

“Dark” Z implications for parity violation, rare meson decays, and Higgs physics

Hooman Davoudiasl,^{*} Hye-Sung Lee,[†] and William J. Marciano[‡]

Department of Physics, Brookhaven National Laboratory, Upton, New York 11973, USA

(Received 30 March 2012; published 25 June 2012)

General consequences of mass mixing between the ordinary Z boson and a relatively light Z_d boson, the “dark” Z , arising from a $U(1)_d$ gauge symmetry, associated with a hidden sector such as dark matter, are examined. New effects beyond kinetic mixing are emphasized. Z - Z_d mixing introduces a new source of low energy parity violation well explored by possible future atomic parity violation and planned polarized electron scattering experiments. Rare $K(B)$ meson decays into $\pi(K)\ell^+\ell^-$ ($\ell = e, \mu$) and $\pi(K)\nu\bar{\nu}$ are found to already place tight constraints on the size of Z - Z_d mixing. Those sensitivities can be further improved with future dedicated searches at K and B factories as well as binned studies of existing data. Z - Z_d mixing can also lead to the Higgs decay $H \rightarrow ZZ_d$, followed by $Z \rightarrow \ell_1^+\ell_1^-$ and $Z_d \rightarrow \ell_2^+\ell_2^-$ or “missing energy,” providing a potential hidden sector discovery channel at the Large Hadron Collider. An illustrative realization of these effects in a 2 Higgs doublet model is presented.

DOI: [10.1103/PhysRevD.85.115019](https://doi.org/10.1103/PhysRevD.85.115019)

PACS numbers: 14.70.Pw

I. INTRODUCTION

The existence of cosmic dark matter is now essentially established. It appears to constitute about 22% of the energy-matter budget of the Universe, significantly more than the 4% attributed to visible matter [1]. Nevertheless, the exact nature of dark matter remains mysterious. Is it mainly a new, cosmologically stable, elementary particle that interacts with our visible world primarily through gravity, or does it have weak interaction properties that allow it to be detected at high energy accelerators or in sensitive underground cryogenic experiments? Both avenues of exploration are currently in progress. A discovery would revolutionize our view of the Universe and the field of elementary particle physics.

Recently, a possible generic new property of dark matter has been postulated [2] to help explain various astrophysical observations of positron excesses [3]. The basic idea is to introduce a new $U(1)_d$ gauge symmetry mediated by a relatively light Z_d boson that couples to the “dark” charge of hidden sector states, an example of which is dark matter. Such a boson has been dubbed the dark photon, secluded or hidden boson, etc. [4]. Within the framework adopted in our work, however, we refer to it as the dark Z because of its close relationship to the ordinary Z of the standard model (SM) via Z - Z_d mixing. Consequences of that mixing will be explored in this paper, where, after describing the basic characteristics of the dark Z , we provide constraints on its properties imposed by low energy parity violating experiments such as atomic parity violation and polarized electron scattering. Future sensitivities are also discussed. We then briefly describe bounds on the mixing currently

obtained from rare K and B decays along with the potential for future improvements.

Perhaps the most novel prediction from Z - Z_d mixing is its implications for high energy experiments. In particular, it leads to a potentially observable new type of Higgs decay, $H \rightarrow ZZ_d$, with pronounced discovery signatures that we describe [5]. We also discuss a 2 Higgs doublet (2HD) model that exhibits all the features of our general Z - Z_d mixing scenario. (Some works of similar spirit but different contexts can be found in, for example, Refs. [6–10].)

II. SETUP

We begin with what might be called the usual dark boson scenario. It is assumed that a new $U(1)_d$ gauge symmetry of the dark matter or any hidden sector interacts with the $SU(3)_C \times SU(2)_L \times U(1)_Y$ of the SM via kinetic mixing between $U(1)_Y$ and $U(1)_d$ [11]. That effect is parametrized by a gauge invariant $B_{\mu\nu}Z_d^{\mu\nu}$ interaction

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \frac{1}{2}\frac{\varepsilon}{\cos\theta_W}B_{\mu\nu}Z_d^{\mu\nu} - \frac{1}{4}Z_{d\mu\nu}Z_d^{\mu\nu}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \quad Z_{d\mu\nu} = \partial_\mu Z_{d\nu} - \partial_\nu Z_{d\mu} \quad (1)$$

with ε a dimensionless parameter that is unspecified (the normalization of the term proportional to ε has been chosen to simplify the notation in the results that follow). At the level of our discussion, ε is a potentially infinite counterterm necessary for renormalization. Its finite renormalized value is to be determined by experiment. In most discussions, ε is assumed to be $\lesssim \mathcal{O}(\text{few} \times 10^{-3})$. It could, of course, be much smaller [12].

After removal of the ε cross term by field redefinitions

$$B_\mu \rightarrow B_\mu + \frac{\varepsilon}{\cos\theta_W}Z_{d\mu} \quad (2)$$

leading to

^{*}hooman@bnl.gov
[†]hlee@bnl.gov
[‡]marciano@bnl.gov

$$A_\mu \rightarrow A_\mu + \varepsilon Z_{d\mu} \quad Z_\mu \rightarrow Z_\mu - \varepsilon \tan\theta_W Z_{d\mu} \quad (3)$$

for the photon and Z boson fields, one is left with an induced coupling of the Z_d to the usual electromagnetic current (with summation over all charged quarks and leptons)

$$\mathcal{L}_{\text{int}} = -e\varepsilon J_{\text{em}}^\mu Z_{d\mu} \quad J_{\text{em}}^\mu = \sum_f Q_f \bar{f} \gamma^\mu f + \dots, \quad (4)$$

where the ellipsis includes W^\pm current terms and Q_f is the electric charge ($Q_e = -1$). (It is generally assumed that $U(1)_d$ is broken and Z_d becomes massive via a scalar Higgs singlet or a Stueckelberg mass generating mechanism [13,14].) Note also that the induced coupling of Z_d to the weak neutral current via Eq. (3) is highly suppressed at low energies in the above basic scenario because of a cancellation between ε dependent field redefinition and Z - Z_d mass matrix diagonalization effects induced by ε (see, for example, Ref. [15] and our Appendixes A and B).

The phenomenology of the interaction in Eq. (4) has been well examined as a function of m_{Z_d} and ε (e.g., Refs. [16–18]). With the assumption $10 \text{ MeV} \lesssim m_{Z_d} \lesssim 10 \text{ GeV}$ and $\varepsilon \lesssim \mathcal{O}(\text{few} \times 10^{-3})$, bounds have been given and new experiments are underway to find the Z_d via its production in high intensity electron scattering [19]. We will consider this same mass range for our phenomenological analysis in this work. The lower bound $m_{Z_d} \gtrsim 10 \text{ MeV}$ is required in order that astrophysical and beam-dump processes do not severely constrain the interactions of dark Z , which, as discussed below, develops an axionlike component for $m_{Z_d} \rightarrow 0$.

Because of its coupling to our particle world via the small electromagnetic current coupling in Eq. (4), Z_d is often called the dark photon (even though that name was originally intended for a new weakly coupled long-range interaction [20]).

Here, we generalize the above $U(1)_d$ kinetic mixing scenario to include Z - Z_d mass mixing by introducing the 2×2 mass matrix

$$M_0^2 = m_Z^2 \begin{pmatrix} 1 & -\varepsilon_Z \\ -\varepsilon_Z & m_{Z_d}^2/m_Z^2 \end{pmatrix}, \quad (5)$$

where m_{Z_d} and m_Z (with $m_{Z_d}^2 \ll m_Z^2$) represent the dark Z and SM Z masses in the limit of no mixing. The Z - Z_d mixing is parametrized by

$$\varepsilon_Z = \frac{m_{Z_d}}{m_Z} \delta, \quad (6)$$

with δ a small model dependent quantity. We ignore the ε contribution from Eq. (2) in the mass matrix, since its inclusion would affect this part of our discussion only at $\mathcal{O}(\varepsilon^2)$ (see Appendix B). The assumed off-diagonal m_{Z_d} dependence in Eq. (6) allows smooth $m_{Z_d} \rightarrow 0$ behavior for all ε_Z -induced amplitudes involving Z_d , even those stemming from nonconserved current interactions. Also, for

simplicity, ordinary fermions are assumed to be neutral under $U(1)_d$; i.e., they do not carry any fundamental dark charge. Their only couplings to Z_d are induced through ε and ε_Z . More general cases are possible and interesting, but beyond the scope of this paper.

So far, δ is rather arbitrary, although $0 \leq \delta^2 < 1$ is required to avoid an infinite-range or tachyonic Z_d . One expects δ to be small because of the disparity of m_Z and m_{Z_d} . We later show that low energy phenomenology actually requires $\delta^2 \lesssim 0.006$, while rare K and B decays have sensitivity to $\delta^2 \lesssim 10^{-4}$ – 10^{-6} for low mass Z_d . We will also demonstrate how the form in Eq. (5) naturally emerges in a simple 2HD extension of the SM, the details of which will be discussed in Appendix B. However, we emphasize that our general results follow from Z - Z_d mixing through a generic mass matrix of the form in Eq. (5) and are not exclusively tied to any specific expanded Higgs sector. That mixing could, for example, potentially arise from loop effects or dynamical symmetry breaking.

Overall, mixing leads to mass eigenstates Z and Z_d

$$Z = Z^0 \cos\xi - Z_d^0 \sin\xi \quad Z_d = Z^0 \sin\xi + Z_d^0 \cos\xi, \quad (7)$$

where (see Appendix B)

$$\tan 2\xi \simeq 2 \frac{m_{Z_d}}{m_Z} \delta = 2\varepsilon_Z. \quad (8)$$

It is expected that $\sin\xi$ is very small (partly because of the assumed smallness of m_{Z_d}/m_Z and partly because of small δ) and does not measurably affect Z pole parameters (such as m_Z and Γ_Z) because these are shifted fractionally at $\mathcal{O}(\varepsilon_Z^2)$ and require only $\varepsilon_Z \lesssim \mathcal{O}(0.01)$. However, it can, nevertheless, lead to other interesting new phenomenology, which overcomes the m_{Z_d}/m_Z suppression in ε_Z .

As the first example, we consider very low Q^2 parity violating effects where the smallness of m_{Z_d}/m_Z in the induced Z_d couplings is offset by the $m_Z^2/m_{Z_d}^2$ enhancement from Z vs Z_d propagators. Then we describe the induced decays $K \rightarrow \pi Z_d$ and $B \rightarrow K Z_d$, as well as the high energy decay $H \rightarrow Z Z_d$, where the small induced coupling factor m_{Z_d}/m_Z is overcome by m_K/m_{Z_d} , m_B/m_{Z_d} , and m_H/m_{Z_d} enhancements, respectively, in the longitudinal polarization component of the Z_d production amplitudes.

III. ATOMIC PARITY VIOLATION AND POLARIZED ELECTRON SCATTERING

We begin our analysis by writing out the full Z_d coupling to fermions from ε as well as ε_Z .

$$\mathcal{L}_{\text{int}} = \left(-e\varepsilon J_\mu^{\text{em}} - \frac{g}{2 \cos\theta_W} \varepsilon_Z J_\mu^{\text{NC}} \right) Z_d^\mu, \quad (9)$$

where J_μ^{em} is given in Eq. (4) and

$$J_{\mu}^{NC} = \sum_f (T_{3f} - 2Q_f \sin^2 \theta_W) \bar{f} \gamma_{\mu} f - T_{3f} \bar{f} \gamma_{\mu} \gamma_5 f \quad (10)$$

with $T_{3f} = \pm 1/2$ ($T_{3e} = -1/2$) and $\sin^2 \theta_W \approx 0.23$ is the weak mixing angle of the SM. The inclusion of Z - Z_d mixing has introduced parity violation. The $J_{\mu}^{NC} Z_d^{\mu}$ coupling is similar to the $J_{\mu}^{NC} Z^{\mu}$ coupling of the SM Z but reduced by ε_Z in magnitude. Hence, the name dark Z , since it is the ε_Z -induced interactions that we primarily address. Note that the effects of ε and ε_Z can be combined into a simple form

$$\mathcal{L}_{\text{int}} = -\frac{g}{2 \cos \theta_W} \varepsilon_Z J_{\mu}^{NC} Z_d^{\mu} \quad (11)$$

by the replacement $J_{\mu}^{NC}(\sin^2 \theta_W) = J_{\mu}^{NC}(\sin^2 \theta'_W)$

$$\sin^2 \theta'_W = \sin^2 \theta_W - \frac{\varepsilon}{\varepsilon_Z} \cos \theta_W \sin \theta_W \quad (12)$$

in Eq. (10). In that format, one can judge the relative importance of ε in low energy Z_d phenomenology. It depends on the size of $(\varepsilon/\varepsilon_Z)(\cos \theta_W/\sin \theta_W)$. For ε very small, it has little effect but will be significant if $\varepsilon \sim \varepsilon_Z$.

The new source of parity violation in Eq. (9) or Eq. (11) is particularly important for experiments at $Q^2 < m_{Z_d}^2$, where the Z_d propagator can provide an enhancement owing to $m_{Z_d}^2 \ll m_Z^2$. The overall effect for parity violating amplitudes $\mathcal{M}_{NC}^{\text{PV}} = (G_F/2\sqrt{2})F(\sin^2 \theta_W)$ in the SM is (in leading order) to replace

$$G_F \rightarrow \rho_d G_F \quad \sin^2 \theta_W \rightarrow \kappa_d \sin^2 \theta_W \quad (13)$$

with [21]

$$\begin{aligned} \rho_d &= 1 + \delta^2 \frac{m_{Z_d}^2}{Q^2 + m_{Z_d}^2} \\ \kappa_d &= 1 - \frac{\varepsilon}{\varepsilon_Z} \delta^2 \frac{\cos \theta_W}{\sin \theta_W} \frac{m_{Z_d}^2}{Q^2 + m_{Z_d}^2} \end{aligned} \quad (14)$$

or from Eq. (6)

$$\kappa_d = 1 - \varepsilon \frac{m_Z}{m_{Z_d}} \delta \frac{\cos \theta_W}{\sin \theta_W} \frac{m_{Z_d}^2}{Q^2 + m_{Z_d}^2}. \quad (15)$$

It is quite plausible that in a more complete theory, $\varepsilon \propto (m_{Z_d}/m_Z)\delta = \varepsilon_Z$. Then, the effects from kinetic mixing and Z - Z_d mixing become similar in form and magnitude. Here, we allow ε to remain a separate independent parameter.

Assuming no accidental cancellation between the ρ_d and κ_d in Eq. (14), cesium atomic parity violation currently provides the best low energy experimental constraint on those parameters over the entire approximate range of interest ($10 \text{ MeV} \lesssim m_{Z_d} \lesssim 10 \text{ GeV}$) since $Q^2 \ll m_{Z_d}^2$. The nuclear weak charge measured in atomic parity violation (to lowest order in the SM) is given by $Q_W = -N + Z(1 - 4\sin^2 \theta_W)$, which when compared with experiment probes new physics. There is excellent agree-

ment between the SM prediction for the weak charge of cesium (including electroweak radiative corrections) [22–24]

$$Q_W^{\text{SM}}(^{133}\text{Cs}) = -73.16(5) \quad (16)$$

and the experimental value [25–27]

$$Q_W^{\text{exp}}(^{133}\text{Cs}) = -73.16(35). \quad (17)$$

Based on the shift due to ε , ε_Z , and δ

$$Q_W^{\text{SM}} \rightarrow -73.16(1 + \delta^2) + 220 \frac{\varepsilon}{\varepsilon_Z} \delta^2 \cos \theta_W \sin \theta_W,$$

the above agreement then implies the following constraints:

$$\left| \delta^2 \left(1 - 1.27 \frac{\varepsilon}{\varepsilon_Z} \right) \right| \leq 0.005 \quad (1\sigma), \quad (18)$$

$$\delta^2 \lesssim 0.006 \quad (\text{one-sided } 90\% \text{ C.L.}), \quad \text{for } \varepsilon \ll \varepsilon_Z. \quad (19)$$

For $\varepsilon \approx \varepsilon_Z$, the constraints on δ^2 become diluted and the possibility of cancellation occurs if one tunes $\varepsilon/\varepsilon_Z \approx 0.8$. (We note that the fine-tuning $\varepsilon/\varepsilon_Z \approx 0.8$ is similar to a relation employed in Ref. [8] to try and reconcile what appears to be discrepancies in dark matter search scattering experiments on heavy nuclei. However, such a scenario is significantly constrained by the bounds on δ described below.)

An independent constraint primarily applicable to κ_d because of its relative insensitivity to ρ_d comes from parity violating polarized electron-electron Moller scattering asymmetries [28,29]. Experiment E158 at SLAC [30] measured the low energy value of $\sin^2 \theta_W(Q^2)$ at $Q^2 \approx (0.16 \text{ GeV})^2$ and compared it with expectations based on running the Z pole value $\sin^2 \theta_W(m_Z)$ down to low Q^2 [29]. The good agreement with SM loop effects leads to (ignoring the small ρ_d effect)

$$\left| \frac{\varepsilon}{\varepsilon_Z} \delta^2 \right| \frac{m_{Z_d}^2}{(0.16 \text{ GeV})^2 + m_{Z_d}^2} \leq 0.006. \quad (20)$$

For $m_{Z_d}^2 \gg (0.16 \text{ GeV})^2$ and $\varepsilon_Z \approx \varepsilon$, the constraints in Eqs. (19) and (20) are essentially the same. However, for a light $m_{Z_d} \lesssim 200 \text{ MeV}$, the bound in Eq. (20) can be somewhat diluted. Nevertheless, for some range of (ε, m_{Z_d}) values, Eq. (20) can provide more restrictive bounds on δ . For example, consider $\varepsilon \approx 2 \times 10^{-3}$ and $m_{Z_d} \approx 100 \text{ MeV}$, which lie in the region favored by the current discrepancy between theory and experimental values of the muon anomalous magnetic moment [31]. In that case, Eq. (20) becomes

$$|\delta| < 0.01, \quad (21)$$

which is considerably tighter than Eq. (19). If the muon anomaly discrepancy is because of a light Z_d and $\varepsilon \sim 10^{-3}$, that boson's effect on the value of $\sin^2 \theta_W$ extracted

from future more precise very low Q^2 parity violating experiments [32] could eventually become observable.

The sensitivity in Eqs. (20) and (21) is expected to improve by up to an order of magnitude from ongoing and proposed polarized ep and ee scattering experiments at JLAB [32] as well as proposed $Q^2 \simeq (0.05 \text{ GeV})^2$ ep studies at MESA in Mainz [33]. Our analysis illustrates the complementarity of direct searches at intense electron scattering facilities in JLAB and Mainz for a light vector particle (the dark photon coupled through kinetic mixing) produced via electron scattering, with low Q^2 measurements of $\sin^2\theta_W$ in parity violating experiments (that probe ε and the mass mixing of the dark Z). We also note that proposed measurements of atomic parity violation for ratios of different nuclear isotopes would eliminate atomic physics uncertainties as well as any dependence on ρ_d [34–37]. They would then be sensitive to $(\varepsilon/\varepsilon_Z)\delta^2$ but with negligible Q^2 dependence (since $Q^2 \simeq 0$). It is amusing to note that in principle, very low energy measurements of $\sin^2\theta_W$ in atomic parity violation and low Q^2 polarized electron scattering experiments could find different $\sin^2\theta_W$ results from one another if a very low mass Z_d is contributing to both because of the Q^2 dependence in Eq. (14).

Our conclusion, based on the above discussion, is that currently $\delta^2 \leq 0.006$ is a modest, reasonably reliable constraint for most values of m_{Z_d} , although fine-tuning of ε and ε_Z could loosen the bound. That constraint can be much stronger for $\varepsilon \sim 10^{-3}$ [see Eq. (21)] and could be further improved significantly by future low energy parity violating experiments. For now, the bound $\delta^2 \leq 0.006$ provides a starting point for comparison with the sensitivity to δ^2 in rare K and B decays, which we next describe.

IV. RARE K AND B DECAYS

Experimental studies of rare flavor-changing weak neutral current decays of K and B mesons have proven to be powerful probes of high and low scale “new” physics phenomena. Here, we illustrate the effect of Z - Z_d mass mixing on the transition amplitudes $s \rightarrow dZ_d$ and $b \rightarrow sZ_d$ induced within the framework of Cabibbo-Kobayashi-Maskawa (CKM) charged current mixing (see Fig. 1). Those loop-induced couplings can lead to decays such as $K \rightarrow \pi Z_d$ and $B \rightarrow KZ_d$ or K^*Z_d characterized by the signature $Z_d \rightarrow \ell^+\ell^-$ ($\ell = e$ or μ) with invariant mass $m_{\ell\ell} = m_{Z_d}$ or $Z_d \rightarrow$ missing energy where Z_d decays into $\nu\bar{\nu}$ or essentially undetectable light hidden sector particles. In all such two-body decays, the monoenergetic outgoing π or K will provide a tight constraint (for a given m_{Z_d}) and a very distinct overall signal.

Here, we note that the phenomenology of Z_d is affected by its lifetime τ_{Z_d} . A sufficiently large value of τ_{Z_d} will allow Z_d to escape the detector and lead to a missing energy signal. However, for smaller values of τ_{Z_d} , a displaced vertex can provide a distinct signature. In Fig. 2, using representative values of δ and ε , we have plotted τ_{Z_d}

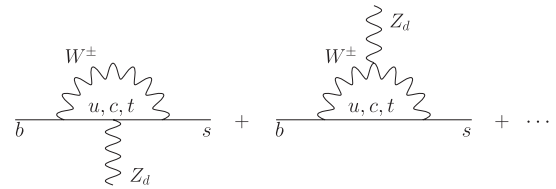


FIG. 1. Examples of diagrams contributing to $b \rightarrow sZ_d$. Similar diagrams give rise to $s \rightarrow dZ_d$.

for $10 \text{ MeV} \leq m_{Z_d} \leq 10 \text{ GeV}$, assuming that Z_d only decays into SM final states. We provide a simple formula for the partial width of Z_d into SM fermions, $\Gamma(Z_d \rightarrow ff)$, in Appendix C.

Of course, the amplitudes for $\bar{d}sZ_d$ and $\bar{s}bZ_d$ being loop induced will in general depend on the details of the complete model considered, including its underlying Higgs flavor symmetry breaking structure. Those details are beyond the scope of this paper where we are primarily interested in the generic effects of Z - Z_d mixing parameterized by $\varepsilon_Z = (m_{Z_d}/m_Z)\delta$ in Eq. (6).

A simple illustrative example of a scenario that leads to Z - Z_d mixing and CKM-induced flavor-changing weak neutral currents is the Type-I 2HD model discussed in Sec. VI and detailed in Appendix B. There, the underlying $U(1)_d$ gauge symmetry naturally forbids tree level flavor-changing neutral currents in the scalar and pseudoscalar Higgs sectors. It also yields, through Higgs doublet and singlet vacuum expectation values, a mechanism to provide mass for Z_d and give rise to a small δ in Eq. (6).

To obtain the induced Z_d flavor-changing amplitudes, we can make use of existing CKM loop-induced calculations for $\bar{d}_L\gamma_\mu s_L Z^\mu$ and $\bar{s}_L\gamma_\mu b_L Z^\mu$ amplitudes [38] and replace $Z \rightarrow \varepsilon_Z Z_d$. (See Fig. 1.) (We ignore kinetic mixing-induced couplings, since their effects are highly suppressed. For example, Ref. [39] found $\text{BR}(B \rightarrow KZ_d) \sim 6 \times 10^{-7} \varepsilon^2$ for $m_{Z_d} \simeq 1 \text{ GeV}$. As we demonstrate, mass mixing, ε_Z , induced rates can be much larger and potentially

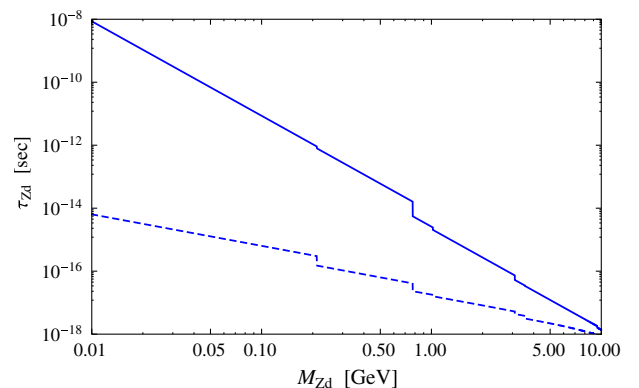


FIG. 2 (color online). Z_d lifetime with Z_d mass for $\delta^2 = 10^{-4}$ with $\varepsilon = 0$ (solid curve) and $\varepsilon = 2 \times 10^{-3}$ (dashed curve) cases. We take ρ , ϕ , J/ψ , Y masses as the representative threshold for decays to mesons.

observable.) As an alternative computational strategy, if we are primarily interested in relatively light Z_d bosons compared to m_K and m_B , we can employ the Goldstone boson equivalence theorem [40] to obtain amplitudes for longitudinally polarized Z_d bosons from flavor-changing axionlike pseudoscalar couplings well documented in the literature. For our purpose, the latter approach will suffice; however, the direct Z calculation provides a nice cross-check. Nevertheless, we note that the results discussed below should be viewed as somewhat incomplete and should be taken as approximate.

The relevant $\bar{d}_L \gamma_\mu s_L \partial^\mu a$ and $\bar{s}_L \gamma_\mu b_L \partial^\mu a$ axion couplings were computed for the 2HD model more than 30 years ago by Hall and Wise [41] and independently by Frere, Vermaseren, and Gavela [42]. More recently, they were checked and applied to the decay $B \rightarrow Ka$, $a \rightarrow \ell^+ \ell^-$ in Ref. [43]. Here, we use those results to estimate the branching ratios for $K \rightarrow \pi Z_d$ (longitudinal) and $B \rightarrow K Z_d$ (longitudinal), which should approximate the full Z_d final state rates up to corrections of $\mathcal{O}(m_{Z_d}^2/m_K^2)$ and $\mathcal{O}(m_{Z_d}^2/m_B^2)$, respectively. Comparison of those estimates with experiments can then be used to constrain δ for the ranges $m_{Z_d}^2 \ll (m_K - m_\pi)^2$ and $m_{Z_d}^2 \ll (m_B - m_K)^2$ modulo regions not covered because of experimental acceptance cuts on the data (which are beyond the scope of this paper). For example, $m_{Z_d} < 140$ MeV is not covered because of $\pi^0 \rightarrow e^+ e^- \gamma$ Dalitz decay background. Similarly, masses of Z_d near charmonium resonance regions are not covered.

We begin with the predicted branching ratio for $K \rightarrow \pi Z_d$ (longitudinal) in the 2HD model. Based on the analysis in Ref. [41], but adjusting for a modern m_t value, since top now dominates the amplitudes in Fig. 1

$$\text{BR}(K^+ \rightarrow \pi^+ Z_d)_{\text{long}} \simeq 4 \times 10^{-4} \delta^2, \quad (22)$$

where the numerical factor in that expression includes QCD suppression effects and depends on the physical charged scalar Higgs mass of the 2HD model. Those uncertainties should be considered part of the overall model dependence of our analysis.

The Z_d produced in Eq. (22) is expected to decay promptly (see, however, Fig. 2) to $\ell^+ \ell^-$ pairs with invariant mass m_{Z_d} or to missing energy that might be $\nu \bar{\nu}$ or light hidden sector particles. Those decays would add to the SM predictions and should be part of the experimentally measured branching ratios [1,44,45]

$$\text{BR}(K^+ \rightarrow \pi^+ e^+ e^-)_{\text{exp}} = (3.00 \pm 0.09) \times 10^{-7}, \quad (23)$$

$$\text{BR}(K^+ \rightarrow \pi^+ \mu^+ \mu^-)_{\text{exp}} = (9.4 \pm 0.6) \times 10^{-8}, \quad (24)$$

$$\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{exp}} = (1.7 \pm 1.1) \times 10^{-10}, \quad (25)$$

unless eliminated by acceptance cuts, which would negate bounds in certain m_{Z_d} regions. For example, the result in

Eq. (23) applied a $m_{ee} > 140$ MeV cut, while Eq. (25) was obtained with a rather stringent cut on E_π . Clearly, a new round of bump hunting in the $\ell^+ \ell^-$ spectrum is warranted. Toward that end, we note that $Z_d \rightarrow \ell^+ \ell^-$ decays will have a characteristic polarized spin-1 $\sin^2 \theta$ distribution relative to the longitudinal polarization of the Z_d . Unlike the spin-0 axion case, where because of chiral conservation the a preferentially decays to the heaviest fermion possible and the distribution is isotropic, we expect $\text{BR}(Z_d \rightarrow e^+ e^-) \simeq \text{BR}(Z_d \rightarrow \mu^+ \mu^-)$ modulo phase space.

With the above caveats, we compare Eq. (22) with (23)–(25), which agree with SM expectations and find rather tight bounds

$$|\delta| \lesssim 0.01 / \sqrt{\text{BR}(Z_d \rightarrow e^+ e^-)}, \quad (26)$$

$$|\delta| \lesssim 0.001 / \sqrt{\text{BR}(Z_d \rightarrow \text{missing energy})} \quad (27)$$

modulo acceptance cut criteria.

Equations (11) and (12) yield [46]

$$\frac{\text{BR}(Z_d \rightarrow e^+ e^-)}{\text{BR}(Z_d \rightarrow \nu \bar{\nu})} \simeq \frac{1}{6} + \frac{1}{2} \left(\frac{\varepsilon}{\varepsilon_Z} \right)^2, \quad (28)$$

where ε from kinetic mixing now comes into play. For $\varepsilon \gg \varepsilon_Z$, the charged lepton decays dominate and Eq. (26) is more applicable. For $\varepsilon \lesssim \varepsilon_Z$, the tighter constraint in Eq. (27) takes precedence. Of course, both should be used cautiously, given their model and experimental acceptance dependence.

For the case of $B \rightarrow K Z_d$ (longitudinal), we can apply a similar approach and find [41–43]

$$\text{BR}(B \rightarrow K Z_d)_{\text{long}} \simeq 0.1 \delta^2. \quad (29)$$

The relatively large coefficient in Eq. (29) results from a factor of m_t^4 in the $b \rightarrow s Z_d$ loop-induced correction from Fig. 1. That factor makes rare B decays a particularly sensitive probe of the Z_d . Employing the recent bounds that follow from the discussion of $B \rightarrow Ka$, with the axion-type particle $a \rightarrow \ell^+ \ell^-$ in Refs. [39,43] implies conservatively $\text{BR}(B \rightarrow K Z_d \rightarrow K \ell^+ \ell^-) < 10^{-7}$, while the bound from B decay containing missing energy are based on [1,47,48]

$$\text{BR}(B^+ \rightarrow K^+ \bar{\nu} \nu)_{\text{exp}} < 1.4 \times 10^{-5}. \quad (30)$$

We then roughly find

$$|\delta| \lesssim 0.001 / \sqrt{\text{BR}(Z_d \rightarrow \ell^+ \ell^-)}, \quad (31)$$

$$|\delta| \lesssim 0.01 / \sqrt{\text{BR}(Z_d \rightarrow \text{missing energy})}. \quad (32)$$

It has been suggested [39] that even tighter bounds may be obtained from dedicated searches for $\ell^+ \ell^-$ pairs in B decays, particularly if displaced vertices result from suppressed decay rates. Nevertheless, even the relatively crude

bounds in Eqs. (31) and (32) are very constraining where applicable and are likely to be significantly improved by future dedicated searches.

On the basis of our analysis, it is clear that rare K and B decays provide sensitive windows to Z - Z_d mass mixing and should be further explored in future high intensity experiments. In fact, for both cases, a more refined binned analysis of existing data would likely result in tighter bounds than those in Eqs. (26) and (31) or even uncover a hint of the Z_d 's presence. Although applicable to a limited range of m_{Z_d} and dependent on the Z_d branching ratios, one can easily conclude $|\delta| \lesssim 0.01 - 0.001$ over some restricted m_{Z_d} domain. In addition, further improvements are possible and warranted. That constraint on δ sets a standard for other rare decay studies. As we show in the next section, it is possible that searches for the rare Higgs decay $H \rightarrow ZZ_d$ have the statistical significance to also explore $|\delta| \lesssim 0.01 - 0.001$ but have the potential advantage of covering a much broader range of m_{Z_d} values including $m_{Z_d} \gtrsim 5$ GeV if backgrounds can be controlled.

V. HIGGS DECAYS

We now address a primary consequence of our paper, the decay $H \rightarrow ZZ_d$ induced by Z - Z_d mass matrix mixing. To put our analysis into a current day perspective, we take $m_H = 125$ GeV, a value roughly suggested by early small excesses at the Large Hadron Collider (LHC) in the expected decay modes $H \rightarrow \gamma\gamma, WW^*$, and ZZ^* [49,50]. We note, however, that our findings regarding the sensitivity of Higgs searches for $H \rightarrow ZZ_d$ are fairly independent of the exact value of m_H .

To set the stage, we estimate that, roughly, one expects each LHC experiment to have about 75 000 Higgs bosons in the existing data before cuts (for the integrated luminosity of $4.7\text{--}4.9 \text{ fb}^{-1}$ with $E_{\text{c.m.}} = 7$ TeV) for $m_H = 125$ GeV in the SM. In Table I, we list the expected Higgs decay branching ratios within the context of the SM. Of particular interest for comparison with $H \rightarrow ZZ_d$ are the SM decays (1) $H \rightarrow ZZ^* \rightarrow \ell_1^+ \ell_1^- \ell_2^+ \ell_2^-$ and

TABLE I. Standard model Higgs decay branching ratios for $m_H = 125$ GeV ($\Gamma_H \simeq 4.1$ MeV) from Ref. [51].

H decay channel	Branching ratio
$b\bar{b}$	0.578
WW^*	0.215
gg	0.086
$\tau^+ \tau^-$	0.063
$c\bar{c}$	0.029
ZZ^*	0.026
$\gamma\gamma$	2.3×10^{-3}
$Z\gamma$	1.5×10^{-3}
$H \rightarrow ZZ^* \rightarrow \ell_1^+ \ell_1^- \ell_2^+ \ell_2^-$	1.2×10^{-4}
$H \rightarrow ZZ^* \rightarrow \ell^+ \ell^- \nu \bar{\nu}$	3.6×10^{-4}

(2) $H \rightarrow ZZ^* \rightarrow \ell^+ \ell^- \nu \bar{\nu}$ where the $*$ signifies a ‘‘virtual,’’ off mass shell boson and $\ell = e, \mu$. The first of these, even at the $\text{BR} \sim 10^{-4}$ level, may have already been seen at the LHC where a handful of candidate events have been reported. If it truly is a Higgs signal, hundreds more 4-lepton $\ell_1^+ \ell_1^- \ell_2^+ \ell_2^-$ events will be clearly observed in the coming years. The second decay, $H \rightarrow \ell^+ \ell^- \nu \bar{\nu}$, is more difficult and to our knowledge has not been experimentally studied.

For the first case, one lepton pair will have an invariant mass of $m_Z \simeq 91$ GeV while the second pair will have an invariant mass ranging from 0 to about 34 GeV with a differential decay rate distribution as depicted in Fig. 3. The second mode $H \rightarrow ZZ^* \rightarrow \ell^+ \ell^- \nu \bar{\nu}$, with the neutrinos identified by missing energy, while experimentally more challenging should be searched for as well, since it can be used to constrain potentially invisible decays of the Z_d , as we subsequently discuss.

As we shall see, the decays $H \rightarrow ZZ_d$ are significantly enhanced beyond naive expectations, even for very small mixing. To appreciate that phenomenon, we remind the reader that for a very heavy Higgs ($m_H^2 \gg m_W^2, m_Z^2$) the decay rates for $H \rightarrow W^+ W^-$ and $H \rightarrow ZZ$ can become enormous, growing like $\sim g^2 m_H^3 / m_V^2$, $V = W, Z$ with increasing m_H . That behavior comes about because the final state W and Z bosons are longitudinally polarized, resulting in a $\sim m_H^2 / m_V^2$ enhancement factor at the decay rate level (for each final state gauge boson).

Such an effect is a manifestation of the Goldstone boson equivalence theorem, which states that at high energies ($s \gg m_V^2$), S -matrix elements involving W^\pm and Z bosons are equivalent, up to $\mathcal{O}(m_V/\sqrt{s})$, to the corresponding amplitudes in the Higgs-Goldstone scalar theory with the Goldstone boson replacing W_L^\pm, Z_L (longitudinal components). In the heavy Higgs limit, the $W^+ W^-$ and ZZ decay products are essentially longitudinally polarized and behave like their Goldstone boson components. The

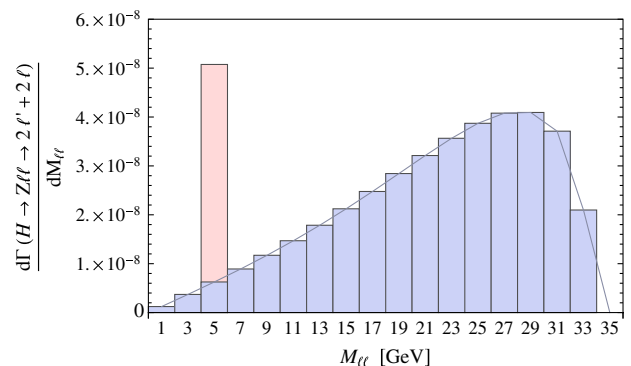


FIG. 3 (color online). Differential decay rate $H \rightarrow ZZ^* \rightarrow Z\ell^+\ell^- \rightarrow 4\ell$ vs $\ell^+\ell^-$ invariant mass with $m_H = 125$ GeV in the SM. For the illustration, $H \rightarrow ZZ_d \rightarrow Z\ell^+\ell^-$ with $m_{Z_d} = 5$ GeV and $\delta^2 \text{BR}(Z_d \rightarrow \ell^+\ell^-) = 10^{-5}$ (which would need $N_{\text{Higgs}} \simeq 10^6$ for 3σ evidence) is also shown (spike at the 5 GeV bin). Bin size is selected to be 2 GeV.

Higgs coupling to Goldstone bosons is of the form $(-ig/2)m_H^2/m_V$, and squaring that coupling and dividing by $1/m_H$ gives the $\Gamma(H \rightarrow VV) \sim g^2 m_H^3/m_V^2$ exhibited by heavy Higgs decays. We note that the longitudinal polarization of the gauge bosons can be very helpful in identifying a Higgs decay since the subsequent decay W or $Z \rightarrow$ leptons have a characteristic angular distribution $\propto \sin^2\theta$ relative to the polarization.

Of course, our example of 125 GeV Higgs is too light to decay into W^+W^- or ZZ pairs. It can, however, decay into one real and one virtual boson with the latter directly producing a lepton pair with an invariant mass distribution as illustrated in Fig. 3 [52]. The integrated partial width for $H \rightarrow ZZ^* \rightarrow \ell_1^+ \ell_1^- \ell_2^+ \ell_2^-$ is, however, suppressed by $\alpha/4\pi$ (from the $Z^* \ell_2^+ \ell_2^-$ coupling and three-body phase space) and the small $\text{BR}(Z \rightarrow \ell^+ \ell^-) \simeq 2 \times 0.034$ for $\ell = e, \mu$. One finds

$$\Gamma(H \rightarrow ZZ^* \rightarrow \ell_1^+ \ell_1^- \ell_2^+ \ell_2^-) \simeq 1.8 \times 10^{-6} \frac{G_F}{8\sqrt{2}\pi} m_H m_W^2 \quad (33)$$

with no significant sign of enhancement for longitudinal polarization, which is not surprising, since $m_H/m_Z \simeq 1.4$ in our example. Nevertheless, even with the 10^{-6} suppression factor in Eq. (33), it is expected that a SM 125 GeV Higgs should be starting to be seen with about several events per experiment in existing data, after acceptance cuts, and with hundreds more to follow in subsequent years. So, Eq. (33) represents a decay rate standard that is easily discernible if backgrounds are in check. We note that the decay rate for $H \rightarrow ZZ^* \rightarrow \ell^+ \ell^- \nu \bar{\nu}$ is expected in the SM to be about 3 times larger than Eq. (33) but more difficult to measure.

Now we come to the decay $H \rightarrow ZZ_d$ owing to Z - Z_d mixing in our dark Z scenario. That mixing, parametrized by $\varepsilon_Z = (m_{Z_d}/m_Z)\delta$, a very small quantity, might naively appear to be negligible since it leads to a tiny HZZ_d coupling $\sim (g/\cos\theta_W)m_Z\varepsilon_Z$. Consequently, the $H \rightarrow ZZ_d$ decay rate will be suppressed by $\varepsilon_Z^2 = (m_{Z_d}/m_Z)^2\delta^2$. However, because of the Goldstone boson equivalence theorem, we gain an enhancement factor of $\sim (m_H/m_{Z_d})^2$ in the decay rate for longitudinally polarized Z_d final states (a feature that may also help in identifying their subsequent $Z_d \rightarrow \ell_2^+ \ell_2^-$ products via angular distribution if statistics suffice). That enhancement negates the small m_{Z_d}/m_Z factor in the HZZ_d coupling. Also, there is no $\alpha/4\pi$ suppression for $H \rightarrow ZZ_d$, only the small $\text{BR}(Z \rightarrow \ell^+ \ell^-) \simeq 2 \times 0.034$ that needs to be included for Z identification. A detailed calculation (see Appendix B) leads to

$$\begin{aligned} \Gamma(H \rightarrow ZZ_d \rightarrow \ell_1^+ \ell_1^- \ell_2^+ \ell_2^-) \\ \simeq 7 \times 10^{-3} \frac{G_F m_H^3}{8\sqrt{2}\pi} \delta^2 \text{BR}(Z_d \rightarrow \ell_2^+ \ell_2^-). \end{aligned} \quad (34)$$

Note the m_H^3 behavior that results from Z and Z_d being produced in their longitudinal polarization modes. A simi-

lar formula with $\text{BR}(Z_d \rightarrow \ell_2^+ \ell_2^-)$ replaced by $\text{BR}(Z_d \rightarrow$ missing energy) applies to the case $Z_d \rightarrow \nu \bar{\nu}$ or invisible dark particles.

In terms of its branching fraction relative to the SM expected width, one finds

$$\frac{\Gamma(H \rightarrow ZZ_d)}{\Gamma_H^{\text{SM}}(125 \text{ GeV})} \simeq 16 \times \delta^2 \lesssim 0.1 \quad (35)$$

with $\Gamma_H^{\text{SM}}(125 \text{ GeV}) \simeq 4.1 \times 10^{-3} \text{ GeV}$ [51] and using the low energy bound in Eq. (19). We see that as much as 10% of all LHC Higgs decays could be producing ZZ_d . With current statistics, even a 10% loss of SM expectations would not be noticed, but eventually it would be uncovered by precision Higgs production and decay studies.

Taking the ratio of Eqs. (33) and (34) gives

$$\frac{\Gamma(H \rightarrow ZZ_d \rightarrow \ell_1^+ \ell_1^- \ell_2^+ \ell_2^-)}{\Gamma(H \rightarrow ZZ^* \rightarrow \ell_1^+ \ell_1^- \ell_2^+ \ell_2^-)} \simeq 10^4 \delta^2 \text{BR}(Z_d \rightarrow \ell_2^+ \ell_2^-) \quad (36)$$

with a similar expression

$$\begin{aligned} \frac{\Gamma(H \rightarrow ZZ_d \rightarrow \ell^+ \ell^- + \text{missing energy})}{\Gamma(H \rightarrow ZZ^* \rightarrow \ell^+ \ell^- + \text{missing energy})} \\ \simeq (1/3) \times 10^4 \delta^2 \text{BR}(Z_d \rightarrow \text{missing energy}) \end{aligned} \quad (37)$$

for invisible Z_d decays. Even for $\delta^2 \simeq 10^{-4}$, well below the atomic parity violation bound of 0.006 in Eq. (19), one would expect $H \rightarrow ZZ_d$ events with $\ell_1^+ \ell_1^- \ell_2^+ \ell_2^-$ or $\ell^+ \ell^- +$ missing energy to be starting to appear or already present in LHC data. If there are no $Z_d \rightarrow$ dark particles decays, we expect the branching fractions of Z_d into $\ell^+ \ell^-$ to be given by Eq. (28). Therefore, in that case, one expects $\text{BR}(Z_d \rightarrow \ell^+ \ell^-)$ to be relatively large, particularly if $(\varepsilon/\varepsilon_Z)^2 \gtrsim 1$. If $Z_d \rightarrow$ dark particles dominates its decay rate and significantly dilutes $\text{BR}(Z_d \rightarrow \ell^+ \ell^-)$, one still has the possibility of seeing $H \rightarrow ZZ_d \rightarrow \ell^+ \ell^- +$ missing energy, although this perhaps is more experimentally challenging. Of course, given the original motivation for introducing a Z_d into astrophysics as a way of explaining positron excesses through its decays, a relatively large $\text{BR}(Z_d \rightarrow \ell^+ \ell^-)$ might be expected.

Returning to Eq. (36), we see that even for a somewhat suppressed $\text{BR}(Z_d \rightarrow \ell^+ \ell^-)$, the LHC experiments should be able to search for a Z_d in the $H \rightarrow \ell_1^+ \ell_1^- \ell_2^+ \ell_2^-$ decay chain down to $\delta^2 \text{BR}(Z_d \rightarrow \ell^+ \ell^-) \sim \mathcal{O}(10^{-5})$, depending on backgrounds. (The domain explored by rare K and B decays for some subset of m_{Z_d} values.) The signature, two isolated lepton pairs $\ell_1^+ \ell_1^- + \ell_2^+ \ell_2^-$ with a total invariant mass of m_H and individual masses of m_Z and m_{Z_d} should stick out as a spike in the invariant mass plot of Fig. 3, as illustrated for $m_{Z_d} = 5 \text{ GeV}$ and $\delta^2 \text{BR}(Z_d \rightarrow \ell^+ \ell^-) = 10^{-5}$. In the bin centered at $M_{\ell\ell} = 5 \text{ GeV}$, the SM expectation from Higgs of $m_H = 125 \text{ GeV}$ is $\sim 6.3 \times 10^{-9} \text{ GeV}$, while the signal associated with $H \rightarrow ZZ_d$ is $\sim 4.5 \times 10^{-8} \text{ GeV}$. With existing data of $N_H \simeq 75\,000$, no

meaningful number of signal or background events are expected, and one would need $N_{\text{Higgs}} \simeq 10^6$ for 3σ evidence (beyond the SM $H \rightarrow ZZ^* \rightarrow 4\ell$ channel) at the LHC experiments. However, this simple estimate ignores other reducible and irreducible backgrounds, and a more reliable statement requires inclusion of such details. Also, the $\ell_2^+ \ell_2^-$ decay pair from Z_d should exhibit an angular distribution consistent with its longitudinal polarization. That sensitivity is potentially orders of magnitude below the $\delta^2 < 0.006$ already established by atomic parity violation. We note that while the Higgs decay constraints on δ may not surpass those derived before from rare K and B decays, they are applicable well beyond the $\mathcal{O}(\text{GeV})$ regime of m_{Z_d} , relevant for the meson decays. They represent a potentially unique broad capability of the LHC unmatched by low energy experiments.

We should point out that current searches for $H \rightarrow ZZ^* \rightarrow 4\ell$ are likely to miss $H \rightarrow ZZ_d$ because they generally cut out a lighter second lepton pair with $M_{\ell\ell} \lesssim 15$ GeV, i.e., the range of interest, in order to avoid $Z\gamma^*$ backgrounds. Hopefully, our results will provide some incentive for revisiting the low mass region in search of Z_d .

In addition to $Z_d \rightarrow \ell^+ \ell^-$, one should mount a search for $H \rightarrow ZZ_d \rightarrow \ell^+ \ell^- + \text{missing energy}$. Here, one might be helped by the fact that the missing energy and missing momentum of the Z_d decay pair are nearly equal. A thorough study of LHC capabilities for uncovering that decay mode is clearly warranted. We also add that the Higgs can have a decay mode $H \rightarrow Z_d Z_d$ in our framework. The rate for this decay is proportional to δ^4 , so, roughly, it is suppressed compared to the ZZ_d mode by $\mathcal{O}(\delta^2)$, which, given our bound in Eq. (19), is a suppression of 0.006 or smaller. The rate for the $Z_d Z_d$ channel could be enhanced if hidden sector scalars that couple directly to Z_d and give it mass are allowed to mix with the SM sector Higgs scalars.

VI. A 2 HIGGS DOUBLET EXAMPLE

In the preceding discussion, we examined the dark Z phenomenology in a general framework. As mentioned before, the main ingredient we introduced was mass mixing between the SM Z and the Z_d , which could be realized in a variety of models. In this section, to demonstrate how our general framework might be realized, we will consider a 2 Higgs doublet extension of the SM. (See Ref. [53] for a recent review on 2HD models.) Here, we assume two $SU(2)_L \times U(1)_Y$ Higgs doublets, H_1 and H_2 , but allow H_2 to carry a dark charge that couples it directly to $U(1)_d$. Note that the assumption of the $U(1)_d$ in our example is well motivated, as it allows the model to evade severe constraints from flavor-changing neutral currents that are often addressed through the introduction of a \mathbb{Z}_2 symmetry in generic 2HD models. We also allow, for generality, a singlet scalar, H_d , that also provides part of the Z_d mass through its dark sector vacuum expectation value v_d .

With the above assumptions, H_2 does not couple directly to ordinary fermions, but does contribute to W^\pm , Z , and Z_d masses as well as Z - Z_d mixing through its vacuum expectation value v_2 . Such a setup is akin to what is often called a Type-I 2HD model [54]. Here, we will take H_1 to be a SM-like Higgs scalar, identified as H in our preceding general analysis. To keep the discussion simple, we ignore scalar mixing among the H_1 , H_2 , and H_d states. The v_1 , v_2 , and v_d vacuum expectation values of H_1 (the SM doublet), H_2 , and H_d give rise to $\delta = \sin\beta \sin\beta_d$, where $\tan\beta = v_2/v_1$ and $\tan\beta_d = v_2/v_d$, as will be shown in Appendix B. The condition of a SM-like H_1 can be satisfied, to a good approximation, for $\tan\beta \lesssim 1/3$ and does not require a large hierarchy of scales in the Higgs sector. The constraints on δ previously discussed will, however, constrain the product $\sin\beta \sin\beta_d$.

There are many additional features of our 2HD model worth studying. For example, nonzero Higgs scalar mixing (which we set to zero) could give rise to enhancements in $H \rightarrow Z_d Z_d$, as mentioned before, or perhaps $H \rightarrow hh$ (h being a lighter Higgs scalar remnant of H_2) [55]. Those possibilities are interesting but more model dependent.

VII. SUMMARY AND CONCLUSION

In this work, we explored the possibility of mass mixing between the Z boson of the SM and a new light vector boson Z_d associated with a hidden or dark sector $U(1)_d$ gauge symmetry. Such a light state has been invoked in discussions of astrophysical anomalies that may originate from cosmic dark matter. We dub this new vector boson the dark Z , as its properties are analogous to that of the SM Z . In particular, the couplings of Z_d can provide new sources of parity violation and measurably affect the decay of the Higgs through novel channels such as $H \rightarrow ZZ_d$. Existing atomic parity violation, polarized e scattering, and rare K and B decay data already place interesting bounds on the degree of Z - Z_d mass mixing, but further improvement is possible and warranted (see Table II).¹

The presence of kinetic mixing affects the phenomenology of Z_d , but much of the main physics discussed in our work persists even in the absence of kinetic mixing. Various experimental efforts are currently devoted to possible signals of the dark photon, based solely on the possibility of kinetic mixing between $U(1)_d$ and the SM photon. Here, we want to emphasize the $m_Z^2/m_{Z_d}^2$

¹One could contemplate searching for Z_d effects in precision neutrino neutral current cross section measurements such as $\nu_\mu e \rightarrow \nu_\mu e$ or deep-inelastic $\nu_\mu N \rightarrow \nu_\mu X$. However, to be competitive with anticipated low energy parity violation polarized electron scattering or atomic experiments, those neutrino studies would have to reach $\sim \pm 0.1\%$ statistical and normalization uncertainties, a challenging task that would likely require a high energy neutrino factory (see Ref. [56]). A detailed discussion of Z_d effects on neutrino cross sections will be given in a separate publication.

TABLE II. Rough ranges of current (future) constraints on δ from various processes examined along with commentary on applicability of the bounds. These processes have negligible sensitivity to pure kinetic mixing effects.

Process	Current (future) bound on δ	Comment
Low energy parity violation	$ \delta \leq 0.08 - 0.01$ (0.001)	Fairly independent of m_{Z_d} . Depends on ε .
Rare K decays	$ \delta \leq 0.01 - 0.001$ (0.0003)	$m_\pi^2 < m_{Z_d}^2 \ll m_K^2$. Depends on $\text{BR}(Z_d)$.
Rare B decays	$ \delta \leq 0.02 - 0.001$ (0.0003)	$m_\pi^2 < m_{Z_d}^2 \ll m_B^2$. Depends on $\text{BR}(Z_d)$. Some mass gap ~ 3 GeV.
$H \rightarrow ZZ_d$	$ \delta \leq (0.003 - 0.001)$	$m_{Z_d}^2 \ll (m_H - m_Z)^2$. Depends on $\text{BR}(Z_d)$ and background.

enhancement factor in low energy parity violation and the longitudinal polarization enhancement E_{Z_d}/m_{Z_d} , with E_{Z_d} the energy of Z_d , in rare meson decays and the Higgs decay $H \rightarrow ZZ_d$. These enhancements make such processes particularly sensitive to very small Z - Z_d mixing. In particular, future polarized ep and ee scattering experiments can provide further probes of the scenario we have considered in this work. These parity violating probes are sensitive to a wide range of Z_d masses, including $m_{Z_d} \lesssim 140$ MeV, where other searches fail because of π^0 Dalitz decays background and are independent of Z_d branching fractions. The rare K and B decays currently provide some of the most stringent bounds on the degree of Z - Z_d mixing; however, they depend on the Z_d branching fractions and also do not apply to m_{Z_d} above the meson mass. In addition, there can be gaps in the bounds, for example, in the m_{Z_d} charmonium mass region.

In the event of the discovery of a SM-like Higgs at the LHC, say at ~ 125 GeV based on current hints, a new front in the search for a dark Z can be established. The Higgs decay data are particularly unique for $m_{Z_d} \gtrsim 5$ GeV and hence probe a part of parameter space that is inaccessible to meson data. The reach for this new physics can be extended well beyond the current limits through precise measurements of Higgs decays, as may be done at an e^+e^- or $\mu^+\mu^-$ collider if high statistics are available. We conclude that pushing the above types of experiments as far as possible is strongly motivated, for they could be windows to the “dark side” of particle physics.

ACKNOWLEDGMENTS

This work was supported in part by the United States Department of Energy under Grant No. DE-AC02-98CH10886. W.M. acknowledges partial support from the Gutenberg Research College.

APPENDIX A: GAUGE KINETIC TERMS

The gauge kinetic terms allowed by the gauge symmetries $SU(2)_L \times U(1)_Y \times U(1)_d$ are

$$\begin{aligned} \mathcal{L}_{\text{gauge}} = & -\frac{1}{4} \hat{B}_{\mu\nu} \hat{B}^{\mu\nu} + \frac{1}{2} \frac{\varepsilon}{\cos\theta_W} \hat{B}_{\mu\nu} \hat{Z}_{d\mu\nu}^0 \\ & - \frac{1}{4} \hat{Z}_{d\mu\nu}^0 \hat{Z}_{d\mu\nu}^0 \end{aligned} \quad (\text{A1})$$

with $F_{\mu\nu} = \partial_\mu F_\nu - \partial_\nu F_\mu$. The hatted quantities are fields before the diagonalization of the gauge kinetic terms. The diagonalization is done by the field redefinition known as a $GL(2, R)$ rotation

$$\begin{pmatrix} Z_{d\mu}^0 \\ B_\mu \end{pmatrix} = \begin{pmatrix} \sqrt{1 - \varepsilon^2/\cos^2\theta_W} & 0 \\ -\varepsilon/\cos\theta_W & 1 \end{pmatrix} \begin{pmatrix} \hat{Z}_{d\mu}^0 \\ \hat{B}_\mu \end{pmatrix}, \quad (\text{A2})$$

after which B gets a \hat{Z}_d component proportional to ε while Z_d does not get any \hat{B} component.

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} Z_{d\mu\nu}^0 Z_{d\mu\nu}^{0\mu\nu}. \quad (\text{A3})$$

We will take $\hat{Z}_{d\mu}^0 = Z_{d\mu}^0$ and $\hat{B}_\mu = B_\mu + (\varepsilon/\cos\theta_W) Z_{d\mu}^0$ and ignore $\mathcal{O}(\varepsilon^2)$ terms from here on. After electroweak mixing with Weinberg angle θ_W

$$\begin{pmatrix} A \\ Z^0 \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B \\ W_3 \end{pmatrix}, \quad (\text{A4})$$

we get

$$\begin{aligned} A_\mu = \hat{A}_\mu - \varepsilon \hat{Z}_{d\mu}^0 & \quad Z_\mu^0 = \hat{Z}_\mu^0 + \varepsilon \tan\theta_W \hat{Z}_{d\mu}^0 \\ Z_{d\mu}^0 = \hat{Z}_{d\mu}^0 & \end{aligned} \quad (\text{A5})$$

as an effect of the gauge kinetic mixing. Thus, Z_d^0 is unaffected to $\mathcal{O}(\varepsilon)$ while both A_μ and Z_μ^0 are shifted by the gauge kinetic mixing followed by the electroweak mixing. However, the bare fields do not take into consideration Z^0 - Z_d^0 mixing via the mass matrix from the Higgs mechanism, which we will deal with in the following appendix.

APPENDIX B: SCALAR KINETIC TERMS

The scalar kinetic term is given by

$$\mathcal{L}_{\text{scalar}} = \sum_i |D_\mu \Phi_i|^2, \quad (\text{B1})$$

where i runs for all Higgs scalars. Considering only neutral components of gauge bosons, we have

$$\begin{aligned} D_\mu \Phi_i = & (\partial_\mu + ig'Y[\Phi_i]\hat{B}_\mu + igT_3[\Phi_i]\hat{W}_{3\mu} \\ & + ig_d Q_d[\Phi_i]\hat{Z}_{d\mu}^0) \Phi_i \end{aligned} \quad (\text{B2})$$

before gauge kinetic diagonalization, where Y , T_3 , and Q_d are hypercharge, isospin, and dark charge, respectively.

After symmetry breaking, the scalars can be written with the vacuum expectation values (v_i).

$$\Phi_i = \frac{1}{\sqrt{2}}(H_i + v_i). \quad (\text{B3})$$

1. Vector boson mass

From Eq. (B1), we can get the relevant vector boson mass terms

$$\mathcal{L}_{\text{scalar}} = \frac{1}{2}m_{Z^0}^2 Z^0 Z^0 - \Delta^2 Z^0 Z_d^0 + \frac{1}{2}m_{Z_d^0}^2 Z_d^0 Z_d^0 + \dots \quad (\text{B4})$$

The mixing of two vector bosons is given by

$$\begin{pmatrix} Z \\ Z_d \end{pmatrix} = \begin{pmatrix} \cos\xi & -\sin\xi \\ \sin\xi & \cos\xi \end{pmatrix} \begin{pmatrix} Z^0 \\ Z_d^0 \end{pmatrix} \quad (\text{B5})$$

with

$$\tan 2\xi = \frac{2\Delta^2}{m_{Z^0}^2 - m_{Z_d^0}^2}. \quad (\text{B6})$$

2HD Model Realization:

We discuss some details in context of the 2HD model example in Sec. VI. We set $U(1)_d$ charges as $Q_d[H_1] = 0$, $Q_d[H_2] = Q_d[H_d] = 1$ for notational convenience. Then the gauge boson mass squared is given by, with $g_Z = g'/\sin\theta_W = g/\cos\theta_W$,

$$\begin{aligned} m_{Z^0}^2 &= \frac{1}{4}g_Z^2(v_1^2 + v_2^2), \\ m_{Z_d^0}^2 &= g_d^2(v_2^2 + v_d^2) + \frac{\varepsilon}{\cos\theta_W}g_d g' v_2^2 \\ &\quad + \frac{1}{4}\left(\frac{\varepsilon}{\cos\theta_W}\right)^2 g'^2(v_1^2 + v_2^2), \\ \Delta^2 &= \frac{1}{2}g_d g_Z v_2^2 + \frac{1}{4}\frac{\varepsilon}{\cos\theta_W}g_Z g'(v_1^2 + v_2^2). \end{aligned} \quad (\text{B7})$$

We assume $m_{Z^0}^2 \ll m_{Z_d^0}^2$, which will be the case as long as $(g_d^2, \varepsilon g_d, \varepsilon^2) \ll g_Z^2$ and v_d is not exceedingly larger than the electroweak scale. We define $\tan\beta \equiv v_2/v_1$, $\tan\beta_d \equiv v_2/v_d$, and $v^2 \equiv v_1^2 + v_2^2 \simeq (246 \text{ GeV})^2$. Then we have

$$\begin{aligned} m_Z^2 &\simeq m_{Z^0}^2 = \frac{1}{4}g_Z^2 v^2, \\ m_{Z_d}^2 &\simeq m_{Z_d^0}^2 - \frac{(\Delta^2)^2}{m_{Z^0}^2} = g_d^2(v_d^2 + v^2 \sin^2\beta \cos^2\beta) \\ &= g_d^2 v^2 \frac{\sin^2\beta}{\sin^2\beta_d} (1 - \sin^2\beta \sin^2\beta_d), \\ \xi &\simeq \frac{\Delta^2}{m_{Z^0}^2} = \frac{2g_d}{g_Z} \sin^2\beta + \varepsilon \tan\theta_W. \end{aligned} \quad (\text{B8})$$

Gauge kinetic mixing ε does not contribute to Z_d mass but it affects the Z - Z_d mixing angle ξ .

- (i) In the $v_2 = 0$ limit (i.e., pure dark photon limit), the Z_d mass is entirely from the Higgs singlet H_d and the Z - Z_d mixing angle is provided entirely by ε . We have

$$m_Z^2 \simeq m_{Z^0}^2 \quad m_{Z_d}^2 \simeq g_d^2 v_d^2 \quad \xi \simeq \varepsilon \tan\theta_W, \quad (\text{B9})$$

which give

$$M_0^2 \simeq \begin{pmatrix} m_Z^2 & -\varepsilon \tan\theta_W m_Z^2 \\ -\varepsilon \tan\theta_W m_Z^2 & m_{Z_d}^2 + \varepsilon^2 \tan^2\theta_W m_Z^2 \end{pmatrix}. \quad (\text{B10})$$

The mixing induced by the mass matrix cancels the effects because of field redefinition in Eq. (A5) for the Z_d -induced neutral current coupling.

- (ii) In the $\varepsilon = 0$ limit (i.e., pure dark Z limit),

$$\begin{aligned} m_{Z_d}^2 &\simeq g_d^2 v^2 \frac{\sin^2\beta}{\sin^2\beta_d} (1 - \sin^2\beta \sin^2\beta_d) \\ \xi &\simeq \frac{2g_d}{g_Z} \sin^2\beta \simeq \frac{m_{Z_d}}{m_Z} \frac{\sin\beta \sin\beta_d}{\sqrt{1 - \sin^2\beta \sin^2\beta_d}}. \end{aligned} \quad (\text{B11})$$

Taking $1 - \sin^2\beta \sin^2\beta_d \simeq 1$ is valid when $|\Delta^2| \ll m_{Z^0} m_{Z_d^0}$. In this limit

$$m_Z^2 \simeq m_{Z^0}^2 \quad m_{Z_d}^2 \simeq m_{Z_d^0}^2 \quad \xi \simeq \varepsilon_Z \quad (\text{B12})$$

with

$$\varepsilon_Z = \frac{m_{Z_d}}{m_Z} \delta \quad \text{and} \quad \delta = \sin\beta \sin\beta_d. \quad (\text{B13})$$

2. Higgs-vector-vector couplings

We assume no mixing among Higgs scalars and refer to the SM-like Higgs as H . From Eq. (B1), we can get the relevant Higgs coupling to vector bosons.

$$\begin{aligned} \mathcal{L}_{\text{scalar}} &= \frac{1}{2}\mathcal{C}_{HZZ}HZZ + \mathcal{C}_{HZZ_d}HZZ_d \\ &\quad + \frac{1}{2}\mathcal{C}_{HZ_dZ_d}HZ_dZ_d + \dots \end{aligned} \quad (\text{B14})$$

The Feynman rules for coupling of H to two vector bosons V_1 and V_2 are then given by $ig_{\mu\nu}\mathcal{C}_{HV_1V_2}$.

In the 2HD example, we get

$$\begin{aligned} \mathcal{C}_{HZZ} &= \mathcal{C}_{HZZ}^{\text{SM}} \cos\beta (\cos\xi + \varepsilon \tan\theta_W \sin\xi)^2 \\ \mathcal{C}_{HZZ_d} &= \mathcal{C}_{HZZ}^{\text{SM}} \cos\beta (\cos\xi + \varepsilon \tan\theta_W \sin\xi) \\ &\quad \times (\sin\xi - \varepsilon \tan\theta_W \cos\xi) \\ \mathcal{C}_{HZ_dZ_d} &= \mathcal{C}_{HZZ}^{\text{SM}} \cos\beta (\sin\xi - \varepsilon \tan\theta_W \cos\xi)^2 \end{aligned} \quad (\text{B15})$$

with $\mathcal{C}_{HZZ}^{\text{SM}} = \frac{1}{2}g_Z^2 v$.

The ratio of couplings is

$$\Theta = \frac{\mathcal{C}_{HZZ_d}}{\mathcal{C}_{HZZ}} = \frac{\mathcal{C}_{HZ_dZ_d}}{\mathcal{C}_{HZZ_d}} = \frac{\sin\xi - \varepsilon \tan\theta_W \cos\xi}{\cos\xi + \varepsilon \tan\theta_W \sin\xi}, \quad (\text{B16})$$

which, with small $|\xi| \ll 1$ from Eq. (B8), yields

$$\Theta \simeq \xi - \varepsilon \tan\theta_W \simeq \frac{2g_d}{g_Z} \sin^2\beta, \quad (\text{B17})$$

showing that Θ is not sensitive to ε .

The relevant Higgs decay rates, for $m_{Z_d} \ll m_H$, are given by

$$\begin{aligned} \Gamma(H \rightarrow ZZ) &= \frac{1}{128\pi} \frac{m_H^3}{m_Z^4} \sqrt{1 - \frac{4m_Z^2}{m_H^2} \left(1 - \frac{4m_Z^2}{m_H^2} + \frac{12m_Z^4}{m_H^4}\right)} \\ &\quad \times (C_{HZZ})^2 \\ \Gamma(H \rightarrow ZZ_d) &\simeq \frac{1}{64\pi} \frac{m_H^3}{m_Z^2 m_{Z_d}^2} \left(1 - \frac{m_Z^2}{m_H^2}\right)^3 (\Theta C_{HZZ})^2 \\ \Gamma(H \rightarrow Z_d Z_d) &\simeq \frac{1}{128\pi} \frac{m_H^3}{m_{Z_d}^4} (\Theta^2 C_{HZZ})^2 \end{aligned} \quad (\text{B18})$$

with couplings given in Eq. (B15). Equation (B18) conveniently shows the effects of phase space and Z-Z_d mixing in the Higgs decay rates. The ratio of Higgs decay rates in the Z_dZ_d and ZZ_d channels is

$$\begin{aligned} \frac{\Gamma(H \rightarrow Z_d Z_d)}{\Gamma(H \rightarrow ZZ_d)} &\simeq \frac{\Theta^2}{2} \frac{m_Z^2}{m_{Z_d}^2} \left(1 - \frac{m_Z^2}{m_H^2}\right)^{-3} \\ &\simeq \frac{1}{2} \sin^2\beta \sin^2\beta_d \left(1 - \frac{m_Z^2}{m_H^2}\right)^{-3} \\ &= \frac{1}{2} \delta^2 \left(1 - \frac{m_Z^2}{m_H^2}\right)^{-3}, \end{aligned} \quad (\text{B19})$$

where Eqs. (B11) and (B12) have been used in the second line.

APPENDIX C: Z_d DECAY WIDTH

Using Eqs. (10) and (11) in the text, we find that the partial decay width of Z_d into the SM fermion pair $f\bar{f}$ is given by, neglecting m_f/m_{Z_d} corrections [46],

$$\Gamma(Z_d \rightarrow f\bar{f}) \simeq \frac{N_C}{48\pi} \varepsilon_Z^2 g_Z^2 (g_{Vf}^2 + g_{Af}^2) m_{Z_d}, \quad (\text{C1})$$

where $g_{Vf}^2 = T_{3f} - 2Q_f(\sin^2\theta_W - (\varepsilon/\varepsilon_Z)\cos\theta_W \sin\theta_W)$ and $g_{Af}^2 = -T_{3f}$. Here, $N_C = 3$ for quarks and $N_C = 1$ for leptons.

-
- [1] K. Nakamura *et al.* (Particle Data Group), *J. Phys. G* **37**, 075021 (2010).
 - [2] P. Fayet, *Phys. Rev. D* **70**, 023514 (2004); D. P. Finkbeiner and N. Weiner, *Phys. Rev. D* **76**, 083519 (2007); N. Arkani-Hamed, D. P. Finkbeiner, T. R. Slatyer, and N. Weiner, *Phys. Rev. D* **79**, 015014 (2009).
 - [3] O. Adriani *et al.* (PAMELA Collaboration), *Nature (London)* **458**, 607 (2009).
 - [4] For very early considerations of a light new gauge boson, see P. Fayet, *Phys. Lett.* **95B**, 285 (1980); **96B**, 83 (1980); P. Fayet and M. Mezard, *Phys. Lett.* **104B**, 226 (1981).
 - [5] Typically, discussions of Z' effects on Higgs physics have involved a heavy Z'. See, for example, K. Agashe *et al.*, *Phys. Rev. D* **76**, 115015 (2007); V. Barger, P. Langacker, and H. S. Lee, *Phys. Rev. Lett.* **103**, 251802 (2009).
 - [6] K. S. Babu, C. F. Kolda, and J. March-Russell, *Phys. Rev. D* **57**, 6788 (1998).
 - [7] C. Bouchiat and P. Fayet, *Phys. Lett. B* **608**, 87 (2005); P. Fayet, *Phys. Rev. D* **74**, 054034 (2006); *Phys. Lett. B* **675**, 267 (2009); *Phys. Rev. D* **81**, 054025 (2010).
 - [8] M. T. Frandsen, F. Kahlhoefer, S. Sarkar, and K. Schmidt-Hoberg, *J. High Energy Phys.* **09** (2011) 128.
 - [9] O. Lebedev, H. M. Lee, and Y. Mambrini, *Phys. Lett. B* **707**, 570 (2012).
 - [10] J. F. Kamenik and C. Smith, *J. High Energy Phys.* **03** (2012) 090; *Phys. Rev. D* **85**, 093017 (2012).
 - [11] B. Holdom, *Phys. Lett.* **166B**, 196 (1986).
 - [12] S. A. Abel, M. D. Goodsell, J. Jaeckel, V. V. Khoze, and A. Ringwald, *J. High Energy Phys.* **07** (2008) 124.
 - [13] E. C. G. Stueckelberg, *Helv. Phys. Acta* **11**, 225 (1938).
 - [14] D. Feldman, Z. Liu, and P. Nath, *Phys. Rev. D* **75**, 115001 (2007).
 - [15] S. Gopalakrishna, S. Jung, and J. D. Wells, *Phys. Rev. D* **78**, 055002 (2008).
 - [16] B. Batell, M. Pospelov, and A. Ritz, *Phys. Rev. D* **79**, 115008 (2009).
 - [17] J. D. Bjorken, R. Essig, P. Schuster, and N. Toro, *Phys. Rev. D* **80**, 075018 (2009).
 - [18] J. Jaeckel and A. Ringwald, *Annu. Rev. Nucl. Part. Sci.* **60**, 405 (2010).
 - [19] S. Abrahamyan *et al.* (APEX Collaboration), *Phys. Rev. Lett.* **107**, 191804 (2011).
 - [20] L. Ackerman, M. R. Buckley, S. M. Carroll, and M. Kamionkowski, *Phys. Rev. D* **79**, 023519 (2009).
 - [21] W. J. Marciano and A. Sirlin, *Phys. Rev. D* **22**, 2695 (1980); **31**, 213(E) (1985).
 - [22] W. J. Marciano and A. Sirlin, *Phys. Rev. D* **27**, 552 (1983); **29**, 75 (1984); **31**, 213(E) (1985).
 - [23] W. J. Marciano and J. L. Rosner, *Phys. Rev. Lett.* **65**, 2963 (1990); **68**, 898(E) (1992).
 - [24] W. J. Marciano, in *21st Annual SLAC Summer Institute on Particle Physics Proceedings* (Stanford University, Stanford, CA, 1993), p. 35.
 - [25] S. G. Porsev, K. Bely, and A. Derevianko, *Phys. Rev. Lett.* **102**, 181601 (2009); *Phys. Rev. D* **82**, 036008 (2010).
 - [26] C. S. Wood, S. C. Bennett, D. Cho, B. P. Masterson, J. L. Roberts, C. E. Tanner, and C. E. Wieman, *Science* **275**, 1759 (1997).
 - [27] S. C. Bennett and C. E. Wieman, *Phys. Rev. Lett.* **82**, 2484 (1999); **82**, 4153(E) (1999); **83**, 889(E) (1999).

- [28] E. Derman and W. J. Marciano, *Ann. Phys. (Leipzig)* **121**, 147 (1979).
- [29] A. Czarnecki and W. J. Marciano, *Phys. Rev. D* **53**, 1066 (1996); *Int. J. Mod. Phys. A* **15**, 2365 (2000); *Nature (London)* **435**, 437 (2005).
- [30] P. L. Anthony *et al.* (SLAC E158 Collaboration), *Phys. Rev. Lett.* **95**, 081601 (2005).
- [31] P. Fayet, *Phys. Rev. D* **75**, 115017 (2007); M. Pospelov, *Phys. Rev. D* **80**, 095002 (2009).
- [32] R. D. McKeown, [arXiv:1109.4855](https://arxiv.org/abs/1109.4855); D. S. Armstrong *et al.*, JLab Proposal Report No. E02-020, 2007.
- [33] K. Aulenbacher, *Hyperfine Interact.* **200**, 3 (2011).
- [34] V. A. Dzuba, V. V. Flambaum, and I. B. Khriplovich, *Z. Phys. D* **1**, 243 (1986).
- [35] E. N. Fortson, Y. Pang, and L. Willets, *Phys. Rev. Lett.* **65**, 2857 (1990).
- [36] C. Monroe, W. Swann, H. Robinson, and C. Wieman, *Phys. Rev. Lett.* **65**, 1571 (1990).
- [37] B. A. Brown, A. Derevianko, and V. V. Flambaum, *Phys. Rev. C* **79**, 035501 (2009).
- [38] A. I. Vainshtein and I. B. Khriplovich, *Pis'ma Zh. Eksp. Teor. Fiz.* **18**, 141 (1973); M. K. Gaillard, B. W. Lee, and R. E. Shrock, *Phys. Rev. D* **13**, 2674 (1976); T. Inami and C. S. Lim, *Prog. Theor. Phys.* **65**, 297 (1981); **65**, 1772(E) (1981).
- [39] B. Batell, M. Pospelov, and A. Ritz, *Phys. Rev. D* **83**, 054005 (2011).
- [40] J. M. Cornwall, D. N. Levin, and G. Tiktopoulos, *Phys. Rev. D* **10**, 1145 (1974); **11**, 972(E) (1975); M. S. Chanowitz and M. K. Gaillard, *Nucl. Phys.* **B261**, 379 (1985); W. J. Marciano and S. S. D. Willenbrock, *Phys. Rev. D* **37**, 2509 (1988).
- [41] L. J. Hall and M. B. Wise, *Nucl. Phys.* **B187**, 397 (1981).
- [42] J. M. Frere, J. A. M. Vermaseren, and M. B. Gavela, *Phys. Lett.* **103B**, 129 (1981).
- [43] M. Freytsis, Z. Ligeti, and J. Thaler, *Phys. Rev. D* **81**, 034001 (2010).
- [44] R. Appel *et al.* (E865 Collaboration), *Phys. Rev. Lett.* **83**, 4482 (1999).
- [45] J. R. Batley *et al.* (NA48/2 Collaboration), *Phys. Lett. B* **677**, 246 (2009); **697**, 107 (2011).
- [46] D. Albert, W. J. Marciano, D. Wyler, and Z. Parsa, *Nucl. Phys.* **B166**, 460 (1980).
- [47] B. Aubert *et al.* (BABAR Collaboration), *Phys. Rev. Lett.* **102**, 091803 (2009).
- [48] J. T. Wei *et al.* (BELLE Collaboration), *Phys. Rev. Lett.* **103**, 171801 (2009).
- [49] G. Aad *et al.* (ATLAS Collaboration), *Phys. Lett. B* **710**, 49 (2012).
- [50] S. Chatrchyan *et al.* (CMS Collaboration), *Phys. Lett. B* **710**, 26 (2012).
- [51] S. Dittmaier *et al.*, [arXiv:1201.3084](https://arxiv.org/abs/1201.3084).
- [52] W. Y. Keung and W. J. Marciano, *Phys. Rev. D* **30**, 248 (1984).
- [53] G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher, and J. P. Silva, [arXiv:1106.0034](https://arxiv.org/abs/1106.0034).
- [54] J. F. Gunion, H. E. Haber, G. Kane, and S. Dawson, *The Higgs Hunter's Guide* (Addison-Wesley, Reading, MA, 1990).
- [55] P. M. Ferreira, R. Santos, M. Sher, and J. P. Silva, *Phys. Rev. D* **85**, 035020 (2012).
- [56] W. J. Marciano and Z. Parsa, *J. Phys. G* **29**, 2629 (2003).