

Large mass splittings for fourth generation fermions allowed by LHC Higgs boson exclusionAmol Dighe,^{1,*} Diptimoy Ghosh,^{1,†} Rohini M. Godbole,^{2,‡} and Arun Prasath^{2,§}¹*Tata Institute of Fundamental Research, 1, Homi Bhabha Road, Mumbai 400 005, India*²*Centre for High Energy Physics, Indian Institute of Science, Bangalore 560 012, India*

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In the context of the standard model with a fourth generation, we explore the allowed mass spectra in the fourth-generation quark and lepton sectors as functions of the Higgs mass. Using the constraints from unitarity and oblique parameters, we show that a heavy Higgs allows large mass splittings in these sectors, opening up new decay channels involving W emission. Assuming that the hints for a light Higgs do not yet constitute an evidence, we work in a scenario where a heavy Higgs is viable. A Higgs heavier than ~ 800 GeV would in fact necessitate either a heavy quark decay channel $t' \rightarrow b'W/b' \rightarrow t'W$ or a heavy lepton decay channel $\tau' \rightarrow \nu'W$ as long as the mixing between the third and fourth generations is small. This mixing tends to suppress the mass splittings and hence the W -emission channels. The possibility of the W -emission channel could substantially change the search strategies of fourth-generation fermions at the LHC and impact the currently reported mass limits.

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I. INTRODUCTION

The number of fermion generations in the standard model (SM) is three, though there is no fundamental principle restricting it to this number. The data on Z decay only puts a lower bound of $m_Z/2$ on their masses [1]. Direct searches at the Tevatron [1–6] and the LHC [7–14] further put lower bounds on the masses of fourth-generation charged fermions, by virtue of them not having been observed at these colliders. These limits are subject to certain assumptions about the decay channels for these quarks. The most conservative, almost model-independent limits for fourth-generation quarks are given in [15]. Indirect limits on the masses and mixing of these fermions are also obtained through the measurements of the oblique parameters S , T , U [16]. Theoretical constraints like the perturbativity of the Yukawa couplings and the perturbative unitarity of heavy fermion scattering amplitudes [17] bound the masses from above. In spite of the rather strong bounds from all the above directions, there is still parameter space available for the fourth generation that is consistent with all the data [18–29]. The fourth-generation scenario (SM4) is thus still viable even after the recent Tevatron and LHC results [30,31].

The discovery of a fourth generation of fermions will have profound phenomenological consequences [32]. Some of the experimental observations that deviate somewhat from the SM expectations—like the CP -violating phases in the neutral B mixing [33–36]—could be interpreted as radiative effects by the fourth-generation fermions. The implications of a fourth family for observables in charmed decays [37] and lepton-flavor violating decays

[38] as well as flavor constraints on the quark sector [39] have also been discussed.

The Cabibbo-Kabayashi-Maskawa (CKM) structure of 4 generations, with 3 observable phases and large Yukawa couplings, may also provide enough source of CP violation for the matter-antimatter asymmetry of the Universe, although the issue of the order of the electroweak phase transition still has to be resolved [40]. The existence of a fourth generation is also intimately connected with the Higgs physics. The Higgs in the SM4 with a mass of about 800 GeV is consistent with precision electroweak (EW) data when $m_{t'}, m_{b'} \sim 500$ GeV, with the mixing between third and fourth-generation quarks of the order of 0.1 [25]. This raises an interesting possibility of Higgs being a composite scalar of fourth-generation quarks [41–43] with interesting phenomenological implications including an enhancement of flavor-changing as well as flavor-diagonal Higgs decays into third and fourth-generation fermions. Implications of a strongly interacting fourth-generation quark sector on LHC Higgs searches has been discussed in [44]. Phenomenology of the lepton sector of the SM4 has been studied in [45–48].

The Higgs production cross section at hadron colliders is affected strongly by the fourth generation through the $gg \rightarrow h$ channel due to the heavy masses of the new fermions [49–52]. The branching ratios of Higgs into different channels are affected too [49,52,53]. As a consequence, the direct search limits on the Higgs mass are stronger in the presence of the fourth generation. Higgs production and decay cross sections in the context of four generations, with next-to-leading order EW and QCD corrections, have been calculated in [50–52]. The production cross section is enhanced for a light Higgs ($m_h < 200$ GeV) by an order of magnitude. However the enhancement may be somewhat reduced for a heavier Higgs [54]. The ATLAS experiment at the LHC has ex-

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cluded the Higgs boson of SM4 with a mass between 119 GeV and 593 GeV [55], while the CMS exclusion limits are from 120 GeV to 600 GeV [56]. These experiments include the one loop EW corrections to Higgs production from the fourth-generation fermions.¹

Recently, experiments at the Tevatron and the LHC have reported an excess of events around $m_h \approx 125$ GeV with a local significance of about $2\sigma - 3\sigma$ in the search for the standard model Higgs boson [63–65]. Reference [66] interprets these “hints” for a light Higgs in the framework of SM4. We on the other hand, assume that these hints do not yet constitute an evidence and *may* well be a statistical fluctuation. Under this assumption, a heavy Higgs with mass ≥ 600 GeV is a viable scenario.

Recent explorations of possible effects of a fourth generation on the Higgs mass, precision observables, quark mixing matrix, and flavor-changing neutral current phenomena have yielded interesting results. It has been pointed out [67] that the existence of a fourth generation allows for a heavier Higgs to be consistent with the precision measurements. The constraints on the mixing between the third and fourth generation have been obtained from the precision EW data [25], and a fit to the flavor-physics data [68]. The latter shows that, while the mixing of the fourth-generation quarks to the three SM generations is consistent with zero and restricted to be small, observable effects on K and B decays are still possible. Large masses of the fourth-generation fermions lead to nonperturbativity of Yukawa couplings at a low scale $\Lambda \ll M_{\text{GUT}}$ as well as instability of the vacuum. This has been investigated in the context of models without supersymmetry (SUSY) [41,67,69–72] and with SUSY [73], after taking into account various bounds from precision EW data as well as collider and direct search experiments on sequential heavy fermions.

In this article, we revisit the electroweak precision constraints from the oblique parameters S , T , U on the fourth generation, taking into account the mixing with the third generation. We perform a χ^2 analysis, varying the fourth-generation quark as well as lepton masses in their experimentally allowed ranges, and obtain a quantitative measure for the fourth-generation fermion mass spectrum preferred by the measurements of these parameters. In the light of the heavy Higgs preferred by the LHC data, we focus on the implications of a heavy Higgs for the mass spectrum. We also study the effect of the mixing between third and fourth generation on this mass spectrum, and try to understand these effects analytically. As we will see later, the correlation between the mass splittings in quark and lepton sectors is strongly influenced by this mixing angle.

¹These bounds can be circumvented either by a suitable extension of the scalar sector [57] or by having Higgs decay to stable invisible particles which could be candidates for dark matter [58–62]. However this is not the minimal SM4 we focus on in this paper.

The paper is organized as follows. In Sec. II, we discuss the bounds on the fourth-generation fermion masses from direct searches and the theoretical requirement of perturbative unitarity. We also analyze the structure of constraints from the measurements of oblique parameters. In Sec. III, we perform a χ^2 -fit to the oblique parameters and obtain constraints on the mass splittings in the quark and lepton sectors, focusing on a heavy Higgs. Sec. IV discusses the collider implications, while Sec. V summarizes our results.

II. CONSTRAINTS ON THE FOURTH-GENERATION FERMION MASSES

A. Lower bounds on masses from direct searches

The direct search constraints presented by CDF [2,3] on the masses of t' and b' quarks have been generalized to more general cases of quark mixing by [15], and a lower bound $m_{t'}, m_{b'} > 290$ GeV has been obtained. The currently quoted exclusion bounds by CDF, DØ, CMS and ATLAS collaborations [6,8–10,12,14] are 400–500 GeV, however as stated earlier, they are based on specific assumptions on branching ratios of the fourth-generation quarks and mass differences between the fourth-generation fermions.

The limits on the masses of heavy charged fermions are obtained from the nonobservation of their expected decay modes. The choice of analyzed decay modes affects the bounds to a large extent. Since we would like our results to be independent of assumptions about the mixing angles, mass differences, and hence branching ratios, in our analysis we shall use the bounds from [15] for the quark masses. For the fourth-generation leptons τ' and ν' , we take the bounds $m_{\tau'} > 101.0$ GeV and $m_{\nu'} > 45.0$ GeV [1]. The mixing of the fourth-generation leptons is restricted to be very small, so it would not affect our analysis.

B. Upper bounds on masses from unitarity

The direct search constraints imply that the fourth-generation quarks are necessarily heavy ($m_F \gg M_W, M_Z$). For such heavy fermions F , the tree-level amplitudes of certain processes like $F\bar{F} \rightarrow F\bar{F}$, WW , ZZ , ZH , HH , in a spontaneously broken $SU(2)_L \times U(1)_Y$ gauge theory, tend to a constant value $G_F m_F^2$ at center-of-mass energies $s \gg m_F^2$. For large value of m_F , the term $G_F m_F^2$ can be $\mathcal{O}(1)$. In that case, the tree-level unitarity of the S -matrix is saturated and in order to regain a unitary S -matrix, higher order amplitudes need to contribute significantly. This necessitates a strong coupling of these fermions to the gauge bosons, which makes the perturbation theory unreliable. This was first studied in the context of the SM with ultraheavy fermions in [17,74]. The corresponding analysis in the context of the minimal supersymmetric standard model with a sequential fourth generation was performed in

[75], in the limit of vanishing mixing between the fourth-generation quarks and the first three generations.

We reevaluate the bounds given in [74], considering the $J = 0$ partial-wave channel of the tree-level amplitudes of the color-neutral and charge-neutral processes $F_i \bar{F}_i \rightarrow F_j \bar{F}_j$. In [74] only the amplitudes involving two heavy fermions of a $SU(2)_L$ doublet were analyzed. The second $SU(2)_L$ doublet of heavy fermions only provided a source for mixing included in the analysis. However, we include all the relevant channels involving all the heavy quarks $-t'$, b' and t , and take into account mixing between the third and fourth generations. The lowest critical value of the fermion mass is obtained by equating the largest eigenvalue of this submatrix to unity. Expressions for the partial-wave matrices are given in the Appendix. The results are shown in Fig. 1 for $\sin\theta_{34} = 0.0, 0.3$. One can easily observe from the figure an improvement of about 6% in the bounds compared to those in [74]. The bounds are affected due to the inclusion of the top quark in the analysis, which introduces more scattering channels. It may be seen from Fig. 1 that the bounds are not very sensitive to the actual value of the mixing.

In the lepton sector, only fourth-generation leptons (τ' and $\nu_{\tau'}$) are relevant for the perturbative unitarity constraints as all the first three generation fermions are light compared to M_W . The mixing between the fourth-generation leptons and the first three generations is constrained by experimental bounds on lepton-flavor violating processes [38,76,77]. The 2σ lower bound on the (4, 4) element of the Pontecarvo-Maki-Nakagawa-Sakata matrix in SM4, $U_{\tau'\nu'}$, is very close to unity: $|U_{\tau'\nu'}| > 0.9934$ [76].

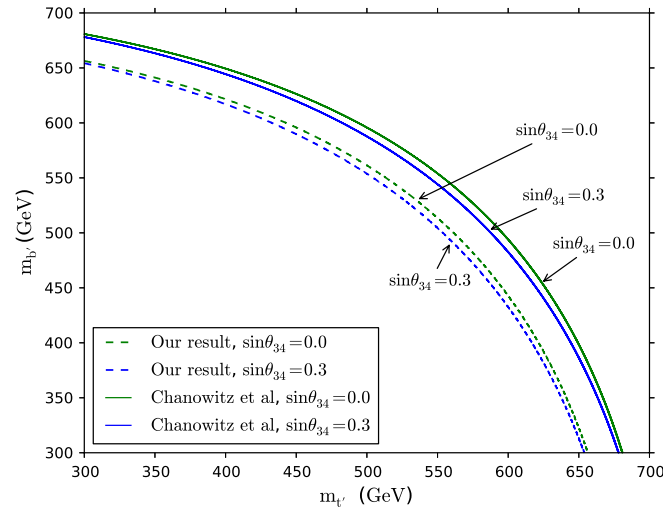


FIG. 1 (color online). The perturbative unitarity bounds on $(m_{t'}, m_{b'})$ are shown above as dashed-lines for $\sin\theta_{34} = 0.0$ (green/light) and 0.3 (blue/dark). The solid lines are obtained from the analytical expression for the unitarity bound given in [74] by substituting the respective values of $\sin\theta_{34}$. The corresponding bounds on the fourth-generation lepton masses are $m_{\tau', \nu'} < 1.2$ TeV [74].

Moreover, we do not have any heavy fermion in the first three generations in contrast to the quark sector which has the top quark. Therefore, we do not have any new channel in addition to those that are considered in [74]. Hence we do not expect any improvement over the bounds given in [74] in the case of leptons.

C. Constraints from oblique parameters

The oblique parameters S, T, U [78] are sensitive probes of the fourth-generation masses and mixing pattern. The definitions of S, T, U parameters are given by

$$S = 16\pi[\Pi'_{33}(0) - \Pi'_{3Q}(0)], \quad (1)$$

$$T = \frac{4\pi}{s_W^2(1 - s_W^2)m_Z^2}[\Pi_{11}(0) - \Pi_{33}(0)], \quad (2)$$

$$U = 16\pi[\Pi'_{11}(0) - \Pi'_{33}(0)], \quad (3)$$

where $s_W^2 = \sin^2\theta_W$, and $\Pi(q^2)$ are the vacuum polarization Π -functions. The suffixes 1, 2, 3 refer to the generators of $SU(2)_L$, and the suffix Q to that of the electromagnetic current. The contribution to these parameters from a fermion doublet (u, d) is [78]

$$S(x_u, x_d) = \frac{N_C}{6\pi} \left[1 - Y \log\left(\frac{x_u}{x_d}\right) \right], \quad (4)$$

$$T(x_u, x_d) = \frac{N_C}{16\pi s_W^2(1 - s_W^2)} \times \left[x_u + x_d - \frac{2x_u x_d}{x_u - x_d} \log\left(\frac{x_u}{x_d}\right) \right], \quad (5)$$

$$U(x_u, x_d) = \frac{N_C}{6\pi} \left[-\frac{5x_u^2 - 22x_u x_d + 5x_d^2}{3(x_u - x_d)^2} + \frac{x_u^3 - 3x_u^2 x_d - 3x_u x_d^2 + x_d^3}{(x_u - x_d)^3} \log\left(\frac{x_u}{x_d}\right) \right], \quad (6)$$

where $x_f \equiv m_f^2/m_Z^2$. Here N_C is the number of colors of the fermions ($N_C = 3$ for quarks and $N_C = 1$ for leptons), and Y is the hypercharge of the fermion doublet. Note that when the quarks in the doublet are almost degenerate, i.e. $\Delta \equiv |m_u - m_d| \ll m_u, m_d$,

$$T(x_u, x_d) = \frac{1}{12\pi s_W^2(1 - s_W^2)} \left(\frac{\Delta^2}{m_Z^2} \right). \quad (7)$$

These expressions can be readily generalized to the case of additional sequential generations.

Following Gfitter [16], we fix the masses of the top quark and the Higgs boson at their reference values of $\tilde{m}_t = 173.1$ GeV and $\tilde{m}_h = 120$ GeV. With these masses, in the SM we have $S = 0, T = 0$ and $U = 0$. The deviation from these values are denoted by $\Delta S, \Delta T, \Delta U$ respectively. The effect of the Higgs mass appears through the dependence [78]

$$\Delta S_H = \frac{1}{6\pi} \log\left(\frac{m_h}{\tilde{m}_h}\right), \quad (8)$$

$$\Delta T_H = -\frac{3}{8\pi(1-s_W^2)} \log\left(\frac{m_h}{\tilde{m}_h}\right), \quad (9)$$

$$\Delta U_H \approx 0. \quad (10)$$

The contribution from the fourth generation of fermions to these parameters, after taking into account the mixing between third and fourth generation of quarks, can be expressed as

$$\Delta S_4 = S(x_{t'}, x_{b'}) + S(x_{\nu'}, x_{\tau'}), \quad (11)$$

$$\begin{aligned} \Delta T_4 = & -s_{34}^2 T(x_{t'}, x_{b'}) + s_{34}^2 T(x_{t'}, x_b) + s_{34}^2 T(x_t, x_{b'}) \\ & + c_{34}^2 T(x_{t'}, x_{b'}) + T(x_{\nu'}, x_{\tau'}), \end{aligned} \quad (12)$$

$$\begin{aligned} \Delta U_4 = & -s_{34}^2 U(x_t, x_b) + s_{34}^2 U(x_{t'}, x_b) + s_{34}^2 U(x_t, x_{b'}) \\ & + c_{34}^2 U(x_{t'}, x_{b'}) + U(x_{\nu'}, x_{\tau'}), \end{aligned} \quad (13)$$

where $s_{34}^2 = \sin^2\theta_{34}$, $c_{34}^2 = \cos^2\theta_{34}$ and θ_{34} is the mixing angle between the third and the fourth-generation quarks. Note that here we neglect the mixing of the fourth-generation quarks with the first two generations, since the bounds on this mixing are rather strong [68]. We also neglect any mixing of the fourth leptonic generation. We work in an approximation in which we neglect the non-oblique corrections to precision EW observables. This allows us to use the S , T , U values provided by fits to the precision EW observables, for example the ones provided by the Gfitter group [16].

When the mixing of the fourth and third generation quarks is nonzero, the decay width for $Z \rightarrow b\bar{b}$ receives contribution from fourth-generation quarks through vertex corrections, in addition to the oblique corrections. To study the effect of fourth-generation fermions on the precision EW observables in general mixing scenarios, one should, in principle include both the vertex and the oblique corrections to precision EW observables [79]. However, as mentioned above, in order to use the Gfitter results on S , T , U which were obtained in the limit of vanishing mixing, we use only the values of the mixing angles that are consistent with the $Z \rightarrow b\bar{b}$ constraints [68].

In our analysis, we evaluate the S , T , U parameters numerically using FeynCalc [80] and LoopTools [81]. Then we take the experimentally measured values of these parameters [16] and perform a χ^2 -fit to six parameters: the four combinations of masses

$$\begin{aligned} m_q &\equiv (m_{t'} + m_{b'})/2, & \Delta_q &\equiv m_{t'} - m_{b'}, \\ m_\ell &\equiv (m_{\nu'} + m_{\tau'})/2, & \Delta_\ell &\equiv m_{\nu'} - m_{\tau'}, \end{aligned}$$

the Higgs mass m_h , and the mixing $\sin\theta_{34}$. We take the ranges of m_q and m_ℓ to be those allowed by the unitarity constraints and the direct search bounds stated above. For

the other parameters, we take $|\Delta_q| < 200$ GeV, $|\Delta_\ell| < 200$ GeV, 100 GeV $< m_h < 800$ GeV, $|\sin\theta_{34}| < 0.3$, unless explicitly specified otherwise.

We present our results in terms of the goodness-of-fit contours for the joint estimates of two parameters at a time, where the other four parameters are chosen to get the minimum of χ^2 . For the purposes of this investigation, we show contours of $p = 0.0455$, which correspond to $\chi^2 = 6.18$, or a confidence level (C.L.) of 95%.

III. CONSTRAINTS ON THE MASS SPLITTINGS Δ_q AND Δ_ℓ

We now focus on the constraints on the mass splittings Δ_q and Δ_ℓ . We scan over the allowed values of the other parameters, and take only those parameter sets that are consistent with all the data currently available.

A. Constraints on Δ_q

In the top left panel of Fig. 2, we show the 95% C.L. contours in the m_h - Δ_q plane marginalizing over other new-physics (NP) parameters (m_q , m_l , Δ_ℓ , θ_{34}). From this panel, it is observed that at large m_h values, the value of $|\Delta_q|$ can exceed M_W .

We now explore the effect of lepton mass splitting (Δ_ℓ) and the fourth-generation quark mixing (θ_{34}) in more detail. The bottom left panel shows the effect of restricting $|\Delta_\ell|$ to M_W , while the top right panel shows the effect of vanishing θ_{34} . It is observed that there is no significant change in the allowed parameter space.

However, when both the conditions of vanishing θ_{34} and $|\Delta_\ell| < M_W$ are imposed, the character of the constraints changes dramatically, as can be seen from the bottom right panel. In this case, not only is $|\Delta_q| > M_W$ allowed at large m_h , one has to have $|\Delta_q| \geq M_W$ for $m_h \gtrsim 800$ GeV. At such large m_h values, then, either $|\Delta_\ell| > m_W$ or $|\Delta_q| > m_W$. At least one of the decays via W emission ($t' \rightarrow b'W$, $b' \rightarrow t'W$, $\tau' \rightarrow \nu'W$ and $\nu' \rightarrow \tau'W$) then must take place.

In most analyses of a fourth-generation scenario with a single higgs doublet, $|\Delta_q|$ had been assumed to be $\lesssim 75$ GeV due to the need to satisfy precision EW constraints. As seen above, the LHC exclusion of Higgs masses up to $m_h \sim 600$ GeV in fact allows larger mass differences in the fourth-generation quark doublet. Since $|\Delta_q| > M_W$ is allowed, the channel $t' \rightarrow b'W$, or $b' \rightarrow t'W$ becomes allowed. This condition will have strong implications for the direct searches of the fourth-generation scenario.

The correlation between Δ_q and Δ_ℓ observed above may be understood analytically as follows. First, note that the functions $T(x_w, x_d)$ in Eq. (5) are positive semidefinite [78]. The only negative contribution to ΔT_4 of fermions is then from the first term of Eq. (12), however it is compensated by the next term which is larger in magnitude and positive. [In particular, when $x_{t'} \gg x_b$, the first two terms in ΔT_4

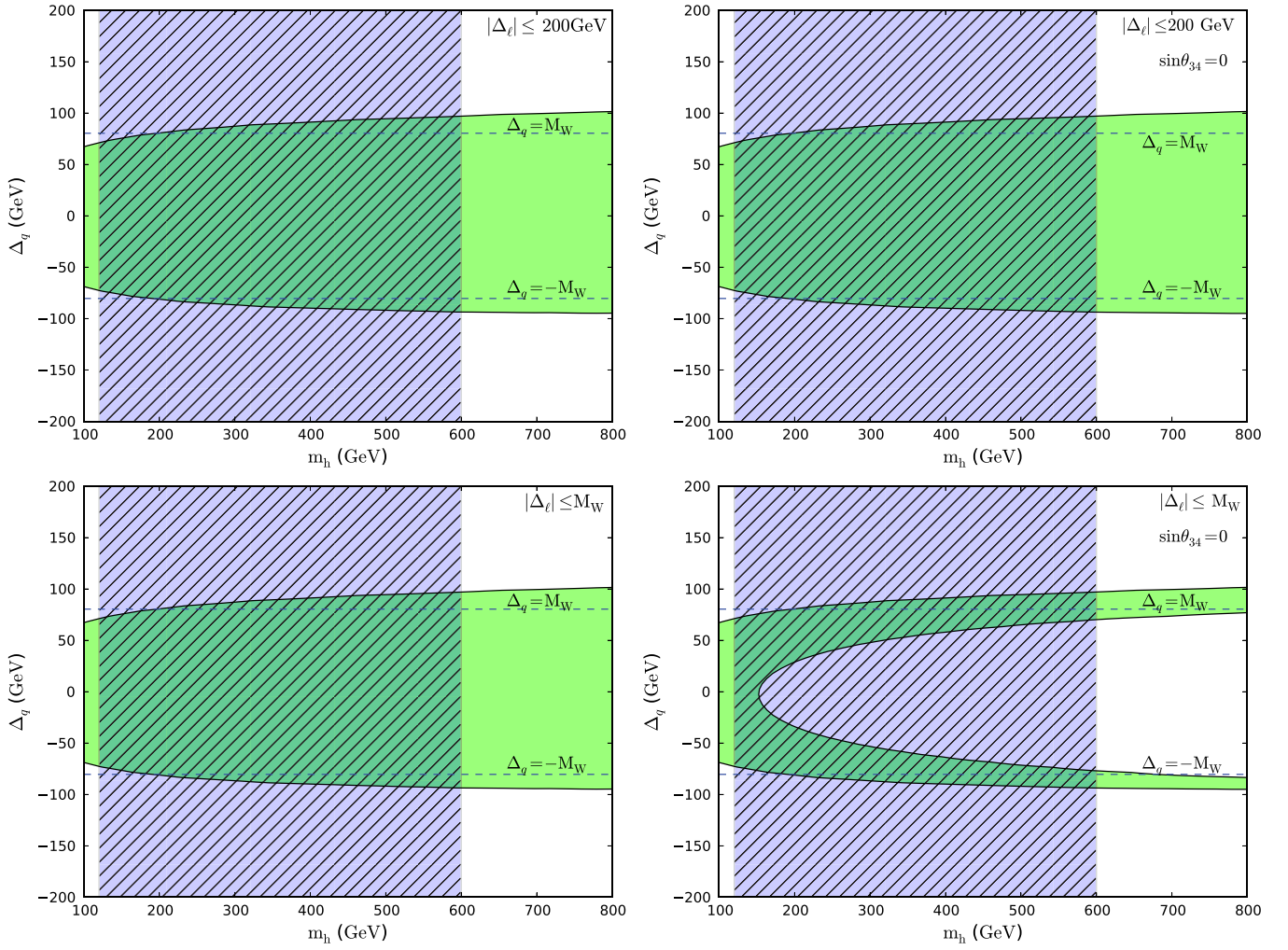


FIG. 2 (color online). The 95% C.L. allowed regions in the m_h - Δ_q plane. In the top two panels, Δ_ℓ is varied over $|\Delta_\ell| \leq 200$ GeV, while in the bottom two panels Δ_ℓ is restricted to be less than M_W . In the left two panels, θ_{34} is varied over $\sin\theta_{34} \leq 0.3$, while in the right two panels, θ_{34} has been fixed to zero (no mixing). The parameters m_q and m_l are varied over their complete allowed range. The grey shaded region is excluded at 95% by the LHC data. The dashed blue lines correspond to $|\Delta_q| = m_W$.

add up to a positive quantity: $3s_{34}^2(x_{t'} - x_t)/(16\pi s_W^2 c_W^2)$.] Thus, the contribution from fermions to ΔT_4 is positive for all values of fermion masses. On the other hand, Eq. (9) shows that the contribution from Higgs to ΔT is negative for $m_h > \tilde{m}_h = 120$ GeV. In order to be consistent with precision electroweak data, the negative contribution to ΔT_H from Higgs should be compensated adequately by the positive contribution ΔT_4 from fermions. (Although we also have S , U parameters, the effect on T dominates the behavior of our results.) This contribution comes from a combination of $T(x_u, x_d)$ in the quark and lepton sectors, leading to a strong correlation between the quark and lepton mass splittings.

When θ_{34} is zero or extremely small, the only contributions to ΔT_4 are from $T(x_{t'}, x_{b'})$ and $T(x_{t'}, x_{\tau'})$. These quantities then have to be sufficiently large to compensate for the large ΔT_H appearing at large m_h , necessitating a large mass splitting either in quark or in lepton sector. As

the Higgs mass increases, the compensating contribution ΔT_4 , and hence the required mass splittings, also increase, exceeding M_W for $m_h \gtrsim 800$ GeV. On the other hand, when θ_{34} is near its maximum allowed value of $\sin\theta_{34} = 0.3$, the first three terms in ΔT_4 also contribute, as a result of which the mass splitting in fourth-generation quarks is restricted and cannot exceed M_W .

Based on the above discussion, the features of Fig. 2 can be easily understood. The insensitivity of the allowed values of $|\Delta_q|$ to $|\sin\theta_{34}|$ in the top panels is mainly due to the fact that the lepton mass-splitting $|\Delta_\ell|$ is varied over a sufficiently large range. This ensures that the contribution of Higgs to ΔT is compensated by the contribution from fermions even in the absence of an enhancement of quark contribution by a nonzero value of $\sin\theta_{34}$. Now in the bottom-right panel, $|\Delta_\ell|$ is restricted to be less than M_W along with $\sin\theta_{34} = 0$. Here the absence of an enhanced quark contribution to ΔT_4 due to $\sin\theta_{34} = 0$, as well as

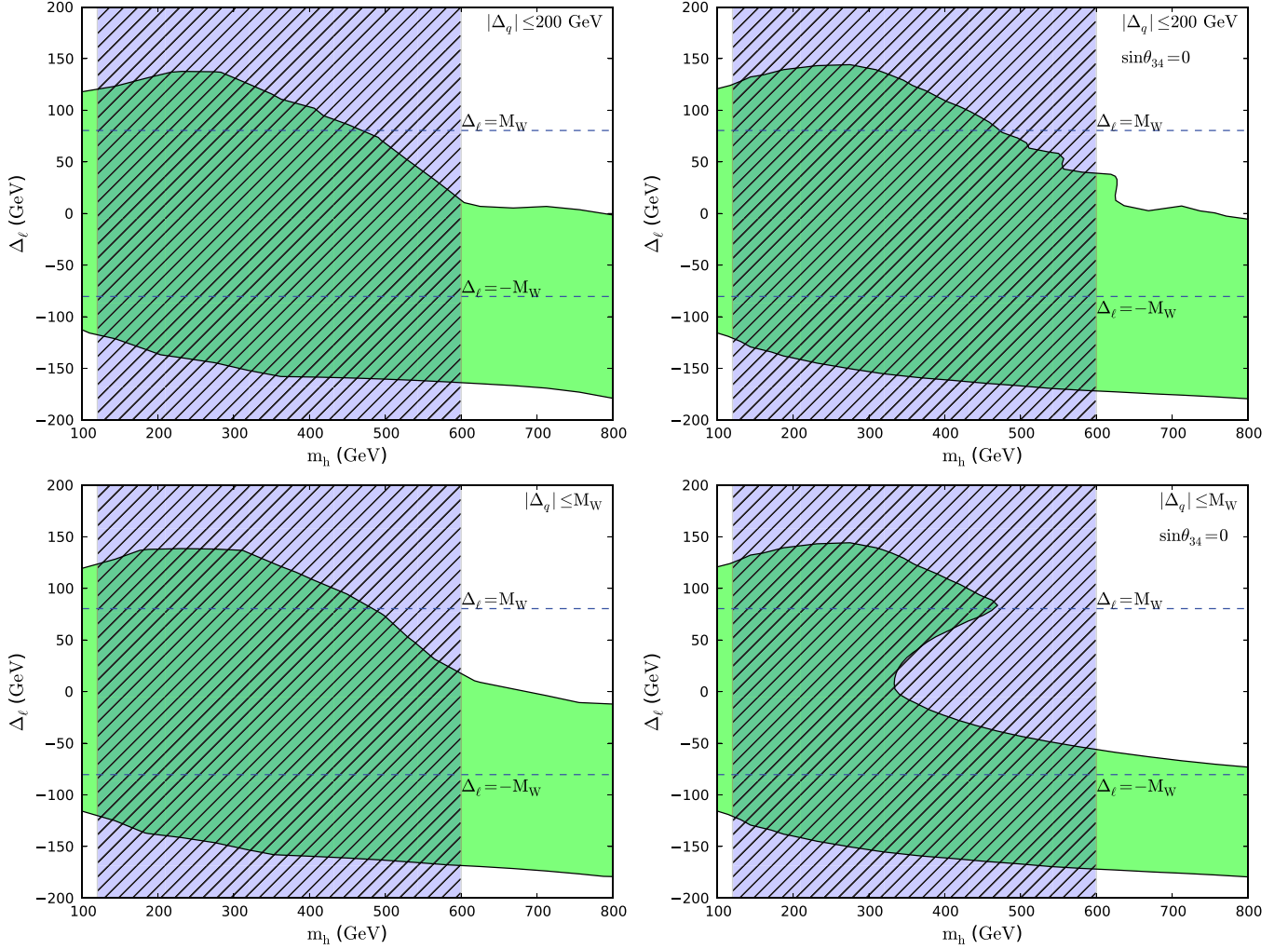


FIG. 3 (color online). The 95% C.L. allowed regions in the m_h - Δ_ℓ plane. In the top two panels, Δ_q is varied over $|\Delta_q| \leq 200$ GeV, while in the bottom two panels Δ_q is restricted to be less than M_W . In the left two panels, θ_{34} is varied over $\sin\theta_{34} \leq 0.3$, while in the right two panels, θ_{34} has been fixed to zero (no mixing). The parameters m_q and m_l are varied over their complete allowed range. The grey shaded region is excluded at 95% by the LHC data. The dashed blue lines correspond to $|\Delta_\ell| = m_W$.

insufficient contribution from leptons to ΔT_4 due to the restriction on $|\Delta_\ell|$, lead to the exclusion of $|\Delta_q| \lesssim M_W$ at large m_h . In the bottom-left panel the excluded regions $|\Delta_q| \lesssim M_W$ become allowed as the contribution from the quarks to ΔT_4 is enhanced by $\sin\theta_{34}$.

Note that earlier works [25,28,67] that had predicted the mass splitting in the quark sector to be less than M_W had focused on a light Higgs. That a lighter Higgs would only allow a small splitting should be clear from our figures and analytical arguments.

B. Constraints on Δ_ℓ

In the top-left panel of Fig. 3, we show the 95% C.L. contours in the m_h - Δ_ℓ plane, marginalizing over other NP parameters ($m_q, m_l, \Delta_q, \theta_{34}$). It is observed that while at low m_h values Δ_ℓ can take any sign, at large m_h values it is necessarily negative, i.e. $m_{\tau'} > m_{\nu'}$. Moreover, $|\Delta_\ell|$ can take values as large as 180 GeV.

The effect of setting $\sin\theta_{34} = 0$ on the allowed values of $|\Delta_\ell|$ is not significant, as can be seen in the top right panel. If $|\Delta_q|$ is restricted to be less than M_W , while allowing any mixing $\sin\theta_{34} \leq 0.3$, the parameter space is again not affected much as the bottom left panel shows. However, if $|\Delta_q|$ is restricted to be less than M_W , along with setting $\sin\theta_{34} = 0$ as shown in the bottom-right panel, $|\Delta_\ell|$ is required to be large in magnitude. For $m_h \gtrsim 800$ GeV, $|\Delta_\ell| > M_W$ and the decay $\tau' \rightarrow \nu' W$ is bound to occur. The features of the four panels regarding the role of $\sin\theta_{34}$ and $|\Delta_q|$ can be understood by the arguments given in the previous section.

The negative sign of Δ_ℓ may be understood as follows. The Higgs contribution ΔS_H to the S parameter is always positive, as can be seen from Eq. (8). It increases as m_h increases. Since leptons have a negative hypercharge, the contribution to S parameter from leptons can be reduced if $m_{\tau'} > m_{\nu'}$ for appropriate quark masses.

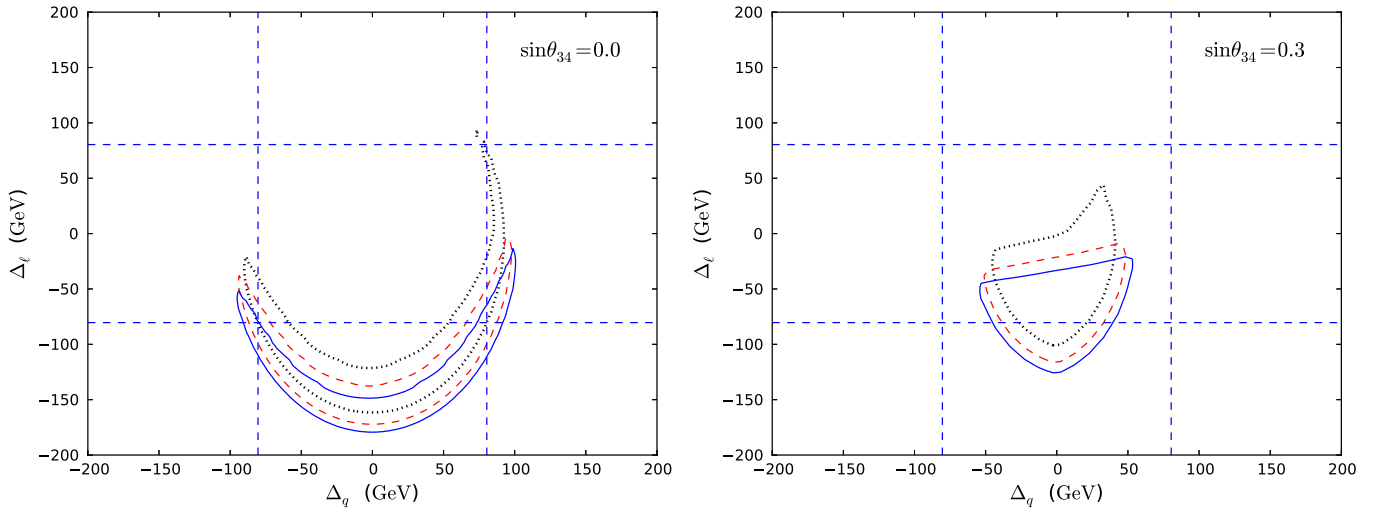


FIG. 4 (color online). The 95% C.L. contours in the Δ_q - Δ_ℓ plane, for $m_h = 400$ GeV (black dotted lines), $m_h = 600$ GeV (red dashed lines) and $m_h = 800$ GeV (blue/solid), respectively. The left (right) panel shows the results when $\sin\theta_{34} = 0.0(0.3)$. All the other parameters are varied over their 2σ allowed ranges. The vertical blue dashed lines correspond to $|\Delta_q| = M_W$, while the horizontal green (dotted) lines correspond to $|\Delta_\ell| = M_W$.

The interplay of the contributions to ΔS from the fourth-generation leptons and from the Higgs is also responsible for the asymmetry in the allowed region about $|\Delta_\ell| = 0$ in the Δ_ℓ - m_h plane. The T parameter is approximately symmetric with respect to the masses of up and down-type fermions when the mass difference of fermions is small compared to their masses, as can be seen from Eq. (7). But for large m_h , minimizing the leptonic contributions to S parameter becomes important for consistency. This causes the allowed regions to prefer $\Delta_\ell < 0$ compared to $\Delta_\ell > 0$, even though the T parameter tends to produce symmetric allowed regions.

In contrast to leptons, for quarks, the T parameter becomes important in constraining $|\Delta_q|$, as the hypercharge of quarks is positive. This makes allowed regions symmetric about $\Delta_q = 0$.

C. Constraints on (Δ_q, Δ_ℓ) and the effect of θ_{34}

The left panel of Fig. 4 shows the allowed parameter space in the Δ_q - Δ_ℓ plane for $\theta_{34} = 0$, for different m_h values. It can be easily seen that with increasing m_h , the allowed difference $m_{t'} - m_{b'}$ increases. This is consistent with the arguments in Sec. III B that used the contribution to ΔS from fermions and Higgs. Also, when the lepton splitting $|\Delta_\ell|$ is small, the quark splitting $|\Delta_q|$ has to be large to compensate for the Higgs contribution to ΔT , as argued earlier in Sec. III A. Indeed at large enough m_h values, the allowed region is outside the central square and hence always corresponds to $|\Delta_q| > M_W$ or $|\Delta_\ell| > m_W$, implying that the W -emission channel is necessarily active. Therefore, in case further direct constraints increase the lower bound on m_h to be ≥ 900 GeV, the W -emission signal is not observed in either quark or lepton channel,

and θ_{34} is restricted by independent experiments to be very small (say $\sin\theta_{34} < 0.05$), then the model with four generations can be ruled out at 95% confidence level.

The scenario when the mixing angle θ_{34} is significant is shown in the right panel of Fig. 4, where the value of θ_{34} corresponds to the current upper bound on it. It can be seen that in such a case, $|\Delta_q| > M_W$ is forbidden, while $|\Delta_\ell| > m_W$ is allowed.

Our results are consistent with those obtained earlier in [82]. In addition, we have shown the pattern of allowed mass differences of fermions as a function of m_h and quark mixing.

IV. COLLIDER IMPLICATIONS

According to our results, $m_{t'} > m_{b'} + M_W$ may be allowed for large m_h values. In that case, the branching ratio $\text{BR}(t' \rightarrow b'W)$ will depend on $\sin\theta_{34}$. Such a scenario was considered in [15] to generalize the direct search experiment limits on $m_{t'}$ and $m_{b'}$ to include the effect of mixing of fourth-generation quarks to the three existing generations. In our case, we have zero mixing between fourth-generation quarks and the first two generations in contrast to the assumption of [15]. But it does not affect the $\text{BR}(t' \rightarrow b'W)$ [although the decays $t' \rightarrow qW$ ($q = d, s$) will be forbidden] as long as the b -quark mass can be neglected. Therefore our use of the results of [15] for the model-independent bounds on the quark masses is still justified. The consideration of $m_{t'} > m_{b'} + M_W$ scenario by [15] was motivated by the result of [82] which stated that in a two-Higgs doublet model with a fourth generation, $m_{t'} - m_{b'}$ can be greater than M_W and also be consistent with precision EW data. Reference [82] also shows that $m_{t'} - m_{b'} > M_W$ is possible with one Higgs doublet of

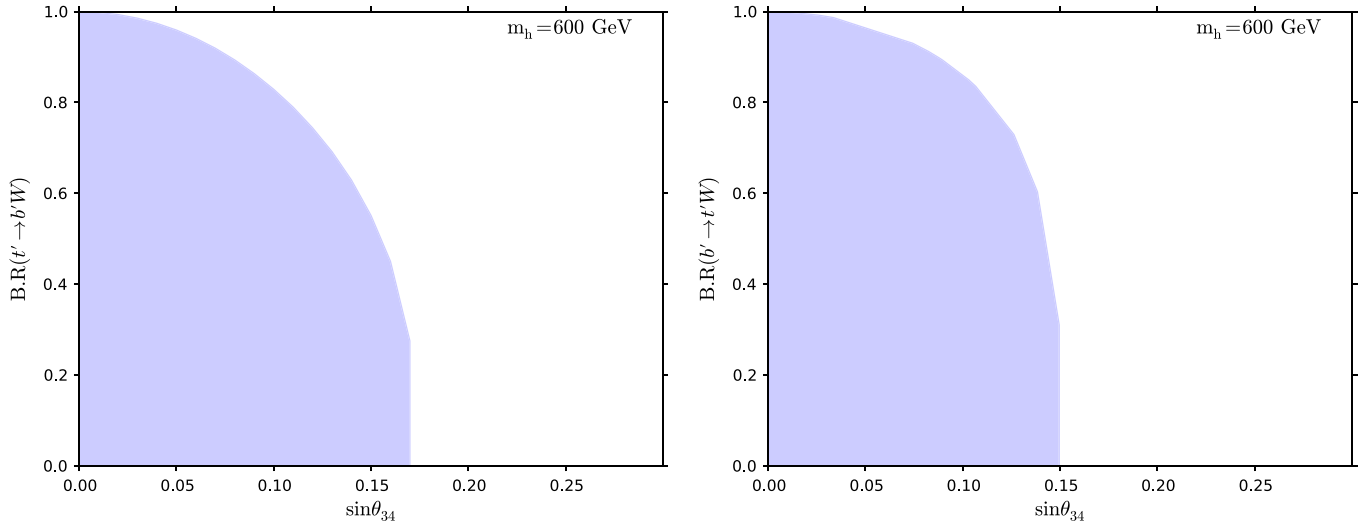


FIG. 5 (color online). Shaded region in the left (right) panel indicates the values of the branching ratios of $t' \rightarrow b'W$ ($b' \rightarrow t'W$) allowed at 95% C.L. by all the constraints considered in this paper.

SM4 when $m_{\tau'} - m_{\nu'} \leq M_W$. However, it assumes no 3–4 mixing in the quark sector. Similar conclusions hold also for the mass difference $m_{b'} - m_{t'}$, which may be greater than M_W , leading to the possibility of the decay channel $b' \rightarrow t'W$.

We have shown that $|m_{t'} - m_{b'}| > M_W$ is allowed even after marginalizing over the lepton masses and θ_{34} . Our result, in addition to the result of [82], justifies considering $|m_{t'} - m_{b'}| > M_W$ for interpreting direct search data on fourth-generation quarks, as was done in [15]. Our result also means that the conditions $|m_{t'} - m_{b'}| > M_W$ can be met even in the case of one Higgs doublet. In Fig. 5, we plot the allowed values of branching ratios of the decays $t' \rightarrow b'W$ and $b' \rightarrow t'W$ as functions of $\sin\theta_{34}$. One can easily see that the branching ratio of the decay of a fourth-generation quark into another fourth-generation

quark can be close 100%. This emphasizes the need to consider these decay modes in direct search experiments which search for fourth-generation quarks.

Figure 5 shows that for $\sin\theta_{34} \geq 0.15$ there exists no point in the parameter space which passes all the constraints (direct search, S , T , U) for which the decay ($b' \rightarrow t'W$ or $t' \rightarrow b'W$) is possible. This can be understood from Fig. 6, where we show the allowed values of Δ_q as a function of θ_{34} . For large θ_{34} values, $\sin\theta_{34} \geq 0.15$, the mass splitting goes below M_W and the W -emission channel is forbidden.

V. SUMMARY AND OUTLOOK

We have explored the allowed mass spectra of fourth-generation fermions, calculating the constraints from direct searches at the colliders, the theoretical requirement of perturbative unitarity, and electroweak precision measurements. We take into account the masses of fourth-generation quarks as well as leptons, and possible mixing of the fourth-generation quarks with the third generation ones. The other mixings of the fourth-generation quarks and leptons are more tightly constrained, and hence neglected.

Our perturbative unitarity calculation with the inclusion of all the $J = 0$ channels for $2 \rightarrow 2$ fermion scattering tightens the earlier upper bounds on fourth-generation quark masses by about 6%, while keeping the constraints on the fourth-generation lepton masses unaffected. These bounds are relatively insensitive to the precision electroweak observables S , T , U . The mixing between the third and fourth-generation quarks is constrained primarily by the flavor-physics data, to $\sin\theta_{34} < 0.3$. The perturbative unitarity bounds depend only weakly on this mixing.

Performing a χ^2 -fit to the measured values of the precision electroweak parameters S , T , U , we find that large

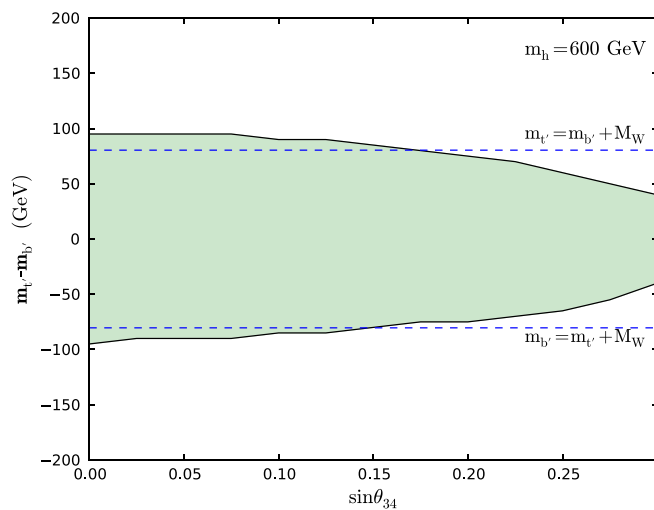


FIG. 6 (color online). Allowed values of $m_{t'} - m_{b'}$ (shaded region) for $m_h = 600$ GeV as a function of $\sin\theta_{34}$ after marginalizing over the other NP parameters.

values of the Higgs mass, $m_h \gtrsim 600$ GeV as indicated by the current LHC data, allow the mass splitting between the fourth-generation quarks or leptons to be greater than M_W . In the case of the quark splitting $\Delta_q = m_{t'} - m_{b'}$, the possibility $|\Delta_q| > M_W$ starts being allowed at 95% C.L. for $m_h \gtrsim 200$ GeV. For the lepton splitting $\Delta_\ell = m_{\nu'} - m_{\tau'}$, the possibility $\Delta_\ell > M_W$ is allowed at 95% for $m_h \lesssim 450$ GeV, while $\Delta_\ell < -M_W$ is allowed for all values of m_h . Moreover, if θ_{34} is small, either $|\Delta_q| > M_W$ or $|\Delta_\ell| > M_W$ is necessary for values of m_h as large as 800 GeV. We present correlations between the values of Δ_q and Δ_ℓ , as well as the constraints on them as functions of m_h and θ_{34} .

Most of the above observations may be explained qualitatively through the analytic expressions for the contribution to the S and T parameters by the Higgs and the fourth-generation fermions, and their interference. In particular, the requirement of $|\Delta_q| > M_W$ or $|\Delta_\ell| > M_W$ at large m_h for small θ_{34} , and the relaxation of this for large θ_{34} , can be easily motivated. These expressions also allow an understanding of the asymmetric bounds on $\pm\Delta_\ell$, and why $m_{\tau'} > m_{\nu'}$ is necessary at large m_h . No such hierarchy of masses can be predicted in the quark sector.

The unique feature of our analysis are the simultaneous consideration of the lepton masses, the quark mixing, and the recent indication of the heavy Higgs. The major consequence of our result is the opening up of the W -emission channels $t' \rightarrow b'W$, $b' \rightarrow t'W$, or $\tau' \rightarrow \nu'W$ for large values of m_h . This will have major implications for the direct collider searches for fourth-generation fermions which currently are performed assuming for example, $t' \rightarrow bW/b' \rightarrow tW$ as the dominant decay modes. Indeed, since the branching ratios of these decay modes, when kinematically allowed, are large, they can have impact on the currently stated exclusion bounds, which have been arrived at by assuming that these decays are not allowed. In order to get model-independent bounds on the masses of the fourth-generation fermions, it is necessary to analyze the data keeping open the possibility of large branching ratios in the W -emission channels.

We also find in our analysis, that in case further direct constraints increase the lower bound on m_h to be $\gtrsim 900$ GeV, the W -emission signal is not observed in either quark or lepton channel, and θ_{34} is restricted by independent experiments to be very small (say $\sin\theta_{34} < 0.05$), then the model with four generations can be ruled out at 95% confidence level.

In conclusion, the fourth generation is currently alive and well. However if the corresponding standard model Higgs is heavy, it presents the possibility of an early direct detection of the fourth-generation fermions and also affects the search strategies as well as possible exclusions of the fourth-generation scenario strongly. Either way will lead to an important step ahead in our understanding of the fundamental particles.

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APPENDIX: PARTIAL-WAVE AMPLITUDES

In this appendix, we give the expressions for the $J = 0$ partial-wave amplitudes of $2 \rightarrow 2$ scattering processes of the heavy quarks of the SM4. Unitarity of the S -matrix and the validity of perturbative expansion of the S -matrix at high center-of-mass energies constrain the behavior of scattering amplitudes at high center-of-mass energies. For example, in the case of $2 \rightarrow 2$ scattering of scalars, the tree-level amplitude is restricted to less than unity, $|M_0| < 1$. If the scattering particles have other quantum numbers, the tree-level amplitudes form a matrix in the space of the quantum numbers. The analogous criterion for the perturbative unitarity of the S -matrix will be that the absolute value of the maximum eigenvalue of the matrix-valued amplitude should be less than unity.

In the case of SM4, the $J = 0$ partial-wave amplitude of $2 \rightarrow 2$ fermion scattering receives contribution from processes of the type $F\bar{F} \rightarrow F'\bar{F}'$ where F and F' are two heavy fermions of the SM4. The scattering amplitudes depend on the helicity configurations of the initial and final state fermions. Since the couplings of fermions to bosons in the SM are of scalar, vector or axial-vector type, the number of helicity configurations at high center-of-mass energies which have nonvanishing $J = 0$ amplitudes reduces to four: $\{++ \rightarrow ++\}$, $\{++ \rightarrow --\}$, $\{-- \rightarrow ++\}$, $\{-- \rightarrow --\}$. The $2 \rightarrow 2$ scattering amplitude of quarks is a matrix with both helicity and color indices. We consider only the amplitudes where the initial and final states are color-neutral and charge-neutral.

Let t , t' , b' be denoted by the indices $i = 1, 2, 3$, respectively, and let m_i be their masses. In the limit where the center-of-mass energy of the scattering $s \gg m_i m_j$, the tree-level amplitudes for the processes $F\bar{F} \rightarrow F'\bar{F}'$ may be written in the form of a 30×30 matrix M . This matrix may be conveniently represented in the basis

$$\begin{aligned} & (t_+^R \bar{t}_+^R, t_-^R \bar{t}_-^R, t_+^R \bar{t}_+^R, t_-^R \bar{t}_-^R, b_+^R \bar{b}_+^R, b_-^R \bar{b}_-^R, \\ & t_+^R \bar{t}_+^R, t_-^R \bar{t}_-^R, t_+^R \bar{t}_+^R, t_-^R \bar{t}_-^R, \\ & t_+^G \bar{t}_+^G, t_-^G \bar{t}_-^G, t_+^G \bar{t}_+^G, t_-^G \bar{t}_-^G, b_+^G \bar{b}_+^G, b_-^G \bar{b}_-^G, \\ & t_+^G \bar{t}_+^G, t_-^G \bar{t}_-^G, t_+^G \bar{t}_+^G, t_-^G \bar{t}_-^G, \\ & t_+^B \bar{t}_+^B, t_-^B \bar{t}_-^B, t_+^B \bar{t}_+^B, t_-^B \bar{t}_-^B, b_+^B \bar{b}_+^B, b_-^B \bar{b}_-^B, \\ & t_+^B \bar{t}_+^B, t_-^B \bar{t}_-^B, t_+^B \bar{t}_+^B, t_-^B \bar{t}_-^B), \end{aligned}$$

where R, G, B represent the three colors. In this basis,

$$M = \begin{bmatrix} A & B & B \\ B & A & B \\ B & B & A \end{bmatrix}. \quad (\text{A1})$$

Where A, B are 10×10 matrices which describe the scattering amplitudes. Taking the mixing between the third and fourth generations into account, the matrices A and B may be written as

$$A = -\frac{\sqrt{2}G_F}{8\pi} \begin{bmatrix} m_{11} & 0 & m_{12} & 0 & 0 & -m_{13}x & 0 & 0 & 0 & 0 \\ 0 & m_{11} & 0 & m_{12} & -m_{13}x & 0 & 0 & 0 & 0 & 0 \\ m_{21} & 0 & m_{22} & 0 & 0 & -m_{23}y & 0 & 0 & 0 & 0 \\ 0 & m_{21} & 0 & m_{22} & -m_{23}y & 0 & 0 & 0 & 0 & 0 \\ 0 & -m_{13}x & 0 & -m_{23}y & m_{33} & 0 & 0 & zm_{13} & 0 & z^*m_{23} \\ -m_{13}x & 0 & -m_{23}y & 0 & 0 & m_{33} & zm_{32} & 0 & z^*m_{31} & 0 \\ 0 & 0 & 0 & 0 & 0 & z^*m_{32} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & z^*m_{31} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & zm_{31} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & zm_{32} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (\text{A2})$$

$$B = -\frac{\sqrt{2}G_F}{8\pi} \begin{bmatrix} m_{11} & 0 & m_{12} & 0 & 0 & -m_{13} & 0 & 0 & 0 & 0 \\ 0 & m_{11} & 0 & m_{12} & -m_{13} & 0 & 0 & 0 & 0 & 0 \\ m_{21} & 0 & m_{22} & 0 & 0 & -m_{23} & 0 & 0 & 0 & 0 \\ 0 & m_{21} & 0 & m_{22} & -m_{23} & 0 & 0 & 0 & 0 & 0 \\ 0 & -m_{13} & 0 & -m_{23} & m_{33} & 0 & 0 & 0 & 0 & 0 \\ -m_{13} & 0 & -m_{23} & 0 & 0 & m_{33} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (\text{A3})$$

where $m_{ij} = m_i m_j$, $x = 1 - |V_{tb'}|^2$, $y = 1 - |V_{t'b'}|^2$, $z = V_{tb'} V_{t'b'}^*$, and $z^* = V_{t'b'}^* V_{tb'}$. Block-diagonalising M , we get

$$M' = \begin{bmatrix} A + 2B & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & A - B & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & A - B \end{bmatrix}. \quad (\text{A4})$$

The maximum eigenvalue is obtained from $A + 2B$ [74], which is given by

$$A + 2B = -\frac{3\sqrt{2}G_F}{8\pi} \begin{bmatrix} m_{11} & 0 & m_{12} & 0 & 0 & -m_{13}\delta_x & 0 & 0 & 0 & 0 \\ 0 & m_{11} & 0 & m_{12} & -m_{13}\delta_x & 0 & 0 & 0 & 0 & 0 \\ m_{21} & 0 & m_{22} & 0 & 0 & -m_{23}\delta_y & 0 & 0 & 0 & 0 \\ 0 & m_{21} & 0 & m_{22} & -m_{23}\delta_y & 0 & 0 & 0 & 0 & 0 \\ 0 & -m_{13}\delta_x & 0 & -m_{23}\delta_y & m_{33} & 0 & 0 & \delta_z m_{13} & 0 & \delta_z^* m_{23} \\ -m_{13}\delta_x & 0 & -m_{23}\delta_y & 0 & 0 & m_{33} & \delta_z m_{32} & 0 & \delta_z^* m_{31} & 0 \\ 0 & 0 & 0 & 0 & 0 & \delta_z^* m_{32} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \delta_z^* m_{31} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \delta_z m_{31} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \delta_z m_{32} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (\text{A5})$$

where $\delta_x = 1 - (1/3)|V_{tb'}|^2$, $\delta_y = 1 - (1/3)|V_{t'b'}|^2$, $\delta_z = (1/3)V_{tb'}V_{t'b'}^*$ and $\delta_z^* = (1/3)V_{t'b'}^*V_{tb'}$.

Note that the presence of a nonzero mixing between the third and fourth-generation quarks is responsible for the appearance of the channels $t\bar{t}' \rightarrow b'\bar{b}'$ and $t'\bar{t} \rightarrow b\bar{b}'$. These channels are directly proportional to the mixing matrix elements in contrast to other channels where the effect of mixing matrix elements is not significant.

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