Decay widths of the $X(1835)$ as an $\bar{N}N$ bound state

D. Samart,[1] Y. Yan,[1,2] Th. Gutsche,[3] and Amand Faessler[3]

[1]*School of Physics, Suranaree University of Technology, 111 University Avenue, Nakhon Ratchasima 30000, Thailand*
[2]*ThEP Center, Commission on Higher Education, Bangkok 10400, Thailand*
[3]*Institut für Theoretische Physik, Universität Tübingen, Auf der Morgenstelle 14, D-72076 Tübingen, Germany*

Partial decay widths of various decay channels of the $X(1835)$ are evaluated in the $^3P_0$ quark model, assuming that the $X(1835)$ is an $N\bar{N}$ bound state with the quantum number assignment $I^G(J^{PC}) = 0^+(0^{-+})$. It is found that the decays to the $\rho\rho$, $\omega\omega$, and $\pi a_0(1450)$ states dominate over other channels, and that the product branching fractions $\mathrm{Br}(J/\psi \to \gamma X)\mathrm{Br}(X \to \pi\pi\eta)$ and $\mathrm{Br}(J/\psi \to \gamma X)\mathrm{Br}(X \to \pi\pi\eta')$ are in the same order. We suggest that the $X(1835)$ may be searched in the $\pi a_0(1450)$ channel.

I. INTRODUCTION

An enhancement was observed by the BES Collaboration [1] in the proton-antiproton ($p\bar{p}$) invariant mass spectrum in the radiative decay $J/\psi \to \gamma p\bar{p}$. It was concluded that the enhancement has properties consistent with either a $J^{PC} = 0^{-+}$ or 0^{++} quantum number assignment and unlikely stems from the effects of any known meson resonance. The mass and width of the resonance are fixed to be $M = 1859^{+3}_{-10}(\text{stat})^{+5}_{-25}(\text{syst})$ MeV and $\Gamma < 30$ MeV if it is interpreted as a single $J^{PC} = 0^{-+}$ resonance. More recently the BES Collaboration [2] analyzed the decay channel $J/\psi \to \gamma\pi^+\pi^-\eta'$ and observed a resonance, the $X(1835)$ with high statistics in the $\pi^+\pi^-\eta'$ invariant mass spectrum, with the product branching fraction $\mathrm{Br}(J/\psi \to \gamma X(1835))\mathrm{Br}(X(1835) \to \pi^+\pi^-\eta') = (2.2 \pm 0.4(\text{stat}) \pm 0.4(\text{syst})) \times 10^{-4}$. The mass and width of the $X(1835)$ are determined to be $1833.7 \pm 6.1(\text{stat}) \pm 2.7(\text{syst})$ MeV and $67.7 \pm 20.3(\text{stat}) \pm 7.7(\text{syst})$ MeV, respectively. The possibility that the $X(1835)$ and the resonance reported in Ref. [1] are the same entity was checked in Ref. [2]. Redoing the S-wave Breit-Wigner fit to the $p\bar{p}$ invariant mass spectrum of Ref. [1], including the effect of final-state interactions on the shape of the $p\bar{p}$ mass spectrum [3,4], yields a mass $M = 1831 \pm 7$ MeV and a width $\Gamma < 153$ MeV that are consistent with the $X(1835)$ observables in Ref. [2]. The $X(1835)$ was confirmed by the BESIII experiment [5] with a statistical significance larger than 20σ. Up to now, however, the spin and parity of the $X(1835)$ has not been well determined.

The nature of the $X(1835)$ is still an open question though a number of theoretical works have been done to interpret this particle. Among the interpretations are the $\bar{N}N$ bound state [6–15], the baryonium with sizable gluon content [16], the pseudoscalar glueball [17–20], the radial excitation of the η' [21–25], and the η_c-glueball mixture [26,27]. The $\bar{N}N$ bound state interpretation has been the most natural one since the $X(1835)$ resonance is a prime candidate for the source of the $p\bar{p}$ invariant mass enhancement in $J/\psi \to \gamma p\bar{p}$ reaction.

The BESII measurement [1] of the photon polar angle distribution in radiative J/ψ decays favors a $J = 0$ $p\bar{p}$ system, but the possibility of a $J = 1$ resonance is not excluded. However, the recent BESIII experiment [5] reveals that the polar angle of the photon in the process $J/\psi \to \gamma X(1835)$ agrees well with the form $1 + \cos\theta_\gamma$, which indicates that the $X(1835)$ is either a scalar or pseudoscalar meson. Therefore, interpreting the $X(1835)$ as an $N\bar{N}$ bound state means that the $X(1835)$ could be an $^{11}S_0$, $^{31}S_0$, $^{13}P_0$, or $^{33}P_0$ state.

The investigations in Refs. [28–30] of $N\bar{N}$ annihilations and $N\bar{N}$ bound states in the $^3P_0$ quark model reveal that the dominant decay modes of $^{31}S_0$, $^{11}S_0$, $^{13}P_0$, and $^{33}P_0$ $N\bar{N}$ bound states are the $\rho\omega$ and $\pi\rho$ channels, the $\rho\rho$ and $\omega\omega$ channels, the $\pi\pi$, $\eta\eta$, $\rho\rho$, and $\omega\omega$ channels, and the $\pi\eta$ and $\rho\omega$ channels, respectively. It may be difficult for the BES detectors to observe resonances in the 4π, 5π, or 6π channel, but we expect that the BES Collaboration is able to retrieve a resonance if it decays mainly to the $\pi\pi$, 3π, $\eta\eta$, or $\pi\eta$ state. Up to now there has been no report from the BES Collaboration of such a resonance from these states. One may conclude that the $X(1835)$ is unlikely to be a $^{31}S_0$, $^{13}P_0$, or $^{33}P_0$ $N\bar{N}$ bound state, and hence that an $^{11}S_0$ $N\bar{N}$ bound state might be the only candidate for the $X(1835)$. Studies of the J/ψ decays $J/\psi \to \gamma\pi^+\pi^-\eta$ and $\gamma\bar{p}p$ in Refs. [6,7] in a semiphenomenological potential model reveal that the explanation of both the reactions may be given by a broad $^{11}S_0$ $\bar{N}N$ bound state near the $\bar{N}N$ threshold. However, the investigations of the spectrum of $\bar{N}N$ bound states in microscopically derived $\bar{N}N$ potentials [31] and in phenomenological potentials [30] have not found such an $^{11}S_0$ $\bar{N}N$ bound state.

In this work we evaluate the partial decay widths for various decay channels of the $X(1835)$ in the $^3P_0$ quark model, assuming that the $X(1835)$ is an $^{11}S_0$ $N\bar{N}$ bound state with the quantum number assignment $I^G(J^{PC}) = 0^+(0^{-+})$. The paper is arranged as follows: In Sec. II we calculate the decay widths for two-body decay channels. The partial decay widths for the $\pi^+\pi^-\eta$ and $\pi^+\pi^-\eta'$

channels are estimated in Sec. III. Discussion and conclusions are given in Sec. IV.

II. $X(1835)$ TO TWO MESONS

Interpreted as an $N\bar{N}$ bound state with the quantum number assignment $I^G(J^{PC}) = 0^+(0^{-+})$, the $X(1835)$ may mainly decay to the two-meson final states $\rho\rho$, $\omega\omega$, $\pi a_0(1450)$, $\eta f_0(1370)$, $\eta' f_0(1370)$, $\pi a_2(1230)$, and $\eta f_2(1270)$ as well as $\pi a_0(980)$, $\eta f_0(600)$, and $\eta' f_0(600)$. The transition amplitude of $X(1835)$ to two mesons takes the form

$$T_{X \to M_1 M_2} = \langle M_1 M_2 | V | \bar{N}N \rangle \langle \bar{N}N | X \rangle$$
$$= \int \frac{d\vec{p}}{(2\pi)^{3/2}} \Phi_X(\vec{p}) T_{\bar{N}N \to M_1 M_2}(\vec{p}, \vec{k}), \quad (1)$$

where $\Phi_X(\vec{p})$ is the $N\bar{N}$ bound state wave function of the $X(1835)$ in momentum space, normalized according to

$$\int \frac{d\vec{p}}{(2\pi)^3} |\Phi_X(\vec{p})|^2 = 1, \quad (2)$$

$\vec{k}$ is the relative momentum between the two final mesons, and $\vec{p}$ the relative momentum between the nucleon and antinucleon of the $X(1835)$ in the center-of-mass system. $T_{\bar{N}N \to M_1 M_2}(\vec{p}, \vec{k})$ is the transition amplitude of a free nucleon-antinucleon pair to two mesons.

In this work we study $N\bar{N}$ annihilations to two mesons in the A2 quark line model, as described in [32], where the effective quark annihilation operator takes the quantum numbers of the vacuum ($^3P_0$, isospin $I = 0$, and color singlet). Meson and baryon decays and $N\bar{N}$ annihilation into two mesons are well described phenomenologically using such an effective quark-antiquark vertex. At least from meson decays, this approximation has been given a rigorous basis in strong-coupling QCD. The nonperturbative $q\bar{q}$ $^3P_0$ vertex is defined according to Ref. [32]

$$V^{ij} = \lambda \sum_\mu \sigma^{ij}_{-\mu} Y_{1\mu}(\vec{q}_i - \vec{q}_j) \delta^{(3)}(\vec{q}_i + \vec{q}_j)(-1)^{1+\mu} 1^{ij}_F 1^{ij}_C, \quad (3)$$

where λ is the effective coupling strength of the $^3P_0$ vertex, $Y_{1\mu}(\vec{q}) = |\vec{q}| \mathcal{Y}_{1\mu}(\hat{q})$ with $\mathcal{Y}_{1\mu}(\hat{q})$ being the spherical harmonics in momentum space, and 1^{ij}_F and 1^{ij}_C are unit operators in flavor and color spaces, respectively. The spin operator $\sigma^{ij}_{-\mu}$ destroys or creates quark-antiquark pairs with spin 1.

The internal spatial wave functions for both the baryons and mesons are taken in the harmonic oscillator approximation in the work. The S-wave meson (originating from a valence $q\bar{q}$ in S wave) wave function can be expressed in terms of the quark momenta as

$$\langle M | \vec{q}_i \vec{q}_j \rangle = N_M \exp\left\{ -\frac{b^2}{8}(\vec{q}_i - \vec{q}_j)^2 \right\} \chi_M, \quad (4)$$

with $N_M = (b^2/\pi)^{3/4}$ and b is the meson radial parameter. The spin-color-flavor wave function is denoted by χ_M. The baryon wave functions are given by

$$\langle B | \vec{q}_i \vec{q}_j \vec{q}_k \rangle = N_B \exp\left\{ -\frac{a^2}{4}\left[(\vec{q}_j - \vec{q}_k)^2 + \frac{(\vec{q}_j + \vec{q}_k - 2\vec{q}_i)^2}{3} \right] \right\} \chi_B, \quad (5)$$

where $N_B = (3a^2/\pi)^{3/2}$ with a being the baryon radial parameter, and χ_B is the spin-color-flavor wave function of baryons.

The transition amplitude of $N\bar{N}$ annihilations to two mesons can be written in terms of the partial waves of the initial and final states,

$$T_{\bar{N}N \to M_1 M_2}(\vec{p}, \vec{k}) = \sum_{LM} \sum_{lm} Y_{LM}(\hat{p}) T_{fi}(p, k) Y^*_{lm}(\hat{k}). \quad (6)$$

In the low-momentum approximation, the partial wave transition amplitude $T_{fi}(p, k)$ is derived in the A2 quark line diagram as

$$T_{fi}(p, k) = \lambda^3 F^L_l(p) G^L_l(k) \langle f | O_{A_2} | i \rangle \cdot \exp\{ -Q^2_p p^2 - Q^2_k k^2 \}. \quad (7)$$

The index i represents the initial state $^{2I+1,2S+1}L_J$ with I, J, L, and S being, respectively, the total isospin, total angular momentum, orbital angular momentum and total spin while the index f stands for the final two meson states with l being the relative orbital angular momentum between the final two mesons. $F^L_l(p)$ is a function of $p = |\vec{p}|$ and of the meson and baryon radial parameters a and b while $G^L_l(k)$ is a function of $k = |\vec{k}|$, a, and b. Q^2_p and Q^2_k are geometrical constants depending on the radial parameters. The matrix elements $\langle f | O_{A_2} | i \rangle$ are the spin-flavor weights for the quark line diagram A_2. For different annihilations of the initial $^{11}S_0$ $N\bar{N}$ state, their values, normalized to the $\rho\rho$ channel, are listed in Table I.

In the low-momentum approximation, $G^L_l(k)$ in Eq. (7) are derived as

$$G^{L=0}_{l=1}(k) = k(1 + Ak^2) \quad (8)$$

with

TABLE I. $\langle f | O_{A_2} | i \rangle$ and partial decay widths for annihilations of the initial $^{11}S_0$ $N\bar{N}$ state to two mesons.

Final states	$\langle f | O_{A_2} | i \rangle$	$\Gamma_i / \Gamma_{X(1835) \to \rho\rho}$
$\rho\rho$	1	1
$\omega\omega$	$1/\sqrt{3}$	0.32
$\pi a_0(1450)$	$\sqrt{2}$	0.25
$\eta f_0(1370)$	1	0.057
$\eta' f_0(1370)$	1	0.052
$\pi a_2(1320)$	$1/\sqrt{6}$	0.055
$\eta f_2(1270)$	$1/(3\sqrt{2})$	0.008

$$A = -\frac{a^2 b^4 (3a^2 + b^2)}{2(3a^2 + 2b^2)(9a^4 + 13b^2 a^2 + 3b^4)} \quad (9)$$

for the annihilation processes of the initial $^{11}S_0$ $N\bar{N}$ state to two s-wave mesons, and

$$G_{l=0}^{L=0}(k) = B_1(1 + Bk^2), \qquad G_{l=2}^{L=0}(k) = C_1 k^2 \quad (10)$$

for the reactions of the initial $^{11}S_0$ $N\bar{N}$ state to the final states of one s-wave and one p-wave meson, where $G_{l=0}^{L=0}(k)$ [$G_{l=2}^{L=0}(k)$] is for the final state with one p-wave meson of spin $J = 0$ ($J = 2$). B, B_1, and C_1 in Eq. (10) are functions of the size parameters a and b, taking the forms

$$B = \frac{a^2(27a^4 + 45b^2 a^2 + 8b^4)}{6(6a^4 + 13b^2 a^2 + 6b^4)},$$

$$B_1 = \frac{b(2a^2 + 3b^2)}{9a^4 + 13b^2 a^2 + 3b^4}, \quad (11)$$

$$C_1 = \frac{a^2 b(27a^4 + 45b^2 a^2 + 11b^4)}{27a^6 + 57b^2 a^4 + 35b^4 a^2 + 6b^6}.$$

Note that we have let $F_l^L(p)$ in Eq. (7) the same for all annihilation channels just for convenience.

The transition amplitude of annihilations of $N\bar{N}$ bound states takes the form

$$T_{f,LSJ}^I(k) = \int p^2 dp\, T_{fi}(p, k)\psi_{LSJ}^I(p), \quad (12)$$

where $\psi_{LSJ}^I(p)$ is the radial wave function of the initial $N\bar{N}$ bound state in momentum space. The partial decay width for the transition of $N\bar{N}$ bound states to two mesons is given by

$$\Gamma_{p\bar{p}\to M_1 M_2} = \frac{1}{2M} \int \frac{d^3 k_1}{2E_1} \frac{d^3 k_2}{2E_2} \delta^{(3)}(\vec{k}_1 + \vec{k}_2)$$
$$\times \delta(E - E_1 - E_2)|T_{f,LSJ}(k)|^2, \quad (13)$$

where M is the mass of the $N\bar{N}$ bound state, and $E_{1,2} = \sqrt{m_{1,2}^2 + \vec{k}_{1,2}^2}$ is the energy of outgoing mesons with mass $m_{1,2}$ and momentum $\vec{k}_{1,2}$. With the explicit form of the transition amplitude given by Eq. (7), the partial decay width for the $X(1835)$ annihilation to two mesons is derived as

$$\Gamma_{X\to M_1 M_2} = \lambda^6 \langle f|O_{A_2}|i\rangle^2 |G_l^L(k)|^2 F_p F_k, \quad (14)$$

with

$$F_p = |F_l^L(p) \int p^2 dp\, \psi_{LSJ}^I(p) \exp\{-Q_P^2 p^2\}|^2 \quad (15)$$

and the kinematical phase-space factor defined by

$$F_k = \frac{k}{8M^2} \exp\{-2Q_k^2 k^2\}. \quad (16)$$

Note that we have assumed that all the $^3P_0$ vertices in the A2 diagram take the same value.

The model dependence in Eq. (14) may be reduced by choosing a simplified phenomenological approach that has been applied in studies of two-meson branching ratios in nucleon-antinucleon annihilation [33] and radiative protonium annihilation [34]. Namely, instead of the phase-space factor of Eq. (16) obtained in the harmonic oscillator approximation for the hadron wave function, we use a kinematical phase-space factor of the phenomenological form

$$f(M_1, M_2) = k \cdot \exp\{-a_s(s - s_{12})^{1/2}\}, \quad (17)$$

where $s_{12} = (m_1 + m_2)^2$ and $\sqrt{s} = (m_1^2 + k^2)^{1/2} + (m_2^2 + k^2)^{1/2}$. The constant $a_s = 1.2$ GeV^{-1} is obtained from the fit to the momentum dependence of the cross section of various $\bar{N}N$ annihilation channels [35].

Partial decay widths are evaluated for the processes of an $^{11}S_0$ state $X(1835)$ to $\rho\rho$, $\omega\omega$, $\pi a_0(1450)$, $\eta f_0(1370)$, $\eta' f_0(1370)$, $\pi a_2(1320)$, and $\eta f_2(1270)$. Here the $a_0(1450)$, $f_0(1370)$, $a_2(1320)$, and $f_2(1270)$ are normal P-wave mesons (originating from a valence $q\bar{q}$ in P wave), and their radial wave functions take the Gaussian form. It is rather difficult to calculate, in the scope of this work and also in any quark model, the partial decay widths to the states $\eta f_0(600)$, $\eta' f_0(600)$, and $\pi a_0(980)$ as the nature of the $f_0(600)$ and $a_0(980)$ mesons is not clear. As the $a_0(980)$ and $f_0(600)$ mesons may have large non-$\bar{q}q$ components [36–38], we may expect that the decays of the $X(1835)$ to the channels $\pi a_0(980)$, $\eta f_0(600)$, and $\eta' f_0(600)$ are less important.

The theoretical results of partial decay widths are very sensitive to the effective coupling constants λ of the $^3P_0$ vertex and the size parameters a and b. The meson size parameter b is determined to be 3.24 GeV^{-1} by the reaction $\rho \to e^+ e^-$ as in Refs. [39,40], which leads to an rms radius $\langle r^2 \rangle^{1/2} = 0.39$ fm for the s-wave mesons, while the optimum meson size parameter derived by fitting to the partial widths of higher quarkonia [41] is $b = 2.5$ GeV^{-1}. However, in studies of $\bar{N}N$ annihilations [28–30,32,42,43], the meson size parameter b is globally adjusted to 4.1 GeV^{-1} ($\langle r^2 \rangle^{1/2} = 0.50$ fm). As for the baryon size parameter a, various values have been employed in different works, ranging from $a = 1.6$ GeV^{-1} to 3.1 GeV^{-1} [28–30,32,42–50].

Theoretical results of partial decay widths are expected to be sensitive to the $N\bar{N}$ bound state wave function $\Phi_X(\vec{p})$ of the $X(1835)$. However, our poor knowledge of $\bar{N}N$ interactions does not allow us to work out a reliable $\Phi_X(\vec{p})$. Therefore, we evaluate in this work the relative partial decay widths to avoid the uncertainties that we have rather poor knowledge of the effective coupling constant λ and of the $\Phi_X(\vec{p})$ of $X(1835)$, and that the meson and nucleon size parameters a and b may range in rather large regions.

It is found that the ratios of partial decay widths are, of course, independent of the effective coupling strength λ of

the $^3P_0$ vertex, insensitive to the $N\bar{N}$ bound state wave function $\Phi_X(\vec{p})$, and insensitive to the meson and baryon size parameters. Indeed, in the low-momentum approximation, the ratios of partial decay widths are independent of the $N\bar{N}$ bound state wave function $\Phi_X(\vec{p})$, as shown in Eq. (14). As an example, we show in Table I the relative partial decay widths, normalized to the $\rho\rho$ channel, of an $^{11}S_0$ state $X(1835)$ to two mesons, with the baryon size parameter $a = 2.0$ GeV^{-1} and meson size parameter $b = 3.24$ GeV^{-1}. In the calculation the η and η' mesons are represented as

$$|\eta'\rangle = \beta|\eta_n\rangle + \alpha|\eta_s\rangle, \qquad |\eta\rangle = \alpha|\eta_n\rangle - \beta|\eta_s\rangle \quad (18)$$

in the basis

$$|\eta_n\rangle = \frac{1}{\sqrt{2}}(|u\bar{u}\rangle + |d\bar{d}\rangle), \qquad |\eta_s\rangle = |s\bar{s}\rangle, \quad (19)$$

where α and β are given in terms of the pseudoscalar mixing angle θ by the relation

$$\alpha = \sqrt{\frac{1}{3}}\cos\theta - \sqrt{\frac{2}{3}}\sin\theta, \qquad \beta = \sqrt{\frac{1}{3}}\cos\theta + \sqrt{\frac{2}{3}}\sin\theta. \quad (20)$$

We take the canonical value $\theta = -10.7°$ derived from the quadratic mass formula, which leads to $\alpha \approx \beta \approx 1/\sqrt{2}$.

For the $X(1835)$ and broad mesons we average over the mass spectrum $f(\mu)$, that is

$$\bar{\Gamma} = \int d\mu_X f_X(\mu_X) \int d\mu_1 f_1(\mu_1)$$
$$\times \int d\mu_2 f_2(\mu_2)\Gamma_{X\to M_1M_2}(\mu_X, \mu_1, \mu_2) \quad (21)$$

with the mass spectrum $f(\mu)$ as in Ref. [32]

$$f_i(\mu) = C\frac{(\Gamma_i/2)^2}{(\mu - M_i)^2 + (\Gamma_i/2)^2}, \quad (22)$$

where C is a normalization constant, and $\Gamma_{X\to M_1M_2}$ are derived in Eq. (14).

One sees in Table I that the $\rho\rho$, $\omega\omega$, and $\pi a_0(1450)$ decay channels dominate over others. After taking into account the large width of $f_0(1370)$, we derive from Eq. (21) almost the same partial decay width for the $\eta f_0(1370)$ and $\eta' f_0(1370)$ channels, as shown in Table I. The decay channels $\pi a_2(1320)$ and $\eta f_2(1270)$ are largely suppressed due to the small spin-flavor weights and limited phase spaces. Note in Eq. (10) that a small phase space strongly suppresses final states with a tensor meson. We have not listed in Table I the channel $\eta' f_2(1270)$ as the partial decay width is even much smaller than the $\eta f_2(1270)$ one due to the very small phase space.

As the branching fractions of the J/ψ one photon radiative decays are in order of 10^{-4} to 10^{-3} [51], the branching fractions Br$(J/\psi \to \gamma X)$Br$(X \to \rho\rho, \omega\omega, \pi a_0(1450))$ are expected to be in order of 10^{-5} to 10^{-3}. In high energy

e^+e^- collisions π mesons are produced in a dominant number, which may make it difficult to retrieve the $\rho\rho$ and $\omega\omega$ channels. However, one may expect that the resonance $X(1835)$ is observed in the $\pi a_0(1450)$ channel given it is an $\bar{N}N$ bound state.

III. $X(1835)$ TO $\eta\pi\pi$ AND $\eta'\pi\pi$

The $X(1835)$ resonance is observed in the $\eta'\pi\pi$ channel with the product branching fraction Br$(J/\psi \to \gamma X)$Br$(\to \pi^+\pi^-\eta') = (2.2 \pm 0.4(\text{stat}) \pm 0.4(\text{syst})) \times 10^{-4}$. In this section we estimate the joint contribution of the $\pi a_0(1450)$ and $f_0(1370)\eta$ channels to the final states $\pi\pi\eta$ and $\pi\pi\eta'$. The broad meson $f_0(1370)$ may decay to 2π, 4π, $\eta\eta$, and $K\bar{K}$, with the 2π channel predominant. The refit in Ref. [52] to five primary sets of data leads to the partial decay widths,

$$\Gamma_{f_0(1370)\to\pi\pi} = 325 \text{ MeV}, \qquad \Gamma_{f_0(1370)\to4\pi} = 54 \text{ MeV},$$
$$\frac{\Gamma_{f_0(1370)\to\eta\eta}}{\Gamma_{f_0(1370)\to\pi\pi}} = 0.19 \pm 0.07. \quad (23)$$

Considering that the $K\bar{K}$ decay channel is usually strongly suppressed, one may estimate $\Gamma_{f_0(1370)\to\pi^+\pi^-}/\Gamma_{\text{tot}} \sim 0.5$ and hence

$$\frac{\Gamma(X \to f_0(1370)\eta \to \eta\pi^+\pi^-)}{\Gamma(X \to \rho\rho)} \sim 0.029,$$
$$\frac{\Gamma(X \to f_0(1370)\eta' \to \eta'\pi^+\pi^-)}{\Gamma(X \to \rho\rho)} \sim 0.026. \quad (24)$$

The $a_0(1450)$ meson decays dominantly to $\pi\eta$, $\pi\eta'$, and $K\bar{K}$ channels, with the experimental values

$$\frac{\Gamma_{a_0(1450)\to\eta'\pi}}{\Gamma_{a_0(1450)\to\eta\pi}} = 0.35 \pm 0.16,$$
$$\frac{\Gamma_{a_0(1450)\to K\bar{K}}}{\Gamma_{a_0(1450)\to\eta\pi}} = 0.88 \pm 0.23. \quad (25)$$

We may estimate the contributions of the $\pi a_0(1450)$ intermediate channel to the final states $\eta\pi^+\pi^-$ and $\eta'\pi^+\pi^-$,

$$\frac{\Gamma(X \to a_0(1450)\pi \to \eta\pi^+\pi^-)}{\Gamma(X \to \rho\rho)} \sim 0.075,$$
$$\frac{\Gamma(X \to a_0(1450)\pi \to \eta'\pi^+\pi^-)}{\Gamma(X \to \rho\rho)} \sim 0.025. \quad (26)$$

Assuming that there is no interference between the contributions of the $\pi a_0(1450)$ and $f_0(1370)\eta$ intermediate channels, one may derive

$$\frac{\Gamma(X \to \eta\pi^+\pi^-)}{\Gamma(X \to \rho\rho)} \sim 0.1, \qquad \frac{\Gamma(X \to \eta'\pi^+\pi^-)}{\Gamma(X \to \rho\rho)} \sim 0.05. \quad (27)$$

Given that the branching ratio Br$(J/\psi \to \gamma X(1835))$ is in order of 10^{-4} to 10^{-3} [51], as other one photon

J/ψ radiative decays, the theoretical estimations in Eq. (27) indicates that the product branching fraction $\mathrm{Br}(J/\psi \to \gamma X(1835))\mathrm{Br}(X(1835) \to \pi\pi\eta')$ is in order of 10^{-5} to 10^{-4}, which is in line with the experimental data.

IV. DISCUSSION AND CONCLUSIONS

Partial decay widths of the $X(1835)$ to various decay channels are evaluated in the $^3P_0$ quark model, assuming that the $X(1835)$ is an $N\bar{N}$ bound state with the quantum number assignment $I^G(J^{PC}) = 0^+(0^{-+})$. We find that the decays of the $X(1835)$ to $\rho\rho$, $\omega\omega$, and $\pi a_0(1450)$ dominate over other channels. Based on the large $\pi a_0(1450)$ partial decay width, we would like to suggest the resonance $X(1835)$ to be searched in the $\pi a_0(1450)$ channel.

The contributions of the $\pi a_0(1450)$, $\eta f_0(1370)$, and $\eta' f_0(1370)$ channels to the final states $\pi\pi\eta$ and $\pi\pi\eta'$ are estimated. It is found that the partial decay widths $\Gamma(X \to \eta\pi^+\pi^-)$ and $\Gamma(X \to \eta'\pi^+\pi^-)$ are in the same order.

The $X(1835)$ may directly decay to a final state of three mesons, like $\pi\pi\eta$ and $\pi\pi\eta'$ although it is found that $\bar{N}N$ annihilations at rest are dominated by two-meson doorway states [32]. In the R3 quark diagram that was applied to study $\bar{N}N$ annihilations at the earliest time, Ref. [53] reveals that the $^{11}S_0$ $\bar{N}N$ channel couples strongly to the $\eta\rho\rho$ and $\rho\rho\rho$ states but very weakly to the $\eta\pi\pi$ one.

Therefore, one may not expect a sizable contribution to the $\eta\pi\pi$ and $\eta'\pi\pi$ partial decay widths by direct three-meson annihilation processes.

Up to now there has been no positive information on the observation of the $X(1835)$ in the $\eta\pi\pi$ state, which makes it natural to consider other interpretations of the $X(1835)$, such as the radial excitation of η'. As a pseudoscalar meson, the $X(1835)$ may decay through the KK^* and K^*K^* channels. However, these decay modes of an $\bar{N}N$ bound state are strongly suppressed [54,55]. It is expected that the product branching fraction $\mathrm{Br}(J/\psi \to \gamma X)\mathrm{Br}(X \to KK^*, K^*K^*)$ is even smaller than $\mathrm{Br}(J/\psi \to \gamma X)\mathrm{Br}(X \to \pi\pi\eta')$ if the $X(1835)$ is interpreted as an $\bar{N}N$ bound state. As the second radial excitation of η', however, a quark model study [24] shows that the $X(1835)$ decays to KK^* and K^*K^* states with large partial decay widths. The product branching fraction $\mathrm{Br}(J/\psi \to \gamma X)\mathrm{Br}(X \to KK^*, K^*K^*)$ shall be much larger than $\mathrm{Br}(J/\psi \to \gamma X)\mathrm{Br}(X \to \pi\pi\eta')$. Whether the $X(1835)$ could be observed in the KK^* and K^*K^* channels with large product branching fractions may tell if it is a radial excitation of η'.

ACKNOWLEDGMENTS

[1] J. Z. Bai *et al.* (BES Collaboration), Phys. Rev. Lett. **91,** 022001 (2003).

[2] M. Ablikim *et al.* (BES Collaboration), Phys. Rev. Lett. **95,** 262001 (2005).

[3] B. S. Zou and H. C. Chiang, Phys. Rev. D **69,** 034004 (2004).

[4] A. Sibirtsev, J. Haidenbauer, S. Krewald, Ulf-G. Meissner, and A. W. Thomas, Phys. Rev. D **71,** 054010 (2005).

[5] M. Ablikim *et al.* (BES Collaboration), Phys. Rev. Lett. **106,** 072002 (2011).

[6] B. Loiseau and S. Wycech, Phys. Rev. C **72,** 011001 (2005).

[7] J.-P. Dedonder, B. Loiseau, B. El-Bennich, and S. Wycech, Phys. Rev. C **80,** 045207 (2009).

[8] A. Datta and P. J. O Donnell, Phys. Lett. B **567,** 273 (2003).

[9] B. S. Zou and H. C. Chiang, Phys. Rev. D **69,** 034004 (2004).

[10] C. H. Chang and H. R. Pang, Commun. Theor. Phys. **43,** 275 (2005).

[11] M. L. Yan, S. Li, B. Wu, and B. Q. Ma, Phys. Rev. D **72,** 034027 (2005).

[12] G. J. Ding and M. L. Yan, Phys. Rev. C **72,** 015208 (2005).

[13] S. L. Zhu and C. S. Gao, Commun. Theor. Phys. **46,** 291 (2006).

[14] Z. G. Wang and S. L. Wan, J. Phys. G **34,** 505 (2007).

[15] Y. L. Ma, J. Phys. G **36,** 055004 (2009).

[16] G. J. Ding, R. G. Ping, and M. L. Yan, Eur. Phys. J. A **28,** 351 (2006).

[17] N. Kochelev and D. P. Min, Phys. Rev. D **72,** 097502 (2005).

[18] B. A. Li, Phys. Rev. D **74,** 034019 (2006).

[19] G. Hao, C. F. Qiao, and A. L. Zhang, Phys. Lett. B **642,** 53 (2006).

[20] X. G. He, X. Q. Li, X. Liu, and J. P. Ma, Eur. Phys. J. C **49,** 731 (2007).

[21] T. Huang and S. L. Zhu, Phys. Rev. D **73,** 014023 (2006).

[22] E. Klempt and A. Zaitsev, Phys. Rep. **454,** 1 (2007).

[23] D. M. Li and B. Ma, Phys. Rev. D **77,** 074004 (2008).

[24] J. F. Liu, G. J. Ding, and M. L. Yan, Phys. Rev. D **82,** 074026 (2010).

[25] J. S. Yu, Z. F. Sun, X. Liu, and Q. Zhao, Phys. Rev. D **83,** 114007 (2011).

[26] N. Kochelev and D. P. Min, Phys. Rev. D **72,** 097502 (2005).

[27] N. Kochelev and D. P. Min, Phys. Lett. B **633,** 283 (2006).

[28] M. Maruyama, S. Furui, and A. Faessler, Nucl. Phys. **A472,** 643 (1987).

[29] M. Maruyama, S. Furui, A. Faessler, and R. Vinh Mau, Nucl. Phys. **A473,** 649 (1987).

[30] C. B. Dover, T. Gutsche, and A. Faessler, Phys. Rev. C **43,** 379 (1991).

[31] Y. Yan, T. Gutsche, R. Thierauf, A. Muhn, and A. Faessler, J. Phys. G **23**, L33 (1997).

[32] C. B. Dover, T. Gutsche, M. Maruyama, and A. Faessler, Prog. Part. Nucl. Phys. **29**, 87 (1992).

[33] A. Kercek, T. Gutsche, and A. Faessler, J. Phys. G **25**, 2271 (1999).

[34] T. Gutsche, R. Vinh Mau, M. Strohmeier-Presicek, and A. Faessler, Phys. Rev. C **59**, 630 (1999).

[35] J. Vandermeulen, Z. Phys. C **37**, 563 (1988).

[36] R. L. Jaffe, Phys. Rev. D **15**, 267 (1977); **15**, 281 (1977).

[37] D. V. Bugg, Eur. Phys. J. C **47**, 57 (2006).

[38] N. A. Törnqvist, Z. Phys. C **68**, 647 (1995).

[39] Y. Yan, C. Kobdaj, P. Suebka, Y. M. Zheng, A. Faessler, T. Gutsche, and V. E. Lyubovitskij, Phys. Rev. C **71**, 025204 (2005).

[40] K. Kittimanapun, K. Khosonthongkee, C. Kobdaj, P. Suebka, and Y. Yan, Phys. Rev. C **79**, 025201 (2009).

[41] T. Barnes, F. E. Close, P. R. Page, and E. S. Swanson, Phys. Rev. D **55**, 4157 (1997).

[42] T. Gutsche, M. Maruyama, and A. Faessler, Nucl. Phys. **A503**, 737 (1989).

[43] A. Muhm, T. Gutsche, R. Thierauf, Y. Yan, and A. Faessler, Nucl. Phys. **A598**, 285 (1996).

[44] S. Mundigl, M. J. Vicente Vacas, and W. Weise, Nucl. Phys. **A523**, 499 (1991).

[45] N. Isgur and G. Karl, Phys. Rev. D **18**, 4187 (1978).

[46] N. Isgur and G. Karl, Phys. Rev. D **20**, 1191 (1979).

[47] S. Capstick and N. Isgur, Phys. Rev. D **34**, 2809 (1986).

[48] C. Chen, X. L. Chen, X. Liu, W. Z. Deng, and S. L. Zhu, Phys. Rev. D **75**, 094017 (2007).

[49] S. Capstick and W. Roberts, Phys. Rev. D **47**, 1994 (1993).

[50] S. Capstick and W. Roberts, Phys. Rev. D **49**, 4570 (1994).

[51] K. Nakamura *et al.* (Particle Data Group), J. Phys. G **37**, 075021 (2010).

[52] D. V. Bugg, Eur. Phys. J. C **52**, 55 (2007).

[53] A. M. Green and J. A. Niskanen, Nucl. Phys. **A412**, 448 (1984).

[54] C. Amsler and F. Myhrer, Annu. Rev. Nucl. Part. Sci. **41**, 219 (1991).

[55] C. Amsler *et al.*, Z. Phys. C **58**, 175 (1993).