51,52]. The Higgs boson mass around 125 GeV gives a very strong constraint on the viable supersymmetry parameter space. Especially in the testable flipped $SU(5)\times U(1)_X$ models with no-scale supersymmetry breaking [30,53] or gauge-mediated supersymmetry breaking [31,54], we definitely need the extra contributions to the lightest *CP*-even Higgs boson mass from the vectorlike particles, which will be studied in detail in this paper.

APPENDIX A: RENORMALIZATION GROUP EQUATIONS IN THE SM WITH VECTORLIKE PARTICLES

When $M_V < M_S$, at the renormalization scale between them, we have the Standard Model plus vectorlike particles, with the RGEs for the gauge couplings and Yukawa couplings as follows [47–50]:

$$(4\pi)^2\frac{d}{dt}g_i = b_i g_i^3 + \frac{g_i^3}{(4\pi)^2}\left[\sum_{j=1}^{3}B_{ij}g_j^2 - \sum_{\alpha=u,d,e,xu,xd} d_i^\alpha \mathrm{Tr}(Y_\alpha^\dagger Y_\alpha)\right], \tag{A1}$$

where $t=\ln\mu$ and μ is the renormalization scale. The g_1, g_2, and g_3 are the gauge couplings for $U(1)_Y$, $SU(2)_L$, and $SU(3)_C$, respectively, where we use the $SU(5)$ normalization $g_1^2\equiv(5/3)g_Y^2$. The beta-function coefficients are

$$b=\left(\tfrac{41}{10}, -\tfrac{19}{6}, -7\right),\qquad B=\begin{pmatrix}\frac{199}{50} & \frac{27}{10} & \frac{44}{5}\\ \frac{9}{10} & \frac{35}{6} & 12\\ \frac{11}{10} & \frac{9}{2} & -26\end{pmatrix}, \tag{A2}$$

$$d^u=\left(\tfrac{17}{10}, \tfrac{3}{2}, 2\right),\qquad d^d=d^{xu}=d^{xd}=\left(\tfrac{1}{2}, \tfrac{3}{2}, 2\right),$$
$$d^e=\left(\tfrac{3}{2}, \tfrac{1}{2}, 2\right). \tag{A3}$$

And

$$\frac{d}{dt}Y_{u,d,e,xu,xd} = \frac{1}{16\pi^2}Y_{u,d,e,xu,xd}\beta^{(1)}_{u,d,e,xu,xd}, \tag{A4}$$

where

$$\beta_u^{(1)} = \frac{3}{2}(Y_u^\dagger Y_u - Y_d^\dagger Y_d) + Y_2 - \left(\frac{17}{20}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2\right), \tag{A5}$$

$$\beta_d^{(1)} = \frac{3}{2}(Y_d^\dagger Y_d - Y_u^\dagger Y_u) + Y_2 - \left(\frac{1}{4}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2\right), \tag{A6}$$

$$\beta_e^{(1)} = \frac{3}{2}Y_e^\dagger Y_e + Y_2 - \frac{9}{4}(g_1^2 + g_2^2), \tag{A7}$$

$$\beta_{xu}^{(1)} = \frac{3}{2}Y_{xu}^\dagger Y_{xu} + Y_2 - \left(\frac{1}{4}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2\right), \tag{A8}$$

$$\beta_{xd}^{(1)} = \frac{3}{2}Y_{xd}^\dagger Y_{xd} + Y_2 - \left(\frac{1}{4}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2\right), \tag{A9}$$

with

$$Y_2 = Tr\{3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d + Y_e^\dagger Y_e\} + 3Y_{xu}^\dagger Y_{xu} + 3Y_{xd}^\dagger Y_{xd}. \tag{A10}$$

APPENDIX B: RENORMALIZATION GROUP EQUATIONS IN MODEL I

In Model I, the two-loop renormalization group equations for the gauge couplings are

$$(4\pi)^2\frac{d}{dt}g_i = b_i g_i^3 + \frac{g_i^3}{(4\pi)^2}\left[\sum_{j=1}^{3}B_{ij}g_j^2 - \sum_{\alpha=u,d,e,xu,xd} d_i^\alpha \mathrm{Tr}(Y_\alpha^\dagger Y_\alpha)\right], \tag{B1}$$

where Y_u, Y_d, Y_e, Y_{xu}, and Y_{xd} are the Yukawa couplings for the up-type quark, down-type quark, lepton, vectorlike particles $\overline{XF}$, and vectorlike particles XF, respectively. The beta-function coefficients are

$$b=\left(\tfrac{33}{5}, 1, -3\right) + \left(\tfrac{3}{5}, 3, 3\right), \tag{B2}$$

$$B=\begin{pmatrix}\frac{199}{25} & \frac{27}{5} & \frac{88}{5}\\ \frac{9}{5} & 25 & 24\\ \frac{11}{5} & 9 & 14\end{pmatrix} + \begin{pmatrix}\frac{3}{25} & \frac{3}{5} & \frac{16}{5}\\ \frac{1}{5} & 21 & 16\\ \frac{2}{5} & 6 & 34\end{pmatrix}, \tag{B3}$$

$$d^u=\left(\tfrac{26}{5}, 6, 4\right),\qquad d^d=\left(\tfrac{14}{5}, 6, 4\right),\qquad d^e=\left(\tfrac{18}{5}, 2, 0\right), \tag{B4}$$

$$d^{xu}=\left(\tfrac{14}{5}, 6, 4\right),\qquad d^{xd}=\left(\tfrac{14}{5}, 6, 4\right). \tag{B5}$$

The two-loop renormalization group equations for Yukawa couplings are

$$(4\pi)^2 \frac{d}{dt} Y_\alpha = \frac{1}{16\pi^2} \beta^{(1)}_{Y_\alpha} + \frac{1}{(16\pi^2)^2} \beta^{(2)}_{Y_\alpha}, \tag{B6}$$

where $\alpha = u, d, e, xu, xd$. In addition, $\beta^{(1)}_{Y_\alpha}$ and $\beta^{(2)}_{Y_\alpha}$ are given as follows:

$$\beta^{(1)}_{Y_u} = Y_u\Big(3\,\mathrm{Tr}(Y_u Y_u^\dagger) + 3Y_u^\dagger Y_u + Y_d^\dagger Y_d + 3Y_{xu}^\dagger Y_{xu} - \tfrac{16}{3}g_3^2 - 3g_2^2 - \tfrac{13}{15}g_1^2\Big), \tag{B7}$$

$$\begin{aligned}\beta^{(2)}_{Y_u} = Y_u\Big(&-3\,\mathrm{Tr}(3Y_uY_u^\dagger Y_uY_u^\dagger + Y_uY_d^\dagger Y_dY_u^\dagger) - 9Y_{xu}^\dagger Y_{xu}Y_{xu}^\dagger Y_{xu} - 9Y_u^\dagger Y_u\mathrm{Tr}(Y_uY_u^\dagger) - 9Y_u^\dagger Y_uY_{xu}^\dagger Y_{xu}\\ &- Y_d^\dagger Y_d\mathrm{Tr}(3Y_dY_d^\dagger + Y_eY_e^\dagger) - 3Y_d^\dagger Y_dY_{xd}Y_{xd}^\dagger - 4Y_u^\dagger Y_uY_u^\dagger Y_u - 2Y_d^\dagger Y_dY_d^\dagger Y_d - 2Y_d^\dagger Y_dY_u^\dagger Y_u\\ &+ \Big(16g_3^2 + \frac{4}{5}g_1^2\Big)\mathrm{Tr}(Y_uY_u^\dagger) + \Big(16g_3^2 - \frac{2}{5}g_1^2\Big)Y_{xu}^\dagger Y_{xu} + \Big(6g_2^2 + \frac{2}{5}g_1^2\Big)Y_u^\dagger Y_u + \frac{2}{5}g_1^2Y_d^\dagger Y_d + \frac{128}{9}g_3^4\\ &+ 8g_3^2g_2^2 + \frac{136}{45}g_3^2g_1^2 + \frac{33}{2}g_2^4 + g_2^2g_1^2 + \frac{2977}{450}g_1^4\Big),\end{aligned} \tag{B8}$$

$$\beta^{(1)}_{Y_d} = Y_d\Big(\mathrm{Tr}(3Y_dY_d^\dagger + Y_eY_e^\dagger) + 3Y_d^\dagger Y_d + Y_u^\dagger Y_u + 3Y_{xd}^\dagger Y_{xd} - \tfrac{16}{3}g_3^2 - 3g_2^2 - \tfrac{7}{15}g_1^2\Big), \tag{B9}$$

$$\beta^{(1)}_{Y_e} = Y_e\Big(\mathrm{Tr}(3Y_dY_d^\dagger + Y_eY_e^\dagger) + 3Y_e^\dagger Y_e + 3Y_{xd}^\dagger Y_{xd} - 3g_2^2 - \tfrac{9}{5}g_1^2\Big), \tag{B11}$$

$$\begin{aligned}\beta^{(2)}_{Y_d} = Y_d\Big(&-3\,\mathrm{Tr}(3Y_dY_d^\dagger Y_dY_d^\dagger + Y_dY_u^\dagger Y_uY_d^\dagger + Y_eY_e^\dagger Y_eY_e^\dagger) - 9Y_{xd}^\dagger Y_{xd}Y_{xd}^\dagger Y_{xd} - 3Y_u^\dagger Y_u\mathrm{Tr}(Y_uY_u^\dagger)\\ &- 3Y_u^\dagger Y_uY_{xu}Y_{xu}^\dagger - 3Y_d^\dagger Y_d\mathrm{Tr}(3Y_dY_d^\dagger + Y_eY_e^\dagger) - 9Y_d^\dagger Y_dY_{xd}^\dagger Y_{xd} - 4Y_d^\dagger Y_dY_d^\dagger Y_d - 2Y_u^\dagger Y_uY_u^\dagger Y_u - 2Y_u^\dagger Y_uY_d^\dagger Y_d\\ &+ \Big(16g_3^2 - \frac{2}{5}g_1^2\Big)\mathrm{Tr}(Y_dY_d^\dagger) + \frac{6}{5}g_1^2\mathrm{Tr}(Y_eY_e^\dagger) + \Big(16g_3^2 - \frac{2}{5}g_1^2\Big)Y_{xd}^\dagger Y_{xd} + \Big(6g_2^2 + \frac{4}{5}g_1^2\Big)Y_d^\dagger Y_d + \frac{4}{5}g_1^2Y_u^\dagger Y_u\\ &+ \frac{128}{9}g_3^4 + 8g_3^2g_2^2 + \frac{8}{9}g_3^2g_1^2 + \frac{33}{2}g_2^4 + g_2^2g_1^2 + \frac{1561}{450}g_1^4\Big),\end{aligned} \tag{B10}$$

$$\begin{aligned}\beta^{(2)}_{Y_e} = Y_e\Big(&-3\,\mathrm{Tr}(3Y_dY_d^\dagger Y_dY_d^\dagger + Y_dY_u^\dagger Y_uY_d^\dagger + Y_eY_e^\dagger Y_eY_e^\dagger) - 9Y_{xd}^\dagger Y_{xd}Y_{xd}^\dagger Y_{xd} - 3Y_e^\dagger Y_e\mathrm{Tr}(3Y_dY_d^\dagger + Y_eY_e^\dagger)\\ &- 9Y_e^\dagger Y_eY_{xd}^\dagger Y_{xd} - 4Y_e^\dagger Y_eY_e^\dagger Y_e + \Big(16g_3^2 - \tfrac{2}{5}g_1^2\Big)\mathrm{Tr}(Y_dY_d^\dagger) + \tfrac{6}{5}g_1^2\mathrm{Tr}(Y_eY_e^\dagger) + \Big(16g_3^2 - \tfrac{2}{5}g_1^2\Big)Y_{xd}^\dagger Y_{xd}\\ &+ 6g_2^2Y_e^\dagger Y_e + \tfrac{33}{2}g_2^4 + \tfrac{9}{5}g_2^2g_1^2 + \tfrac{729}{50}g_1^4\Big),\end{aligned} \tag{B12}$$

$$\beta^{(1)}_{Y_{xu}} = Y_{xu}\Big(3\,\mathrm{Tr}(Y_uY_u^\dagger) + 6Y_{xu}^\dagger Y_{xu} - \tfrac{16}{3}g_3^2 - 3g_2^2 - \tfrac{7}{15}g_1^2\Big), \tag{B13}$$

$$\begin{aligned}\beta^{(2)}_{Y_{xu}} = Y_{xu}\Big(&-3\,\mathrm{Tr}(3Y_uY_u^\dagger Y_uY_u^\dagger + Y_uY_d^\dagger Y_dY_u^\dagger) - 22Y_{xu}^\dagger Y_{xu}Y_{xu}^\dagger Y_{xu} - 9Y_{xu}^\dagger Y_{xu}\mathrm{Tr}(Y_uY_u^\dagger) + \Big(16g_3^2 + \tfrac{4}{5}g_1^2\Big)\mathrm{Tr}(Y_uY_u^\dagger)\\ &+ \Big(16g_3^2 + 6g_2^2 + \tfrac{2}{5}g_1^2\Big)Y_{xu}^\dagger Y_{xu} + \tfrac{128}{9}g_3^4 + 8g_3^2g_2^2 + \tfrac{8}{9}g_3^2g_1^2 + \tfrac{33}{2}g_2^4 + g_2^2g_1^2 + \tfrac{1561}{450}g_1^4\Big),\end{aligned} \tag{B14}$$

$$\beta^{(1)}_{Y_{xd}} = Y_{xd}\Big(\mathrm{Tr}(3Y_dY_d^\dagger + Y_eY_e^\dagger) + 6Y_{xd}^\dagger Y_{xd} - \tfrac{16}{3}g_3^2 - 3g_2^2 - \tfrac{7}{15}g_1^2\Big), \tag{B15}$$

$$\begin{aligned}\beta^{(2)}_{Y_{xd}} = Y_{xd}\Big(&-3\,\mathrm{Tr}(3Y_dY_d^\dagger Y_dY_d^\dagger + Y_dY_u^\dagger Y_uY_d^\dagger + Y_eY_e^\dagger Y_eY_e^\dagger) - 22Y_{xd}^\dagger Y_{xd}Y_{xd}^\dagger Y_{xd} - 3Y_{xd}^\dagger Y_{xd}\mathrm{Tr}(3Y_dY_d^\dagger + Y_eY_e^\dagger)\\ &+ \Big(16g_3^2 - \tfrac{2}{5}g_1^2\Big)\mathrm{Tr}(Y_dY_d^\dagger) + \tfrac{6}{5}g_1^2\mathrm{Tr}(Y_eY_e^\dagger) + \Big(16g_3^2 + 6g_2^2 + \frac{2}{5}g_1^2\Big)Y_{xd}^\dagger Y_{xd} + \tfrac{128}{9}g_3^4 + 8g_3^2g_2^2 + \tfrac{8}{9}g_3^2g_1^2\\ &+ \tfrac{33}{2}g_2^4 + g_2^2g_1^2 + \tfrac{1561}{450}g_1^4\Big).\end{aligned} \tag{B16}$$

APPENDIX C: RENORMALIZATION GROUP EQUATIONS IN MODEL II

In Model II, below the intermediate scale $M_I = 1.0 \times 10^{11}$ GeV, we have the same RGEs as in Model I. Above M_I, we have additional vectorlike particles $(Xf, \overline{Xf})$. Thus, we need to add extra contributions to b and B from the vectorlike particles $(Xf, \overline{Xf})$. Comparing to the RGEs in Model I, we also need to change the coefficients of the g_3^4, g_2^4 and g_1^4 terms in $\beta^{(2)}_{Y_u}$, $\beta^{(2)}_{Y_d}$, $\beta^{(2)}_{Y_{xu}}$, and $\beta^{(2)}_{Y_{xd}}$, and change the coefficients of the g_2^4 and g_1^4 terms in $\beta^{(2)}_{Y_e}$. In short, comparing to the RGEs in Model I, the coefficients in the RGEs above M_I, which need to be changed, are the following:

$$b = \left(\tfrac{33}{5}, 1, -3\right) + \left(\tfrac{3}{5}, 3, 3\right) + \left(\tfrac{11}{5}, 1, 1\right), \tag{C1}$$

$$B = \begin{pmatrix} \frac{199}{25} & \frac{27}{5} & \frac{88}{5} \\ \frac{9}{5} & 25 & 24 \\ \frac{11}{5} & 9 & 14 \end{pmatrix} + \begin{pmatrix} \frac{3}{25} & \frac{3}{5} & \frac{16}{5} \\ \frac{1}{5} & 21 & 16 \\ \frac{2}{5} & 6 & 34 \end{pmatrix} + \begin{pmatrix} \frac{31}{15} & \frac{9}{5} & \frac{128}{15} \\ \frac{3}{5} & 7 & 0 \\ \frac{16}{15} & 0 & \frac{34}{3} \end{pmatrix}, \tag{C2}$$

$$\begin{aligned}\beta^{(2)}_{Y_u} = Y_u\Big(&-3\,\mathrm{Tr}(3Y_uY_u^\dagger Y_uY_u^\dagger + Y_uY_d^\dagger Y_dY_u^\dagger) - 9Y_{xu}^\dagger Y_{xu}Y_{xu}^\dagger Y_{xu} - 9Y_u^\dagger Y_u\mathrm{Tr}(Y_uY_u^\dagger) - 9Y_u^\dagger Y_uY_{xu}^\dagger Y_{xu} \\ &- Y_d^\dagger Y_d\mathrm{Tr}(3Y_dY_d^\dagger + Y_eY_e^\dagger) - 3Y_d^\dagger Y_dY_{xd}Y_{xd}^\dagger - 4Y_u^\dagger Y_uY_u^\dagger Y_u - 2Y_d^\dagger Y_dY_d^\dagger Y_d - 2Y_d^\dagger Y_dY_u^\dagger Y_u + \Big(16g_3^2 + \tfrac{4}{5}g_1^2\Big)\mathrm{Tr}(Y_uY_u^\dagger) \\ &+ \Big(16g_3^2 - \tfrac{2}{5}g_1^2\Big)Y_{xu}^\dagger Y_{xu} + \Big(6g_2^2 + \tfrac{2}{5}g_1^2\Big)Y_u^\dagger Y_u + \tfrac{2}{5}g_1^2Y_d^\dagger Y_d + \tfrac{176}{9}g_3^4 + 8g_3^2g_2^2 + \tfrac{136}{45}g_3^2g_1^2 + \tfrac{39}{2}g_2^4 + g_2^2g_1^2 + \tfrac{767}{90}g_1^4\Big),\end{aligned} \tag{C3}$$

$$\begin{aligned}\beta^{(2)}_{Y_d} = Y_d\Big(&-3\,\mathrm{Tr}(3Y_dY_d^\dagger Y_dY_d^\dagger + Y_dY_u^\dagger Y_uY_d^\dagger + Y_eY_e^\dagger Y_eY_e^\dagger) - 9Y_{xd}^\dagger Y_{xd}Y_{xd}^\dagger Y_{xd} - 3Y_u^\dagger Y_u\mathrm{Tr}(Y_uY_u^\dagger) - 3Y_u^\dagger Y_uY_{xu}Y_{xu}^\dagger \\ &- 3Y_d^\dagger Y_d\mathrm{Tr}(3Y_dY_d^\dagger + Y_eY_e^\dagger) - 9Y_d^\dagger Y_dY_{xd}^\dagger Y_{xd} - 4Y_d^\dagger Y_dY_d^\dagger Y_d - 2Y_u^\dagger Y_uY_u^\dagger Y_u - 2Y_u^\dagger Y_uY_d^\dagger Y_d \\ &+ \Big(16g_3^2 - \tfrac{2}{5}g_1^2\Big)\mathrm{Tr}(Y_dY_d^\dagger) + \tfrac{6}{5}g_1^2\mathrm{Tr}(Y_eY_e^\dagger) + \Big(16g_3^2 - \tfrac{2}{5}g_1^2\Big)Y_{xd}^\dagger Y_{xd} + (6g_2^2 + \tfrac{4}{5}g_1^2)Y_d^\dagger Y_d \\ &+ \tfrac{4}{5}g_1^2Y_u^\dagger Y_u + \tfrac{176}{9}g_3^4 + 8g_3^2g_2^2 + \tfrac{8}{9}g_3^2g_1^2 + \tfrac{39}{2}g_2^4 + g_2^2g_1^2 + \tfrac{2023}{450}g_1^4\Big),\end{aligned} \tag{C4}$$

$$\begin{aligned}\beta^{(2)}_{Y_e} = Y_e\Big(&-3\,\mathrm{Tr}(3Y_dY_d^\dagger Y_dY_d^\dagger + Y_dY_u^\dagger Y_uY_d^\dagger + Y_eY_e^\dagger Y_eY_e^\dagger) - 9Y_{xd}^\dagger Y_{xd}Y_{xd}^\dagger Y_{xd} - 3Y_e^\dagger Y_e\mathrm{Tr}(3Y_dY_d^\dagger + Y_eY_e^\dagger) \\ &- 9Y_e^\dagger Y_eY_{xd}^\dagger Y_{xd} - 4Y_e^\dagger Y_eY_e^\dagger Y_e + \Big(16g_3^2 - \tfrac{2}{5}g_1^2\Big)\mathrm{Tr}(Y_dY_d^\dagger) + \tfrac{6}{5}g_1^2\mathrm{Tr}(Y_eY_e^\dagger) + \Big(16g_3^2 - \tfrac{2}{5}g_1^2\Big)Y_{xd}^\dagger Y_{xd} \\ &+ 6g_2^2Y_e^\dagger Y_e + \tfrac{39}{2}g_2^4 + \tfrac{9}{5}g_2^2g_1^2 + \tfrac{927}{50}g_1^4\Big),\end{aligned} \tag{C5}$$

$$\begin{aligned}\beta^{(2)}_{Y_{xu}} = Y_{xu}\Big(&-3\,\mathrm{Tr}(3Y_uY_u^\dagger Y_uY_u^\dagger + Y_uY_d^\dagger Y_dY_u^\dagger) - 22Y_{xu}^\dagger Y_{xu}Y_{xu}^\dagger Y_{xu} - 9Y_{xu}^\dagger Y_{xu}\mathrm{Tr}(Y_uY_u^\dagger) + \Big(16g_3^2 + \tfrac{4}{5}g_1^2\Big)\mathrm{Tr}(Y_uY_u^\dagger) \\ &+ \Big(16g_3^2 + 6g_2^2 + \tfrac{2}{5}g_1^2\Big)Y_{xu}^\dagger Y_{xu} + \tfrac{176}{9}g_3^4 + 8g_3^2g_2^2 + \tfrac{8}{9}g_3^2g_1^2 + \tfrac{39}{2}g_2^4 + g_2^2g_1^2 + \tfrac{2023}{450}g_1^4\Big),\end{aligned} \tag{C6}$$

$$\begin{aligned}\beta^{(2)}_{Y_{xd}} = Y_{xd}\Big(&-3\,\mathrm{Tr}(3Y_dY_d^\dagger Y_dY_d^\dagger + Y_dY_u^\dagger Y_uY_d^\dagger + Y_eY_e^\dagger Y_eY_e^\dagger) - 22Y_{xd}^\dagger Y_{xd}Y_{xd}^\dagger Y_{xd} - 3Y_{xd}^\dagger Y_{xd}\mathrm{Tr}(3Y_dY_d^\dagger + Y_eY_e^\dagger) \\ &+ \Big(16g_3^2 - \tfrac{2}{5}g_1^2\Big)\mathrm{Tr}(Y_dY_d^\dagger) + \tfrac{6}{5}g_1^2\mathrm{Tr}(Y_eY_e^\dagger) + \Big(16g_3^2 + 6g_2^2 + \tfrac{2}{5}g_1^2\Big)Y_{xd}^\dagger Y_{xd} \\ &+ \tfrac{176}{9}g_3^4 + 8g_3^2g_2^2 + \tfrac{8}{9}g_3^2g_1^2 + \tfrac{39}{2}g_2^4 + g_2^2g_1^2 + \tfrac{2023}{450}g_1^4\Big).\end{aligned} \tag{C7}$$

APPENDIX D: RENORMALIZATION GROUP EQUATIONS IN MODEL III

In Model III, below the intermediate scale $M_I = 1.0 \times 10^{11}$ GeV, we have the same RGEs as in Model I. Above M_I, we have additional vectorlike particles $(Xf, \overline{Xf})$ and $(Xl, \overline{Xl})$. Thus, we need to add extra contributions to b and B from the vectorlike particles $(Xf, \overline{Xf})$ and $(Xl, \overline{Xl})$. Comparing to the RGEs in Model I, we also need to change the coefficients of the g_3^4, g_2^4 and g_1^4 terms in $\beta_{Y_u}^{(2)}$, $\beta_{Y_d}^{(2)}$, $\beta_{Y_{xu}}^{(2)}$, and $\beta_{Y_{xd}}^{(2)}$, and change the coefficients of the g_2^4 and g_1^4 terms in $\beta_{Y_e}^{(2)}$. In short, comparing to the RGEs in Model I, the coefficients in the RGEs above M_I, which need to be changed, are the following:

$$b = \left(\tfrac{33}{5}, 1, -3\right) + \left(\tfrac{3}{5}, 3, 3\right) + \left(\tfrac{17}{5}, 1, 1\right), \tag{D1}$$

$$B = \begin{pmatrix} \frac{199}{25} & \frac{27}{5} & \frac{88}{5} \\ \frac{9}{5} & 25 & 24 \\ \frac{11}{5} & 9 & 14 \end{pmatrix} + \begin{pmatrix} \frac{3}{25} & \frac{3}{5} & \frac{16}{5} \\ \frac{1}{5} & 21 & 16 \\ \frac{2}{5} & 6 & 34 \end{pmatrix} + \begin{pmatrix} \frac{371}{15} & \frac{9}{5} & \frac{128}{15} \\ \frac{3}{5} & 7 & 0 \\ \frac{16}{15} & 0 & \frac{34}{3} \end{pmatrix}, \tag{D2}$$

$$\begin{aligned} \beta_{Y_u}^{(2)} = Y_u\Big(&-3\,\mathrm{Tr}(3Y_uY_u^\dagger Y_uY_u^\dagger + Y_uY_d^\dagger Y_dY_u^\dagger) - 9Y_{xu}^\dagger Y_{xu}Y_{xu}^\dagger Y_{xu} - 9Y_u^\dagger Y_u\mathrm{Tr}(Y_uY_u^\dagger) - 9Y_u^\dagger Y_uY_{xu}^\dagger Y_{xu} \\ &- Y_d^\dagger Y_d\mathrm{Tr}(3Y_dY_d^\dagger + Y_eY_e^\dagger) - 3Y_d^\dagger Y_dY_{xd}Y_{xd}^\dagger - 4Y_u^\dagger Y_uY_u^\dagger Y_u - 2Y_d^\dagger Y_dY_d^\dagger Y_d - 2Y_d^\dagger Y_dY_u^\dagger Y_u + \left(16g_3^2 + \tfrac{4}{5}g_1^2\right)\mathrm{Tr}(Y_uY_u^\dagger) \\ &+ \left(16g_3^2 - \tfrac{2}{5}g_1^2\right)Y_{xu}^\dagger Y_{xu} + \left(6g_2^2 + \tfrac{2}{5}g_1^2\right)Y_u^\dagger Y_u + \tfrac{2}{5}g_1^2Y_d^\dagger Y_d + \tfrac{176}{9}g_3^4 + 8g_3^2g_2^2 + \tfrac{136}{45}g_3^2g_1^2 + \tfrac{39}{2}g_2^4 + g_2^2g_1^2 + \tfrac{4303}{450}g_1^4\Big), \end{aligned} \tag{D3}$$

$$\begin{aligned} \beta_{Y_d}^{(2)} = Y_d\Big(&-3\,\mathrm{Tr}(3Y_dY_d^\dagger Y_dY_d^\dagger + Y_dY_u^\dagger Y_uY_d^\dagger + Y_eY_e^\dagger Y_eY_e^\dagger) - 9Y_{xd}^\dagger Y_{xd}Y_{xd}^\dagger Y_{xd} - 3Y_u^\dagger Y_u\mathrm{Tr}(Y_uY_u^\dagger) - 3Y_u^\dagger Y_uY_{xu}Y_{xu}^\dagger \\ &- 3Y_d^\dagger Y_d\mathrm{Tr}(3Y_dY_d^\dagger + Y_eY_e^\dagger) - 9Y_d^\dagger Y_dY_{xd}^\dagger Y_{xd} - 4Y_d^\dagger Y_dY_d^\dagger Y_d - 2Y_u^\dagger Y_uY_u^\dagger Y_u - 2Y_u^\dagger Y_uY_d^\dagger Y_d \\ &+ \left(16g_3^2 - \tfrac{2}{5}g_1^2\right)\mathrm{Tr}(Y_dY_d^\dagger) + \tfrac{6}{5}g_1^2\mathrm{Tr}(Y_eY_e^\dagger) + \left(16g_3^2 - \tfrac{2}{5}g_1^2\right)Y_{xd}^\dagger Y_{xd} + \left(6g_2^2 + \tfrac{4}{5}g_1^2\right)Y_d^\dagger Y_d \\ &+ \tfrac{4}{5}g_1^2Y_u^\dagger Y_u + \tfrac{176}{9}g_3^4 + 8g_3^2g_2^2 + \tfrac{8}{9}g_3^2g_1^2 + \tfrac{39}{2}g_2^4 + g_2^2g_1^2 + \tfrac{91}{18}g_1^4\Big), \end{aligned} \tag{D4}$$

$$\begin{aligned} \beta_{Y_e}^{(2)} = Y_e\Big(&-3\,\mathrm{Tr}(3Y_dY_d^\dagger Y_dY_d^\dagger + Y_dY_u^\dagger Y_uY_d^\dagger + Y_eY_e^\dagger Y_eY_e^\dagger) - 9Y_{xd}^\dagger Y_{xd}Y_{xd}^\dagger Y_{xd} - 3Y_e^\dagger Y_e\mathrm{Tr}(3Y_dY_d^\dagger + Y_eY_e^\dagger) \\ &- 9Y_e^\dagger Y_eY_{xd}^\dagger Y_{xd} - 4Y_e^\dagger Y_eY_e^\dagger Y_e + \left(16g_3^2 - \tfrac{2}{5}g_1^2\right)\mathrm{Tr}(Y_dY_d^\dagger) + \tfrac{6}{5}g_1^2\mathrm{Tr}(Y_eY_e^\dagger) + \left(16g_3^2 - \tfrac{2}{5}g_1^2\right)Y_{xd}^\dagger Y_{xd} \\ &+ 6g_2^2Y_e^\dagger Y_e + \tfrac{39}{2}g_2^4 + \tfrac{9}{5}g_2^2g_1^2 + \tfrac{207}{10}g_1^4\Big), \end{aligned} \tag{D5}$$

$$\begin{aligned} \beta_{Y_{xu}}^{(2)} = Y_{xu}\Big(&-3\,\mathrm{Tr}(3Y_uY_u^\dagger Y_uY_u^\dagger + Y_uY_d^\dagger Y_dY_u^\dagger) - 22Y_{xu}^\dagger Y_{xu}Y_{xu}^\dagger Y_{xu} - 9Y_{xu}^\dagger Y_{xu}\mathrm{Tr}(Y_uY_u^\dagger) + \left(16g_3^2 + \tfrac{4}{5}g_1^2\right)\mathrm{Tr}(Y_uY_u^\dagger) \\ &+ \left(16g_3^2 + 6g_2^2 + \tfrac{2}{5}g_1^2\right)Y_{xu}^\dagger Y_{xu} + \tfrac{176}{9}g_3^4 + 8g_3^2g_2^2 + \tfrac{8}{9}g_3^2g_1^2 + \tfrac{39}{2}g_2^4 + g_2^2g_1^2 + \tfrac{91}{18}g_1^4\Big), \end{aligned} \tag{D6}$$

$$\begin{aligned} \beta_{Y_{xd}}^{(2)} = Y_{xd}\Big(&-3\,\mathrm{Tr}(3Y_dY_d^\dagger Y_dY_d^\dagger + Y_dY_u^\dagger Y_uY_d^\dagger + Y_eY_e^\dagger Y_eY_e^\dagger) - 22Y_{xd}^\dagger Y_{xd}Y_{xd}^\dagger Y_{xd} - 3Y_{xd}^\dagger Y_{xd}\mathrm{Tr}(3Y_dY_d^\dagger + Y_eY_e^\dagger) \\ &+ \left(16g_3^2 - \tfrac{2}{5}g_1^2\right)\mathrm{Tr}(Y_dY_d^\dagger) + \tfrac{6}{5}g_1^2\mathrm{Tr}(Y_eY_e^\dagger) + \left(16g_3^2 + 6g_2^2 + \tfrac{2}{5}g_1^2\right)Y_{xd}^\dagger Y_{xd} \\ &+ \tfrac{176}{9}g_3^4 + 8g_3^2g_2^2 + \tfrac{8}{9}g_3^2g_1^2 + \tfrac{39}{2}g_2^4 + g_2^2g_1^2 + \tfrac{91}{18}g_1^4\Big). \end{aligned} \tag{D7}$$

APPENDIX E: RENORMALIZATION GROUP EQUATIONS IN MODEL IV

In Model IV, we have additional vectorlike particles Xl and $\overline{Xl}$. Thus, we need to add extra contributions to b and B from the vectorlike particles Xl and $\overline{Xl}$. Comparing to the RGEs in Model I, we also need to change the coefficients of the g_1^4 terms in $\beta^{(2)}_{Y_u}$, $\beta^{(2)}_{Y_d}$, $\beta^{(2)}_{Y_e}$, $\beta^{(2)}_{Y_{xu}}$, and $\beta^{(2)}_{Y_{xd}}$. In short, comparing to the RGEs in Model I, the corresponding coefficients of the RGEs, which need to be changed, are the following:

$$b = \left(\tfrac{33}{5}, 1, -3\right) + \left(\tfrac{3}{5}, 3, 3\right) + \left(\tfrac{6}{5}, 0, 0\right), \tag{E1}$$

$$B = \begin{pmatrix} \frac{199}{25} & \frac{27}{5} & \frac{88}{5} \\ \frac{9}{5} & 25 & 24 \\ \frac{11}{5} & 9 & 14 \end{pmatrix} + \begin{pmatrix} \frac{3}{25} & \frac{3}{5} & \frac{16}{5} \\ \frac{1}{5} & 21 & 16 \\ \frac{2}{5} & 6 & 34 \end{pmatrix} + \begin{pmatrix} \frac{36}{25} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{E2}$$

$$\begin{aligned}\beta^{(2)}_{Y_u} = Y_u\Big(&-3\,\mathrm{Tr}(3Y_uY_u^\dagger Y_uY_u^\dagger + Y_uY_d^\dagger Y_dY_u^\dagger) - 9Y_{xu}^\dagger Y_{xu}Y_{xu}^\dagger Y_{xu} - 9Y_u^\dagger Y_u\mathrm{Tr}(Y_uY_u^\dagger) - 9Y_u^\dagger Y_uY_{xu}^\dagger Y_{xu} \\ &- Y_d^\dagger Y_d\mathrm{Tr}(3Y_dY_d^\dagger + Y_eY_e^\dagger) - 3Y_d^\dagger Y_dY_{xd}Y_{xd}^\dagger - 4Y_u^\dagger Y_uY_u^\dagger Y_u - 2Y_d^\dagger Y_dY_d^\dagger Y_d - 2Y_d^\dagger Y_dY_u^\dagger Y_u + \left(16g_3^2 + \tfrac{4}{5}g_1^2\right)\mathrm{Tr}(Y_uY_u^\dagger) \\ &+ \left(16g_3^2 - \tfrac{2}{5}g_1^2\right)Y_{xu}^\dagger Y_{xu} + \left(6g_2^2 + \tfrac{2}{5}g_1^2\right)Y_u^\dagger Y_u + \tfrac{2}{5}g_1^2Y_d^\dagger Y_d + \tfrac{128}{9}g_3^4 + 8g_3^2g_2^2 + \tfrac{136}{45}g_3^2g_1^2 + \tfrac{33}{2}g_2^4 + g_2^2g_1^2 + \tfrac{689}{90}g_1^4\Big),\end{aligned} \tag{E3}$$

$$\begin{aligned}\beta^{(2)}_{Y_d} = Y_d\Big(&-3\,\mathrm{Tr}(3Y_dY_d^\dagger Y_dY_d^\dagger + Y_dY_u^\dagger Y_uY_d^\dagger + Y_eY_e^\dagger Y_eY_e^\dagger) - 9Y_{xd}^\dagger Y_{xd}Y_{xd}^\dagger Y_{xd} - 3Y_u^\dagger Y_u\mathrm{Tr}(Y_uY_u^\dagger) - 3Y_u^\dagger Y_uY_{xu}Y_{xu}^\dagger \\ &- 3Y_d^\dagger Y_d\mathrm{Tr}(3Y_dY_d^\dagger + Y_eY_e^\dagger) - 9Y_d^\dagger Y_dY_{xd}^\dagger Y_{xd} - 4Y_d^\dagger Y_dY_d^\dagger Y_d - 2Y_u^\dagger Y_uY_u^\dagger Y_u - 2Y_u^\dagger Y_uY_d^\dagger Y_d \\ &+ \left(16g_3^2 - \tfrac{2}{5}g_1^2\right)\mathrm{Tr}(Y_dY_d^\dagger) + \tfrac{6}{5}g_1^2\mathrm{Tr}(Y_eY_e^\dagger) + \left(16g_3^2 - \tfrac{2}{5}g_1^2\right)Y_{xd}^\dagger Y_{xd} + \left(6g_2^2 + \tfrac{4}{5}g_1^2\right)Y_d^\dagger Y_d \\ &+ \tfrac{4}{5}g_1^2Y_u^\dagger Y_u + \tfrac{128}{9}g_3^4 + 8g_3^2g_2^2 + \tfrac{8}{9}g_3^2g_1^2 + \tfrac{33}{2}g_2^4 + g_2^2g_1^2 + \tfrac{1813}{450}g_1^4\Big),\end{aligned} \tag{E4}$$

$$\begin{aligned}\beta^{(2)}_{Y_e} = Y_e\Big(&-3\,\mathrm{Tr}(3Y_dY_d^\dagger Y_dY_d^\dagger + Y_dY_u^\dagger Y_uY_d^\dagger + Y_eY_e^\dagger Y_eY_e^\dagger) - 9Y_{xd}^\dagger Y_{xd}Y_{xd}^\dagger Y_{xd} - 3Y_e^\dagger Y_e\mathrm{Tr}(3Y_dY_d^\dagger + Y_eY_e^\dagger) \\ &- 9Y_e^\dagger Y_eY_{xd}^\dagger Y_{xd} - 4Y_e^\dagger Y_eY_e^\dagger Y_e + \left(16g_3^2 - \tfrac{2}{5}g_1^2\right)\mathrm{Tr}(Y_dY_d^\dagger) + \tfrac{6}{5}g_1^2\mathrm{Tr}(Y_eY_e^\dagger) + \left(16g_3^2 - \tfrac{2}{5}g_1^2\right)Y_{xd}^\dagger Y_{xd} \\ &+ 6g_2^2Y_e^\dagger Y_e + \tfrac{33}{2}g_2^4 + \tfrac{9}{5}g_2^2g_1^2 + \tfrac{837}{50}g_1^4\Big),\end{aligned} \tag{E5}$$

$$\begin{aligned}\beta^{(2)}_{Y_{xu}} = Y_{xu}\Big(&-3\,\mathrm{Tr}(3Y_uY_u^\dagger Y_uY_u^\dagger + Y_uY_d^\dagger Y_dY_u^\dagger) - 22Y_{xu}^\dagger Y_{xu}Y_{xu}^\dagger Y_{xu} - 9Y_{xu}^\dagger Y_{xu}\mathrm{Tr}(Y_uY_u^\dagger) + \left(16g_3^2 + \tfrac{4}{5}g_1^2\right)\mathrm{Tr}(Y_uY_u^\dagger) \\ &+ \left(16g_3^2 + 6g_2^2 + \tfrac{2}{5}g_1^2\right)Y_{xu}^\dagger Y_{xu} + \tfrac{128}{9}g_3^4 + 8g_3^2g_2^2 + \tfrac{8}{9}g_3^2g_1^2 + \tfrac{33}{2}g_2^4 + g_2^2g_1^2 + \tfrac{1813}{450}g_1^4\Big),\end{aligned} \tag{E6}$$

$$\begin{aligned}\beta^{(2)}_{Y_{xd}} = Y_{xd}\Big(&-3\,\mathrm{Tr}(3Y_dY_d^\dagger Y_dY_d^\dagger + Y_dY_u^\dagger Y_uY_d^\dagger + Y_eY_e^\dagger Y_eY_e^\dagger) - 22Y_{xd}^\dagger Y_{xd}Y_{xd}^\dagger Y_{xd} - 3Y_{xd}^\dagger Y_{xd}\mathrm{Tr}(3Y_dY_d^\dagger + Y_eY_e^\dagger) \\ &+ \left(16g_3^2 - \tfrac{2}{5}g_1^2\right)\mathrm{Tr}(Y_dY_d^\dagger) + \tfrac{6}{5}g_1^2\mathrm{Tr}(Y_eY_e^\dagger) + \left(16g_3^2 + 6g_2^2 + \tfrac{2}{5}g_1^2\right)Y_{xd}^\dagger Y_{xd} \\ &+ \tfrac{128}{9}g_3^4 + 8g_3^2g_2^2 + \tfrac{8}{9}g_3^2g_1^2 + \tfrac{33}{2}g_2^4 + g_2^2g_1^2 + \tfrac{1813}{450}g_1^4\Big).\end{aligned} \tag{E7}$$

APPENDIX F: RENORMALIZATION GROUP EQUATIONS IN MODEL V

In Model V, below the intermediate scale $M_I = 1.0 \times 10^{11}$ GeV, we have the same RGEs as in Model IV. Above M_I, we have extra vectorlike particles $(Xf, \overline{Xf})$, and we have the same RGEs as in Model III.

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