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$B_s^0(\bar{B}_s^0) \to D_{CP}^0 K\bar{K}$: Detecting and discriminating new physics in B_s^0-$\bar{B}_s^0$ mixing

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If the weak phase of B_s^0-$\bar{B}_s^0$ mixing ($2\beta_s$) is found to be significantly different from zero, this is a clear signal of new physics (NP). However, if such a signal is found, we would like an unambiguous determination of $2\beta_s$ in order to ascertain which NP models could be responsible. In addition, in the presence of NP, the width difference $\Delta\Gamma_s$ between the two B_s mass eigenstates can be positive or negative, and ideally this sign ambiguity should be resolved experimentally. Finally, in order to see if the NP is contributing to Γ_{12}^s in addition to M_{12}^s, the precise measurement of $|\Gamma_{12}^s|$ is crucial. In this paper, we consider several different methods of measuring $2\beta_s$ using penguin-free two- and three-body decays with $\bar{b} \to \bar{c}u\bar{s}$ and $\bar{b} \to \bar{u}c\bar{s}$ transitions. We find that the most promising of these is a time-dependent Dalitz-plot analysis of $B_s^0(\bar{B}_s^0) \to D_{CP}^0 K\bar{K}$. With this decay, the unambiguous measurements of $2\beta_s$ and $\Delta\Gamma_s$ are possible, and the weak phase γ can also be extracted.

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I. INTRODUCTION

Over the past several years, a number of discrepancies with the predictions of the standard model (SM) have been observed in B decays, intriguingly all in $\bar{b} \to \bar{s}$ transitions. Some examples are: (i) in $B \to \pi K$ decays, it is difficult to account for all the experimental measurements within the SM [1,2], (ii) the values of the B_d^0-$\bar{B}_d^0$ mixing phase $\sin 2\beta$ obtained from different penguin-dominated $\bar{b} \to \bar{s}$ channels tend to be systematically smaller than that obtained from $B_d^0 \to J/\psi K_S$ [3], (iii) the fractions of transversely and longitudinally polarized decays in $B \to \phi K^*$ (f_T and f_L, respectively) are observed to be roughly equal [4], in contrast to the naive expectation that $f_T/f_L \ll 1$, (iv) the differential forward-backward asymmetry of leptons in the exclusive decay $\bar{B} \to \bar{K}^* \mu^+\mu^-$ is found to differ from the SM expectations in both the low- and high-q^2 regions (q^2 is the dilepton invariant mass) [5,6].

In light of this, it is particularly important to study $\bar{b} \to \bar{s}$ transitions and look for new-physics (NP) effects. Now, if NP is present in $\Delta B = 1$ $\bar{b} \to \bar{s}$ decays, it would be highly unnatural for it not to also affect the $\Delta B = 2$ transition, in particular, B_s^0-$\bar{B}_s^0$ mixing. In order to see where NP can enter, we briefly review the mixing. In the B_s system, the mass eigenstates B_L and B_H (L and H indicate the light and heavy states, respectively) are admixtures of the flavor eigenstates B_s^0 and $\bar{B}_s^0$:

$$|B_L\rangle = p|B_s^0\rangle + q|\bar{B}_s^0\rangle, \qquad |B_H\rangle = p|B_s^0\rangle - q|\bar{B}_s^0\rangle, \quad (1)$$

with $|p|^2 + |q|^2 = 1$. As a result, the initial flavor eigenstates oscillate into one another according to the Schrödinger equation

$$i\frac{d}{dt}\begin{pmatrix} |B_s^0(t)\rangle \\ |\bar{B}_s^0(t)\rangle \end{pmatrix} = \left(M^s - i\frac{\Gamma^s}{2}\right)\begin{pmatrix} |B_s^0(t)\rangle \\ |\bar{B}_s^0(t)\rangle \end{pmatrix}, \quad (2)$$

where $M = M^\dagger$ and $\Gamma = \Gamma^\dagger$ correspond, respectively, to the dispersive and absorptive parts of the mass matrix. The off-diagonal elements, $M_{12}^s = M_{21}^{s*}$ and $\Gamma_{12}^s = \Gamma_{21}^{s*}$, are generated by B_s^0-$\bar{B}_s^0$ mixing. We define

$$\Gamma_s \equiv \frac{\Gamma_H + \Gamma_L}{2}, \qquad \Delta M_s \equiv M_H - M_L, \qquad \Delta\Gamma_s \equiv \Gamma_L - \Gamma_H. \quad (3)$$

Expanding the mass eigenstates, we find, to a very good approximation [7],

$$\Delta M_s = 2|M_{12}^s|, \qquad \Delta\Gamma_s = 2|\Gamma_{12}^s|\cos\phi_s, \qquad \frac{q}{p} = e^{-2i\beta_s}\left[1 - \frac{a}{2}\right], \quad (4)$$

where $\phi_s \equiv \arg(-M_{12}^s/\Gamma_{12}^s)$ is the CP phase in $\Delta B = 2$ transitions. In Eq. (4) the small expansion parameter a is given by

$$a = \frac{\Gamma_{12}^s}{M_{12}^s}\sin\phi_s. \quad (5)$$

This is expected to be $\ll 1$, and hence can be neglected in the definition of q/p.

The weak phase $2\beta_s$ appears in the indirect (mixing-induced) CP asymmetries. For example, in $B_s^0 \to J/\psi\phi$,

$$2\beta_s^{\psi\phi} = \arg\left(\frac{q}{p}\frac{\bar{A}}{A}\right), \quad (6)$$

where A and $\bar{A}$ are, respectively, the amplitudes for $B_s^0 \to J/\psi\phi$ and $\bar{B}_s^0 \to J/\psi\phi$. Now, the weak phases ϕ_s and $2\beta_s$ are independent—we have $\phi_s = 2\beta_s - \arg(-\Gamma_{12}^s)$ [7]. It is often said that, in the SM, $\phi_s = -2\beta_s$. However, strictly speaking, this is not true—it holds only in the limit of ϕ_s and $-2\beta_s \simeq 0$ [8].

The precise measurement of ΔM_s determines $|M_{12}^s|$ [9]. However, because of hadronic uncertainties, the SM prediction for ΔM_s is not very precise—in Ref. [10], it is noted that the theoretical uncertainties still allow NP contributions to $|M_{12}^s|$ of order 20%. In addition, Γ_{12}^s can be calculated from the absorptive part of the B_s^0-$\bar{B}_s^0$ mixing box diagram, leading to $\Delta\Gamma_s$. Unlike the B_d system, where $\Delta\Gamma_d$ is negligibly small, in the B_s system $\Delta\Gamma_s$ is expected to be reasonably large, which leads to certain advantages for the search for CP-violating effects in the B_s system over that of B_d system. The updated SM predictions of the width difference and the CP phase ϕ_s are given by [10]

$$\Delta\Gamma_s^{\rm SM} \simeq 2|\Gamma_{12}^s| = 0.087 \pm 0.021\ {\rm ps}^{-1}, \qquad \phi_s \approx 0.22°. \tag{7}$$

The CDF [11] and DØ [12] collaborations have measured the CP asymmetry in $B_s^0 \to J/\psi\phi$, and found a hint for indirect CP violation. In general, this result is interpreted as evidence for a nonzero value of $2\beta_s^{\psi\phi}$, and the contributions of various NP models to the B_s mixing phase have been explored [13–19]. It has also been pointed out that NP in the decay $\bar{b} \to \bar{s}c\bar{c}$ could also play an important role [20]. Recently CDF and DØ updated their measurements of the CP-violating phase. The 68% C.L. allowed ranges are [21,22][1]

$$\begin{aligned} 2\beta_s^{\psi\phi} &\in [2.3°, 59.6°] \cup [123.8°, 177.6°], \qquad \text{CDF}, \\ &\in [9.7°, 52.1°] \cup [127.9°, 170.3°], \qquad \text{DØ}. \end{aligned} \tag{8}$$

Most of the values of $2\beta_s^{\psi\phi}$ here suggest NP. Now, $2\beta_s^{\psi\phi}$ is obtained with a twofold ambiguity, i.e. the measurement is insensitive to the transformation $(2\beta_s^{\psi\phi}, \Delta\Gamma_s) \leftrightarrow (\pi - 2\beta_s^{\psi\phi}, -\Delta\Gamma_s)$. The problem here is that, in the presence of NP (either in M_{12}^s or Γ_{12}^s or in both of them), the sign of $\Delta\Gamma_s$ can be positive or negative (in the SM, $\Delta\Gamma_s > 0$). The sign ambiguity in $\Delta\Gamma_s$ will then always lead to a twofold ambiguity in the extraction of $2\beta_s^{\psi\phi}$. The complete differential decay rate for the process $B_s^0 \to J/\psi\phi(\to K^+K^-)$ including both the s-and p-wave angular momentum states for the K^+K^- pair allows an unbiased[2] measurement of $2\beta_s^{\psi\phi}$ [24]. In addition, the interference of the s-and p-wave amplitudes is helpful for removing the twofold ambiguity in the measurement of $2\beta_s^{\psi\phi}$. On the other hand, as mentioned above, the possibility of NP in the decay $\bar{b} \to \bar{s}c\bar{c}$ cannot be ruled out, so that the phase $2\beta_s^{\psi\phi}$ extracted from $B_s^0 \to J/\psi\phi$ should not necessarily be taken as purely a mixing phase. It is therefore worthwhile to look for a process in which NP in the decay can essentially be neglected, and which permits the determination of $2\beta_s$ without any ambiguity. If the measured value of $2\beta_s$ is found to be significantly different from that in $B_s^0 \to J/\psi\phi$, it will be clear signal of NP in $\bar{b} \to \bar{s}c\bar{c}$.

In addition, the DØ Collaboration recently found a large CP asymmetry in the like-sign dimuon signal, which they attribute primarily to $a_{\rm SL}^s$, the semileptonic CP asymmetry in $B_s^0 \to X_s\mu\nu$ [25,26]. Now, the DØ result is less than 2σ away from zero and consequently to an excellent approximation also about 2σ away from the SM prediction ($a_{\rm SL}^{s,{\rm SM}} \approx 2 \times 10^{-5}$) [10]. Still, NP in B_s^0-$\bar{B}_s^0$ mixing can explain the result (for example, see Ref. [27]). However, if one wishes to reproduce the central value of $a_{\rm SL}^s$, one requires NP specifically in Γ_{12}^s [28,29]. There are NP models that can contribute to Γ_{12}^s through the decay $b \to s\tau^+\tau^-$ [30,31], and a significant enhancement of its magnitude over that of the SM [10] is possible. Furthermore, the possibility of NP effects in Γ_{12}^s through the decay $\bar{b} \to \bar{s}c\bar{c}$ cannot be ruled out [29,30].

We therefore see that there are some hints of NP in the B_s system, but nothing definitive yet. Thus, it is important to look for additional methods of probing NP in B_s^0-$\bar{B}_s^0$ mixing. Ideally, the new method(s) would allow an unambiguous determination of the mixing phase $2\beta_s$, even taking into account the possible sign ambiguity in $\Delta\Gamma_s$. Also useful are methods which remove this sign ambiguity even without providing any direct information on the CP phases ϕ_s or $2\beta_s$. Finally, if NP is present in the mixing, we would like to know if it contributes to Γ_{12}^s in addition to M_{12}^s. Hence, along with the measurement of $2\beta_s$, independent and unbiased measurements of $|\Gamma_{12}^s|$ and ϕ_s are essential.

To be specific: several years ago, the two-body decays $B_s^0(\bar{B}_s^0) \to D_s^{\pm}K^{\mp}, D_s^{*\pm}K^{\mp}, \ldots$ were examined with the idea of extracting weak phases [32]. Because the final state is accessible to both B_s^0 and $\bar{B}_s^0$ mesons, a mixing-induced indirect CP asymmetry occurs. Using this, and assuming that $\Delta\Gamma_s$ is sizeable, the conclusion of Ref. [32] is that one can measure the phase $2\beta_s + \gamma$ with a twofold discrete ambiguity, and that this ambiguity can be removed if factorization is assumed. However, if there is NP in B_s^0-$\bar{B}_s^0$ mixing, $\Delta\Gamma_s < 0$ is allowed as well. This implies that, in fact, $2\beta_s + \gamma$ can be obtained with a fourfold discrete ambiguity (or twofold if factorization is assumed).

In Ref. [33] it was shown that the sign ambiguity in $\Delta\Gamma_s$ can be removed using $B_s^0 \to D_s^{\pm}K^{\mp}$ decays. Although the method does not allow a direct determination of the phase ϕ_s, it does discriminate between the two solutions with $\cos\phi_s > 0$ and $\cos\phi_s < 0$, which then determines the sign of $\Delta\Gamma_s$. However, the method is based on several assumptions: (i) the weak phase γ is taken from the B-factory measurements, (ii) factorization is assumed, i.e. the strong phase is taken to be $\simeq 0$, and (iii) the SM-predicted value of Γ_{12}^s has been used in the analysis.

In 1991, the decays $B_s^0(\bar{B}_s^0) \to D_{CP}^0\phi$, where D_{CP}^0 is a neutral D-meson CP-eigenstate, were proposed to extract the CKM angle γ with a twofold ambiguity [34,35]. However, these methods assumed that the phase $2\beta_s$

[1]In Ref. [23], LHCb present their first CP-asymmetry measurement in $B_s^0 \to J/\psi\phi$ decays. With 0.37 fb^{-1} of data, and considering only the positive solution for $\Delta\Gamma_s$, LHCb find $2\beta_s^{\psi\phi} = 0.15 \pm 0.18({\rm stat}) + 0.06({\rm syst})$ and $\Delta\Gamma_s = 0.123 \pm 0.029({\rm stat}) \pm 0.011({\rm syst})\ {\rm ps}^{-1}$. This is a substantial improvement over the previous measurements of Refs. [21,22].

[2]If the s-wave components are neglected, the measurement of $2\beta_s^{\psi\phi}$ would be biased by 7%–17% towards zero [24].

is approximately zero (or known). The current experimental data [see Eq. (8)] is not completely in favor of this assumption—there is the possibility that $2\beta_s$ can be significantly different from zero. In addition, at present $2\beta_s$ is measured with a twofold ambiguity, which adds a further discrete ambiguity to the determination of γ.

We therefore see that previous analyses of two-body B decays only partially probe NP in B_s^0-$\bar{B}_s^0$ mixing—$2\beta_s$ is, in general, not determined unambiguously, the sign ambiguity in $\Delta\Gamma_s$ is generally unresolved, and the possibility of NP affecting Γ_{12}^s has not been considered. In this paper we go beyond the previous analyses to explore all of these issues.

The measurement of $\Delta\Gamma_s$ can be combined with the measurement of the semileptonic asymmetry $a_{\rm sl}^s$ as follows to obtain ϕ_s and $|\Gamma_{12}^s|$. The expression for $\Delta\Gamma_s$ is given in Eq. (4); the semileptonic asymmetry is defined as

$$a_{\rm sl}^s = {\rm Im}\left[\frac{\Gamma_{12}^s}{M_{12}^s}\right] = \frac{2|\Gamma_{12}^s|}{\Delta M_s}\sin\phi_s. \tag{9}$$

Combining Eqs. (4) and (9) we obtain

$$\tan\phi_s = \frac{a_{\rm sl}^s \Delta M_s}{\Delta\Gamma_s}, \qquad |\Gamma_{12}^s| = \frac{\sqrt{\Delta\Gamma_s{}^2 + a_{\rm sl}^s{}^2 \Delta M_s{}^2}}{2}. \tag{10}$$

Now, ΔM_s is known very precisely—$\Delta M_s = 17.77 \pm 0.12$ [9,36]—and so the precise measurements of $a_{\rm sl}^s$ and $\Delta\Gamma_s$ (without sign ambiguity) allow one to extract ϕ_s without any ambiguity,[3] as well as $|\Gamma_{12}^s|$.

The comparison of the measured values of ϕ_s and $|\Gamma_{12}^s|$ with those predicted by the SM [Eq. (7)] can reveal the presence of NP. CDF and DØ have measured $a_{\rm sl}^s$ directly and the average of their measurements is given by [36]

$$a_{\rm sl}^s = -0.0115 \pm 0.0061. \tag{11}$$

If we take $|\Delta\Gamma_s| = 0.075 \pm 0.04$, as given by CDF [21], we obtain

$$\tan\phi_s = -2.72 \pm 2.05, \qquad |\Gamma_{12}^s| = 0.11 \pm 0.05 \ {\rm ps}^{-1}, \tag{12}$$

while $|\Delta\Gamma_s| = 0.163^{+0.065}_{-0.064}$, as given by DØ [22], yields

$$\tan\phi_s = -1.25 \pm 0.83, \qquad |\Gamma_{12}^s| = 0.13 \pm 0.05 \ {\rm ps}^{-1}. \tag{13}$$

Although $|\Gamma_{12}^s|$ and ϕ_s can significantly deviate from their SM predictions [Eq. (7)], both of them are consistent with the SM within the error bar. Note that in the above numerical analysis we do not consider the negative solution for $\Delta\Gamma_s$, which introduces a sign ambiguity in the extraction of ϕ_s. It is clear that improved measurements of both $a_{\rm sl}^s$ and $\Delta\Gamma_s$ are essential in order to understand the underlying physics of B_s^0-$\bar{B}_s^0$ mixing and the width difference. In this paper we focus on the measurement of $\Delta\Gamma_s$ from the Dalitz-plot analysis of the three-body decays.

In Sec. II we review the two-body decays. In particular, in Sec. II C, we update the analysis of $B_s^0(\bar{B}_s^0) \to D_{CP}^0\phi$, considering both $\Delta\Gamma_s > 0$ and $\Delta\Gamma_s < 0$. In Sec. III, we present the Dalitz-plot analyses of three-body decays. In particular, in Sec. III C, we focus on $B_s^0(\bar{B}_s^0) \to D_s^\pm K^\mp \pi^0, D_s^\pm \pi^\mp K^0, \ldots$, using the interference between the different intermediate resonant decays to provide additional information. And in Sec. III D, we show that a much greater improvement can be obtained by performing a time-dependent Dalitz-plot analysis of the decay $B_s^0(\bar{B}_s^0) \to D_{CP}^0 K\bar{K}$. Finally, in Sec. IV, we present a possible way to determine $\Delta\Gamma_s$, or equivalently $|\Gamma_{12}^s|$ and ϕ_s, using three-body decays. We conclude in Sec. V.

II. TWO-BODY DECAYS

A. $B_s^0(\bar{B}_s^0) \to f, \bar{f}$

Consider a final state f, not necessarily a CP eigenstate, to which both B_s^0 and $\bar{B}_s^0$ can decay. In the presence of B_s^0-$\bar{B}_s^0$ mixing, the time-dependent decay rates are given by [37]

$$\begin{aligned}
\Gamma(B_s^0(t) \to f) \sim \frac{1}{2}e^{-\Gamma_s t}\Big\{&(|A_f|^2 + |\bar{A}_f|^2)\cosh(\Delta\Gamma_s t/2) \\
&+ (|A_f|^2 - |\bar{A}_f|^2)\cos\Delta m_s t \\
&- 2\sinh(\Delta\Gamma_s t/2){\rm Re}\left[\frac{q}{p}A_f^*\bar{A}_f\right] \\
&- 2\sin\Delta m_s t\,{\rm Im}\left[\frac{q}{p}A_f^*\bar{A}_f\right]\Big\}, \\
\Gamma(\bar{B}_s^0(t) \to f) \sim \frac{1}{2}e^{-\Gamma_s t}\Big\{&(|A_f|^2 + |\bar{A}_f|^2)\cosh(\Delta\Gamma_s t/2) \\
&- (|A_f|^2 - |\bar{A}_f|^2)\cos\Delta m_s t \\
&- 2\sinh(\Delta\Gamma_s t/2){\rm Re}\left[\frac{q}{p}A_f^*\bar{A}_f\right] \\
&+ 2\sin\Delta m_s t\,{\rm Im}\left[\frac{q}{p}A_f^*\bar{A}_f\right]\Big\},
\end{aligned} \tag{14}$$

where $A_f \equiv A(B_s^0 \to f)$, $\bar{A}_f \equiv A(\bar{B}_s^0 \to f)$, and $q/p = e^{-2i\beta_s}$. This yields

$$\begin{aligned}
&\Gamma(B_s^0(t) \to f) - \Gamma(\bar{B}_s^0(t) \to f) \\
&\quad \sim (|A_f|^2 + |\bar{A}_f|^2)e^{-\Gamma_s t}[C\cos\Delta m_s t - S\sin\Delta m_s t], \\
&\Gamma(B_s^0(t) \to f) + \Gamma(\bar{B}_s^0(t) \to f) \\
&\quad \sim (|A_f|^2 + |\bar{A}_f|^2)e^{-\Gamma_s t}[\cosh(\Delta\Gamma_s t/2) \\
&\qquad - \mathcal{A}_{\Delta\Gamma}\sinh(\Delta\Gamma_s t/2)],
\end{aligned} \tag{15}$$

where

[3] Knowledge of $\tan\phi_s$ gives ϕ_s with a twofold ambiguity, $\phi_s \leftrightarrow \pi + \phi_s$. However, $a_{\rm sl}^s$ determines $\sin\phi_s$, which allows one to differentiate ϕ_s and $\pi + \phi_s$.

$$C \equiv \frac{1-|\lambda|^2}{1+|\lambda|^2}, \qquad S \equiv \frac{2\,\mathrm{Im}\lambda}{1+|\lambda|^2},$$
$$\mathcal{A}_{\Delta\Gamma} \equiv \frac{2\,\mathrm{Re}\lambda}{1+|\lambda|^2}, \qquad \lambda \equiv \frac{q}{p}\frac{\bar{A}_f}{A_f}. \tag{16}$$

The idea is that, by fitting the data corresponding to the difference ("tagged") and sum ("untagged") of decay rates to the four time-dependent functions given on the right-hand side of the equations in Eq. (15), the coefficients of these functions can be obtained, from which C, S, and $\mathcal{A}_{\Delta\Gamma}$ can be derived. However, there is a complication—in the presence of NP in $\Delta B = 2$ transitions, $\Delta\Gamma_s$ is unknown (though it is assumed to be reasonably large). Therefore, for the untagged combination, both $\Delta\Gamma_s$ and $\mathcal{A}_{\Delta\Gamma}$ must be found in the fit. Still, though this will determine $|\Delta\Gamma_s|$, its sign will remain unknown. The reason is that only the function $\sinh(\Delta\Gamma_s t/2)$ is sensitive to the sign of $\Delta\Gamma_s$, and it is multiplied by $\mathcal{A}_{\Delta\Gamma}$. Thus, any change in the sign of $\Delta\Gamma_s$ can be compensated for by changing the sign of $\mathcal{A}_{\Delta\Gamma}$. The bottom line is that any analysis which uses $\mathcal{A}_{\Delta\Gamma}$ will have a discrete ambiguity due to the unknown sign of $\Delta\Gamma_s$.

Similarly,

$$\begin{aligned}
\Gamma(B_s^0(t)\to\bar{f}) \sim \frac{1}{2}e^{-\Gamma_s t}\Big\{&(|A_{\bar{f}}|^2+|\bar{A}_{\bar{f}}|^2)\cosh(\Delta\Gamma_s t/2)\\
&+(|A_{\bar{f}}|^2-|\bar{A}_{\bar{f}}|^2)\cos\Delta m_s t\\
&-2\sinh(\Delta\Gamma_s t/2)\mathrm{Re}\Big[\frac{p}{q}\bar{A}_{\bar{f}}^* A_{\bar{f}}\Big]\\
&+2\sin\Delta m_s t\,\mathrm{Im}\Big[\frac{p}{q}\bar{A}_{\bar{f}}^* A_{\bar{f}}\Big]\Big\},\\
\Gamma(\bar{B}_s^0(t)\to\bar{f}) \sim \frac{1}{2}e^{-\Gamma_s t}\Big\{&(|A_{\bar{f}}|^2+|\bar{A}_{\bar{f}}|^2)\cosh(\Delta\Gamma_s t/2)\\
&-(|A_{\bar{f}}|^2-|\bar{A}_{\bar{f}}|^2)\cos\Delta m_s t\\
&-2\sinh(\Delta\Gamma_s t/2)\mathrm{Re}\Big[\frac{p}{q}\bar{A}_{\bar{f}}^* A_{\bar{f}}\Big]\\
&-2\sin\Delta m_s t\,\mathrm{Im}\Big[\frac{p}{q}\bar{A}_{\bar{f}}^* A_{\bar{f}}\Big]\Big\},
\end{aligned} \tag{17}$$

where $A_{\bar{f}} \equiv A(B_s^0\to\bar{f})$ and $\bar{A}_{\bar{f}} \equiv A(\bar{B}_s^0\to\bar{f})$. Then

$$\frac{\Gamma(B_s^0(t)\to\bar{f})-\Gamma(\bar{B}_s^0(t)\to\bar{f})}{\Gamma(B_s^0(t)\to\bar{f})+\Gamma(\bar{B}_s^0(t)\to\bar{f})} = \frac{\bar{C}\cos\Delta m_s t+\bar{S}\sin\Delta m_s t}{\cosh(\Delta\Gamma_s t/2)-\bar{\mathcal{A}}_{\Delta\Gamma}\sinh(\Delta\Gamma_s t/2)}, \tag{18}$$

where

$$\bar{C} \equiv \frac{1-|\bar{\lambda}|^2}{1+|\bar{\lambda}|^2}, \qquad \bar{S} \equiv \frac{2\,\mathrm{Im}\bar{\lambda}}{1+|\bar{\lambda}|^2},$$
$$\bar{\mathcal{A}}_{\Delta\Gamma} \equiv \frac{2\,\mathrm{Re}\bar{\lambda}}{1+|\bar{\lambda}|^2}, \qquad \bar{\lambda} \equiv \frac{p}{q}\frac{A_{\bar{f}}}{\bar{A}_{\bar{f}}}. \tag{19}$$

B. $B_s^0(\bar{B}_s^0)\to D_s^\pm K^\mp$

Consider the decay $B_s^0(\bar{B}_s^0)\to PP$ (P is a pseudoscalar), in which the final state contains a single c quark.[4] Excluding those final states involving η's, there are only two decays in which the B_s^0 and $\bar{B}_s^0$ amplitudes are of comparable size: $B_s^0(\bar{B}_s^0)\to D_s^- K^+$ and $B_s^0(\bar{B}_s^0)\to D_s^+ K^-$. The B_s^0 decays are mediated by color-allowed tree-level transitions $\bar{b}\to\bar{c}u\bar{s}$ and $\bar{b}\to\bar{u}c\bar{s}$. Within the SM, the amplitudes take the form[5] (there is a minus sign associated with the $\bar{u}$ quark)

$$\begin{aligned}
&A(B_s^0\to D_s^- K^+) = T', \qquad A(B_s^0\to D_s^+ K^-) = -\tilde{T}' e^{i\gamma},\\
&A(\bar{B}_s^0\to D_s^- K^+) = \tilde{T}' e^{-i\gamma}, \qquad A(\bar{B}_s^0\to D_s^+ K^-) = -T'.
\end{aligned} \tag{20}$$

We have explicitly written the weak-phase dependence, while the diagrams contain strong phases. The magnitudes of the Cabbibo-Kobayashi-Maskawa (CKM) matrix elements $|V_{cb}^* V_{us}|$ and $|V_{ub}^* V_{cs}|$ have been absorbed into the diagrams T' and $\tilde{T}'$, respectively. (As this is a $\bar{b}\to\bar{s}$ transition, the diagrams are written with primes.)

Using the amplitudes of Eq. (20), one obtains [see Eqs. (16) and (19)]

$$\begin{aligned}
C &= \frac{1-|\lambda|^2}{1+|\lambda|^2}, \qquad S = -\frac{2|\lambda|}{1+|\lambda|^2}\sin(2\beta_s+\gamma-\delta),\\
\mathcal{A}_{\Delta\Gamma} &= \frac{2|\lambda|}{1+|\lambda|^2}\cos(2\beta_s+\gamma-\delta),\\
\bar{C} &= \frac{1-|\lambda|^2}{1+|\lambda|^2}, \qquad \bar{S} = \frac{2|\lambda|}{1+|\lambda|^2}\sin(2\beta_s+\gamma+\delta),\\
\bar{\mathcal{A}}_{\Delta\Gamma} &= \frac{2|\lambda|}{1+|\lambda|^2}\cos(2\beta_s+\gamma+\delta),
\end{aligned} \tag{21}$$

where $|\lambda| = \tilde{T}'/T'$ (defined to be positive) and δ is the strong-phase difference between $\tilde{T}'$ and T'. $|\lambda|$ can be obtained from the measurement of C. Using this, S and $\mathcal{A}_{\Delta\Gamma}$ give $\sin(2\beta_s+\gamma-\delta)$ and $\cos(2\beta_s+\gamma-\delta)$, respectively. Thus, one obtains $2\beta_s+\gamma-\delta$ with no discrete ambiguity. Similarly, $2\beta_s+\gamma+\delta$ can be obtained with no discrete ambiguity from $\bar{S}$ and $\bar{\mathcal{A}}_{\Delta\Gamma}$. These can be combined to give the phases $(2\beta_s+\gamma, \delta)$ with a twofold ambiguity $[(2\beta_s+\gamma, \delta)$ or $(2\beta_s+\gamma+\pi, \delta+\pi)]$. This discrete ambiguity can be removed if one assumes factorization, which predicts δ to be near 0.

In fact, this is not quite correct. As discussed below Eq. (16), in the presence of NP in B_s^0-$\bar{B}_s^0$ mixing there is an additional discrete ambiguity due to the unknown sign of $\Delta\Gamma_s$. Thus, the two-body decays $B_s^0(\bar{B}_s^0)\to D_s^\pm K^\mp$

[4]Much of the discussion in this subsection can be found in Ref. [32], except that here NP in $\Delta\Gamma_s$ is considered.

[5]In Ref. [29], it is shown that NP in the decays $\bar{b}\to\bar{c}u\bar{s}$ and $\bar{b}\to\bar{u}c\bar{s}$ are strongly constrained. Such NP contributions are therefore neglected throughout this paper.

permit the extraction of $2\beta_s + \gamma$ with a fourfold ambiguity (or twofold if factorization is assumed).

Now, the value of γ can be taken from the independent measurements at the B-factories. One then obtains $2\beta_s$ with a fourfold ambiguity. Alternatively, since γ has not been measured in B_s decays, it can be kept with the aim of determining its value independently (this was the original purpose of Ref. [32].) We adopt this latter approach in much of the paper.

We therefore see that this method permits the extraction of $2\beta_s + \gamma$ with a fourfold ambiguity (or twofold if factorization is assumed). It does not resolve the sign ambiguity in $\Delta\Gamma_s$, and says nothing about the possibility of NP affecting Γ_{12}^s. In order to address these remaining points, it is necessary to examine other methods. A first step involves the decays $B_s^0(\bar{B}_s^0) \to D_{CP}^0\phi$, discussed in the next subsection.

C. $B_s^0(\bar{B}_s^0) \to D_{CP}^0\phi$

Another pair of decays to which the method of the previous subsection can be applied is $B_s^0(\bar{B}_s^0) \to D^0\phi$, $\bar{D}^0\phi$. Here the decays are mediated by color-suppressed tree-level transitions. The amplitudes (of comparable size) are given by

$$\begin{aligned} A(B_s^0 \to D^0\phi) &= -C_1^\phi e^{i\gamma}, \quad & A(B_s^0 \to \bar{D}^0\phi) &= C_2^\phi, \\ A(\bar{B}_s^0 \to \bar{D}^0\phi) &= C_1^\phi e^{-i\gamma}, \quad & A(\bar{B}_s^0 \to D^0\phi) &= -C_2^\phi. \end{aligned} \tag{22}$$

By measuring the time dependence of the decays, one can obtain S, $\bar{S}$, $A_{\Delta\Gamma}$, and $\bar{A}_{\Delta\Gamma}$ as given in Eqs. (16) and (19). Using these observables we define

$$\begin{aligned} \sin(2\beta_s + \gamma + \delta_\phi) &= -\frac{1+|\lambda|^2}{2|\lambda|} S \equiv S_D, \\ \sin(2\beta_s + \gamma - \delta_\phi) &= \frac{1+|\lambda|^2}{2|\lambda|} \bar{S} \equiv \bar{S}_D, \\ \cos(2\beta_s + \gamma + \delta_\phi) &= \frac{1+|\lambda|^2}{2|\lambda|} A_{\Delta\Gamma} \equiv A_{\Delta\Gamma}^D, \\ \cos(2\beta_s + \gamma - \delta_\phi) &= \frac{1+|\lambda|^2}{2|\lambda|} \bar{A}_{\Delta\Gamma} \equiv \bar{A}_{\Delta\Gamma}^D, \end{aligned} \tag{23}$$

with $\delta_\phi = \arg(C_1^\phi/C_2^\phi)$. The method of the previous subsection then allows us to obtain $2\beta_s + \gamma$ with a twofold ambiguity (for the moment, we put aside the ambiguity due to the sign of $\Delta\Gamma_s$).

The advantage of these decays is that there is a third decay which is related: $B_s^0(\bar{B}_s^0) \to D_{CP}^0\phi$, where D_{CP}^0 is a CP eigenstate (either CP-odd or CP-even). In our analysis we consider D_{CP}^0 as the CP-even superposition $(D^0 + \bar{D}^0)/\sqrt{2}$. The amplitudes for the decays are then given by

$$\begin{aligned} \sqrt{2}A(B_s^0 \to D_{CP}^0\phi) &= -C_1^\phi e^{i\gamma} + C_2^\phi, \\ \sqrt{2}A(\bar{B}_s^0 \to D_{CP}^0\phi) &= C_1^\phi e^{-i\gamma} - C_2^\phi. \end{aligned} \tag{24}$$

By measuring the time-dependent decay amplitudes of $B_s^0(\bar{B}_s^0) \to D\phi$ ($D = D^0$, $\bar{D}^0$, D_{CP}^0), one can extract the magnitudes $|C_1^\phi|$, $|C_2^\phi|$, $|A_{D_{CP}}| = |A(B_s^0 \to D_{CP}^0\phi)|$ and $|\bar{A}_{D_{CP}}| = |A(\bar{B}_s^0 \to D_{CP}^0\phi)|$ (they are combinations of the overall normalizations and the C parameters [Eq. (16)]).

Using the first equation of Eq. (24), we define

$$\cos(\gamma + \delta_\phi) = \frac{2|A_{D_{CP}}|^2 - |C_1^\phi|^2 - |C_2^\phi|^2}{2|C_1^\phi||C_2^\phi|} \equiv \Sigma^+. \tag{25}$$

Similarly, from the second equation of Eq. (24), we get

$$\cos(\gamma - \delta_\phi) = \frac{2|\bar{A}_{D_{CP}}|^2 - |C_1^\phi|^2 - |C_2^\phi|^2}{2|C_1^\phi||C_2^\phi|} \equiv \Sigma^-. \tag{26}$$

Therefore, in the case of the $B_s^0(\bar{B}_s^0) \to D\phi$ decays, we have two more observables, Σ^+ and Σ^-. Combining Eqs. (23), (25), and (26), it is straightforward to find expressions for $\sin 2\beta_s$, $\cos 2\beta_s$, $\sin(2\beta_s + 2\gamma)$, and $\cos(2\beta_s + 2\gamma)$ in terms of the above observables:

$$\begin{aligned} \sin 2\beta_s &= \frac{S_D^2 - \bar{S}_D^2 + \Sigma^{+2} - \Sigma^{-2}}{2(S_D\Sigma^+ - \bar{S}_D\Sigma^-)}, \\ \sin(2\beta_s + 2\gamma) &= \frac{S_D^2 - \bar{S}_D^2 - \Sigma^{+2} + \Sigma^{-2}}{2(S_D\Sigma^- - \bar{S}_D\Sigma^+)}, \\ \cos 2\beta_s &= \frac{S_D^2 - \bar{S}_D^2 - \Sigma^{+2} + \Sigma^{-2}}{2(\bar{A}_{\Delta\Gamma}^D\Sigma^- - A_{\Delta\Gamma}^D\Sigma^+)}, \\ \cos(2\beta_s + 2\gamma) &= \frac{S_D^2 - \bar{S}_D^2 + \Sigma^{+2} - \Sigma^{-2}}{2(\bar{A}_{\Delta\Gamma}^D\Sigma^+ - A_{\Delta\Gamma}^D\Sigma^-)}, \end{aligned} \tag{27}$$

with

$$S_D^2 - \bar{S}_D^2 = -(A_{\Delta\Gamma}^D)^2 + (\bar{A}_{\Delta\Gamma}^D)^2. \tag{28}$$

Many years ago, $B_s^0(\bar{B}_s^0) \to D\phi$ decays were studied [34], but without the dependence on $\Delta\Gamma_s$. It was found that $\sin 2\beta_s$ and $\sin(2\beta_s + 2\gamma)$ could be obtained, which correspond to determining $2\beta_s$ with a twofold ambiguity and 2γ with a fourfold ambiguity. In the present case, the dependence on $\Delta\Gamma_s$ is included. This allows us to obtain $A_{\Delta\Gamma}^D$ and $\bar{A}_{\Delta\Gamma}^D$, which then permits us to measure $\cos 2\beta_s$ and $\cos(2\beta_s + 2\gamma)$, in addition to $\sin 2\beta_s$ and $\sin(2\beta_s + 2\gamma)$ [Eq. (27)]. These measurements allow an unambiguous determination of $2\beta_s$ and 2γ. We therefore see that a nonzero $\Delta\Gamma_s$ helps quite a bit in determining the weak phases. As has been discussed above, the sign of $\Delta\Gamma_s$ is not known, which implies that $A_{\Delta\Gamma}^D$ and $\bar{A}_{\Delta\Gamma}^D$ also have a sign ambiguity. This means that, in fact, $2\beta_s$ and γ are determined up to a twofold and fourfold[6] ambiguity, respectively. Therefore, once we are able to fix the sign of $\Delta\Gamma_s$,

[6]Using Eq. (27), we can determine $\cos 2\gamma$ without any ambiguity, whereas, due to the unknown sign of $A_{\Delta\Gamma}^D$ or $\bar{A}_{\Delta\Gamma}^D$, $\sin 2\gamma$ can be determined only with a twofold ambiguity. Combining these two results, 2γ can therefore be determined with a twofold ambiguity (or γ with a fourfold ambiguity).

the $B_s^0(\bar{B}_s^0) \to D\phi$ decays might be considered as an alternative mode to probe simultaneously γ and $2\beta_s$.

We therefore see that two-body $\bar{b} \to \bar{c}u\bar{s}/\bar{b} \to \bar{u}c\bar{s}$ decays do not provide sufficient information to measure the CP phases $2\beta_s$ and 2γ in an unambiguous manner. In the next section we show that there are several ways to improve upon the two-body decay methods by using a Dalitz-plot analysis of the corresponding three-body decays.

III. THREE-BODY DECAYS

A. $B_s^0(\bar{B}_s^0) \to f, \bar{f}$

In recent years, it has been shown that one can get useful information from three-body B decays. For instance, time-integrated Dalitz-plot analyses of $B_s^0 \to K\pi\pi$ and $B_s^0 \to \pi K\bar{K}$ decays have been proposed as a probe of γ [38]. And various tests of the SM, as well as the extraction of weak phases, have been examined in the context of $B \to K\pi\pi$, $B \to K\bar{K}K$, $B \to \pi\bar{K}K$, and $B \to \pi\pi\pi$ decays [39].

In the previous section we discussed two-body $\bar{b} \to \bar{c}u\bar{s}/\bar{b} \to \bar{u}c\bar{s}$ decays; in this section we examine the corresponding three-body decays. In $B_s^0(\bar{B}_s^0) \to PPP$ decays which receive a tree contribution, there are 5 final-state $(f, \bar{f})$ pairs: $(D_s^- K^+\pi^0,\ D_s^+ K^-\pi^0)$, $(D_s^- K^0\pi^+,\ D_s^+\bar{K}^0\pi^-)$, $(D^- K^+\bar{K}^0,\ D^+ K^0 K^-)$, $(\bar{D}^0 K^+ K^-,\ D^0 K^+ K^-)$, and $(\bar{D}^0 K^0\bar{K}^0,\ D^0 K^0\bar{K}^0)$. The CKM matrix elements of these decays are the same as in the corresponding two-body decay modes, and will therefore exhibit very similar time-dependent CP asymmetries.

The decay amplitude of $B_s^0(\bar{B}_s^0) \to PPP$ receives several different contributions, both resonant and nonresonant. In the following, we perform a time-dependent Dalitz-plot analysis of the three-body decays. This permits the measurement of each of the contributing amplitudes, as well as their relative phases. As we will see below, the Dalitz-plot analysis reduces the ambiguity in the measurement of γ and $2\beta_s$ compared to the corresponding two-body decays. We also show how this analysis resolves the sign ambiguity in $\Delta\Gamma_s$.

B. Dalitz-plot analysis

Here we review certain aspects of the Dalitz-plot analysis. We focus on the general three-body decay $B \to P_1P_2P_3$. We define the Dalitz-plot variables

$$s_{12} \equiv (p_1 + p_2)^2, \qquad s_{13} \equiv (p_1 + p_3)^2, \qquad s_{23} \equiv (p_2 + p_3)^2, \tag{29}$$

which are related by the conservation law

$$s_{12} + s_{13} + s_{23} = m_B^2 + m_1^2 + m_2^2 + m_3^2. \tag{30}$$

This shows that there are only two independent variables (below, we use s_{12} and s_{13}).

$B \to P_1P_2P_3$ can take place either via intermediate resonances or nonresonant contributions. A widely used approximation in the parametrization of the decay amplitude is the isobar model. In this model, the individual terms are interpreted as complex production amplitudes for two-body resonances, and one also includes a term describing the nonresonant component. The amplitude is then written as

$$\mathcal{A}(s_{12}, s_{13}) = \sum_j a_j F_j(s_{12}, s_{13}), \tag{31}$$

where the sum is over all decay modes (resonant and nonresonant). Here, the a_j are the complex coefficients describing the magnitudes and phases of different decay channels, while the $F_j(s_{12}, s_{13})$ contain the strong dynamics. The CP-conjugate amplitude is given by

$$\bar{\mathcal{A}}(s_{12}, s_{13}) = \sum_j \bar{a}_j \bar{F}_j(s_{13}, s_{12}), \tag{32}$$

where $\bar{F}_j(s_{13}, s_{12}) = F_j(s_{12}, s_{13})$.

Now, in the experimental analysis, the $F_j(s_{12}, s_{13})$ take different (known) forms for the various contributions. By performing a maximum likelihood fit over the entire Dalitz plot, one can obtain the magnitudes and relative phases of the a_j, and similarly for the $\bar{a}_j$. Thus, the full decay amplitudes can be obtained.

C. $B_s^0(\bar{B}_s^0) \to D_s^\pm K^\mp \pi^0$

In this subsection we focus specifically on the decay $B_s^0(\bar{B}_s^0) \to D_s^\pm K^\mp \pi^0$, and use a modification of the method elaborated in Ref. [40]. The Dalitz-plot variables are

$$\begin{aligned} s^+ &\equiv (p_{D_s} + p_\pi)^2, \\ s^- &\equiv (p_K + p_\pi)^2, \\ s^0 &\equiv (p_{D_s} + p_K)^2. \end{aligned} \tag{33}$$

The amplitudes are written as

$$\begin{aligned} \mathcal{A}(s^+, s^-) &= \sum_j a_j F_j(s^+, s^-), \\ \bar{\mathcal{A}}(s^+, s^-) &= \sum_j \bar{a}_j \bar{F}_j(s^-, s^+). \end{aligned} \tag{34}$$

The time-dependent decay rates for the oscillating $B_s^0(t)$ and $\bar{B}_s^0(t)$ mesons, decaying to the same final state f, are given by

$$\begin{aligned} \Gamma(B_s^0(t) \to f) \sim \frac{1}{2} e^{-\Gamma_s t}[&A_{\rm ch}(s^+, s^-)\cosh(\Delta\Gamma_s t/2) \\ &- A_{\rm sh}(s^+, s^-)\sinh(\Delta\Gamma_s t/2) + A_c(s^+, s^-) \\ &\times \cos(\Delta m_s t) - A_s(s^+, s^-)\sin(\Delta m_s t)], \\ \Gamma(\bar{B}_s^0(t) \to f) \sim \frac{1}{2} e^{-\Gamma_s t}[&A_{\rm ch}(s^-, s^+)\cosh(\Delta\Gamma_s t/2) \\ &- A_{\rm sh}(s^-, s^+)\sinh(\Delta\Gamma_s t/2) - A_c(s^-, s^+) \\ &\times \cos(\Delta m_s t) + A_s(s^-, s^+)\sin(\Delta m_s t)]. \end{aligned} \tag{35}$$

Here

$$\begin{aligned} A_{\rm ch}(s^+,s^-) &= |\mathcal{A}(s^+,s^-)|^2 + |\bar{\mathcal{A}}(s^+,s^-)|^2, \\ A_c(s^+,s^-) &= |\mathcal{A}(s^+,s^-)|^2 - |\bar{\mathcal{A}}(s^+,s^-)|^2, \\ A_{\rm sh}(s^+,s^-) &= 2\,{\rm Re}(e^{-2i\beta_s}\bar{\mathcal{A}}(s^+,s^-)\mathcal{A}^*(s^+,s^-)), \\ A_s(s^+,s^-) &= 2\,{\rm Im}(e^{-2i\beta_s}\bar{\mathcal{A}}(s^+,s^-)\mathcal{A}^*(s^+,s^-)), \end{aligned} \quad (36)$$

where $\mathcal{A}(s^+,s^-)$ and $\bar{\mathcal{A}}(s^+,s^-)$ are given in Eq. (34).

Now, there are a number of resonances which contribute to $B_s^0(\bar{B}_s^0) \to D_s^\pm K^\mp \pi^0$. For illustrative purposes, we consider just two of them: $D_s^\pm K^{*\mp}(892)$ and $D_s^{*\pm} K^\mp$. The decays $K^{*\pm} \to K^\pm \pi^0$ and $D_s^{*\pm} \to D_s^\pm \pi^0$ have already been observed: $B(K^{*\pm}(892) \to K^\pm \pi^0) = 50\%$, $B(D_s^{*\pm} \to D_s^\pm \pi^0) = (5.8 \pm 0.7)\%$ [41]. $B_s^0(\bar{B}_s^0) \to D_s^\pm K^{*\mp}$ and $B_s^0(\bar{B}_s^0) \to D_s^{*\pm} K^\mp$ are the $B_s^0(\bar{B}_s^0) \to VP$ equivalents of the decay discussed in Sec. II, $B_s^0(\bar{B}_s^0) \to D_s^\pm K^\mp$. The additional ingredient here is that we also consider the decay products of the V, so that we have the full decay chain $B_s^0(\bar{B}_s^0) \to VP \to PPP$.

For these two resonances, we have

$$\begin{aligned} \mathcal{A}_{K^*}(B_s^0 \to D_s^- K^{*+} \to D_s^- K^+ \pi^0) &= a_1^{K^*} e^{i\gamma} F_{K^*}, \\ \bar{\mathcal{A}}_{K^*}(\bar{B}_s^0 \to D_s^- K^{*+} \to D_s^- K^+ \pi^0) &= a_2^{K^*} F_{K^*}, \\ \mathcal{A}_{D_s^*}(B_s^0 \to D_s^{*-} K^+ \to D_s^- K^+ \pi^0) &= a_1^{D_s^*} e^{i\gamma} F_{D_s^*}, \\ \bar{\mathcal{A}}_{D_s^*}(\bar{B}_s^0 \to D_s^{*-} K^+ \to D_s^- K^+ \pi^0) &= a_2^{D_s^*} F_{D_s^*}. \end{aligned} \quad (37)$$

Including both resonances, the amplitudes of $B_s^0(\bar{B}_s^0) \to D_s^- K^+ \pi^0$ are

$$\begin{aligned} \mathcal{A} &= e^{i\gamma}(a_1^{K^*} F_{K^*} + a_1^{D_s^*} F_{D_s^*}), \\ \bar{\mathcal{A}} &= (a_2^{K^*} F_{K^*} + a_2^{D_s^*} F_{D_s^*}). \end{aligned} \quad (38)$$

With these amplitudes, $A_{\rm ch}$, A_c, $A_{\rm sh}$ and A_s [Eq. (36)] take the forms

$$\begin{aligned} A_{\rm ch}^{D_s K\pi} &= (|a_1^{K^*}|^2 + |a_2^{K^*}|^2)|F_{K^*}|^2 + (|a_1^{D_s^*}|^2 + |a_2^{D_s^*}|^2)|F_{D_s^*}|^2 + 2\,{\rm Re}((a_1^{K^*}F_{K^*})^*(a_1^{D_s^*}F_{D_s^*})) + 2\,{\rm Re}((a_2^{K^*}F_{K^*})^*(a_2^{D_s^*}F_{D_s^*})), \\ A_c^{D_s K\pi} &= (|a_1^{K^*}|^2 - |a_2^{K^*}|^2)|F_{K^*}|^2 + (|a_1^{D_s^*}|^2 - |a_2^{D_s^*}|^2)|F_{D_s^*}|^2 + 2\,{\rm Re}((a_1^{K^*}F_{K^*})^*(a_1^{D_s^*}F_{D_s^*})) - 2\,{\rm Re}((a_2^{K^*}F_{K^*})^*(a_2^{D_s^*}F_{D_s^*})), \\ A_{\rm sh}^{D_s K\pi} &= \cos(2\beta_s + \gamma + \delta_{K^*})|a_1^{K^*}||a_2^{K^*}||F_{K^*}|^2 + \cos(2\beta_s + \gamma + \delta_{D_s^*})|a_1^{D_s^*}||a_2^{D_s^*}||F_{D_s^*}|^2 + \cos(2\beta_s + \gamma + \delta)|a_1^{K^*}||a_2^{D_s^*}|{\rm Re} \\ &\quad \times (F_{K^*}^* F_{D_s^*}) - \sin(2\beta_s + \gamma + \delta)|a_1^{K^*}||a_2^{D_s^*}|{\rm Im}(F_{K^*}^* F_{D_s^*}) + {\rm Re}[e^{-i(2\beta_s+\gamma)}(a_1^{D_s^*}F_{D_s^*})^*(a_2^{K^*}F_{K^*})], \\ A_s^{D_s K\pi} &= -\sin(2\beta_s + \gamma + \delta_{K^*})|a_1^{K^*}||a_2^{K^*}||F_{K^*}|^2 - \sin(2\beta_s + \gamma + \delta_{D_s^*})|a_1^{D_s^*}||a_2^{D_s^*}||F_{D_s^*}|^2 - \sin(2\beta_s + \gamma + \delta)|a_1^{K^*}||a_2^{D_s^*}|{\rm Re} \\ &\quad \times (F_{K^*}^* F_{D_s^*}) + \cos(2\beta_s + \gamma + \delta)|a_1^{K^*}||a_2^{D_s^*}|{\rm Im}(F_{K^*}^* F_{D_s^*}) + {\rm Im}[e^{-i(2\beta_s+\gamma)}(a_1^{D_s^*}F_{D_s^*})^*(a_2^{K^*}F_{K^*})], \end{aligned} \quad (39)$$

with $\delta_{K^*} = -\arg((a_1^{K^*})^* a_2^{K^*})$, $\delta_{D_s^*} = -\arg((a_1^{D_s^*})^* a_2^{D_s^*})$, and $\delta = -\arg((a_1^{K^*})^* a_2^{D_s^*})$.

Above, in the discussion of the time-independent Dalitz-plot analysis, we noted that the magnitudes and relative phases of the a_j can be obtained from a maximum likelihood fit over the entire Dalitz plot, given assumed forms for the F_j's. The same holds true for the time-dependent Dalitz-plot analysis—the magnitudes and relative phases of the contributing resonances, i.e. $a_1^{K^*}$, $a_2^{K^*}$, $a_1^{D_s^*}$, and $a_2^{D_s^*}$, can all be obtained. Indeed, such an analysis has been performed by the *BABAR* and Belle collaborations for the decay $B_d^0(t) \to K_S \pi^+ \pi^-$ [42]. In particular, all the coefficients that multiply the $F_i^* F_j$ [Eq. (39)] bilinears can be obtained from a maximum likelihood fit to the corresponding Dalitz-plot PDFs.

This permits the extraction of the weak phases. For example, we can extract $2\beta_s + \gamma + \delta$ without any ambiguity from the third and fourth terms of $A_s^{D_s K\pi}$. In a similar manner, the time-dependent Dalitz-plot analysis of $B_s^0(\bar{B}_s^0) \to D_s^+ K^- \pi^0$ allows the extraction of the phase $2\beta_s + \gamma - \delta$. The combination of these two results yields $2\beta_s + \gamma$ and δ with a twofold ambiguity. And if factorization is imposed, the discrete ambiguity is removed entirely (only the solution with $\delta \simeq 0$ is kept). The key point here is that we do not use $A_{\rm sh}^{D_s K\pi}$ at all. As a consequence, there is no discrete ambiguity due to the sign ambiguity of $\Delta\Gamma_s$ [see the discussion following Eq. (36)]. This is to be contrasted with two-body decays. There $2\beta_s + \gamma$ can also be obtained with a twofold ambiguity. However, because $\mathcal{A}_{\Delta\Gamma}$ and $\bar{\mathcal{A}}_{\Delta\Gamma}$ are used [Eq. (21)], there is an additional discrete ambiguity due to the unknown sign of $\Delta\Gamma_s$.

We note that one can extract different trigonometric functions such as $|\sin(2\beta_s + \gamma + \delta)|$, $|\cos(2\beta_s + \gamma + \delta)|$, $|\cos(2\beta_s + \gamma + \delta_i)|$, etc., from $A_{\rm sh}^{D_s K\pi}$ [Eq. (39)]. Because of the sign ambiguity of $\Delta\Gamma_s$, which can be viewed as the sign ambiguity in $A_{\rm sh}^{D_s K\pi}$, the sign of these trigonometric functions cannot be determined. Depending on the sign of $\Delta\Gamma_s$, their sign could be positive or negative. Therefore, we can determine the sign of $\Delta\Gamma_s$ if we are able to fix the sign of these trigonometric functions. Now, the functions $\sin(2\beta_s + \gamma + \delta)$ and $\cos(2\beta_s + \gamma + \delta)$ can be extracted without ambiguity from $A_s^{D_s K\pi}$, which fixes the sign of $\Delta\Gamma_s$ and hence removes the discrete ambiguity in $A_{\rm sh}^{D_s K\pi}$. Note that this can be done without measuring ϕ_s. This method can therefore be used to determine the sign of $\cos\phi_s$.

In the above, we have concentrated on the decay $B_s^0(\bar{B}_s^0) \to D_s^\pm K^\mp \pi^0$. However, any of the decay pairs discussed in Sec. III can be used. All that is necessary is that there be at least two resonances contributing to the decay. We therefore see that, by using such three-body decays, one can obtain $2\beta_s + \gamma$ (with a twofold ambiguity if factorization is not assumed), as well as resolve the sign ambiguity in $\Delta\Gamma_s$. The resolution of the $\Delta\Gamma_s$ sign ambiguity determines the sign of $\cos\phi_s$. The precise knowledge of γ from other measurements allows one to obtain $2\beta_s$ with a twofold ambiguity (since $2\beta_s + \gamma$ can itself be extracted with a twofold ambiguity), which can be compared with the measurement of $2\beta_s$ from $B_s^0 \to J/\psi\phi$ [Eq. (8)].

Still, it is preferable to have a method that allows the direct determination of $2\beta_s$ and γ individually. This can be done by measuring the decay $B_s^0(\bar{B}_s^0) \to D_{CP}^0 K\bar{K}$, which is discussed in the next subsection.

D. $B_s^0(\bar{B}_s^0) \to D_{CP}^0 K\bar{K}$

In Sec. II C we discussed the two-body decays $B_s^0(\bar{B}_s^0) \to D\phi$ ($D = D^0, \bar{D}^0, D_{CP}^0$), and showed that it is possible to extract $2\beta_s$ and 2γ with a twofold ambiguity due to the unknown sign of $\Delta\Gamma_s$. The time-dependent Dalitz-plot analysis of $B_s^0(\bar{B}_s^0) \to D^0 K\bar{K}$, $\bar{D}^0 K\bar{K}$ is similar to that of the previous subsection, with the intermediate resonances $\phi(1020)$ or $f_0(1500)$ decaying to the final state $K\bar{K}$. In this subsection we consider in addition the related three-body decays $B_s^0(\bar{B}_s^0) \to D_{CP}^0 K\bar{K}$, with $D_{CP}^0 \equiv 1/\sqrt{2}(D^0 \pm \bar{D}^0)$.

$B_s^0(\bar{B}_s^0) \to D_{CP}^0 K\bar{K}$ receives contributions from several different intermediate resonances: $\phi(1020)$, $\phi(1680)$, $f_0(1500)$, $f_0(1710)$, $D_{s_j}^{*\pm}$, etc., which follow the decay chains $B_s^0(\bar{B}_s^0) \to D_{CP}^0\phi \to D_{CP}^0 K^+K^-$, $B_s^0(\bar{B}_s^0) \to D_{CP}^0 f_0 \to D_{CP}^0 K^+K^-$, $B_s^0(\bar{B}_s^0) \to D_{s_j}^{*\pm} K^\mp \to D_{CP}^0 K^\pm K^\mp$. To simplify our analysis, we consider only the $\phi(1020)$ and $f_0(1500)$ resonances. The amplitude with an intermediate ϕ resonance can be written as

$$\begin{aligned}
&\sqrt{2}A_\phi(B_s^0 \to D_{CP}^0\phi(\to K^+K^-)) \\
&\quad = A(B_s^0 \to D^0K^+K^-) + A(B_s^0 \to \bar{D}^0K^+K^-) \\
&\sqrt{2}\bar{A}_\phi(\bar{B}_s^0 \to D_{CP}^0\phi(\to K^+K^-)) \\
&\quad = A(\bar{B}_s^0 \to D^0K^+K^-) + A(\bar{B}_s^0 \to \bar{D}^0K^+K^-),
\end{aligned} \tag{40}$$

where

$$\begin{aligned}
A(B_s^0 \to D^0\phi \to D^0K^+K^-) &= -C_1^\phi e^{i\gamma} F_\phi, \\
A(\bar{B}_s^0 \to D^0\phi \to D^0K^+K^-) &= -C_2^\phi F_\phi, \\
A(B_s^0 \to \bar{D}^0\phi \to D^0K^+K^-) &= C_2^\phi F_\phi, \\
A(\bar{B}_s^0 \to \bar{D}^0\phi \to D^0K^+K^-) &= C_1^\phi e^{-i\gamma} F_\phi.
\end{aligned} \tag{41}$$

The amplitude with an intermediate f_0 resonance is given by a similar expression, with the replacement $\phi \to f_0$. Including the contributions from these two resonances, the total amplitude can be written as

$$\begin{aligned}
\mathcal{A}(B_s^0 \to D_{CP}^0K^+K^-) &= A_\phi(B_s^0 \to D_{CP}^0K^+K^-) \\
&\quad + A_{f_0}(B_s^0 \to D_{CP}^0K^+K^-), \\
\bar{\mathcal{A}}(\bar{B}_s^0 \to D_{CP}^0K^+K^-) &= \bar{A}_\phi(\bar{B}_s^0 \to D_{CP}^0K^+K^-) \\
&\quad + \bar{A}_{f_0}(\bar{B}_s^0 \to D_{CP}^0K^+K^-).
\end{aligned} \tag{42}$$

With these, A_c^{DKK}, $A_{\rm ch}^{DKK}$ and A_s^{DKK} can be computed similarly to Eq. (39). First, we have

$$\begin{aligned}
A_c^{DKK} &= \sum_{i=\phi,f_0} [(|A_i|^2 - |\bar{A}_i|^2) + 2\,{\rm Re}(A_\phi A_{f_0}^* - \bar{A}_\phi \bar{A}_{f_0}^*)], \\
A_{\rm ch}^{DKK} &= \sum_{i=\phi,f_0} [(|A_i|^2 + |\bar{A}_i|^2) + 2\,{\rm Re}(A_\phi A_{f_0}^* + \bar{A}_\phi \bar{A}_{f_0}^*)],
\end{aligned} \tag{43}$$

in which

$$\begin{aligned}
{\rm Re}(A_\phi A_{f_0}^* - \bar{A}_\phi \bar{A}_{f_0}^*) &= |C_2^{f_0}||C_2^\phi| \sin\gamma[r_\phi\{\sin\delta_\phi\,{\rm Re}(F_\phi F_{f_0}^*) + \cos\delta_\phi\,{\rm Im}(F_\phi F_{f_0}^*)\} \\
&\quad + r_{f_0}\{\sin\delta_{f_0}\,{\rm Re}(F_\phi F_{f_0}^*) - \cos\delta_{f_0}\,{\rm Im}(F_\phi F_{f_0}^*)\}], \\
{\rm Re}(A_\phi A_{f_0}^* + \bar{A}_\phi \bar{A}_{f_0}^*) &= |C_2^{f_0}||C_2^\phi|[{\rm Re}(F_\phi F_{f_0}^*) - r_\phi \cos\gamma\{\cos\delta_\phi\,{\rm Re}(F_\phi F_{f_0}^*) - \sin\delta_\phi\,{\rm Im}(F_\phi F_{f_0}^*)\} \\
&\quad - r_{f_0}\cos\gamma\{\cos\delta_{f_0}\,{\rm Re}(F_\phi F_{f_0}^*) + \sin\delta_{f_0}\,{\rm Im}(F_\phi F_{f_0}^*)\} \\
&\quad + r_\phi r_{f_0}\{\cos(\delta_\phi - \delta_{f_0}){\rm Re}(F_\phi F_{f_0}^*) - \sin(\delta_\phi - \delta_{f_0}){\rm Im}(F_\phi F_{f_0}^*)\}],
\end{aligned} \tag{44}$$

where $r_i = |C_1^i|/|C_2^i|$ and $\delta_i = \arg(C_1^i/C_2^i)$ ($i = \phi$, f_0). Using Eq. (44) in Eq. (35), a maximum likelihood fit to the Dalitz-plot PDFs allows one to extract

$$\begin{aligned}
|C_2^{f_0}||C_2^\phi| r_i \cos\gamma \cos\delta_i &\equiv \sigma_c^i, \\
|C_2^{f_0}||C_2^\phi| r_i \sin\gamma \cos\delta_i &\equiv \sigma_s^i.
\end{aligned} \tag{45}$$

This gives the ratio

$$\frac{\sigma_s^i}{\sigma_c^i} = \tan\gamma. \tag{46}$$

Since the hadronic uncertainties cancel in the ratio, it yields a theoretically clean determination of the angle γ with a twofold ambiguity, even without the knowledge of the strong phases.

Second, we have

$$A_s^{DKK} = \mathrm{Im}[e^{-2i\beta_s}\mathcal{A}^*\bar{\mathcal{A}}] = \mathrm{Im}[e^{-2i\beta_s}(A_\phi^*\bar{A}_\phi + A_\phi^*\bar{A}_{f_0} + A_{f_0}^*\bar{A}_\phi + A_{f_0}^*\bar{A}_{f_0})]. \quad (47)$$

The first and fourth terms of A_s^{DKK} are given by

$$\mathrm{Im}[e^{-2i\beta_s}A_i^*\bar{A}_i] = \frac{1}{2}\mathrm{Im}[e^{-2i\beta_s}|C_2^i|^2|F_i|^2(1 + r_i^2 e^{-2i\gamma} + r_i(e^{i(\delta_\phi-\gamma)} + e^{-i(\delta_\phi+\gamma)}))], \quad (48)$$

which allows the extraction of $\sin 2\beta_s$, $\sin(2\beta_s + 2\gamma)$, $\sin(2\beta_s + \gamma - \delta_{\phi/f_0})$ and $\sin(2\beta_s + \gamma + \delta_{\phi/f_0})$. The ϕ-f_0 interference terms are given by

$$\begin{aligned}\mathrm{Im}[e^{-2i\beta_s}(A_\phi^*\bar{A}_{f_0} + A_{f_0}^*\bar{A}_\phi)] = \tfrac{1}{2}\mathrm{Im}[e^{-2i\beta_s}|C_2^\phi||C_2^{f_0}|\{&-(F_\phi^*F_{f_0} + F_{f_0}^*F_\phi) + r_\phi(e^{-i(\gamma+\delta_\phi)}F_\phi^*F_{f_0} + e^{-i(\gamma-\delta_\phi)}F_{f_0}^*F_\phi)\\ &+ r_{f_0}(e^{-i(\gamma-\delta_{f_0})}F_\phi^*F_{f_0} + e^{-i(\gamma+\delta_{f_0})}F_{f_0}^*F_\phi) - r_\phi r_{f_0}(e^{-i(2\gamma+\delta_\phi-\delta_{f_0})}F_\phi^*F_{f_0}\\ &+ e^{-i(2\gamma-\delta_\phi+\delta_{f_0})}F_{f_0}^*F_\phi)\}]. \end{aligned} \quad (49)$$

This yields

$$\begin{aligned}\mathrm{Im}[e^{-2i\beta_s}(A_\phi^*\bar{A}_{f_0} + A_{f_0}^*\bar{A}_\phi)] = \tfrac{1}{2}|C_2^\phi||C_2^{f_0}|[&\sin 2\beta_s\{\mathrm{Re}(F_\phi^*F_{f_0}) + \mathrm{Re}(F_{f_0}^*F_\phi)\} - r_\phi\{\sin(2\beta_s + \gamma + \delta_\phi)\mathrm{Re}(F_\phi^*F_{f_0})\\ &- \cos(2\beta_s + \gamma + \delta_\phi)\mathrm{Im}(F_\phi^*F_{f_0}) + \sin(2\beta_s + \gamma - \delta_\phi)\mathrm{Re}(F_{f_0}^*F_\phi)\\ &- \cos(2\beta_s + \gamma - \delta_\phi)\mathrm{Im}(F_{f_0}^*F_\phi)\} - r_{f_0}\{\sin(2\beta_s + \gamma + \delta_{f_0})\mathrm{Re}(F_{f_0}^*F_\phi)\\ &- \cos(2\beta_s + \gamma + \delta_{f_0})\mathrm{Im}(F_{f_0}^*F_\phi) + \sin(2\beta_s + \gamma - \delta_{f_0})\mathrm{Re}(F_\phi^*F_{f_0})\\ &- \cos(2\beta_s + \gamma - \delta_{f_0})\mathrm{Im}(F_\phi^*F_{f_0})\} + r_\phi r_{f_0}\{\sin(\delta_\phi - \delta_{f_0} + 2\beta_s + 2\gamma)\mathrm{Re}(F_\phi^*F_{f_0})\\ &- \cos(\delta_\phi - \delta_{f_0} + 2\beta_s + 2\gamma)\mathrm{Im}(F_\phi^*F_{f_0}) + \sin(\delta_{f_0} - \delta_\phi + 2\beta_s + 2\gamma)\mathrm{Re}(F_{f_0}^*F_\phi)\\ &- \cos(\delta_{f_0} - \delta_\phi + 2\beta_s + 2\gamma)\mathrm{Im}(F_{f_0}^*F_\phi)\}]. \end{aligned} \quad (50)$$

From the above, we can extract

$$\begin{aligned} -r_i\sin(2\beta_s + \gamma \pm \delta_i) &\equiv S_{DKK}^{i\pm},\\ r_i\cos(2\beta_s + \gamma \pm \delta_i) &\equiv C_{DKK}^{i\pm},\\ \sin 2\beta_s &\equiv S_{DKK},\\ r_{ij}\sin(2\beta_s + 2\gamma \pm \delta_{ij}) &\equiv S_{DKK}^{ij\pm},\\ -r_{ij}\cos(2\beta_s + 2\gamma \pm \delta_{ij}) &\equiv C_{DKK}^{ij\pm}, \end{aligned} \quad (51)$$

where $r_{ij} \equiv r_i r_j$ and the corresponding $\delta_{ij} \equiv \delta_i - \delta_j$ (i, $j = \phi$, f_0). It is straightforward to find expressions for $\tan(2\beta_s + \gamma)$ and $\tan(2\beta_s + 2\gamma)$ in terms of the above observables:

$$\begin{aligned} \tan(2\beta_s + \gamma) &= -\frac{S_{DKK}^{i+} + S_{DKK}^{i-}}{C_{DKK}^{i+} + C_{DKK}^{i-}},\\ \tan(2\beta_s + 2\gamma) &= -\frac{S_{DKK}^{ij+} + S_{DKK}^{ij-}}{C_{DKK}^{ij+} + C_{DKK}^{ij-}}. \end{aligned} \quad (52)$$

With these, one can obtain the expression for $\tan\gamma$ in terms of the extracted observables:

$$\tan\gamma = \frac{\tan(2\beta_s + 2\gamma) - \tan(2\beta_s + \gamma)}{1 - \tan(2\beta_s + \gamma)\tan(2\beta_s + 2\gamma)}. \quad (53)$$

This way of getting $\tan\gamma$ uses A_s^{DKK} [see also Eq. (46)].

Combining Eqs. (52) and (53), we obtain

$$\tan 2\beta_s = \frac{\tan(2\beta_s + \gamma) - \tan\gamma}{1 - \tan(2\beta_s + \gamma)\tan\gamma}. \quad (54)$$

This determines $2\beta_s$ with the twofold ambiguity $2\beta_s \to \pi + 2\beta_s$. However, as we note in Eq. (51), we can extract $\sin 2\beta_s$ without any sign ambiguity. This determines $2\beta_s$ with the twofold ambiguity $2\beta_s \to \pi - 2\beta_s$, which is different from that obtained in $\tan 2\beta_s$. Therefore, the combined measurements of $\tan 2\beta_s$ and $\sin 2\beta_s$ allow us to extract $2\beta_s$ without any ambiguity. The sign ambiguity in $\Delta\Gamma_s$ can be resolved in a similar way to that discussed in Sec. III C.

Above, we discussed the interference between the two resonance states $\phi(1020)$ and $f_0(1500)$. However, the analysis would hold equally for the interference between any two resonances decaying to the same final state. Similar information can also be obtained from the time-dependent Dalitz-plot analysis of $B_s^0(\bar{B}_s^0) \to D_{CP}^0 K^0 \bar{K}^0$.

IV. EXTRACTION OF $\Delta\Gamma_s$

In the previous section(s) we examined methods for extracting the CP phase $2\beta_s$ using various two- and three-body decays. The idea is that if a nonzero value of $2\beta_s$ is found, this will be clear evidence of NP in B_s^0-$\bar{B}_s^0$ mixing. In addition, if such a value of $2\beta_s$ is obtained, we will want to know its exact value in order to ascertain which different models of NP could generate such mixing. To this end, the best method will be that for which the discrete ambiguity in $2\beta_s$ is minimized. However, there is one question which has not yet been addressed: if NP in the mixing is found, does it contribute to Γ_{12}^s in addition to M_{12}^s?

In this paper we have focused on methods for measuring $\Delta\Gamma_s$ using three-body decays. In practice, this will be carried out as follows. For definitiveness, consider the decays $B_s^0(\bar{B}_s^0) \to D_{CP}^0 K^+ K^-$. Generalizing Eq. (35) to $B_s^0(\bar{B}_s^0) \to D_{CP}^0 K^+ K^-$, the time-dependent untagged differential decay distribution is given by

$$\begin{aligned}\Gamma_{\text{untagged}}(D_{CP}^0 K^+ K^-, t) &= \frac{d^2\Gamma(B_s^0 \to D_{CP}^0 K^+ K^-)}{ds^+ ds^-} \\ &\quad + \frac{d^2\Gamma(\bar{B}_s^0 \to D_{CP}^0 K^+ K^-)}{ds^+ ds^-} \\ &\equiv e^{-\Gamma_s t}[A_{\text{ch}}^{DKK} \cosh(\Delta\Gamma_s t/2) \\ &\quad + A_{\text{sh}}^{DKK} \sinh(\Delta\Gamma_s t/2)], \qquad (55)\end{aligned}$$

where A_{ch}^{DKK} and A_{sh}^{DKK} are defined in Eq. (36). Neglecting terms of order $(\Delta\Gamma_s/\Gamma_s^2)^2$ and higher, the time-integrated differential untagged decay distribution is given by

$$\begin{aligned}&\int_0^\infty dt \Gamma_{\text{untagged}}(D_{CP}^0 K^+ K^-, t) \\ &\quad = \frac{1}{4\Gamma_s}\left[A_{\text{ch}}^{DKK} + 2A_{\text{sh}}^{DKK}\frac{\Delta\Gamma_s}{\Gamma_s}\right]. \qquad (56)\end{aligned}$$

For a single resonance, say ϕ,

$$\begin{aligned}A_{\text{ch}}^{DKK} &= A_\phi^2 + \bar{A}_\phi^2, \\ A_{\text{sh}}^{DKK} &= \text{Re}[e^{-2i\beta_s}|C_2^\phi|^2|F_\phi|^2\{1 + r_\phi^2 e^{-2i\gamma} \\ &\quad + r_\phi(e^{-i(\gamma+\delta_\phi)} + e^{-i(\gamma-\delta_\phi)})\}]. \qquad (57)\end{aligned}$$

As discussed in the previous section, A_{ch}^{DKK} is fully known from the CP-averaged branching fraction of the intermediate resonance ϕ. Once we have enough precision, a fit to the distribution given by Eq. (55) or (56) allows one to obtain $\Delta\Gamma_s$ and the various coefficients of $|F_\phi|^2$ (which yields $2\beta_s$). Such a fit will not allow the determination of the sign of $\Delta\Gamma_s$ or $\cos\phi_s$, but Eq. (10) can still be used to obtain ϕ_s (with a twofold ambiguity) and $|\Gamma_{12}^s|$.

However, the above fit, though possible, is made difficult due to the requirement of having to simultaneously extract $\Delta\Gamma_s$ and the components of A_{ch}^{DKK}. Given this, we would rather propose an alternative procedure. Referring again to Eq. (35), the time-dependent tagged differential decay distribution is given by

$$\begin{aligned}\Gamma_{\text{tagged}}(D_{CP}^0 K^+ K^-, t) &= \frac{d^2\Gamma(B_s^0 \to D_{CP}^0 K^+ K^-)}{ds^+ ds^-} \\ &\quad - \frac{d^2\Gamma(\bar{B}_s^0 \to D_{CP}^0 K^+ K^-)}{ds^+ ds^-} \\ &\equiv e^{-\Gamma_s t}[A_c^{DKK} \cos(\Delta m_s t/2) \\ &\quad - A_s^{DKK} \sin(\Delta m_s t/2)], \qquad (58)\end{aligned}$$

where A_c^{DKK} and A_s^{DKK} are defined in Eqs. (36), (43), and (50). From a fit to the above distribution, one can extract only the coefficients of different bilinears in A_s^{DKK} and A_c^{DKK}, since Δm_s is known. Thus, this fit straightforwardly gives information regarding A_c^{DKK} and A_s^{DKK}. As discussed in the previous section, from A_s^{DKK} alone we can extract $2\beta_s$ and $\gamma + 2\beta_s \pm \delta_\phi$ without any ambiguity, and γ with the ambiguity $[\gamma, \pi + \gamma]$. This permits the reconstruction of A_{sh}^{DKK} [Eq. (57)]. That is, all the coefficients of $|F_\phi|^2$ in A_{sh}^{DKK} can be obtained from a fit to Eq. (58). With this knowledge, there is only one unknown in Eq. (55) or (56)—$\Delta\Gamma_s$—and this can be determined by a fit. This may be a somewhat simpler procedure. Once we are able to measure $\Delta\Gamma_s$, then, as discussed in the introduction, along with a_{sl}^s and ΔM_s, Eq. (10) can be used to obtain the CP phase ϕ_s and $|\Gamma_{12}^s|$. This can then reveal the presence of NP in the mixing through a comparison with Eq. (7).

The above analysis is also applicable to the decays $B_s^0(\bar{B}_s^0) \to D_s^\pm K^\mp \pi^0$. However, as was discussed in Sec. III C, for such decays all the trigonometric functions in $A_{\text{sh}}^{D_s K\pi}$ are not fully known—the only known functions are those appearing as the coefficients of $\text{Im}(F_i F_j^*)$ or $\text{Re}(F_i F_j^*)$ $(i \neq j)$. Therefore, for such decays we can use Eq. (55) to fit $\Delta\Gamma_s$, but we need at least two interfering resonances, and only the terms proportional to $\text{Im}(F_i F_j^*)$ or $\text{Re}(F_i F_j^*)$ are useful [see $A_{\text{ch}}^{D_s K\pi}$ in Eq. (39)].

V. CONCLUSIONS

It is well known that the weak phase of B_s^0-$\bar{B}_s^0$ mixing is very small in the SM: $2\beta_s \simeq 0$. If this quantity is measured to be significantly different from zero, this is a smoking-gun signal of NP. However, in general we would like more information from such a measurement. For instance, in order to distinguish among potential NP models, it is important to have an unambiguous determination of $2\beta_s$. Similarly, although the width difference $\Delta\Gamma_s$ between the two B_s mass eigenstates is positive in the SM, it can take either sign in the presence of NP. Ideally, a method probing B_s^0-$\bar{B}_s^0$ mixing which relies on a nonzero $\Delta\Gamma_s$ should be able to remove its sign ambiguity. To date, $2\beta_s$ has been extracted from the measurement of the indirect CP asymmetry in $B_s^0 \to J/\psi\phi$ by the CDF, DØ, and LHCb Collaborations. However, the possibility of NP in $\bar{b} \to \bar{s}c\bar{c}$ decays cannot be ruled out, and it is hard to estimate the size of the penguin pollution in such decays. It is therefore important to have an independent measurement of $2\beta_s$ from processes in which NP effects in the decay can be neglected, and which are not polluted by incalculable hadronic contributions. Finally, although it is usually assumed that NP contributes only to M_{12}^s, it has been shown that NP contributions to Γ_{12}^s can also be important. In order to explore this possibility, it is necessary to measure the CP phase ϕ_s and $|\Gamma_{12}^s|$.

In this paper, we examine a variety of methods of measuring B_s^0-$\bar{B}_s^0$ mixing with an eye to addressing the above issues. We look at penguin-free two- and three-body B_s decays with $\bar{b} \to \bar{c}u\bar{s}$ and $\bar{b} \to \bar{u}c\bar{s}$ transitions, concentrating on those final states which are accessible to both B_s^0 and $\bar{B}_s^0$ mesons (so that there is indirect CP violation). The time-dependent decay rates include both $\Delta m_s t$ and $\Delta\Gamma_s t$ terms.

We begin with a review of $B_s^0(\bar{B}_s^0) \to D_s^\pm K^\mp$ decays. Considering sizeable $\Delta\Gamma_s$, we find that this method allows the extraction of $2\beta_s + \gamma$ with a fourfold ambiguity. We then turn to $B_s^0(\bar{B}_s^0) \to D_{CP}^0\phi$ decays, where D_{CP}^0 is a CP eigenstate. Here we find that $2\beta_s$ and 2γ can each be determined up to a twofold ambiguity. Here, the ambiguity is due to the unknown sign of $\Delta\Gamma_s$. Therefore, once we are able to resolve the sign ambiguity in $\Delta\Gamma_s$ by some other means, the $B_s^0(\bar{B}_s^0) \to D_{CP}^0\phi$ decays are useful to measure $2\beta_s$ and 2γ without any ambiguity.

For unambiguous measurements of $2\beta_s$ and γ, it is necessary to turn to Dalitz-plot analyses of three-body decays. We begin with $B_s^0(\bar{B}_s^0) \to D_s^\pm K^\mp \pi^0$. We find that it is possible to obtain $2\beta_s + \gamma$ with a twofold ambiguity, and to remove the sign ambiguity in $\Delta\Gamma_s$ (for this, it is not necessary to determine ϕ_s). The most promising method involves the decays $B_s^0(\bar{B}_s^0) \to D_{CP}^0 K\bar{K}$, in which all issues can be resolved. We find that $2\beta_s$ can be obtained without any ambiguity, and at the same we can remove the sign ambiguity in $\Delta\Gamma_s$. In addition, γ can be determined up to a twofold ambiguity.

Finally, all such decays allow the extraction of $\Delta\Gamma_s$ directly from a fit to the time-dependent untagged differential decay rate distribution. Given the measurements of ΔM_s, the semileptonic asymmetry $a_{\rm sl}^s$, and $\Delta\Gamma_s$, the CP phase ϕ_s and $|\Gamma_{12}^s|$ can be obtained, which can then reveal the presence of NP in the mixing. In the case of three-body decays the coefficients of $\sinh[\Delta\Gamma_s t/2]$ and $\cosh[\Delta\Gamma_s t/2]$ can be found, either fully or partially, from a fit to the time-dependent tagged differential decay rate distribution. (Of the several three-body decays that we discuss, the decays $B_s^0(\bar{B}_s^0) \to D_{CP}^0 K\bar{K}$ are the most promising, since in such decays these coefficients can be fully reconstructed from this fit.) Therefore, in three-body decays the only unknown in the untagged rate distribution is $\Delta\Gamma_s$. This makes the fit considerably simpler.

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Note added: recently, the LHCb Collaboration measured the sign of $\Delta\Gamma_s$ to be positive, and was therefore able to measure $2\beta_s^{\psi\phi}$ unambiguously [43]. In light of this, the method for resolving the sign ambiguity in $\Delta\Gamma_s$ described in this paper can be considered as an independent cross-check.

[1] S. Nandi and A. Kundu, arXiv:hep-ph/0407061; S. Mishima and T. Yoshikawa, Phys. Rev. D **70**, 094024 (2004); C. S. Kim, S. Oh, and C. Yu, Phys. Rev. D **72**, 074005 (2005).

[2] In the latest update of the πK puzzle, it was seen that, although NP was hinted at in $B \to \pi K$ decays, it could be argued that the SM can explain the data, see S. Baek, C. W. Chiang, and D. London, Phys. Lett. B **675**, 59 (2009).

[3] H. Y. Cheng, C. K. Chua, and A. Soni, Phys. Rev. D **72**, 094003 (2005); G. Buchalla, G. Hiller, Y. Nir, and G. Raz, J. High Energy Phys. 09 (2005) 074; E. Lunghi and A. Soni, J. High Energy Phys. 08 (2009) 051.

[4] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **91**, 171802 (2003); K. F. Chen *et al.* (Belle Collaboration), Phys. Rev. Lett. **91**, 201801 (2003).

[5] A. Ishikawa *et al.* (Belle Collaboration), Phys. Rev. Lett. **96**, 251801 (2006); J. T. Wei *et al.* (BELLE Collaboration), Phys. Rev. Lett. **103**, 171801 (2009).

[6] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. D **73**, 092001 (2006); **79**, 031102 (2009).

[7] I. Dunietz, R. Fleischer, and U. Nierste, Phys. Rev. D **63**, 114015 (2001).

[8] A. Lenz, Nucl. Phys. B, Proc. Suppl. **177–178**, 81 (2008) (Note that this paper assumes negligible NP contributions to Γ_{12}^s).

[9] V. M. Abazov *et al.* (DØ Collaboration), Phys. Rev. Lett. **97**, 021802 (2006); A. Abulencia *et al.* (CDF-Run II Collaboration), Phys. Rev. Lett. **97**, 062003 (2006).

[10] A. Lenz and U. Nierste, arXiv:1102.4274.

[11] T. Aaltonen *et al.* (CDF Collaboration), Phys. Rev. Lett. **100**, 161802 (2008).

[12] V. M. Abazov *et al.* (DØ Collaboration), Phys. Rev. Lett. **101**, 241801 (2008).

[13] S. Nandi and J. P. Saha, Phys. Rev. D **74**, 095007 (2006); A. Kundu and S. Nandi, Phys. Rev. D **78**, 015009 (2008); G. Bhattacharyya, K. B. Chatterjee, and S. Nandi, Phys. Rev. D **78**, 095005 (2008).

[14] V. Barger, L. Everett, J. Jiang, P. Langacker, T. Liu, and C. Wagner, Phys. Rev. D **80**, 055008 (2009);

[15] Some aspects of the 2HDM are discussed in A. S. Joshipura and B. P. Kodrani, Phys. Rev. D **81**, 035013 (2010); See also A. Datta and P. J. O'Donnell, Phys. Rev. D **72**, 113002 (2005); A. Datta, Phys. Rev. D **74**, 014022 (2006).
[16] A. Datta and S. Khalil, Phys. Rev. D **80**, 075006 (2009).
[17] M. Blanke, A. J. Buras, S. Recksiegel and C. Tarantino, arXiv:0805.4393.
[18] A. Soni, A. K. Alok, A. Giri, R. Mohanta, and S. Nandi, Phys. Lett. B **683**, 302 (2010)arXiv:0807.1971; M. Bobrowski, A. Lenz, J. Riedl, and J. Rohrwild, Phys. Rev. D **79**, 113006 (2009); A. Soni, A. K. Alok, A. Giri, R. Mohanta, and S. Nandi, Phys. Rev. D **82**, 033009 (2010); A. J. Buras, B. Duling, T. Feldmann, T. Heidsieck, C. Promberger, and S. Recksiegel, J. High Energy Phys. 09 (2010) 106;
[19] J. K. Parry and H. h. Zhang, Nucl. Phys. **B802**, 63 (2008); B. Dutta and Y. Mimura, Phys. Rev. D **78**, 071702 (2008); Phys. Lett. B **677**, 164 (2009); J. h. Park and M. Yamaguchi, Phys. Lett. B **670**, 356 (2009); P. Ko and J. h. Park, Phys. Rev. D **80**, 035019 (2009); N. Kifune, J. Kubo, and A. Lenz, Phys. Rev. D **77**, 076010 (2008); K. Kawashima, J. Kubo, and A. Lenz, Phys. Lett. B **681**, 60 (2009); F. J. Botella, G. C. Branco, and M. Nebot, Phys. Rev. D **79**, 096009 (2009).
[20] C. W. Chiang, A. Datta, M. Duraisamy, D. London, M. Nagashima, and A. Szynkman, J. High Energy Phys. 04 (2010) 031.
[21] T. Aaltonen *et al.* (CDF Collaboration), Phys. Rev. D **85**, 072002 (2012).
[22] V. M. Abazov *et al.* (DØ Collaboration), Phys. Rev. D **85**, 032006 (2012).
[23] R. Aaij *et al.* (LHCb Collaboration), Phys. Rev. Lett. **108**, 101803 (2012).
[24] Y. Xie, P. Clarke, G. Cowan, and F. Muheim, J. High Energy Phys. 09 (2009) 074.
[25] V. M. Abazov *et al.* (DØ Collaboration), Phys. Rev. D **82**, 032001 (2010).
[26] V. M. Abazov *et al.* (DØ Collaboration), Phys. Rev. Lett. **105**, 081801 (2010).
[27] B. A. Dobrescu, P. J. Fox, and A. Martin, Phys. Rev. Lett. **105**, 041801 (2010); C. H. Chen, C. Q. Geng, and W. Wang, J. High Energy Phys. 11 (2010) 089; P. Ko and J. h. Park, Phys. Rev. D **82**, 117701 (2010); A. Lenz, U. Nierste, J. Charles, S. Descotes-Genon, A. Jantsch, C. Kaufhold, H. Lacker, S. Monteil, V. Niess, and S. T'Jampens, Phys. Rev. D **83**, 036004 (2011); S. Nandi and A. Soni, Phys. Rev. D **83**, 114510 (2011).
[28] A. Dighe, A. Kundu, and S. Nandi, Phys. Rev. D **82**, 031502 (2010).
[29] C. W. Bauer and N. D. Dunn, Phys. Lett. B **696**, 362 (2011).
[30] A. Dighe, A. Kundu, and S. Nandi, Phys. Rev. D **76**, 054005 (2007).
[31] N. G. Deshpande, X. G. He, and G. Valencia, Phys. Rev. D **82**, 056013 (2010); A. K. Alok, S. Baek, and D. London, J. High Energy Phys. 07 (2011) 111.A. Datta, M. Duraisamy, and S. Khalil, Phys. Rev. D **83**, 094501 (2011).
[32] R. Fleischer, Nucl. Phys. **B671**, 459 (2003).
[33] S. Nandi and U. Nierste, Phys. Rev. D **77**, 054010 (2008); S. Nandi, Nucl. Phys. B, Proc. Suppl. **209**, 164 (2010).
[34] M. Gronau and D. London, Phys. Lett. B **253**, 483 (1991).
[35] R. Fleischer, Phys. Lett. B **562**, 234 (2003); Nucl. Phys. **B659**, 321 (2003).
[36] D. Asner *et al.* (Heavy Flavor Averaging Group Collaboration), arXiv:1010.1589.
[37] I. Dunietz, Phys. Rev. D **52**, 3048 (1995).
[38] M. Ciuchini, M. Pierini, and L. Silvestrini, Phys. Lett. B **645**, 201 (2007); M. Gronau, D. Pirjol, A. Soni, and J. Zupan, Phys. Rev. D **75**, 014002 (2007).
[39] N. R.-L. Lorier, M. Imbeault, and D. London, Phys. Rev. D **84**, 034040 (2011); M. Imbeault, N. R.-L. Lorier, and D. London, Phys. Rev. D **84**, 034041 (2011).
[40] F. Polci, M. H. Schune, and A. Stocchi, arXiv:hep-ph/0605129.
[41] K. Nakamura *et al.* (Particle Data Group), J. Phys. G **37**, 075021 (2010).
[42] B. Aubert *et al.* (*BABAR* Collaboration), arXiv:0708.2097; J. Dalseno *et al.* (Belle Collaboration), Phys. Rev. D **79**, 072004 (2009).
[43] R. Aaij *et al.* (LHCb Collaboration), arXiv:1202.4717.