PHYSICAL REVIEW D **85,** 114008 (2012)

Strong *CP* violation and chiral symmetry breaking in hot and dense quark matter

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(Received 20 December 2011; published 4 June 2012)

We investigate chiral symmetry breaking and strong *CP* violation effects in the phase diagram of strongly interacting matter. We demonstrate the effect of strong *CP* violating terms on the phase structure at finite temperature and densities in a three-flavor Nambu-Jona-Lasinio model including the Kobayashi-Maskawa-t' Hooft determinant term. This is investigated using an explicit structure for the ground state in terms of quark-antiquark condensates for both the scalar and the pseudoscalar channels. *CP* restoring transition with temperature at zero baryon density is found to be a second order transition at $\theta = \pi$ while the same at finite chemical potential and small temperature turns out to be a first order transition. Within the model, the tricritical point is found to be $(\mu_c, T_c) \simeq (273, 94)$ MeV at $\theta = \pi$ for such a transition.

DOI: 10.1103/PhysRevD.85.114008 PACS numbers: 12.38.Mh, 11.30.Er, 11.30.Rd, 12.39.−x

I. INTRODUCTION

Strong interaction is known to respect space and time reflection symmetry to a very high degree. However, this is not a direct consequence of laws of quantum chromodynamics (QCD), which, in principle permit a parity violating term or the so-called θ term given as

$$\mathcal{L}_\theta = \frac{\theta}{64\pi^2} g^2 F^a_{\mu\nu} \tilde{F}^{a\mu\nu}. \tag{1}$$

In the above, $F^a_{\mu\nu}$ is the gluon field strength with $\tilde{F}^{\mu\nu}$ being its dual. This term, while being consistent with Lorentz invariance and gauge invariance, violates charge conjugation and parity unless $\theta = 0 \bmod \pi$. However, *CP* symmetry conserving nature of QCD has been established by precise experiments that set limit on the intrinsic electric dipole moment of a neutron. The current experimental limit on this leads to a limit on the coefficient of the *CP* violating term of the QCD Lagrangian density as $\theta < 0.7 \times 10^{-11}$ [1]. This smallness of the *CP* violation term or its complete absence is not understood completely though a possible explanation is given in terms of spontaneous breaking of a new symmetry: the Peccei-Quinn symmetry [2] that could give rise to axions. For zero temperature and zero density, spontaneous parity violation does not arise for $\theta = 0$ by the well-known Vafa-Witten theorem [3]. On the other hand for $\theta = \pi$ there could be spontaneous *CP* violation by the so-called Dashen phenomena [4]. Because of the nonperturbative nature of this θ term in QCD, the problem has been studied extensively in low energy effective theories like chiral perturbation theory [5], linear sigma model [6], and the Nambu-Jona-Lasinio (NJL) model and its different extensions [7–10].

Even if *CP* is not violated for QCD vacuum, it is possible that it can be violated for QCD matter at finite temperature or density. It has been proposed that hot matter produced in heavy ion collision experiments can give rise to domains of metastable states that violate *CP* [11]. Experimental signatures for the existence of local *CP* violation has been based on charge separation of hadronic matter due to the strong magnetic field produced in noncentral heavy ion collision experiments by a mechanism called chiral magnetic effect [12]. This mechanism may explain the charge separation in the recent STAR (solenoidal tracker At RHIC) results [13]. On the other hand, for central collision, it has been recently suggested that local parity violation leading to a formation of pseudoscalar condensates can have possible consequences regarding excess dilepton production in such collisions [14].

In the present work we focus our attention on how chiral transition is affected when there is a *CP* violating term in the Lagrangian. For this purpose, we adopt the three-flavor NJL model as an effective theory for chiral symmetry breaking in strong interaction [15,16]. The *CP* violating parameter θ is included in the Kobayashi-Maskawa-t' Hooft (KMT) determinant term. As we shall see, a nonzero theta term leads to the possibility of a uniform *P*-violating quark-antiquark condensate in the pseudoscalar channel. In this context we note that the two-flavor scenario for spontaneous *CP* violation for $\theta = \pi$ has been studied in this model [8]. This has been further extended to study the restoration of *CP* at finite temperature [9]. The effect of the theta vacuum on the deconfinement and chiral transition has also been analyzed within a two flavor NJL model with Polyakov loop [10].

We organize the present work as follows. In the following section we discuss the three-flavor NJL model with a

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CP violating term. We consider a variational ground state with quark-antiquark pairs that is related to chiral symmetry breaking. The ansatz functions are to be determined through minimization of the thermodynamic potential. The ansatz is general enough to include both scalar as well as pseudoscalar condensates. As we shall see the pseudoscalar condensates develop for nonzero values of θ in the KMT determinant term. In Sec. III we discuss the resulting phase diagram at finite temperature as well as finite density for different values of the *CP* violating parameter in the Lagrangian. In Sec. IV we summarize our results and give a possible outlook.

II. NJL MODEL WITH *CP* VIOLATION AND AN ANSATZ FOR THE GROUND STATE

To describe the chiral phase structure of strong interaction including the *CP* violating effects, we use the three-flavor NJL model along with the flavor mixing KMT determinant interaction term. The Lagrangian density is given by

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + G\sum_{A=0}^{8}[(\bar{\psi}\lambda^A\psi)^2 + (\bar{\psi}i\gamma^5\lambda^A\psi)^2] - K[e^{i\theta}\det\{\bar{\psi}(1+\gamma^5)\psi\} + e^{-i\theta}\det\{\bar{\psi}(1-\gamma^5)\psi\}], \tag{2}$$

where $\psi^{i,a}$ denotes a quark field with color a ($a = r, g, b$), and flavor i ($i = u, d, s$), indices. The matrix of current quark masses is given by $\hat{m} = \mathrm{diag}_f(m_u, m_d, m_s)$ in the flavor space. In the present investigation, we shall assume isospin symmetry with $m_u = m_d$. In Eq. (2), λ^A, $A = 1, \cdots 8$ denote the Gell-Mann matrices acting in the flavor space and $\lambda^0 = \sqrt{\frac{2}{3}}\mathbb{1}_f$, where $\mathbb{1}_f$ is the unit matrix in the flavor space. The four point interaction term $\sim G$ is symmetric under $SU(3)_V \times SU(3)_A \times U(1)_V \times U(1)_A$. The determinant term $\sim K$, which generates a six point interaction for the case of three flavors, breaks $U(1)_A$ symmetry. The effect of the topological term of Eq. (1) is simulated by the determinant term of Eq. (2) in the quark sector. This can be easily seen by taking the divergence of the flavor singlet axial current

$$\partial_\mu J_5^\mu = 2i\bar{\psi}m\gamma^5\psi + 2iN_f K(e^{i\theta}\det\bar{\psi}(1+\gamma^5)\psi - \mathrm{H.c.}), \tag{3}$$

where $J_5^\mu = \bar{\psi}\gamma^\mu\gamma^5\psi$ summed over all the flavors. This equation may be compared with the usual anomaly equation written in terms of the topological term for the gluon field arising from Eq. (1)

$$\partial_\mu J_5^\mu = 2i\bar{\psi}m\gamma^5\psi + 2N_f\frac{\theta}{32\pi^2}g^2F^a_{\mu\nu}\tilde{F}^{a\mu\nu}. \tag{4}$$

Thus the effect of gluon operator $\frac{\theta}{32\pi^2}g^2F^a_{\mu\nu}\tilde{F}^{a\mu\nu}$ is simulated by the imaginary part of the determinant term in the quark sector. Such a term can lead to formation of condensates in the pseudoscalar channel as we investigate in the following.

We shall next consider an ansatz for the ground state with quark-antiquark condensates that includes both the scalar and the*CP* violating pseudoscalar channel. To make notations clear, we first write down the field operator expansion for the quark fields as [17,18]

$$\psi(\mathbf{x}, t=0) \equiv \frac{1}{(2\pi)^{3/2}}\int \tilde{\psi}(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}d\mathbf{k} = \frac{1}{(2\pi)^{3/2}}\int[U_0(\mathbf{k})q^0(\mathbf{k}) + V_0(-\mathbf{k})\tilde{q}^0(-\mathbf{k})]e^{i\mathbf{k}\cdot\mathbf{x}}d\mathbf{k}, \tag{5}$$

where $U_0(\mathbf{k})$ and $V_0(-\mathbf{k})$ are the four component spinors that can be explicitly written as

$$U_0(\mathbf{k}) = \begin{pmatrix} \cos\left(\frac{\chi^0}{2}\right) \\ \sigma\cdot\hat{k}\sin\left(\frac{\chi^0}{2}\right)\end{pmatrix} \quad \text{and} \quad V_0(-\mathbf{k}) = \begin{pmatrix} -\sigma\cdot\hat{k}\sin\left(\frac{\chi^0}{2}\right) \\ \cos\left(\frac{\chi^0}{2}\right)\end{pmatrix}. \tag{6}$$

The superscript 0 indicates that the operators $q^0 = q^0_{Ir}u_{Ir}$ and $\tilde{q}^0 = \tilde{q}^0_{Ir}v_{Ir}$ are the two component operators for the quark annihilation and antiquark creation corresponding to the perturbative or the chiral vacuum $|0\rangle$. Here we have suppressed the color and flavor indices of the quark field operators. The function $\chi^0(\mathbf{k})$ in the spinors in Eq. (6) are given as $\cot\chi^0_i = m_i/|\mathbf{k}|$, for free massive fermion fields, i being the flavor index. For massless fields $\chi^0(|\mathbf{k}|) = \pi/2$.

We next consider an ansatz of the ground state at zero temperature as

$$|\Omega\rangle = U_q|0\rangle, \tag{7}$$

where, $U_q = U_{qI}U_{qII}$ is a unitary operator. U_{qI} and U_{qII} are unitary operators described in terms of quark-antiquark creation and annihilation operators. Explicitly they are given as

$$U_{qI} = \exp\left(\int d\mathbf{k}\, q^0_{Ir}(\mathbf{k})^\dagger(\boldsymbol{\sigma}\cdot\hat{\mathbf{k}})_{rs}f(k)\tilde{q}^0_{Is}(-\mathbf{k}) - \mathrm{H.c}\right), \tag{8}$$

and

$$U_{qII} = \exp\left(\int d\mathbf{k}\, q_{Ir}(\mathbf{k})^\dagger r g(k)\tilde{q}_{I-r}(-\mathbf{k}) - \mathrm{H.c}\right), \tag{9}$$

where $f(k)$ and $g(k)$ are the ansatz functions that are to be determined later from the extremization of the thermodynamic potential.

Finally, to include the effect of temperature and baryon density, we use the techniques of thermofield dynamics

(TFD) that is quite convenient while dealing with operators and states [19,20]. Here, the statistical average of an operator is given as an expectation value over a "thermal vacuum." The methodology of TFD involves the doubling of the Hilbert space [19]. Explicitly, the thermal vacuum is constructed from the ground state at zero temperature and density through a thermal Bogoliubov transformation given as

$$|\Omega(\beta, \mu)\rangle = \mathcal{U}_F|\Omega\rangle = e^{\mathcal{B}(\beta,\mu)^\dagger - \mathcal{B}(\beta,\mu)}|\Omega\rangle \tag{10}$$

with

$$\mathcal{B}^\dagger(\beta, \mu) = \int [\theta_-(\mathbf{k}, \beta, \mu) q'(\mathbf{k})^\dagger \underline{q}'(-\mathbf{k})^\dagger + \theta_+(\mathbf{k}, \beta, \mu)\tilde{q}'(\mathbf{k})\tilde{\underline{q}}'(-\mathbf{k})] d\mathbf{k}. \tag{11}$$

In Eq. (11) the ansatz functions $\theta_\pm(\mathbf{k}, \beta, \mu)$ are related to the quark and antiquark thermal distributions, respectively, as can be seen from the extremization of the thermodynamic potential, and the underlined operators are the operators in the extended Hilbert space associated with thermal doubling in the TFD method.

In the following section, we shall compute the thermodynamic potential that involves calculating the thermal average of different operators given by the expectation values of the corresponding operator with respect to the state given in Eq. (10). This can be evaluated directly by realizing that the state $|\Omega(\beta, \mu)\rangle$ is obtained from the state $|0\rangle$ by successive Bogoliubov transformations. We have, for example,

$$\langle\Omega(\beta\mu)|\psi_\alpha^{\dagger ia}(\mathbf{x})\psi_\beta^{jb}(\mathbf{y})|\rangle\Omega(\beta\mu)\rangle = \delta^{ij}\delta^{ab}\int \frac{d\mathbf{k}}{(2\pi)^3} e^{-i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}\Lambda^i(\mathbf{k}, \beta, \mu)_{\beta\alpha} \tag{12}$$

where,

$$\Lambda^i(\mathbf{k}, \beta, \mu) = \frac{1}{2}[(\cos^2\theta_+^i + \sin^2\theta_-^i) + (\sin^2\theta_-^i - \sin^2\theta_+^i) \times (\gamma^0 \cos\phi^i \cos 2g^i + \alpha\cdot\hat{\mathbf{k}}\sin\phi^i\cos 2g^i - i\gamma^0\gamma^5\sin 2g^i] \tag{13}$$

where, we have introduced a new function $\phi^i(\mathbf{k}) = \chi_0 + 2f^i(\mathbf{k})$ in terms of the condensate function $f^i(\mathbf{k})$ of Eq. (8). From Eq. (12), it is easy to calculate the scalar and pseudoscalar condensates. In terms of the ansatz functions $\phi^i(\mathbf{k})$ and $g^i(\mathbf{k})$, the scalar and pseudoscalar condensates for the ith flavor can be written as

$$\langle\bar{\psi}\psi\rangle_i = -\frac{2N_c}{(2\pi)^3}\int d\mathbf{k}\cos\phi^i\cos 2g^i(1 - n_-^i - n_+^i) \equiv -I_s^i, \tag{14}$$

$$\langle\bar{\psi}\gamma_5\psi\rangle_i = -i\frac{2N_c}{(2\pi)^3}\int d\mathbf{k}\sin 2g^i(1 - n_-^i - n_+^i) \equiv -iI_p^i, \tag{15}$$

respectively, where $n_\mp^i = \sin^2\theta_\mp^i$. Thus a nonvanishing I_s^i will imply a chiral symmetry breaking phase, while a nonvanishing I_p^i or equivalently $g^i(\mathbf{k})$ will indicate a CP violating phase. The condensate functions $\phi^i(\mathbf{k})$, $g^i(\mathbf{k})$ as well as the thermal functions $\theta_\mp^i(\mathbf{k}, \beta, \mu)$ shall be determined by extremization of the thermodynamic potential with respect to the corresponding functions. We shall carry out these extremizations in the following section.

III. EVALUATION OF THERMODYNAMIC POTENTIAL AND GAP EQUATIONS

As was already mentioned, we shall be considering the chiral phase structure in the presence of the CP violating terms within the framework of the NJL model given by Eq. (2). The energy density is given by the expectation value of the Hamiltonian corresponding to the Lagrangian given in Eq. (2) with respect to the thermal ansatz of Eq. (10). The energy density can be written as

$$\epsilon = T + V = T + V_S + V_D, \tag{16}$$

where T is the expectation value of the kinetic term in Eq. (2) and can be calculated as

$$T = \langle\psi^\dagger(-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + \beta m)\psi\rangle = -\frac{2N_c}{(2\pi)^3}\sum_i\int d\mathbf{k} \times (m^i\cos\phi^i + |\mathbf{k}|\sin\phi^i)\cos 2g^i(1 - n_-^i - n_+^i) \tag{17}$$

by using Eq. (12). V_S is the contribution from the four point interaction term in Eq. (2) to the energy density and is given as

$$V_S = -G\left\langle\sum_{A=0}^{8}[(\bar{\psi}\lambda^A\psi)^2 + (\bar{\psi}i\gamma_5\lambda^A\psi)^2]\right\rangle = -2G\sum_i[I_s^{i2} + I_p^{i2}]. \tag{18}$$

Finally, V_D denotes the contribution from the determinant interaction term in Eq. (2) and is given as

$$V_D = K\langle e^{i\theta}\det\{\bar{\psi}(1+\gamma_5)\psi\} + e^{-i\theta}\det\{\bar{\psi}(1-\gamma_5)\psi\}\rangle = 2K\left[\cos\theta\left\{-\prod_{i=1}^{3}I_s^i + \frac{1}{2}|\epsilon_{ijk}|I_s^iI_p^jI_p^k\right\} + \sin\theta\left\{-\prod_{i=1}^{3}I_p^i + \frac{1}{2}|\epsilon_{ijk}|I_s^iI_s^jI_p^k\right\}\right]. \tag{19}$$

The thermodynamic potential is given as

$$\Omega = \epsilon - \mu\rho - \frac{S}{\beta}, \tag{20}$$

where μ is the quark chemical potential corresponding to the quark number density ρ given as

$$\rho = \sum_{i=u,d,s} \langle \psi^\dagger \psi \rangle_i = \frac{2N_c}{(2\pi)^3} \sum_{i=u,d,s} \int d\mathbf{k}(1 - \sin^2\theta_+^i + \sin^2\theta_-^i), \quad (21)$$

and S is the entropy density given as

$$S = \frac{2N_c}{(2\pi)^3} \sum_{i=u,d,s} \int d\mathbf{k}(\cos^2\theta_-^i \ln\cos^2\theta_-^i + \sin^2\theta_-^i \ln\sin^2\theta_-^i + \cos^2\theta_+^i \ln\cos^2\theta_+^i + \sin^2\theta_+^i \ln\sin^2\theta_+). \quad (22)$$

Thus the thermodynamic potential given by Eq. (20) is known in terms of the ansatz functions of Eq. (10). Extremizing the thermodynamic potential with respect to $\phi^i(\mathbf{k})$ and $g^i(\mathbf{k})$, respectively, leads to

$$\tan\phi^i = \frac{|\mathbf{k}|}{M_s^i} \quad \text{and} \quad \tan 2g^i = \frac{M_p^i}{\sqrt{M_s^{i2} + |\mathbf{k}|^2}}, \quad (23)$$

where M_s^i and M_p^i are, respectively, the contributions to the constituent quark mass (for ith flavor) from the scalar and pseudoscalar condensates, respectively, and they are given by

$$M_s^i = m_i + 4GI_s^i + K|\epsilon_{ijk}|\{\cos\theta(I_s^j I_s^k - I_p^j I_p^k) - \sin\theta(I_s^j I_p^k + I_p^j I_s^k)\}, \quad (24)$$

$$M_p^i = 4GI_p^i - K|\epsilon_{ijk}|\{\cos\theta(I_s^j I_p^k + I_p^j I_s^k) - \sin\theta(I_p^j I_p^k - I_s^j I_s^k)\}. \quad (25)$$

Finally extremizing the thermodynamic potential with respect to the ansatz functions $\theta_\mp^i$ leads to

$$\sin^2\theta_\pm^i = \frac{1}{\exp(\omega^i \mp \mu^i) + 1}, \quad (26)$$

where $\omega^i(\mathbf{k}) = \sqrt{\mathbf{k}^2 + (M_s^{i2} + M_p^{i2})}$. Thus it is observed that the constituent quark masses get contribution from both the scalar and the pseudoscalar condensates.

Substituting the extremized solution for the condensate functions $\tan\phi^i$ and $\tan 2g^i$ in Eqs. (14) and (15), we have the self-consistent equations for the scalar and pseudoscalar condensates,

$$I_s^i \equiv -\langle \bar\psi \psi \rangle_i = \frac{2N_c}{(2\pi)^3} \int d\mathbf{k}(1 - n_-^i - n_+^i)\frac{M_s^i}{\omega^i}, \quad (27)$$

$$I_p^i \equiv i\langle \bar\psi \gamma_5 \psi \rangle_i = \frac{2N_c}{(2\pi)^3} \int d\mathbf{k}(1 - n_-^i - n_+^i)\frac{M_p^i}{\omega^i}. \quad (28)$$

Thus with the scalar and the pseudoscalar condensates given as above, Eqs. (24) and (25) are actually coupled self-consistent equations for M_s^i and M_p^i.

Substituting the extremized solutions for the condensate functions and using the gap equations Eqs. (24) and (25) in Eq. (20), the thermodynamic potential becomes

$$\Omega = -\frac{2N_c}{(2\pi)^3} \sum_i \int d\mathbf{k}(\omega^i - |\mathbf{k}|) + 2G_s \sum_i [I_s^{i2} - I_p^{i2}] + \sum_i M_p^i I_p^i + 4K\cos\theta \prod_{i=1}^3 I_s^i - 2K\sin\theta\Big[\prod_{i=1}^3 I_p^1 + \frac{1}{2}|\epsilon_{ijk}| I_s^i I_s^j I_p^k\Big] - \frac{2N_c}{\beta(2\pi)^3} \sum_i \int d\boldsymbol{k}[\ln\{1 + e^{-\beta(\omega^i - \mu^i)}\} + \ln\{1 + e^{-\beta(\omega^i + \mu^i)}\}]. \quad (29)$$

In the above we have subtracted the perturbative vacuum energy density contribution. Let us note that the effective potential has been calculated using an explicit ansatz for the condensate and not evaluating the effective potential at a mean field level after performing a chiral transformation for the quarks so as to remove it from the determinant term as it has been computed in Refs. [8,10].

Equation (29) for the thermodynamic potential and the gap equations for the scalar and pseudoscalar masses, i.e. Eqs. (24) and (25), shall be the focus of our numerical analysis that we discuss in the following section.

IV. RESULTS AND DISCUSSIONS

For numerical calculations, we have taken the values of the parameters of the NJL model as follows. The coupling constant G_s has the dimension of $[\text{Mass}]^{-2}$ while the six-fermion coupling K has a dimension $[\text{Mass}]^{-5}$. To regularize the divergent integrals we use a sharp cutoff Λ in three-momentum space. Thus we have five parameters in total, namely, the current quark masses for the nonstrange and strange quarks, m_q and m_s, the two couplings G_s, K, and the three-momentum cutoff Λ. We have chosen here $\Lambda = 0.6023$ GeV, $G_s\Lambda^2 = 1.835$, $K\Lambda^5 = 12.36$, $m_q = 5.5$ MeV, and $m_s = 0.1407$ GeV as was used in Ref. [16]. After choosing $m_q = 5.5$ MeV, the remaining four parameters are fixed by fitting to the pion decay constant and the masses of pion, kaon, and η'. With this set of parameters the mass of η is underestimated by about 6% and the constituent masses of the light quarks turn out to be $M^{u,d} = 0.368$ GeV for u-d quarks, while the same for strange quark turns out as $M^s = 0.549$ GeV, at zero temperature and zero density.

For given temperature and chemical potential, we first solve the coupled self-consistent gap equations (24) and (25) with the parameters of the model as above. Since we have assumed isospin symmetry and have $m_u = m_d$, these are actually four coupled equations: two for the scalar condensates related to the two masses $M_s^u = M_s^d$, M_s^s and two equations for the pseudoscalar condensate related to the corresponding mass parameters $M_p^u = M_p^d$, M_p^s. The solutions to these equations are then substituted in Eq. (29) and checked whether they correspond to the minimum of the thermodynamic potential. If there are more solutions to the gap equation, the one with the minimum thermodynamic potential is chosen.

Let us first discuss the ground state structure at zero temperature and zero density. In Fig. [1(a)] we show the θ dependence of contributions to the mass of up quark from the scalar as well as the pseudoscalar condensates. As is clearly seen as θ increases, the condensates in the two channels behave in a complimentary manner. While the magnitude of scalar condensates decreases with θ (till $\theta = \pi$), the magnitude of the pseudoscalar condensate increases so that the total constituent quark mass $M = \sqrt{M_s{}^2 + M_p{}^2}$ remains almost the same. Spontaneous *CP* violation is clearly seen for $\theta = \pi$ with two degenerate solutions for M_p^u differing by a sign. In Fig. [1(b)] we show the effective potential as calculated above as a function of θ. The effective potential is normalized with respect to the same at $\theta = 0$. The minimum of the potential is at $\theta = 0$, which is consistent with the Vafa-Witten theorem and has a cusp at $\theta = \pi$ that has also been observed in the two-flavor NJL model [8].

Next we consider the effects of nonzero values of density and temperature. In Fig. 2, we show the variation of masses of the quarks with chemical potential at zero temperature. In Fig. [2(a)] we show the variation of masses for the case $\theta = 0$. In this case the pseudoscalar condensates vanish and the contribution to the masses of the quarks are from the scalar condensates only. The (approximate) first order chiral transition takes place at $\mu \sim 361$ MeV for u and d quarks with their masses decreasing discontinuously to about $M^{u,d} \sim 52$ MeV from their vacuum value of $M^{u,d} = 368$ MeV. Because of the flavor mixing KMT term, this decrease is reflected also in the drop of the strange quark mass to $M^s = 464$ MeV from its vacuum value of $M^s = 549$ MeV. This result is similar to the results obtained in the context of color superconductivity in the NJL model with a determinant term [21] and in the context of chiral symmetry breaking in a similar model [22]. Similarly in Fig. [2(b)] we show the variation of masses of up and strange quarks as well as the variation of the contributions from the scalar and the pseudoscalar condensates to the constituent mass of up quarks for $\theta = \pi/2$. Here the critical chemical potential for chiral transition is $\mu_c \sim 375$ MeV where the mass contributions from the scalar and the pseudoscalar condensates are modified to 24 and 12 MeV, respectively, from their vacuum values of 266 and 248 MeV. The total mass for the u and d quarks drops to 27 MeV from its vacuum value of 364 MeV. On the other hand, the contribution to the strange mass from the pseudoscalar condensate is negligible ($\sim$12 MeV) compared to the contribution from the scalar condensate ($\sim$548 MeV). Such a behavior can be easily understood when one examines the two mass gap equations (24) and (25) for strange quarks. Because of the large current quark mass of the strange quarks, the dominant contribution to the thermodynamic potential and consequently to the mass gap equations arises from the strange quark condensate I_s^s in the scalar channel. The pseudoscalar condensate for the strange quarks gets contributions from the light quark condensates both in the scalar and pseudoscalar channel that is subdominant. This results in a small magnitude for M_p^s compared to all other condensates. As the quark chemical potential increases, because of flavor mixing again, the strange quark mass also decrease to 463 MeV at $\mu_c = 375$ MeV. For $\theta = \pi$ the scalar condensate almost vanishes but for the nonzero current quark masses while the contribution to the constituent quark mass arises from the pseudoscalar condensate as

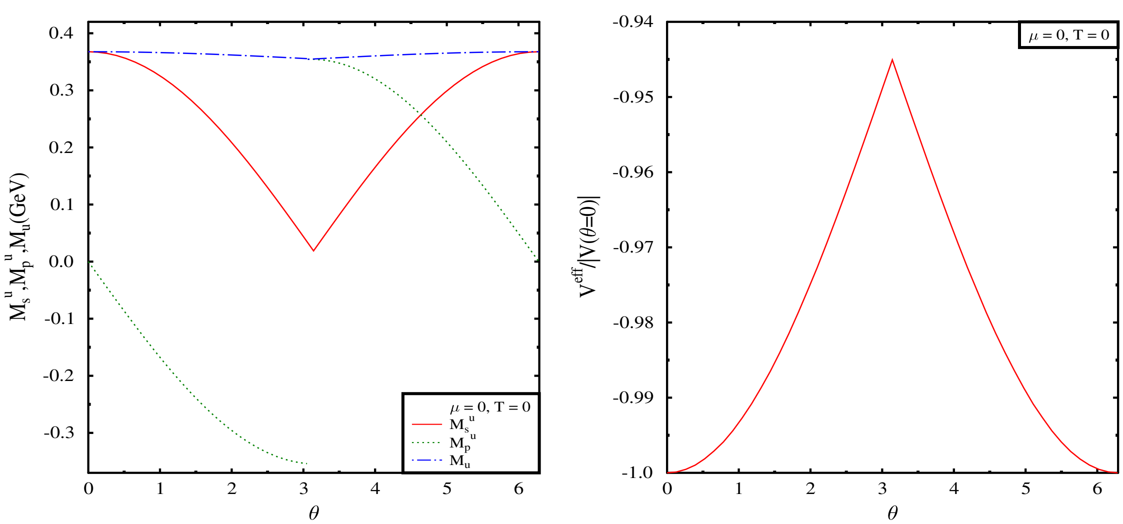

FIG. 1 (color online). θ dependence of the condensates (a) and the effective potential at zero temperature and zero baryon density (b).

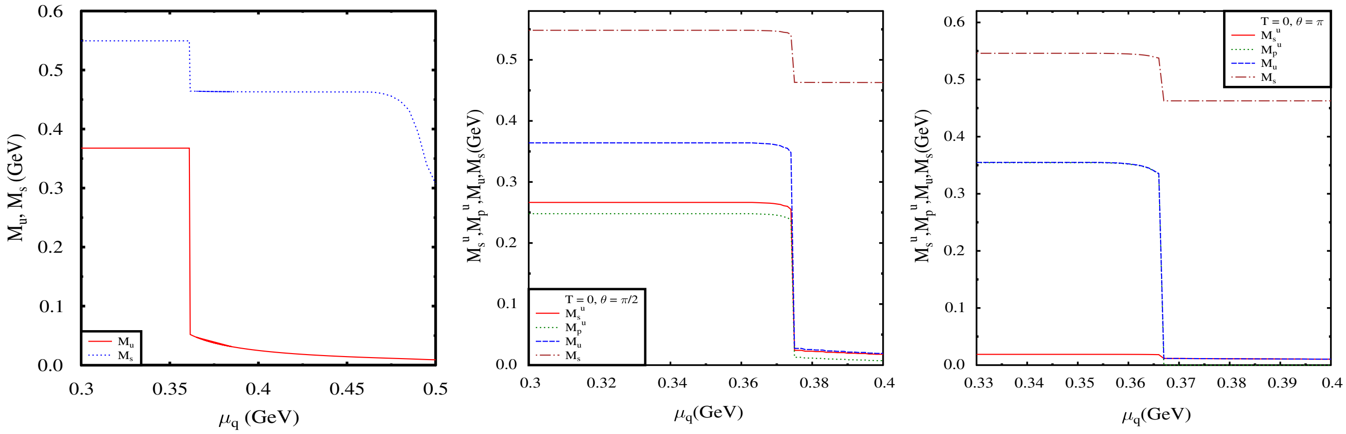

FIG. 2 (color online). Quark masses as a function of quark chemical potential at zero temperature for $\theta = 0$ (a), $\theta = \pi/2$ (b), and $\theta = \pi$ (c). The pseudoscalar contribution for strange quarks is negligible in this range of chemical potentials.

shown in Fig. [2(c)]. As the quark chemical potential is increased there is a first order transition at $\mu_c \sim 368$ MeV. At μ_c the pseudoscalar condensate vanishes and the contribution to quark mass arises solely from the scalar condensate that is nonvanishing because of the nonzero current quark masses.

Next we discuss the condensate variations with temperature. For $\theta = 0$, at zero baryon density the chiral crossover transition takes place for temperature about 200 MeV as may be seen in Fig. [3(a)]. As θ increases, the pseudoscalar condensate starts becoming nonzero and increase with θ. For $\theta = \pi/2$, the masses arising from both types of condensates are shown in Fig. [3(b)]. Here, both the scalar and pseudoscalar masses show a crossover transition as temperature is increased. In Fig. [3(c)], we show the behavior of the masses for $\theta = \pi$. The transition for the pseudoscalar mass becomes a second order transition at $\theta = \pi$ instead of a crossover that was the nature of transition for lower θ. This feature is elaborated in Fig. 4 where θ dependence of the nature of transition of scalar and pseudoscalar condensates with temperature is shown for zero baryon density. Fig. [4(a)] shows the θ dependence of the transitions for the scalar condensate. The transition is always a crossover for scalar condensate. Figure [4(b)] shows the transitions of the pseudoscalar condensates for different θ. We can see that the transition is a second order transition for $\theta = \pi$ whereas it is a crossover for other values of θ. Similar kinds of results have been obtained in sigma model calculations [6] where the transition at $\theta = \pi$ is found to be a first order transition instead of a second order transition. The *CP* restoring transition temperature for the zero baryon density case turns out to be about 192 MeV. However, the total constituent mass is nonzero as the scalar condensate is nonvanishing due to the nonzero current quark masses. This high temperature restoration of *CP* is expected as the instanton effects responsible for the *CP* violating phase become suppressed exponentially at high temperature [23]. Let us, however, note that what we have considered here is the equilibrium uniform *CP* violating phase that is restored at high temperature. However, in the context of heavy ion collisions, a *local* parity violating phase can also exist due to fluctuations of topological

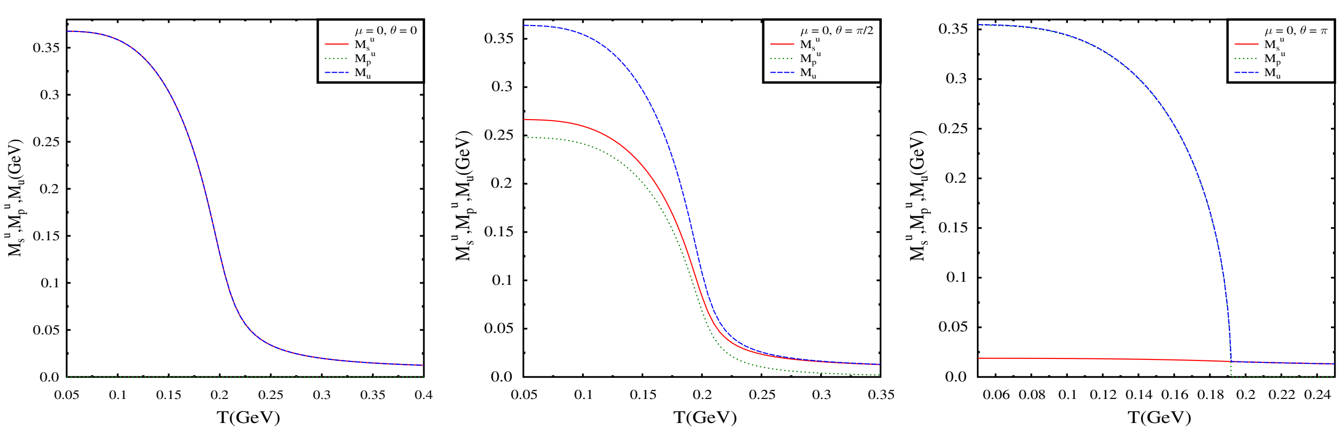

FIG. 3 (color online). Quark masses as a function of temperature at zero quark chemical potential for $\theta = 0$ (a), $\theta = \pi/2$ (b), and $\theta = \pi$.

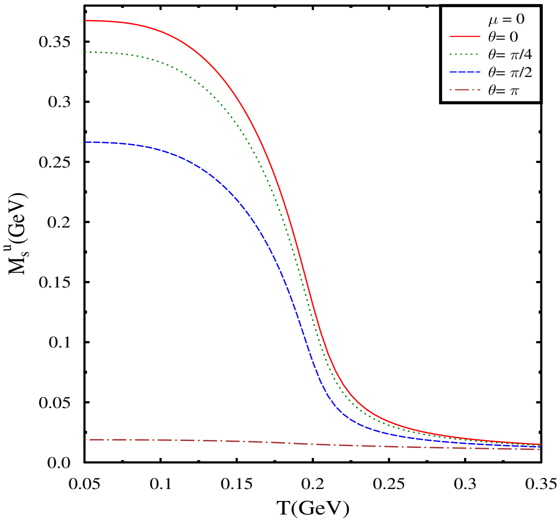

FIG. 4 (color online). The nature of transition of the scalar (a) and pseudoscalar (b) condensates with temperature at zero baryon density for different values of θ.

charges that are induced through sphaleron configurations that unlike instantons are not exponentially suppressed [11]. Further, such domains can also arise depending upon the kinetics of the transition, in particular, in non-equilibrium situations in heavy ion collisions [24]. Such *CP*-odd domains can decay via *CP*-odd processes and can have observable effects like chiral magnetic effect for noncentral heavy ion collisions [12] as well as lead to possible excess dilepton production for central collisions [14].

The *CP* restoring transition in the present three-flavor NJL model turns out to be second order for the zero chemical potential case similar to the results for the two-flavor case [9] unlike the case of the linear sigma model coupled to quarks [6]. The reason behind such different behavior regarding the order of the transition is due to the nonanalytic vacuum term in the NJL model [9]. However, a first order *CP* transition is observed with finite chemical potential and small temperature. This is clearly shown in Fig. 5 where we have shown the dynamical mass, M_p^u arising from the pseudoscalar condensate, for different temperatures as a function of the quark chemical potential for $\theta = \pi$. While at zero temperature, the order parameter decreases discontinuously; as the temperature increases, it becomes less sharp and finally results in a second order transition at high temperature.

In Fig. 6, we show the phase diagram in the plane of a quark chemical potential and temperature for the *CP* violating transition. Since the transition is first order at zero temperature and second order at zero chemical potential, there is a tricritical point for this transition in this plane. This turns out to be $(\mu_c, T_c) = (273, 94)$ MeV. Including Polyakov loop for the two flavor NJL model, such a tricritical point occurs at $(\mu_c, T_c) = (209, 165)$ MeV [10]. First order transitions are associated with the existence of metastable states. *CP* is restored in these metastable states and these are the nontrivial solutions of the gap equation (25), however, with lower pressure than the stable solutions. In the phase diagram of Fig. 6, such solutions may exist in the region between the solid line and the dotted line. A presence of such metastable states with parity violating condensates can have experimental consequences, since in the presence of such condensates there could be mixing between states of different parities. This can lead to decay of mesons to *both* even and odd number of pions. Apart from this, the properties regarding the dispersion relations for the in-medium mesons itself can change in presence of a *P*-violating condensate. This may result into excess production of dileptons in heavy ion collision experiments [14].

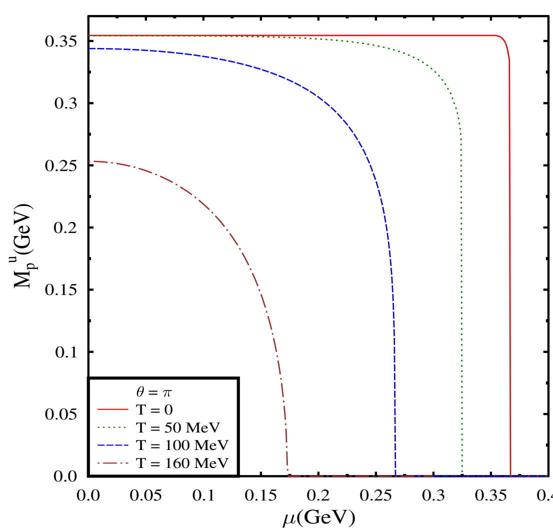

FIG. 5 (color online). Contribution from the pseudoscalar condensates to the u quark mass as a function of quark chemical potential for different temperatures. Here we have taken $\theta = \pi$.

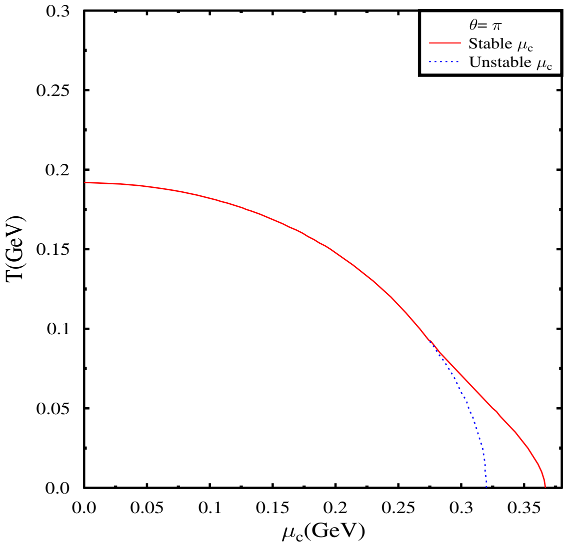

FIG. 6 (color online). The phase diagram for *CP* transition in the T-μ plane. The region between the solid line and the dotted line have solutions to the pseudoscalar mass gap equation but with higher thermodynamic potential. Here we have taken $\theta = \pi$.

V. SUMMARY

To summarize, we have investigated the effect of θ vacuum on the chiral transition in the T-μ phase diagram of strong interaction. This is studied within the framework of a three-flavor NJL model. The effect of a *CP* violating θ term of QCD is incorporated through the KMT determinant interaction term in the quark space that is induced by instantons [25]. In chiral perturbation theory, such effects can be included through a log determinant interaction term [5]. Spontaneous *CP* violation at $\theta = \pi$ occurs in such theory depending upon the ratio of the strength of such a log determinant interaction term to the number of flavors as compared to the quark masses. In the two-flavor NJL model, it was observed that spontaneous *CP* violation for $\theta = \pi$ occurs depending on the magnitude of current quark masses as well as the strength of the determinant coupling [8]. In the present case of the three-flavor NJL model, we observe that spontaneous *CP* violation occurs for $\theta = \pi$ for the phenomenologically consistent parameters [16] for the current quark masses as well as the strength of the determinant coupling.

To calculate the thermodynamical potential in the presence of a θ term, instead of performing a chiral rotation of the quark field operators [8,10], we have used a variational approach using an explicit construct for the ground state. We have considered an ansatz state general enough to have condensates both in the scalar as well as the pseudoscalar channel. The ansatz functions are determined through minimization of the thermodynamic potential.

In the three-flavor case considered here, we observed that at zero baryon density, the uniform *CP* violating phase is restored beyond a critical temperature of about 200 MeV for $\theta = \pi$. This is expected as the *CP* violation effects here arise from instanton induced interactions whose effect is suppressed exponentially with temperature [23]. It might be noted, however, that there could be *local* violation of *CP* due to sphaleron induced interactions that do not have such exponential suppression [11].

Apart from the effect of the temperature that has been considered earlier for two flavor cases [6,9], we have also considered the effect of the finite quark chemical potential and discussed the phase diagram in the T-μ plane. Within the model, the *CP* restoring transition is a first order transition at low temperature and high baryon chemical potential while it is a second order transition at high T and low μ. The existence of a first order transition leads to a possibility of formation of metastable *CP*-odd domains that could be of relevance for the planned heavy ion collision experiments at the Facility for Antiproton and Ion Research and the Nucleotron Based Ion Collider Facility. Such domains here arise from the instanton induced determinant interaction in the flavor space. Such parity violating condensates can result in mixing of parity partners, and the mass eigenstates will then be mixtures of different parity states. Therefore, low energy theorems based on field-current type identities with definite parity will not hold and need to be modified in the presence of P-violating condensates [26]. Further, it turns out that for the range of temperature and the chemical potentials we have considered, the strange quark condensates do not get dynamically generated in the *CP* violating pseudoscalar channel even for nonvanishing θ. Nonetheless, the strange quark antiquark condensates in the scalar channel do affect the pseudoscalar light quark condensates through the flavor mixing coupled gap equations.

ACKNOWLEDGMENTS

H. M. would like to thank Herman Warringa for discussions during the initial stages of this investigation. He would also like to thank Institut fuer Theoretische Physik, University of Frankfurt for warm hospitality, and Alexander von Humboldt Foundation, Germany for support. A. M. would like to acknowledge financial support from Department of Science and Technology, Government of India, through the project No. SR/S2/HEP-031/2010.

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