

Transverse Energy Density Fluctuations in Heavy-Ion Collisions in a Gaussian Model

Berndt Müller

Department of Physics, Duke University, Durham, North Carolina 27708, USA

Andreas Schäfer

Institut für Theoretische Physik, Universität Regensburg, 93040 Regensburg, Germany

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We calculate the transverse correlation of fluctuations of the deposited energy density in nuclear collisions in the framework of a Gaussian model similar to the color glass condensate model.

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I. EVENT-BY-EVENT FLUCTUATIONS

The event-by-event fluctuations of the transverse emission pattern of hadrons in high-energy collisions of identical heavy nuclei have recently attracted much interest experimentally [1–7] and theoretically [8–22]. When averaged over collision events, the azimuthal angular distribution of emitted hadrons around the beam axis is symmetric with respect to the plane perpendicular to the impact parameter vector \mathbf{b} between the two nuclei [Au + Au or Cu + Cu at the Relativistic Heavy Ion Collider (RHIC) or Pb + Pb at the Large Hadron Collider (LHC)]. The event averaged angular distribution

$$\frac{dN}{d^2p_T} = \frac{dN}{\pi dp_T^2} \left(1 + \sum_{n=1}^{\infty} v_n(p_T) \cos(\phi_p) \right), \quad (1)$$

where ϕ_p is the angle between \mathbf{p}_T and \mathbf{b} , is therefore completely characterized by the even Fourier coefficients v_n . The dominant coefficient, v_2 , is called elliptic flow.

Owing to quantum fluctuations in the density distributions of the colliding nuclei and finite particle number effects on the distribution of emitted particles, the left-right symmetry is broken in individual collision events. The angular distribution can then be written in the form

$$\frac{dN}{d^2p_T} = \frac{dN}{\pi dp_T^2} \left(1 + \sum_{n=1}^{\infty} v_n(p_T) \cos(\phi_p + \psi_n) \right), \quad (2)$$

where ψ_n describes the tilt angle of the “event plane” for each Fourier coefficient with respect to the reaction plane defined by the vector \mathbf{b} . For even n , ψ_n is peaked around zero; for odd n , ψ_n is randomly distributed. The dominant odd coefficient, v_3 , is known as triangular flow. The event averages of the amplitude of the coefficients v_n are found to be constant over a rather large pseudorapidity range ($|\eta| \leq 2$) in Pb + Pb collisions at the LHC [5], indicating an approximately boost invariant origin.

The main dynamical source of event-by-event fluctuations in the coefficients v_n are believed to be nearly boost invariant fluctuations in the transverse distribution of the energy density at the beginning of the hydrodynamic expansion of the quark-gluon plasma formed in the nuclear

collisions. The geometric anisotropy of these fluctuations then translates in an anisotropic transverse collective flow pattern, which manifests itself in anisotropic particle emission. In the color glass condensate model of energy deposition there are two obvious sources of fluctuations in the deposited energy density. One is geometric fluctuations of the position of nucleons in the colliding nuclei at the moment of impact, leading to transverse fluctuations in the density of field generating color charges. This mechanism has been studied widely and is usually described geometrically by the Monte-Carlo Glauber model [23]. The transverse correlation length of the fluctuations generated by this mechanism will be of the order of the nucleon radius.

The other source of energy density fluctuations are fluctuations in the color field strength for a given density of color charges. This mechanism has not been investigated quantitatively up to now. The transverse correlation length generated by color field fluctuations will be dictated by the single scale governing the physics of the color glass condensate, the saturation scale Q_s . Since Q_s^{-1} is much smaller than the nucleon radius, the color field fluctuations can be expected to govern the microscopic structure of the transverse energy density fluctuations, which then is modulated on longer transverse scales by fluctuations in the nucleon density in the colliding nuclei.

Here we calculate the transverse correlation function of the deposited energy density in nuclear collisions in the framework of the Gaussian approximation similar, but not equivalent, to the color glass condensate model originally proposed by McLerran and Venugopalan [24,25]. Transverse correlations of the produced gluon density over a long rapidity range have been studied in the Gaussian color glass condensate model in connection with the “ridge” phenomenon. For example, Dumitru *et al.* [26] calculated the transverse momentum correlation of produced gluons; a formal expression for the correlated production of an arbitrary number of gluons was derived by Gelis *et al.* [27]. More recently (after the original submission of this manuscript), a Monte-Carlo calculation of the transverse fluctuations of the deposited energy density including local fluctuations in the color charge density of the colliding nuclei was published [28].

Our results derived below give an analytical representation of the contribution of color charge fluctuations to the fluctuations in the transverse energy density, which enable systematic studies of their parameter dependence and of the contribution of fluctuating gluon fields in the colliding nuclei to the energy density fluctuations at the beginning of the hydrodynamic expansion of the quark-gluon plasma in relativistic heavy-ion collisions. In Secs. II and III we derive the two-point correlator of the energy density following the collision of two nuclei. In Sec. IV we evaluate the resulting integrals and present numerical results for the correlation function for a representative choice of parameters.

II. ENERGY DENSITY FLUCTUATIONS IN THE COLOR GLASS CONDENSATE MODEL

In the Gaussian random source approximation to the color glass condensate model of small- x gluon structure of atomic nuclei [24,25] the probability distribution of color charge density $\rho^a(\mathbf{x})$ in the transverse plane is assumed to be of the form

$$P[\rho^a] = \exp\left(-\frac{1}{g^2\mu^2} \int d^2x \rho^a(\mathbf{x})^2\right). \quad (3)$$

Here μ^2 represents the area density of color charges in the colliding nuclei, and $Q_s = g^2\mu$ is called the saturation scale, because it represents the scale at which the small- x evolution of the gluon density becomes nonlinear due to saturation effects [29,30]. Owing to the independent contributions of several nucleons to the color field, the Gaussian approximation is expected to provide a good description to the color source distribution in colliding nuclei at small x [31]. In the light-cone gauge, the Gaussian color charge distribution translates into a Gaussian distribution of transverse gauge field strengths. Here we will follow the work of Lappi [32].

To calculate the initial state density fluctuations

$$\langle \varepsilon(\mathbf{x})\varepsilon(\mathbf{y}) \rangle - \langle \varepsilon(\mathbf{x}) \rangle \langle \varepsilon(\mathbf{y}) \rangle, \quad (4)$$

where \mathbf{x}, \mathbf{y} denote vectors in the transverse plane, we start from the expression for the deposited energy density of the gauge field given in Eq. (10) in Ref. [32]:

$$\varepsilon(\mathbf{x}) = \frac{1}{4} F_{ij}^c(\mathbf{x}) F_{ij}^c(\mathbf{x}) + 2A^{\eta c}(\mathbf{x}) A^{\eta c}(\mathbf{x}) \quad (5)$$

with transverse vector indices $i, j, m, n, \dots = 1, 2$. Immediately after the collision, the field strength tensor in the region between the receding nuclei only receives contributions from the mixed terms, as the color field of each individual nucleus is a pure gauge and the field strength tensor of each individual nucleus is thus zero outside the nuclear volume [33]:

$$F_{ij}^c(\mathbf{x}) = g f_{abc} (A_i^a(1; \mathbf{x}) A_j^b(2; \mathbf{x}) + A_i^a(2; \mathbf{x}) A_j^b(1; \mathbf{x})) \quad (6)$$

$$A^{\eta c}(\mathbf{x}) A^{\eta c}(\mathbf{x}) = \frac{g^2}{4} f_{abc} f_{a'b'c} A_i^a(1; \mathbf{x}) A_i^b(2; \mathbf{x}) A_j^{a'}(1; \mathbf{x}) A_j^{b'}(2; \mathbf{x}). \quad (7)$$

Here “1” and “2” denote the gauge fields carried by nucleus 1 and 2, respectively. The field correlator in the color glass condensate model is given by

$$\begin{aligned} \langle A_i^a(n; \mathbf{x}) A_j^b(m; \mathbf{y}) \rangle &= \frac{1}{2} \langle A_i^a(n; \mathbf{x}) A_j^b(m; \mathbf{y}) \rangle \\ &+ \frac{1}{2} \langle A_j^b(m; \mathbf{y}) A_i^a(n; \mathbf{x}) \rangle \\ &= \delta_{mn} \delta_{ab} \int \frac{d^2p}{(2\pi)^2} \\ &\times \cos[\mathbf{p} \cdot (\mathbf{x} - \mathbf{y})] \frac{p_i p_j}{\mathbf{p}^2} G(|\mathbf{p}|) \end{aligned} \quad (8)$$

where $G(|\mathbf{p}|)$ is the Fourier transform of the function

$$G(|\mathbf{x}|) = \frac{4}{g^2 N |\mathbf{x}|^2} \left[1 - \exp\left(\frac{g^2 N}{8\pi} g^2 \mu^2 |\mathbf{x}|^2 \ln(\Lambda |\mathbf{x}|)\right) \right] \times \Theta(1 - \Lambda |\mathbf{x}|) \quad (9)$$

with the IR cut-off parameter Λ . It is convenient to decompose the momentum quadrupole tensor as follows:

$$p_i p_j = \frac{p_1^2 + p_2^2}{2} \delta_{ij} + \frac{p_1^2 - p_2^2}{2} \sigma_{ij}^3 + p_1 p_2 \sigma_{ij}^1, \quad (10)$$

where σ^1, σ^3 are the familiar Pauli matrices. We thus obtain

$$\begin{aligned} \langle A_i^a(n; \mathbf{x}) A_j^b(m; \mathbf{y}) \rangle &= \frac{1}{2} \delta_{mn} \delta_{ab} \int \frac{d^2p}{(2\pi)^2} \left(\cos[p_1(x_1 - y_1)] \cos[p_2(x_2 - y_2)] \delta_{ij} G(|\mathbf{p}|) + \cos[p_1(x_1 - y_1)] \right. \\ &\times \cos[p_2(x_2 - y_2)] \sigma_{ij}^3 \frac{p_1^2 - p_2^2}{\mathbf{p}^2} G(|\mathbf{p}|) - \sin[p_1(x_1 - y_1)] \sin[p_2(x_2 - y_2)] \sigma_{ij}^1 \frac{p_1 p_2}{\mathbf{p}^2} G(|\mathbf{p}|) \left. \right) \\ &= \frac{1}{2} \delta_{mn} \delta_{ab} (\delta_{ij} D(\mathbf{x} - \mathbf{y}) + \sigma_{ij}^3 E(\mathbf{x} - \mathbf{y}) - \sigma_{ij}^1 F(\mathbf{x} - \mathbf{y})) \equiv \delta_{mn} \delta_{ab} \mathcal{S}_{ij}(\mathbf{x} - \mathbf{y}). \end{aligned} \quad (11)$$

For later use, we will note the values of the individual correlation functions D, E, F at the origin:

$$D(0) = \int \frac{d^2 p}{(2\pi)^2} G(|\mathbf{p}|) = \lim_{|\mathbf{x}| \rightarrow 0} G(|\mathbf{x}|); \quad E(0) = F(0) = 0. \quad (12)$$

The expression for $D(0)$ diverges logarithmically for the function $G(|\mathbf{x}|)$ given in (9), if the gauge coupling g is taken as a constant. However, as pointed out by Kovchegov and Weigert [34], the ultraviolet divergence can be removed by including effects from the running of the coupling constant by means of the substitution

$$g^4 \rightarrow g^2(\mu^2)g^2(1/|\mathbf{x}|^2) \quad (13)$$

in the exponent of (9). The specific structure of this substitution, sometimes called the ‘‘triumvirate’’ structure of the running coupling, is motivated by the form of next-to-leading order corrections to the small- x evolution of the BFKL kernel in the color dipole approach to parton saturation [35,36].

We first evaluate the expectation value of the deposited energy density:

$$\begin{aligned} \langle \varepsilon(\mathbf{x}) \rangle &= \frac{g^2}{2} f_{abc} f_{a'b'c} \langle A_i^a(1; \mathbf{x}) A_j^b(2; \mathbf{x}) A_i^{a'}(1; \mathbf{x}) A_j^{b'}(2; \mathbf{x}) + A_i^a(1; \mathbf{x}) A_j^b(2; \mathbf{x}) A_i^{a'}(2; \mathbf{x}) A_j^{b'}(1; \mathbf{x}) \rangle \\ &\quad + \frac{g^2}{2} f_{abc} f_{a'b'c} \langle A_i^a(1; \mathbf{x}) A_i^b(2; \mathbf{x}) A_i^{a'}(1; \mathbf{x}) A_j^{b'}(2; \mathbf{x}) \rangle \\ &= \frac{g^2}{2} f_{abc} f_{a'b'c} D(0)^2 (\delta_{aa'} \delta_{bb'} + \delta_{ab'} \delta_{ba'} / 2 + \delta_{aa'} \delta_{bb'} / 2) \\ &= \frac{g^2}{2} N(N^2 - 1) D^2(0), \end{aligned} \quad (14)$$

recovering the result given in Eq. (14) of Ref. [32].

Next we evaluate the two-point correlator of the energy density:

$$\begin{aligned} \langle \varepsilon(\mathbf{x}) \varepsilon(\mathbf{y}) \rangle &= \frac{g^4}{4} f_{abc} f_{a'b'c} f_{efd} f_{e'f'd'} \langle (A_i^a(1; \mathbf{x}) A_j^b(2; \mathbf{x}) A_i^{a'}(1; \mathbf{x}) A_j^{b'}(2; \mathbf{x}) + A_i^a(1; \mathbf{x}) A_j^b(2; \mathbf{x}) A_i^{a'}(2; \mathbf{x}) A_j^{b'}(1; \mathbf{x}) \\ &\quad + A_i^a(1; \mathbf{x}) A_i^b(2; \mathbf{x}) A_j^{a'}(1; \mathbf{x}) A_j^{b'}(2; \mathbf{x})) (A_m^e(1; \mathbf{y}) A_n^f(2; \mathbf{y}) A_m^{e'}(1; \mathbf{y}) A_n^{f'}(2; \mathbf{y}) \\ &\quad + A_m^e(1; \mathbf{y}) A_n^f(2; \mathbf{y}) A_m^{e'}(2; \mathbf{y}) A_n^{f'}(1; \mathbf{y}) + A_m^e(1; \mathbf{y}) A_m^f(2; \mathbf{y}) A_n^{e'}(1; \mathbf{y}) A_n^{f'}(2; \mathbf{y})) \rangle \end{aligned} \quad (15)$$

We again make use of the fact that only correlators among fields in the same nucleus are nonzero, which allows us to suppress the labels 1 and 2:

$$\begin{aligned} \langle \varepsilon(\mathbf{x}) \varepsilon(\mathbf{y}) \rangle &= \frac{g^4}{4} f_{abc} f_{a'b'c} f_{efd} f_{e'f'd'} \langle (A_i^a(\mathbf{x}) A_i^{a'}(\mathbf{x}) A_m^e(\mathbf{y}) A_m^{e'}(\mathbf{y})) \langle A_j^b(\mathbf{x}) A_j^{b'}(\mathbf{x}) A_n^f(\mathbf{y}) A_n^{f'}(\mathbf{y}) \rangle \\ &\quad + \langle A_i^a(\mathbf{x}) A_i^{a'}(\mathbf{x}) A_m^e(\mathbf{y}) A_n^{f'}(\mathbf{y}) \rangle \langle A_j^b(\mathbf{x}) A_j^{b'}(\mathbf{x}) A_n^f(\mathbf{y}) A_m^{e'}(\mathbf{y}) \rangle + \langle A_i^a(\mathbf{x}) A_i^{a'}(\mathbf{x}) A_m^e(\mathbf{y}) A_n^{e'}(\mathbf{y}) \rangle \langle A_j^b(\mathbf{x}) A_j^{b'}(\mathbf{x}) A_m^f(\mathbf{y}) A_n^{f'}(\mathbf{y}) \rangle \\ &\quad + \langle A_i^a(\mathbf{x}) A_j^{b'}(\mathbf{x}) A_m^e(\mathbf{y}) A_m^{e'}(\mathbf{y}) \rangle \langle A_j^b(\mathbf{x}) A_i^{a'}(\mathbf{x}) A_n^f(\mathbf{y}) A_n^{f'}(\mathbf{y}) \rangle + \langle A_i^a(\mathbf{x}) A_j^{b'}(\mathbf{x}) A_m^e(\mathbf{y}) A_n^{f'}(\mathbf{y}) \rangle \langle A_j^b(\mathbf{x}) A_i^{a'}(\mathbf{x}) A_n^f(\mathbf{y}) A_m^{e'}(\mathbf{y}) \rangle \\ &\quad + \langle A_i^a(\mathbf{x}) A_j^{b'}(\mathbf{x}) A_m^e(\mathbf{y}) A_n^{e'}(\mathbf{y}) \rangle \langle A_j^b(\mathbf{x}) A_i^{a'}(\mathbf{x}) A_m^f(\mathbf{y}) A_n^{f'}(\mathbf{y}) \rangle + \langle A_i^a(\mathbf{x}) A_j^{b'}(\mathbf{x}) A_m^e(\mathbf{y}) A_m^{e'}(\mathbf{y}) \rangle \langle A_j^b(\mathbf{x}) A_j^{b'}(\mathbf{x}) A_n^f(\mathbf{y}) A_n^{f'}(\mathbf{y}) \rangle \\ &\quad + \langle A_i^a(\mathbf{x}) A_j^{b'}(\mathbf{x}) A_m^e(\mathbf{y}) A_n^{f'}(\mathbf{y}) \rangle \langle A_j^b(\mathbf{x}) A_j^{b'}(\mathbf{x}) A_n^f(\mathbf{y}) A_m^{e'}(\mathbf{y}) \rangle + \langle A_i^a(\mathbf{x}) A_i^{a'}(\mathbf{x}) A_m^e(\mathbf{y}) A_n^{e'}(\mathbf{y}) \rangle \langle A_j^b(\mathbf{x}) A_j^{b'}(\mathbf{x}) A_m^f(\mathbf{y}) A_n^{f'}(\mathbf{y}) \rangle \rangle \\ &\equiv \frac{g^4}{4} f_{abc} f_{a'b'c} f_{efd} f_{e'f'd'} \sum_{\alpha=1}^9 M_{\alpha}. \end{aligned} \quad (16)$$

III. ENERGY DENSITY FLUCTUATIONS IN A MODIFIED GAUSSIAN MODEL

In principle, the expression (16) can be evaluated within the color glass condensate (CGC) model. However, the nonlinear connection between the gauge potential and the color charge density makes such a calculation extremely demanding. Here we take the view that the CGC model is only exact asymptotically but receives substantial unknown corrections for realistic, i.e. not asymptotically large, collision energies. We also note that the physics encoded in the McLerran-Venugopalan model of the saturated gluon density is basically encoded in just two numbers, namely, the size of the typical gauge fields A_i^a and the scale of its transverse correlations $1/Q_s$. For realistic, presently accessible collision energies, the effort of an exact evaluation of (16) in the Gaussian CGC model may thus not be justified.

Therefore, we here propose a model which drastically simplifies the evaluation of Eq. (16). This model assumes Gaussian correlations for the transverse gauge field components in the light-cone gauge rather than for the color charges, as assumed in the CGC model. For non-Abelian gauge fields both approximations are not equivalent, because the connection

between $\rho(\mathbf{x})$ and $A_i^a(\mathbf{x})$ is nonlinear [25,37]. For the Gaussian source charge distribution that forms the basis of the McLerran-Venugopalan model, these nonlinearities will lead to corrections to the energy density correlator obtained here. These corrections, originating from transverse gauge links connecting \mathbf{x} and \mathbf{y} , vanish in the limit $|\mathbf{x} - \mathbf{y}| \rightarrow 0$ and we will assume that they are small in general. As we only want to know the approximate properties of the energy density fluctuations (size and typical length scale) we thus will combine (9) and (17). To explore the size of the uncertainties introduced by this approximation we will study below several model variants. We will find that the quantitative differences among the predictions for $\Delta\varepsilon/\varepsilon$ are, indeed, very small.

The Gaussian approximation we advocate implies the factorize of the correlators of four gauge fields into products of correlators among two gauge fields, e.g.:

$$\langle A_j^b(\mathbf{x})A_i^{a'}(\mathbf{x})A_n^f(\mathbf{y})A_m^{e'}(\mathbf{y}) \rangle = \langle A_j^b(\mathbf{x})A_i^{a'}(\mathbf{x}) \rangle \langle A_n^f(\mathbf{y})A_m^{e'}(\mathbf{y}) \rangle + \langle A_j^b(\mathbf{x})A_n^f(\mathbf{y}) \rangle \langle A_i^{a'}(\mathbf{x})A_m^{e'}(\mathbf{y}) \rangle + \langle A_j^b(\mathbf{x})A_m^{e'}(\mathbf{y}) \rangle \langle A_n^f(\mathbf{y})A_i^{a'}(\mathbf{x}) \rangle \quad (17)$$

which represents the crucial simplification of our model, allowing for a nearly complete analytic treatment.

To proceed further we use the symmetry with respect to the color indices e' and f' to combine, e.g., the second and third term in the large brackets of (16):

$$M_2 + M_3 = \langle A_i^a(\mathbf{x})A_i^{a'}(\mathbf{x})A_m^e(\mathbf{y})A_n^{f'}(\mathbf{y}) \rangle \langle A_j^b(\mathbf{x})A_j^{b'}(\mathbf{x})(A_n^f(\mathbf{y})A_m^{e'}(\mathbf{y}) - A_m^f(\mathbf{y})A_n^{e'}(\mathbf{y})) \rangle \quad (18)$$

The second factor is easily shown to vanish:

$$\begin{aligned} \langle \dots \rangle &= \langle A_j^b(\mathbf{x})A_n^f(\mathbf{y}) \rangle \langle A_j^{b'}(\mathbf{x})A_m^{e'}(\mathbf{y}) \rangle \langle A_j^{b'}(\mathbf{x})A_n^f(\mathbf{y}) \rangle \langle A_j^b(\mathbf{x})A_m^{e'}(\mathbf{y}) \rangle - (m \leftrightarrow n) \\ &= \delta_{bf}\delta_{b'e'}S_{jn}(\mathbf{x} - \mathbf{y})S_{jm}(\mathbf{x} - \mathbf{y}) + \delta_{b'f'}\delta_{be'}S_{jn}(\mathbf{x} - \mathbf{y})S_{jm}(\mathbf{x} - \mathbf{y}) - (m \leftrightarrow n) = 0 \end{aligned} \quad (19)$$

The same holds true for the fourth and seventh term, M_4 and M_7 . After considerable algebra, the fifth, sixth, eighth and ninth terms combine to

$$\begin{aligned} M_5 + M_6 + M_8 + M_9 &= \frac{g^4}{4} f_{abc}f_{a'b'c'}f_{efd}f_{e'f'd'} \langle A_i^a(\mathbf{x})A_j^{a'}(\mathbf{x})A_m^e(\mathbf{y})A_n^{f'}(\mathbf{y}) \rangle \langle [A_i^b(\mathbf{x})A_j^{b'}(\mathbf{x}) \\ &\quad - A_j^b(\mathbf{x})A_i^{b'}(\mathbf{x})][A_n^f(\mathbf{y})A_m^{e'}(\mathbf{y}) - A_m^f(\mathbf{y})A_n^{e'}(\mathbf{y})] \rangle \\ &= \frac{g^4}{4} f_{abc}f_{a'b'c'}f_{efd}f_{e'f'd'} (\delta_{ae}\delta_{a'f'}S_{im}S_{jn} + \delta_{af'}\delta_{a'e}S_{in}S_{jm}) (\delta_{bf}\delta_{b'e'}2[S_{in}S_{jm} - S_{im}S_{jn}] \\ &\quad + \delta_{be'}\delta_{b'f'}2[S_{im}S_{jn} - S_{in}S_{jm}]) \\ &= \frac{g^4}{16} N^2(N^2 - 1)[D(\mathbf{x} - \mathbf{y})^2 + E(\mathbf{x} - \mathbf{y})^2 + F(\mathbf{x} - \mathbf{y})^2]^2, \end{aligned} \quad (20)$$

where we made use of the relation

$$f_{abc}f_{a'b'c'}f_{a'bd} = \frac{N}{2} f_{ab'd} \quad (21)$$

which follows from the Jacobi identity.

Finally we evaluate the first term:

$$\begin{aligned} M_1 &= \frac{g^4}{4} f_{abc}f_{a'b'c'}f_{efd}f_{e'f'd'} \langle A_i^a(\mathbf{x})A_i^{a'}(\mathbf{x})A_m^e(\mathbf{y})A_m^{e'}(\mathbf{y}) \rangle \langle A_j^b(\mathbf{x})A_j^{b'}(\mathbf{x})A_n^f(\mathbf{y})A_n^{f'}(\mathbf{y}) \rangle \\ &= \frac{g^4}{4} N^2(N^2 - 1)^2 D(0)^4 + \frac{g^4}{2} N^2(N^2 - 1) D(0)^2 [D(\mathbf{x} - \mathbf{y})^2 + E(\mathbf{x} - \mathbf{y})^2 + F(\mathbf{x} - \mathbf{y})^2] \\ &\quad + \frac{3g^4}{8} N^2(N^2 - 1) [D(\mathbf{x} - \mathbf{y})^2 + E(\mathbf{x} - \mathbf{y})^2 + F(\mathbf{x} - \mathbf{y})^2]^2. \end{aligned} \quad (22)$$

Combining these equations we finally obtain

$$\begin{aligned} \langle \varepsilon(\mathbf{x})\varepsilon(\mathbf{y}) \rangle - \langle \varepsilon(\mathbf{x}) \rangle \langle \varepsilon(\mathbf{y}) \rangle &= \frac{g^4}{2} N^2(N^2 - 1) D(0)^2 [D(\mathbf{x} - \mathbf{y})^2 + E(\mathbf{x} - \mathbf{y})^2 + F(\mathbf{x} - \mathbf{y})^2] \\ &\quad + \frac{7g^4}{16} N^2(N^2 - 1) [D(\mathbf{x} - \mathbf{y})^2 + E(\mathbf{x} - \mathbf{y})^2 + F(\mathbf{x} - \mathbf{y})^2]^2. \end{aligned} \quad (23)$$

Since the functions D , E , F always appear in the same combination, it makes sense to introduce the abbreviation

$$K(\mathbf{x} - \mathbf{y}) = D(\mathbf{x} - \mathbf{y})^2 + E(\mathbf{x} - \mathbf{y})^2 + F(\mathbf{x} - \mathbf{y})^2, \quad (24)$$

In terms of which the average deposited energy density and its fluctuation can be expressed as:

$$\varepsilon_0 = \langle \varepsilon \rangle = \frac{g^2}{2} N(N^2 - 1) K(0); \quad (25)$$

$$\begin{aligned} \Delta \varepsilon(\mathbf{x} - \mathbf{y})^2 &= \langle \varepsilon(\mathbf{x}) \varepsilon(\mathbf{y}) \rangle - \langle \varepsilon(\mathbf{x}) \rangle \langle \varepsilon(\mathbf{y}) \rangle \\ &= \frac{g^4}{2} N^2 (N^2 - 1) \left[K(0) K(\mathbf{x} - \mathbf{y}) + \frac{7}{8} K(\mathbf{x} - \mathbf{y})^2 \right]. \end{aligned} \quad (26)$$

IV. EVALUATION OF INTEGRALS

Next we simplify the integrals $D(\mathbf{x} - \mathbf{y})$, $E(\mathbf{x} - \mathbf{y})$ and $F(\mathbf{x} - \mathbf{y})$. We abbreviate $\mathbf{z} = \mathbf{x} - \mathbf{y}$ and $z = |\mathbf{z}|$. We begin with $D(\mathbf{z})$.

$$\begin{aligned} D(\mathbf{z}) &= \int_0^\infty \frac{p dp}{4\pi^2} G(p) \int_0^{2\pi} d\phi \cos(pz_1 \cos\phi) \\ &\quad \times \cos(pz_2 \sin\phi) \\ &= \int_0^\infty \frac{p dp}{4\pi^2} G(p) \int_0^\pi d\phi (\cos[p(z_1 \cos\phi + z_2 \sin\phi)] \\ &\quad + \cos[p(z_1 \cos\phi - z_2 \sin\phi)]) \end{aligned} \quad (27)$$

We substitute $\phi \rightarrow \pi - \phi$ in the last term and introduce the notation $z_1 = z \cos\psi$, $z_2 = z \sin\psi$:

$$\begin{aligned} D(\mathbf{z}) &= \int_0^\infty \frac{p dp}{4\pi^2} G(p) \int_0^{2\pi} d\phi \cos[pz \cos(\phi - \psi)] \\ &= \int_0^\infty \frac{p dp}{2\pi} G(p) J_0(pz) = \int_0^\infty \frac{d^2 p}{4\pi^2} G(p) e^{i\mathbf{p} \cdot \mathbf{z}} \\ &\equiv G(z), \end{aligned} \quad (28)$$

where we have used Eq. (3.715.18) from [38]. Similarly we obtain

$$\begin{aligned} E(\mathbf{z}) &= \int_0^\infty \frac{p dp}{4\pi^2} G(p) \int_0^{2\pi} d\phi \cos(pz_1 \cos\phi) \\ &\quad \times \cos(pz_2 \sin\phi) (\cos^2\phi - \sin^2\phi) \\ &= \int_0^\infty \frac{dp}{4\pi^2} G(p) \int_0^\pi d\phi (\cos[p(z_1 \cos\phi + z_2 \sin\phi)] \\ &\quad + \cos[p(z_1 \cos\phi - z_2 \sin\phi)]) \cos(2\phi). \end{aligned} \quad (29)$$

Using the same substitutions we find:

$$\begin{aligned} E(\mathbf{z}) &= \int_0^\infty \frac{p dp}{4\pi^2} G(p) \int_0^{2\pi} d\phi \cos[pz \cos(\phi - \psi)] \\ &\quad \times \cos(2\phi) \\ &= -\cos(2\psi) \int_0^\infty \frac{p dp}{2\pi} G(p) J_2(pz) \end{aligned} \quad (30)$$

where we used Eqs. (3.715.18) and (3.715.7) from [38]. Finally, a similar calculation yields:

$$\begin{aligned} F(\mathbf{z}) &= \int_0^\infty \frac{p dp}{4\pi^2} G(p) \int_0^{2\pi} d\phi \sin(pz_1 \cos\phi) \\ &\quad \times \sin(pz_2 \sin\phi) \cos\phi \sin\phi \\ &= \sin(2\psi) \int_0^\infty \frac{p dp}{2\pi} G(p) J_2(pz). \end{aligned} \quad (31)$$

We conclude that the function $K(z)$ only depends on the distance z between the points \mathbf{x} and \mathbf{y} .

We can express $D(z)$ and $E(z)^2 + F(z)^2$ in terms of the following integrals:

$$C_n(z) = \int_0^\infty \frac{p dp}{2\pi} G(p) J_n(pz), \quad (32)$$

for $n = 0, 2$, namely:

$$D(z) = C_0(z); \quad (33)$$

$$E(z)^2 + F(z)^2 = C_2(z)^2. \quad (34)$$

We rewrite the integrals as follows:

$$C_n(z) = \int \frac{d^2 p}{(2\pi)^2} \int d^2 x e^{-i\mathbf{p} \cdot \mathbf{x}} G(\mathbf{x}) J_n(pz). \quad (35)$$

We have to evaluate integrals of the type

$$B_n(x, z) = \int_0^\infty \frac{p dp}{2\pi} J_0(px) J_n(pz). \quad (36)$$

This integral can be evaluated for $n = 0$ using formula (6.633.2) in [38]:

$$\int_0^\infty p dp e^{-c^2 p^2} J_0(px) J_0(pz) = \frac{e^{-(x^2+z^2)/4c^2}}{2c^2} I_0\left(\frac{xz}{2c^2}\right). \quad (37)$$

We are interested in the limit $c \rightarrow 0$, which means that we can apply the limit of $I_0(z)$ for large arguments:

$$I_0(z) \rightarrow \frac{e^z}{\sqrt{2\pi z}}. \quad (38)$$

This yields

$$\begin{aligned} B_0(x, z) &= \lim_{c \rightarrow 0} \frac{1}{4\pi c} \frac{1}{\sqrt{\pi x z}} \exp\left(-\frac{(x-z)^2}{4c^2}\right) \\ &= \frac{1}{2\pi z} \delta(x-z). \end{aligned} \quad (39)$$

For $n = 2$ we use the recursion relation for Bessel functions:

$$J_2(z) = \frac{2}{z} J_1(z) - J_0(z), \quad (40)$$

and apply formula (6.512.3) from [38]:

$$\int_0^\infty dp J_0(px) J_1(pz) = \frac{1}{z} \theta(z-x), \quad (41)$$

with the convention $\theta(0) = 1/2$. This implies:

$$B_2(x, z) = \frac{1}{\pi z^2} \theta(z-x) - \frac{1}{2\pi z} \delta(x-z). \quad (42)$$

When we insert these results into the desired integrals, we find:

$$C_0(z) = G(z); \quad (43)$$

$$C_2(z) = \frac{2}{z^2} \int_0^z x dx G(\mathbf{x}) - G(z). \quad (44)$$

V. NUMERICAL RESULTS

We now evaluate the average energy density and its fluctuations for a choice of the parameters that is motivated by the initial conditions at which thermal QCD matter is formed in heavy-ion collisions at RHIC and LHC:

$$\begin{aligned} Q_s^2 &= (g^2 \mu)^2 = 2 \text{ GeV}^2; & g^2(\mu^2) &= 3.785; \\ g^2(1/x^2) &= \frac{16\pi^2}{9 \ln(1/(\Lambda^2 x^2))}. \end{aligned} \quad (45)$$

Note that the result is independent of the value of Λ . The functions $C_0(z)$ and $C_2(z)$ are shown in Figs. 1 and 2 for these parameter values.

For these parameters, the initial value of the deposited energy density is $\varepsilon_0 \approx 240 \text{ GeV}/\text{fm}^3$. This very large energy density quickly decreases due to the longitudinal expansion and reaches much smaller values by the time of thermalization. What matters for us is not the absolute value of the initial energy density, but the relative size and spatial correlation of its fluctuations, $\Delta\varepsilon(\mathbf{x} - \mathbf{y})/\varepsilon_0$. This function is shown in Fig. 3 for the parameters listed above.

As the figure shows, the fluctuations of the initial energy density are locally of similar magnitude as the energy density itself and fall over distances of the inverse saturation scale, here assumed as $Q_s^{-1} \approx 0.14 \text{ fm}$. This result is in accord with the intuitive picture of the field configuration immediately after the collision as a bundle of longitudinally stretching random color flux tubes with characteristic transverse width $1/Q_s$.

In order to explore the sensitivity of our result for the relative energy density fluctuations, we consider two other

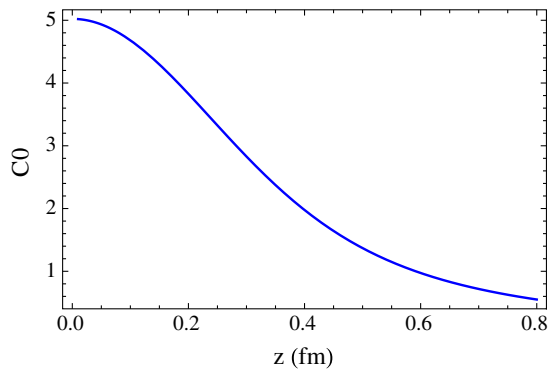


FIG. 1 (color online). The function $C_0(z)$ for the selected parameters.

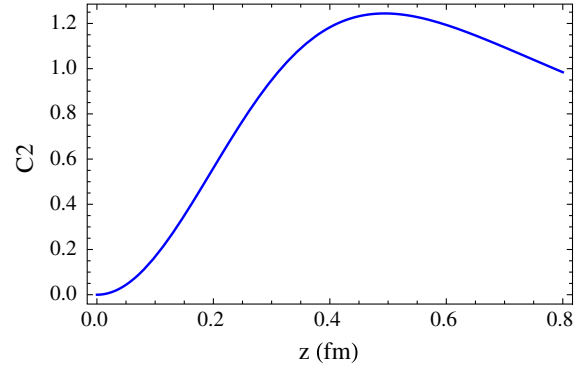


FIG. 2 (color online). The function $C_2(z)$ for the selected parameters.

forms of the color potential correlation function $G(|\mathbf{x}|)$. The correlation function (9) of the McLerran-Venugopalan (MV) model is characterized by two parameters: its normalization $G_0 = G(0)$, which is related to the average value of the energy density, and its second derivative at $\mathbf{x} = 0$ which defines the correlation length ξ . For the two-scale running coupling (13) we can write (9) as

$$G_{\text{MV}}(|\mathbf{x}|) = G_0 \phi_{\text{MV}}(\mathbf{x}^2/\xi^2) \quad (46)$$

with $G_0 = 4\pi\mu^2/9$ and $1/\xi^2 = N_c \pi(g\mu)^2/9$ and the dimensionless correlation function

$$\phi_{\text{MV}}(u) = (1 - e^{-u})/u. \quad (47)$$

Because the behavior of $G(\mathbf{x})$ is essentially Abelian at short distances and non-Abelian corrections are small, we require other model functions to have the same behavior in the limit $|\mathbf{x}| \rightarrow 0$. The two functions we consider here are:

$$\begin{aligned} \phi_1(u) &= e^{-u/2} & (\text{Model 1}); \\ \phi_2(u) &= \left(1 + \frac{u}{2}\right)^{-1} & (\text{Model 2}). \end{aligned} \quad (48)$$

The correlation functions of the color potential is then obtained as

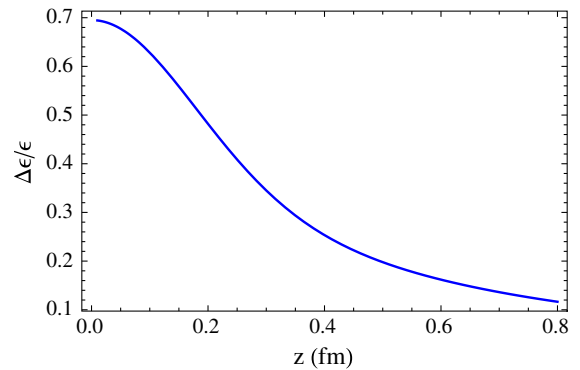


FIG. 3 (color online). The function $\Delta\varepsilon(z)/\varepsilon_0$ for the selected parameters.

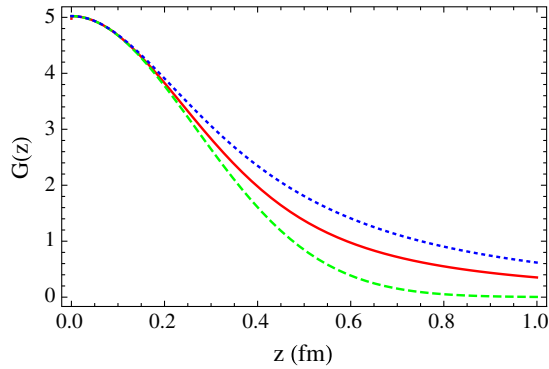


FIG. 4 (color online). The correlation function $G(|\mathbf{x}|)$ for the three models considered here: the MV model is shown as solid line (red), model 1 as dashed line (green) and model 2 as dotted line (blue).

$$G_{1/2}(|\mathbf{x}|) = G_0 \phi_{1/2}(\mathbf{x}^2/\xi^2) \quad (49)$$

with the same parameters G_0 and ξ as in the MV model.

In the following figures the results for the MV model are shown as solid lines (red), those for Model 1 as dashed lines (green) and those for Model 2 as dotted lines (blue). The three correlation functions $G(|\mathbf{x}|)$ are compared in Fig. 4, where it is seen that they exhibit the same behavior for $|\mathbf{x}| \rightarrow 0$, but fall off differently at large distances. The results for the correlation function of the relative fluctuation of the energy density $\delta\varepsilon/\varepsilon$ are shown in Fig. 5. The figure shows that the differences are reduced when compared with those of the field correlator $G(z)$. In particular, the MV model and the Gaussian Model 1 are nearly indistinguishable. Overall, it seems unlikely that the differences between the energy density fluctuations of the three models could lead to observable differences in the final state of a relativistic heavy-ion collision.

VI. SUMMARY

We have calculated the initial energy density fluctuations in high-energy heavy-ion collisions using a Gaussian model approximation for the gauge fields. The fluctuations turn out to be very large with a transverse profile determined by the saturation scale Q_s . A finite result is only

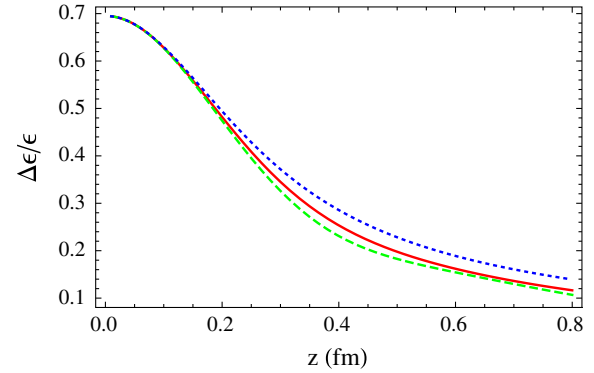


FIG. 5 (color online). The relative energy density fluctuation $\Delta\varepsilon(z)/\varepsilon_0$ for the three models considered here: the MV model is shown as solid line (red), model 1 as dashed line (green) and model 2 as dotted line (blue).

obtained when the triumvirate running coupling is used, giving additional support to the correctness of Eq. (13). The fluctuation probabilities thus derived can serve as input for any calculation aiming at the investigation of early fluctuations, in particular, for calculations which study the fate of such fluctuations during thermalization. For example, it is possible to investigate the problem of event-by-event fluctuations in heavy-ion collisions within the AdS/CFT paradigm using methods similar to those employed in [39]. For such investigations of the generic thermalization process in strongly coupled gauge theories the precise properties of these fluctuations should be irrelevant such that our approximate treatment should be fully sufficient.

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- [1] B. Alver *et al.* (PHOBOS Collaboration), *Phys. Rev. Lett.* **104**, 142301 (2010).
 - [2] P. Sorensen (STAR Collaboration), *J. Phys. G* **35**, 104102 (2008).
 - [3] ALICE Collaboration, *Phys. Rev. Lett.* **107**, 032301 (2011).
 - [4] A. Adare *et al.* (PHENIX Collaboration), *Phys. Rev. Lett.* **107**, 252301 (2011).
 - [5] J.J. Jia (ATLAS Collaboration), *J. Phys. G* **38**, 124012 (2011).
 - [6] P. Sorensen (STAR Collaboration), *J. Phys. G* **38**, 124029 (2011).
 - [7] E. Appelt (CMS Collaboration), [arXiv:1110.1026](https://arxiv.org/abs/1110.1026).
 - [8] M. Miller and R. Snellings, [arXiv:nucl-ex/0312008](https://arxiv.org/abs/nucl-ex/0312008).
 - [9] W. Broniowski, P. Bozek, and M. Rybczynski, *Phys. Rev. C* **76**, 054905 (2007).

- [10] R. P. G. Andrade, F. Grassi, Y. Hama, T. Kodama, and W. L. Qian, *Phys. Rev. Lett.* **101**, 112301 (2008).
- [11] E. Shuryak, *Phys. Rev. C* **80**, 054908 (2009).
- [12] B. Alver and G. Roland, *Phys. Rev. C* **81**, 054905 (2010).
- [13] H. Petersen, G. -Y. Qin, S. A. Bass, and B. Müller, *Phys. Rev. C* **82**, 041901 (2010).
- [14] B. H. Alver, C. Gombeaud, M. Luzum, and J. -Y. Ollitrault, *Phys. Rev. C* **82**, 034913 (2010).
- [15] P. Staig and E. Shuryak, *J. Phys. G* **38**, 124039 (2011); *Phys. Rev. C* **84**, 034908 (2011); **84**, 044912 (2011).
- [16] H. Holopainen, H. Niemi, and K. J. Eskola, *Phys. Rev. C* **83**, 034901 (2011).
- [17] G.-Y. Qin, H. Petersen, S. A. Bass, and B. Müller, *Phys. Rev. C* **82**, 064903 (2010).
- [18] B. Schenke, S. Jeon, and C. Gale, *Phys. Rev. Lett.* **106**, 042301 (2011).
- [19] Z. Qiu and U. W. Heinz, *Phys. Rev. C* **84**, 024911 (2011).
- [20] R. S. Bhalerao, M. Luzum, and J.-Y. Ollitrault, *Phys. Rev. C* **84**, 034910 (2011); *Phys. Rev. C* **84**, 054901 (2011).
- [21] G.-Y. Qin and B. Müller, [arXiv:1109.5961](https://arxiv.org/abs/1109.5961).
- [22] R. D. de Souza, J. Takahashi, T. Kodama, and P. Sorensen, *Phys. Rev. C* **85**, 054909 (2012).
- [23] H.-J. Drescher and Y. Nara, *Phys. Rev. C* **75**, 034905 (2007); **76**, 041903 (2007).
- [24] L. D. McLerran and R. Venugopalan, *Phys. Rev. D* **49**, 2233 (1994).
- [25] L. D. McLerran and R. Venugopalan, *Phys. Rev. D* **49**, 3352 (1994).
- [26] A. Dumitru, F. Gelis, L. McLerran, and R. Venugopalan, *Nucl. Phys.* **A810**, 91 (2008).
- [27] F. Gelis, T. Lappi, and L. McLerran, *Nucl. Phys. A* **828**, 149 (2009).
- [28] B. Schenke, P. Tribedy, and R. Venugopalan, [arXiv:1202.6646](https://arxiv.org/abs/1202.6646).
- [29] A. H. Mueller, *Nucl. Phys.* **B558**, 285 (1999).
- [30] J. Jalilian-Marian, A. Kovner, A. Leonidov, and H. Weigert, *Phys. Rev. D* **59**, 014014 (1998).
- [31] Y. V. Kovchegov, *Phys. Rev. D* **54**, 5463 (1996).
- [32] T. Lappi, *Phys. Lett. B* **643**, 11 (2006).
- [33] We note that the expressions (6,8) are valid only in the comoving time limit $\tau \rightarrow 0^+$ and represent the initial conditions for the nonlinear field evolution in the CGC model. The ultraviolet divergence of the energy density, which is removed by using a certain running coupling, is an artifact of this limit.
- [34] Y. V. Kovchegov and H. Weigert, *Nucl. Phys.* **A807**, 158 (2008).
- [35] Y. V. Kovchegov and H. Weigert, *Nucl. Phys.* **A789**, 260 (2007).
- [36] The running coupling also removes the infrared divergence at $x \rightarrow \Lambda^{-1}$ due to the logarithm in the exponent of (9). While this is convenient, it is also obscuring the fact that the Gaussian model does not correctly treat correlations and other effects due to color confinement.
- [37] C. Marquet and H. Weigert, *Nucl. Phys.* **A843**, 68 (2010).
- [38] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products* (Academic Press, Elsevier, 2007).
- [39] V. Balasubramanian *et al.*, *Phys. Rev. Lett.* **106**, 191601 (2011); *Phys. Rev. D* **84**, 026010 (2011).