Perturbative QCD for $B \to a_1(b_1)\rho(\omega, \phi)$ and $B_s \to a_1(b_1)P(V)$ decays

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Within the framework of the perturbative QCD approach, we study the two-body charmless decays $B \to a_1(1260)(b_1(1235))\rho(\omega, \phi)$ and $B_s \to a_1(1260)(b_1(1235))P(V)$, where, P, V stand for any light pseudoscalar meson and vector meson, respectively. We find the following results: (a) With the exception of the decays $\bar{B}^0 \to a_1^0 \rho^0(\omega)$, other tree-dominated decays $B \to a_1 \rho(\omega)$ have larger branching ratios of order 10⁻⁵. With the exception of the decays $\bar{B} \to b_1^+ \rho^-$ and $B^- \to b_1^0 \rho^-$, other $B \to b_1 \rho(\omega)$ decays have smaller branching ratios of order 10^{-6} . The decays $B \to a_1(b_1)\phi$ are highly suppressed and have very small branching ratios of order 10⁻⁹. (b) The decays $\bar{B}_s^0 \to a_1^- K^+ (K^{*+})$ have contributions from the factorization emission diagrams with a large Wilson coefficient $C_2 + C_1/3$, so they have the largest branching ratios and arrive at the order of 10^{-5} . For the decays $\bar{B}_s^0 \to b_1 K(K^*)$, $a_1 \pi$, all of their branching ratios are of order a few times 10^{-6} . The branching ratios of other decays fall in the order of $10^{-7} - 10^{-9}$. (c) For the decays $\bar{B}^0 \to a_1^0 \rho^0$ and $B^- \to b_1^- \rho^0$, their two transverse polarizations are larger than their longitudinal polarizations, which are about 43.3% and 44.9%, respectively. (d) With the exception of the decays $\bar{B}_s^0 \to a_1^0 K^{*0}$, $a_1^0 \omega$, $b_1^0 \omega$, the longitudinal polarization fractions of other $\bar{B}_s^0 \to a_1(b_1)V$ decays are very large and more than 90%. (e) For the decays $B^- \to a_1^0 \rho^-$, $b_1^0 \rho^-$ and $\bar{B}^0 \to b_1^0 \rho^0$, $b_1^0 \omega$, where the transverse polarization fractions range from 4.7 to 7.5%, we calculate their direct CP-violating asymmetries, neglecting the transverse polarizations and find that those for two charged decays have smaller values, which are about 11.8% and -3.7% , respectively. Compared with the decays $\bar{B}_s^0 \to a_1(b_1)P$, most of the $\bar{B}_s^0 \to a_1(b_1)V$ decays have smaller direct CP asymmetries.

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I. INTRODUCTION

In general, the mesons are classified in J^{PC} multiplets. There are two types of orbitally excited axial-vector mesons, namely, 1^{++} and 1^{+-} . The former includes $a_1(1260)$, $f_1(1285)$, $f_1(1420)$, and K_{1A} , which compose the ${}^{3}P_1$ nonet, and the latter includes $b_1(1235)$, $h_1(1170)$, $h_1(1380)$, and K_{1B} , which compose the ${}^{1}P_1$ nonet. There is an important characteristic of these axial-vector mesons, with the exception of $a_1(1260)$ and $b_1(1235)$, that is, each different flavor state can mix with one another, which comes from the other nonet meson or the same nonet. There is not a mix between $a_1(1260)$ and $b_1(1235)$ because of the opposite C parities. They also do not mix with others. So compared with other axial-vector mesons, these two mesons should have less uncertainties regarding their inner structures.

Like the decay modes $B \rightarrow VV$, the charmless decays $B \rightarrow AV$ also have three polarization states and so are expected to have rich physics. In many $B \to VV$ decays, the information on branching ratios and polarization fractions among various helicity amplitudes have been studied by many authors [\[1–](#page-11-0)[4](#page-11-1)]. Through polarization studies, some underling helicity structures of the decay mechanism are proclaimed. They find that the polarization fractions follow the naive counting rule, that is, $f_L \sim 1 - O(m_V^2/m_B^2)$, $f_{\parallel} \sim$ $f_{\perp} \sim O(m_V^2/m_B^2)$. In the tree-dominated decay modes,

such as $B^0 \rightarrow \rho^+ \rho^-$, f_L is more than 90%. But if the contribution from the factorizable emission amplitudes is suppressed for some decay modes, this counting rule might be modified to some extent even more dramatically by other contributions. For example, the polarization fractions of the decay $B \to \phi K^*$ are modified by its annihilation contribution. Whether a similar situation also occurs in the $B \rightarrow AV$ decay modes is worth researching. We know that $a_1(1260)$ has some similar behaviors as the vector meson, so one can expect that there should exist some similar characteristics in the branching ratios and the polarization fractions between the decays $B \to a_1(1260)V$ and $B \to$ ρV , where $a_1(1260)$ is replaced by its scalar partner ρ , while this is not the case for $b_1(1235)$ because of its different characteristics in the decay constant and lightcone distribution amplitude (LCDA) compared with those of $a_1(1260)$. For example, the longitude decay constant is very small for the charged $b_1(1235)$ states and vanishes under the SU(3) limit. It is zero for the neutral $b_1^0(1235)$ state, while the transverse decay constant of $a_1(1260)$ vanishes under the SU(3) limit. In the isospin limit, the chiral-odd (-even) LCDAs of meson $b_1(1235)$ are symmetric (antisymmetric) under the exchange of quark and antiquark momentum fractions. It is just contrary to the symmetric behavior for $a_1(1260)$. In view of these differences, one can expect that there should exist very different results between $B \to a_1(1260)V$ and $B \to b_1(1235)V$. On the experimental side, a few of the $B \rightarrow AV$ decays are [*z](#page-0-1)hangzhiqing@haut.edu.cn studied, such as $B \to J/\psi K_1(1270)$ [\[5](#page-11-2)], $B^0 \to D^{*-}a_1^+$ [[6\]](#page-11-3),

 $B^0 \rightarrow a_1 \rho$ [\[7](#page-11-4)], $B \rightarrow b_1 \rho$, $b_1 K^*$ [[8](#page-11-5)]. In most of them only the upper limits for the branching ratios can be available. On the theoretical side, many charmless $B \to AV$ decays have been studied by Cheng and Yang in Ref. [\[9\]](#page-11-6) where the branching ratios are very different with those calculated by the naive factorization approach [\[10\]](#page-11-7). In most cases, the former are larger than the latter. To clarify such large differences is another motivation of this work. To our knowledge the study of charmless decays $B_s \to AP$, AV is still lacking both in experiments and theories, so we are also going to fill this gap and provide a ready reference to the forthcoming experiments to compare their data with the predictions in the perturbative quantum chromodynamics (pQCD) approach. It can be noted that we do not include the decays $B \to a_1(1260)(b_1(1235))K^*$ and $B \to a_2(1260)(b_1(1235))$ $a_1(1260)(b_1(1235))P$, which have been discussed in other works [[11](#page-11-8)[,12](#page-11-9)].

In the following $a_1(1260)$ and $b_1(1235)$ are denoted as a_1 and b_1 in some places for convenience. This paper is organized as follows. In Sec. [II](#page-1-0), decay constants and lightcone distribution amplitudes of the relevant mesons are introduced. In Sec. [III](#page-2-0), we then analyze these decay channels using the pQCD approach. The numerical results and a discussion are given in Secs. [IV](#page-3-0)and [V.](#page-7-0) The conclusions are presented in Sec. [VI](#page-10-0).

II. DECAY CONSTANTS AND DISTRIBUTION AMPLITUDES

For the wave function of the heavy $B_{(s)}$ meson, we take

$$
\Phi_{B_{(s)}}(x,b) = \frac{1}{\sqrt{2N_c}} (\not{P}_{B_{(s)}} + m_{B_{(s)}}) \gamma_5 \phi_{B_{(s)}}(x,b). \tag{1}
$$

Here, only the contribution of Lorentz structure $\phi_{B(s)}(x, b)$ is taken into account, since the contribution of the second Lorentz structure $\bar{\phi}_{B_{(s)}}$ is numerically small [[13](#page-11-10)] and has been neglected. For the distribution amplitude $\phi_{B_{(s)}}(x, b)$ in Eq. ([1\)](#page-1-1), we adopt the following model:

$$
\phi_{B_{(s)}}(x,b) = N_{B_{(s)}} x^2 (1-x)^2 \exp\left[-\frac{M_{B_{(s)}}^2 x^2}{2\omega_b^2} - \frac{1}{2}(\omega_b b)^2\right],\tag{2}
$$

where ω_b is a free parameter and we take $\omega_b = 0.4 \pm$ 0.04(0.5 \pm 0.05) Gev for $B(B_s)$ in numerical calculations, and $N_B = 91.745(N_{B_s} = 63.671)$ is the normalization factor for $\omega_b = 0.4(0.5)$. For the B_s meson, the SU(3) breaking effects are taken into consideration.

In these decays, both the longitudinal and the transverse polarizations are involved with the vector mesons. Their distribution amplitudes are defined as

$$
\langle V(P, \epsilon_L^*) | \bar{q}_{2\beta}(z) q_{1\alpha}(0) | 0 \rangle
$$

\n
$$
= \frac{1}{\sqrt{2N_c}} \int_0^1 dx e^{ixp \cdot z} [m_V \mathbf{\ell}_L^* \phi_V(x)
$$

\n
$$
+ \mathbf{\ell}_L^* \mathbf{P} \phi_V^t(x) + m_V \phi_V^s(x)]_{\alpha\beta},
$$

\n
$$
\langle V(P, \epsilon_T^*) | \bar{q}_{2\beta}(z) q_{1\alpha}(0) | 0 \rangle
$$

\n
$$
= \frac{1}{\sqrt{2N_c}} \int_0^1 dx e^{ixp \cdot z} [m_V \mathbf{\ell}_T^* \phi_V^v(x) + \mathbf{\ell}_T^* \mathbf{P} \phi_V^T(x)]
$$

\n
$$
+ m_V i \epsilon_{\mu\nu\rho\sigma} \gamma_5 \gamma^\mu \epsilon_T^{* \nu} n^\rho \nu^\sigma \phi_V^a(x)]_{\alpha\beta},
$$
\n(3)

where $n(v)$ is the unit vector having the same (opposite) direction with the moving of the vector meson and x is the momentum fraction of the q_2 quark. The distribution amplitudes of the axial vectors have the same format as those of the vectors, with the exception of the factor $i\gamma_5$ from the left-hand side of the following equation:

$$
\langle A(P, \epsilon_L^*) | \bar{q}_{2\beta}(z) q_{1\alpha}(0) | 0 \rangle
$$

\n
$$
= \frac{i\gamma_5}{\sqrt{2N_c}} \int_0^1 dx e^{ixp \cdot z} [m_A \mathbf{\ell}_L^* \phi_A(x)
$$

\n
$$
+ \mathbf{\ell}_L^* \mathbf{P} \phi_A^t(x) + m_A \phi_A^s(x)]_{\alpha\beta},
$$

\n
$$
\langle A(P, \epsilon_T^*) | \bar{q}_{2\beta}(z) q_{1\alpha}(0) | 0 \rangle
$$

\n
$$
= \frac{i\gamma_5}{\sqrt{2N_c}} \int_0^1 dx e^{ixp \cdot z} [m_A \mathbf{\ell}_T^* \phi_A^v(x) + \mathbf{\ell}_T^* \mathbf{P} \phi_A^T(x)
$$

\n
$$
+ m_A i \epsilon_{\mu\nu\rho\sigma} \gamma_5 \gamma^\mu \epsilon_T^{* \nu} n^\rho \nu^\sigma \phi_A^a(x)]_{\alpha\beta}.
$$

\n(4)

The upper twist-2 and twist-3 distribution functions of the final state mesons, $\phi_{V(A)}, \phi_{V(A)}^t$, $\phi_{V(A)}^s$, $\phi_{V(A)}^r$, $\phi_{V(A)}^v$, and $\phi_{V(A)}^a$ can be calculated by using the light-cone QCD sum rule. We list the distribution functions of the vector (V) mesons, namely, $\rho(\omega, K^*, \phi)$, as follows:

TABLE I. Decay constants and Gegenbauer moments for each meson (in MeV). The values are taken at $\mu = 1$ GeV.

f_{K^*} 209 ± 2	$f_{K^*}^T$ 165 ± 9	f_{ϕ} 231 ± 4	f^I_{ϕ}
			186 ± 9
f_K	f_{π}	f_{a_1}	$f_{b_1}^T$
160	130	238 ± 10	-180 ± 8
f_{ρ}	f_{ρ}^T	f_{ω}	f_{ω}^T
209 ± 2	165 ± 9	195 ± 3	151 ± 9
a_{1K}	$a_{1\pi}$	a_{2K}	$a_{2\pi}$
0.17	Ω	0.2	0.44
$a_1^{\parallel}(K^*)$	$a_1^{\perp}(K^*)$	$a_2^{\parallel}(K^*)$	$a_2^{\perp}(K^*)$
0.03 ± 0.02	0.04 ± 0.03	0.11 ± 0.09	0.10 ± 0.08
$a_2^{\parallel}(\rho, \omega)$	$a_2^{\perp}(\rho,\omega)$	$a_2^{\parallel}(\phi)$	$a_2^{\perp}(\phi)$
0.15 ± 0.07	0.14 ± 0.06	0.18 ± 0.08	0.14 ± 0.07
$a_2^{\parallel}(a_1(1260))$	$a_1^{\perp}(a_1(1260))$	$a_1^{\parallel}(b_1(1235))$	$a_2^{\perp}(b_1(1235))$
-0.02 ± 0.02	-1.04 ± 0.34	-1.95 ± 0.35	0.03 ± 0.19

$$
\phi_V(x) = \frac{f_V}{2\sqrt{2N_c}} \phi_{\parallel}(x), \quad \phi_V^T(x) = \frac{f_V^T}{2\sqrt{2N_c}} \phi_{\perp}(x), \n\phi_V^t(x) = \frac{f_V^T}{2\sqrt{2N_c}} h_{\parallel}^{(t)}(x), \quad \phi_V^s(x) = \frac{f_V^T}{2\sqrt{4N_c}} \frac{d}{dx} h_{\parallel}^{(s)}(x), \n\phi_V^v(x) = \frac{f_V}{2\sqrt{2N_c}} g_{\perp}^{(v)}(x), \quad \phi_V^a(x) = \frac{f_V}{8\sqrt{2N_c}} \frac{d}{dx} g_{\perp}^{(a)}(x). \quad (5)
$$

Here, the axial-vector (A) mesons, a_1 and b_1 , can be obtain by replacing all the ϕ_V with ϕ_A , by replacing $f_V^T(f_V)$ with f in Eq. [\(5](#page-2-1)). Here, we use f to present both longitudinally and transversely polarized mesons $a_1(b_1)$ by assuming $f_{a_1}^T = f_{a_1} = f$ for a_1 and $f_{b_1} = f_{b_1}^T = f$ for b_1 . In Eq. ([5](#page-2-1)), the twist-2 distribution functions are in the first line and can be expanded as

$$
\phi_{\parallel, \perp} = 6x(1-x)\left[1 + a_2^{\parallel, \perp} \frac{3}{2}(5t^2 - 1)\right], \text{ for } V \text{ mesons}; \quad (8)
$$

$$
\phi_{\parallel, \perp} = 6x(1-x)\bigg[a_0^{\parallel, \perp} + 3a_1^{\parallel, \perp}t + a_2^{\parallel, \perp}\frac{3}{2}(5t^2 - 1)\bigg],
$$

for *A* mesons, (9)

where the zeroth Gegenbauer moments are $a_0^{\perp}(a_1) =$ $a_0^{\parallel}(b_1) = 0$ and $a_0^{\parallel}(a_1) = a_0^{\perp}(b_1) = 1$.

As for twist-3 LCDAs, we use the asymptotic forms for V mesons:

$$
h_{\parallel}^{(t)}(x) = 3t^2, \qquad h_{\parallel}^{(s)}(x) = 6x(1-x),
$$

\n
$$
g_{\perp}^{(a)}(x) = 6x(1-x), \qquad g_{\perp}^{(v)}(x) = \frac{3}{4}(1+t^2). \qquad (10)
$$

And we use the following forms for A mesons:

$$
h_{\parallel}^{(t)}(x) = 3a_0^{\perp}t^2 + \frac{3}{2}a_1^{\perp}t(3t^2 - 1),
$$

\n
$$
h_{\parallel}^{(s)}(x) = 6x(1 - x)(a_0^{\perp} + a_1^{\perp}t),
$$

\n
$$
g_{\perp}^{(a)}(x) = 6x(1 - x)(a_0^{\parallel} + a_1^{\parallel}t),
$$

\n
$$
g_{\perp}^{(v)}(x) = \frac{3}{4}a_0^{\parallel}(1 + t^2) + \frac{3}{2}a_1^{\parallel}t^3.
$$
\n(11)

The wave functions for the pseudoscalar (P) mesons K , π are given as

$$
\phi_{\parallel}^{b_1} = f_{b_1} 6x \bar{x} \bigg[1 + \mu_{b_1} \sum_{i=1}^2 a_i^{\parallel b_1} C_i^{3/2} (2x - 1) \bigg], \qquad (6)
$$

where $\mu_{b_1} = 1/a_0^{\parallel, b_1}$, which is infinite in the *SU*(3) limit. So it is convenient to use the following format:

$$
\phi_{\parallel}^{b_1} = f_{b_1}^T 6x \bar{x} [a_0^{\parallel, b_1} + a_1^{\parallel, b_1} C_1^{3/2} (2x - 1) + a_2^{\parallel, b_1} C_2^{3/2} (2x - 1)],
$$
\n(7)

where we have the relation $f_{b_1} = f_{b_1}^T(\mu) a_0^{\parallel b_1}(\mu)$. This amounts to treating the decay constant of b_1 as $f_{b_1}^T$, but it does not mean that f_{b_1} is equal to $f_{b_1}^T$. It is similar for $f_{a_1}^T$ and $f_{a_1}^T$.

$$
\Phi_P(P, x, \zeta) \equiv \frac{1}{\sqrt{2N_C}} \gamma_5 [\not\!\!\!\!/ \phi^A(x) + m_0 \phi^P(x) + \zeta m_0 (\not\!\!\!\!/ \psi - v \cdot n) \phi^T(x)], \tag{12}
$$

where the parameter ζ is either $+1$ or -1 , depending on the assignment of the momentum fraction x . The chiral scale parameter m_0 is defined as $m_0 = \frac{M_p^2}{m_{q_1} + m_{q_2}}$. The distribution amplitudes are expanded as

$$
\phi_{K,\pi}^A(x) = \frac{3f_{K,\pi}}{\sqrt{6}} x(1-x)[1 + a_{1(K,\pi)} C_1^{3/2}(t) + a_{2(K,\pi)} C_2^{3/2}(t)],
$$
\n(13)

$$
\phi_K^p(x) = \frac{3f_K}{2\sqrt{6}} [1 + 0.43 C_2^{1/2}(t)];
$$

$$
\phi_\pi^p(x) = \frac{3f_\pi}{2\sqrt{6}} [1 + 0.24 C_2^{1/2}(t)],
$$
 (14)

$$
\phi_K^T(x) = \frac{-f_K}{2\sqrt{6}} [C_1^{1/2}(t) + 0.35 C_3^{1/2}(t)];
$$

\n
$$
\phi_\pi^T(x) = \frac{-f_\pi}{2\sqrt{6}} [C_1^{1/2}(t) + 0.55 C_3^{1/2}(t)],
$$
\n(15)

with the Gegenbauer polynomials defined as

$$
C_1^{3/2}(t) = 3t
$$
, $C_2^{3/2}(t) = 1.5(5t^2 - 1)$, (16)

$$
C_1^{1/2}(t) = t, \t C_2^{1/2}(t) = 0.5(3t^2 - 1),
$$

\n
$$
C_3^{1/2}(t) = 0.5t(5t^2 - 3).
$$
\n(17)

As for the distribution amplitudes of the pseudoscalar mesons η and η' , we use the quark flavor basis mixing mechanism proposed by Ref. [\[14\]](#page-11-11) and take the same formulae and parameter values as those in Ref. [[15](#page-11-12)].

In Eqs. ([8](#page-2-2))–[\(11\)](#page-2-3) and [\(13\)](#page-2-4)–([17](#page-2-5)), the function $t = 2x - 1$. As in Ref. [[16](#page-11-13)], the decay constants and the Gegenbauer moments $a_n^{\parallel \perp}$ for each meson are quoted the numerical results [[17](#page-11-14)[–22\]](#page-11-15) and listed in Table [I](#page-1-2).

III. THE PERTURBATIVE QCD CALCULATION

The pQCD approach is an effective theory to handle hadronic *B* decays [\[23–](#page-11-16)[25](#page-11-17)]. Because it takes into account the transverse momentum of the valence quarks in the hadrons, one will encounter double logarithm divergences when the soft and the collinear momenta overlap. Fortunately, these large double logarithms can be resummed into the Sudakov factor [\[26\]](#page-11-18). There are also other types of double logarithms which arise from the loop corrections to the weak decay vertex. These double logarithms can also be resummed and result in the threshold factor [[27](#page-11-19)]. This factor decreases faster than any other power of the momentum fraction in the threshold region, which removes the endpoint singularity. This factor is

¹As is usual, one employ the decay constant f_{b_1} to define the longitudinal LCDAs of the b_1 meson as

often parameterized into a simple form that is independent on channels, twists, and flavors [\[28\]](#page-11-20). Certainly, when the higher order diagrams only suffer from soft or collinear infrared divergence, it is ease to cure by using the eikonal approximation [[29](#page-11-21)]. Controlling these kinds of divergences reasonably makes the pQCD approach more selfconsistent.

Here, we take the decay $\bar{B}^0 \rightarrow a_1^0 \rho^0$ as an example, whose diagrams are shown in Fig. [1](#page-3-1). These eight Feynman diagrams belong to the condition of the a_1^0 meson being at the emission position. Another eight Feynman diagrams obtained by exchanging the positions of a_1^0 and ρ^0 in Fig. [1](#page-3-1) also contribute to the decay. All of these single hard gluon exchange diagrams contain all of the leading order contributions to $\bar{B}^0 \to a_1^0 \rho^0$ in the pQCD approach. Similar to the $B \to VV$ decay modes, such as $B \to \rho \rho$ [\[1\]](#page-11-0) and $B \to K^* \rho(\omega)$ [\[2\]](#page-11-22), both longitudinal and transverse polarizations can contribute to the decay width. So we can get three kinds of polarization amplitudes, M_L (longitudinal) and $M_{N,T}$ (transverse), by calculating these diagrams. Because of the aforementioned distribution amplitudes of the axial vectors having the same format as those of the vectors with the exception of a factor, the formulas of the decays considered here can be obtained from those of the $B \to VV$ decays by some replacements. Certainly, there also exists a difference: if the emitted meson is b_1 for the factorizable emission diagrams, the

amplitudes contributed by the $(V - A)(V \pm A)$ operators would be zero due to the vanishing decay constant f_{b_1} . For the tree-dominated decays, the contributions from the factorizable emission diagrams, namely, Figs. [1\(a\)](#page-3-2) and [1\(b\)](#page-3-2), are very important. In the pQCD approach, the form factor can be extracted from the amplitudes obtained by calculating such diagrams, where the two transverse amplitudes are highly suppressed by the factor $r_{a_1(b_1)}\cdot r_{\rho(\omega)}$ compared with the longitudinal amplitudes. Here, $r_{a_1(b_1)} = \frac{m_{a_1(b_1)}}{m_B}$ and $r_{\rho(\omega)} = \frac{m_{\rho(\omega)}}{m_B}$. To some decays, the nonfactorizable emission diagrams, namely, Figs. $1(c)$ and $1(d)$, play a more important role, where the contributions from the transverse polarizations are not suppressed. Certainly, the contributions from the nonfactorizable and the factorizable annihilation diagrams, which are Figs. $1(g)$, $1(h)$, $1(e)$, and $1(f)$ can also not be neglected for some decays.

IV. NUMERICAL RESULTS AND DISCUSSIONS FOR $B_{u,d}$ DECAYS

We use the following input parameters in the numerical calculations [[30](#page-11-23),[31](#page-11-24)]:

$$
f_B = 190
$$
 MeV, $f_{B_s} = 230$ MeV,
\n $M_B = 5.28$ GeV, $M_{B_s} = 5.37$ GeV, (18)

$$
\tau_{B^{\pm}} = 1.638 \times 10^{-12} \text{ s}, \qquad \tau_{B^0} = 1.525 \times 10^{-12} \text{ s},
$$

$$
\tau_{B_s} = 1.472 \times 10^{-12} \text{ s}, \tag{19}
$$

$$
|V_{ud}| = 0.974, \t |V_{td}| = 8.58 \times 10^{-3},
$$

$$
|V_{ub}| = 3.54 \times 10^{-3}, \t |V_{tb}| = 0.999,
$$
 (20)

$$
|V_{ts}| = 0.039\,96, \t |V_{us}| = 0.225\,39,\n\alpha = (91.0 \pm 3.9)^\circ, \t \gamma = 67.2^\circ.
$$
\t(21)

In the B-rest frame, the decay rates of $B \to a_1(b_1)V$, where V represents ρ , ω , ϕ , can be written as

$$
\Gamma = \frac{G_F^2 (1 - r_{a_1(b_1)}^2)}{32\pi M_B} \sum_{\sigma = L, N, T} \mathcal{M}^{\sigma \dagger} \mathcal{M}^{\sigma},
$$
 (22)

where \mathcal{M}^{σ} is the total decay amplitude of each decay considered. The subscript σ is the helicity states of the two final mesons with one longitudinal component and two transverse ones. The decay amplitude can be decomposed FIG. 1. Diagrams contributing to the decay $\bar{B}^0 \to a_1^0 \rho^0$. into three scalar amplitudes a, b, c according to

$$
\mathcal{M}^{\sigma} = \epsilon_{2\mu}^{*}(\sigma)\epsilon_{3\nu}^{*}(\sigma)\left[ag^{\mu\nu} + \frac{b}{M_{2}M_{3}}P_{B}^{\mu}P_{B}^{\nu}\right] \n+ i\frac{c}{M_{2}M_{3}}\epsilon^{\mu\nu\alpha\beta}P_{2\alpha}P_{3\beta}\right] \n= \mathcal{M}_{L} + \mathcal{M}_{N}\epsilon_{2}^{*}(\sigma = T) \cdot \epsilon_{3}^{*}(\sigma) \n= T) + i\frac{\mathcal{M}_{T}}{M_{B}^{2}}\epsilon^{\alpha\beta\gamma\rho}\epsilon_{2\alpha}^{*}(\sigma)\epsilon_{3\beta}^{*}(\sigma)P_{2\gamma}P_{3\rho}, \quad (23)
$$

where M_2 and M_3 are the masses of the two final mesons $a_1(b_1)$ and $\rho(\omega, \phi)$, respectively. The amplitudes \mathcal{M}_L , \mathcal{M}_N , \mathcal{M}_T can be expressed as

$$
\mathcal{M}_L = a\epsilon_2^*(L) \cdot \epsilon_3^*(L) + \frac{b}{M_2 M_3} \epsilon_2^*(L) \cdot P_3 \epsilon_3^*(L) \cdot P_2,
$$

$$
\mathcal{M}_N = a, \qquad \mathcal{M}_T = \frac{M_B^2}{M_2 M_3} c.
$$
 (24)

We can use the amplitudes with different Lorentz structures to define the helicity amplitudes, one longitudinal amplitude H_0 and two transverse amplitudes H_{\pm} :

$$
H_0 = M_B^2 \mathcal{M}_L, \quad H_{\pm} = M_B^2 \mathcal{M}_N \mp M_2 M_3 \sqrt{r^2 - 1} \mathcal{M}_T, \tag{25}
$$

where the ratio $r = P_2 \cdot P_3/(M_2M_3)$. After the helicity summation, we can get the relation

$$
\sum_{\sigma=L,N,T} \mathcal{M}^{\sigma \dagger} \mathcal{M}^{\sigma} = |\mathcal{M}_L|^2 + 2(|\mathcal{M}_N|^2 + |\mathcal{M}_T|^2)
$$

$$
= |H_0|^2 + |H_+|^2 + |H_-|^2. \tag{26}
$$

Certainly another equivalent set of helicity amplitudes are often used, that is,

$$
A_0 = -M_B^2 \mathcal{M}_L, \qquad A_{\parallel} = \sqrt{2} M_B^2 \mathcal{M}_N, A_{\perp} = M_2 M_3 \sqrt{2(r^2 - 1)} \mathcal{M}_T.
$$
 (27)

Using this set of helicity amplitudes, we can define three polarization fractions $f_{0,\parallel,\perp}$:

$$
f_{0, \parallel, \perp} = \frac{|A_{0, \parallel, \perp}|^2}{|A_0|^2 + |A_\parallel|^2 + |A_\perp|^2}.
$$
 (28)

The matrix elements \mathcal{M}_i of the operators in the weak Hamiltonian can be calculated by using the pQCD approach, which are written as

$$
M_j = V_{ub} V_{ud}^* T_j - V_{tb} V_{td}^* P_j = V_{ub} V_{ud}^* T_j (1 + z_j e^{i(\alpha + \delta_j)}),
$$
\n(29)

where $j = L$, N, T, and α is the Cabibbo-Kobayashi-Maskawa weak phase angle, defined via $\alpha =$ $\arg[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}].$ Here, we leave this angle as a free parameter. δ_i is the relative strong phase between the tree and the penguin amplitudes, which are denoted as " T_i " and " P_i ," respectively. The term z_i describes the ratio of penguin to tree contributions and is defined as

$$
z_j = \left| \frac{V_{tb} V_{td}^*}{V_{ub} V_{ud}^*} \right| \left| \frac{P_j}{T_j} \right|.
$$
 (30)

In the same way, it is easy to write the decay amplitude $\overline{\mathcal{M}}_i$ for the corresponding conjugated decay mode:

$$
\bar{M}_{j} = V_{ub}^{*} V_{ud} T_{j} - V_{tb}^{*} V_{td} P_{j}
$$

= $V_{ub}^{*} V_{ud} T_{j} (1 + z_{j} e^{i(-\alpha + \delta_{j})}).$ (31)

So the CP-averaged branching ratio for each decay considered is defined as

$$
\mathcal{B} = (|\mathcal{M}_j|^2 + |\bar{\mathcal{M}}_j|^2)/2
$$

= $|V_{ub}V_{ud}^*|^2 \Big[T_L^2 (1 + 2z_L \cos \alpha \cos \delta_L + z_L^2) + 2 \sum_{j=N,T} T_j^2 (1 + 2z_j \cos \alpha \cos \delta_j + z_j^2) \Big].$ (32)

Like the decays of B to two vector mesons, there are also 3 types of helicity amplitudes, therefore corresponding to 3 types of z_j and δ_j , respectively. It is easy to see that the dependence of decay width on δ and α is more complicated compared with that for the decays of B to a pseudoscalar meson (P) and a axial-vector meson (A) .

Using the input parameters and the wave functions as specified in this section and Sec. [II,](#page-1-0) it is easy to get the branching ratios for the decays considered which are listed in Table [II,](#page-5-0) where the first error comes from the uncertainty in the B meson shape parameter $\omega_b = 0.40 \pm 0.04$ GeV, the second one is from the threshold resummation parameter c , and it varies from 0.3 to 0.4.

From Table [II,](#page-5-0) one can find that with the exception of the decays $\bar{B}^0 \to a_1^0 \rho^0$, $a_1^0 \omega$, the branching ratios of other treedominated decays $B \to a_1 \rho(\omega)$ are all of order 10^{-5} . Most of the contributions to such larger branching ratios are from the factorizable emission diagrams (a) and (b) , which contribute to the $B \to \rho(\omega)$ $(B \to a_1)$ form factors. Because of the large Wilson coefficients $C_2 + C_1/3$ in the amplitudes contributed by the tree operators O_1 and O_2 , the branch ratios are almost proportionate to the corresponding form factors. Certainly, they are also related to the decay constants f_{a_1} ($f_{\rho,\omega}$). As the basic input values, they are the same in many factorization approaches; for example, the pQCD and quantum chromodynamics factorization (QCDF) approaches, while for the form factors, there exist some differences between these two approaches. For the QCDF approach, the form factors are used as the input values, which are obtained from light-cone sum rules. In Ref. [[9](#page-11-6)], the form factors $A_0^{B\rho}$ and $V_0^{Ba_1}$ are both about 0.30, and $V_0^{Bb_1}$ is about -0.39, where the authors put an additional minus sign by taking the convention of the decay constants of a_1 and b_1 as the same sign. In this convention, the corresponding form factors have opposite signs. For the pQCD approach, the form factors can be calculated perturbatively. From our

TABLE II. Branching ratios (in units of 10^{-6}) for the decays $B \to a_1(1260)\rho(\omega, \phi)$ and $B \to b_1(1235)\rho(\omega, \phi)$. In our results, the errors for these entries correspond to the uncertainties from ω_B and threshold resummation parameter c, respectively. For comparison, we also listed the results predicted by the QCDF approach [[9\]](#page-11-6) and the naive factorization approach [[10](#page-11-7)].

	This work	[9]	[10]
$\bar{B}^0 \rightarrow a_1^+ \rho^-$	$33.6^{+9.9+15.8}_{-7.4-15.8}$	$23.9^{+10.5+3.2}_{-9.2-0.4}$	4.3
$\bar{B}^0 \rightarrow a_1^- \rho^+$	$27.1_{-6.0-9.2}^{+8.0+9.2}$	$36.0^{+3.5+3.5}_{-4.0-0.7}$	4.7
$\bar{B}^0 \rightarrow a_1^0 \rho^0$	$0.64_{-0.10-0.04}^{+0.12+0.04}$	$1.2^{+2.0+5.1}_{-0.7-0.3}$	0.01
$B^- \rightarrow a_1^0 \rho^-$	$27.7^{+7.8+7.9}_{-5.9-7.9}$	$17.8^{+10.1+3.1}_{-6.4-0.2}$	2.4
$B^{-} \to a_{1}^{-} \rho^{0}$	$21.9^{+5.9+9.3}_{-4.6-9.3}$	$23.2^{+3.6+4.8}_{-2.9-0.1}$	3.0
$\bar{B}^0 \rightarrow a_1^0 \omega$	$0.83^{+0.27+0.40}_{-0.20-0.40}$	$0.2^{+0.1+0.4}_{-0.1-0.0}$	0.003
$B^{-} \rightarrow a_{1}^{-} \omega$	$14.4^{+4.8+6.0}_{-3.5-6.0}$	$22.5^{+3.4+3.0}_{-2.7-0.7}$	2.2
$\bar{B}^0 \rightarrow a_1^0 \phi$	$0.0029_{-0.0006}^{+0.0007}$ + 0.0006	$0.002_{-0.001}^{+0.002}{}_{-0.001}^{+0.009}$	0.0005
$B^- \rightarrow a_1^- \phi$	$0.0058^{+0.0015+0.0013}_{-0.0013-0.0013}$	$0.01_{-0.00}^{+0.01}{}_{-0.00}^{+0.04}$	0.001
$\bar{B}^0 \rightarrow b_1^+ \rho^-$	$46.8^{+15.6+19.1}_{-11.3-19.1}$	$32.1_{-14.7-4.6}^{+16.5+12.0}$	1.6
$\bar{B}^0 \rightarrow b_1^- \rho^+$	$2.2_{-0.3-0.1}^{+0.3+0.1}$	$0.6^{+0.6+1.9}_{-0.3-0.2}$	0.55
$\bar{B}^0 \rightarrow b_1^0 \rho^0$	$3.4^{+0.4+0.4}_{-0.5-0.4}$	$3.2^{+5.2+1.7}_{-2.0-0.4}$	0.002
$B^{-} \to b_1^0 \rho^{-}$	$22.9^{+8.7+24.3}_{-6.3-24.3}$	$29.1_{-10.6-5.9}^{+16.2+5.4}$	0.86
$B^{-} \to b_{1}^{-} \rho^{0}$	$1.4^{+0.2+0.3}_{-0.2-0.3}$	$0.9_{-0.6-0.5}^{+1.7+2.6}$	0.36
$\bar{B}^0 \rightarrow b_1^0 \omega$	$2.8^{+0.7+0.2}_{-0.6-0.2}$	$0.1_{-0.0}^{+0.2+1.6}$	0.004
$B^{-} \rightarrow b_{1}^{-} \omega$	$2.1_{-0.2-0.7}^{+0.4+0.7}$	$0.8^{+1.4+3.1}_{-0.5-0.3}$	0.38
$\bar{B}^0 \rightarrow b_1^0 \phi$	$0.003_{-0.001}^{+0.001}$ + 0.000	$0.01_{-0.00-0.00}^{+0.01+0.01}$	0.0002
$B^- \rightarrow b_1^- \phi$	$0.006_{-0.002-0.001}^{+0.003+0.001}$	$0.02_{-0.01}^{+0.02}{}_{-0.00}^{+0.03}$	0.0004

calculations, we find that the values of $A_0^{B\rho}$, $V_0^{Ba_1}$, and $V_0^{Bb_1}$ are about 0.25, 0.33, and 0.44, respectively. If the decay is governed by the form factor $A_0^{B\rho}$, its branching ratio predicted by the pQCD approach would be smaller than that obtained by the QCDF approach; for example, $\bar{B}^0 \rightarrow$ $a_1^- \rho^+$. On the contrary, if the decay is governed by the form factor $V_0^{Ba_1}$, the result for the pQCD approach would have a larger value. So to accurately determine these form factors is very important. The branching ratio of $B^- \rightarrow$ $a_1^- \rho^0$ is larger than that of $B^- \to a_1^- \omega$. One reason is that the form factor $A_0^{B\rho}$ is a little larger than $A_0^{B\omega}$, which is about 0.23. The other reason is the different interferences from $d\bar{d}$ and $u\bar{u}$: the constructive interference between $-dd$ and $u\bar{u}$ which compose ρ , and the destructive interference between $d\bar{d}$ and $u\bar{u}$ which compose ω . But there is a contrary situation for the QCDF approach between these two decays. Although the neutral decays $\bar{B}^0 \to a_1^0 \rho^0$, $a_1^0 \omega$ are also tree dominant, their tree operator contributions are highly suppressed compared with the two charged decays $B^- \to a_1^- \rho^0$, $a_1^- \omega$ (shown in Table [III\)](#page-5-1). So their branching ratios are small and of order 10^{-7} . Certainly, we only give the leading order results and they might like decays $B \rightarrow$ $\rho^0 \rho^0$, $\rho^0 \omega$, which are sensitive to the next leading order contributions.

As to the tree-dominated decays $B \to b_1^+ \rho^-$, $b_1^0 \rho^-$, which are governed by the decay constant f_{ρ} and the form factor of $B \rightarrow b_1$, they also have large branching ratios. Although $B \to b_1^- \rho^+$ is a color allowed decay, its branching ratio is highly suppressed due to the decay constant f_{b_1} being very small and vanishing under the isospin limit. One should admit that each amplitude for the decays $B^- \to b_1^-(b_1^0)\rho^0$ has near value in magnitude with the corresponding one for the decays $B^- \rightarrow$ $b_1^-(b_1^0)\omega$, but the sign differences before $d\bar{d}$ in the mesons ρ and ω will induce some discrepancies in the branching ratios. Like the decays $B \to \pi \phi$, $a_0(1450) \phi$ [[32,](#page-11-25)[33\]](#page-11-26), whose branching ratios are of order $10^{-8} \sim 10^{-9}$, the decays $B \to a_1(b_1)\phi$ are induced by the flavor-changing neutral current interactions and highly suppressed by the small Wilson coefficients for penguin operators. Moreover, there is no the contribution from the annihilation

Decay mode	Pol. amp.	(a) and (b)	(c) and (d)	(e) and (f)	(g) and (h)
	$A(T_L)$	-219.2	$8.1 - 3.8i$	$-1.2 + 9.0i$	$-0.5 - 0.1i$
	$A(T_N)$	22.8	$-7.1 + 5.2i$	$-0.2 - 0.2ii$	$0.6 + 0.03i$
$a_1^+ \rho^-$	$A(T_T)$	-57.3	$-12.9 + 3.2i$	$0.1 - 0.4i$	$-1.0 - 0.2i$
	$A(P_L)$	8.8	$-0.09 + 0.13i$	$0.59 + 1.7i$	$-1.7 - 3.4i$
	$A(P_N)$	0.9	$0.26 - 0.17i$	$-0.03 - 0.01i$	$0.7 + 3.4i$
	$A(P_T)$	2.2	$0.49 - 0.05i$	$-0.03 + 0.01i$	$1.1 + 6.6i$
	$A(T_L)$	-5.7	$18.4 - 7.3i$	$1.1 - 4.7i$	$1.4 - 1.0i$
	$A(T_N)$	0.5	$-11.0 + 6.5i$	$0.06 + 0.06i$	$0.54 + 0.05i$
$a_1^0\rho^0$	$A(T_T)$	-0.1	$-20.8 + 5.2i$	$0.00 + 0.17i$	$-1.1 - 0.15i$
	$A(P_L)$	0.8	$0.36 + 0.12i$	$0.33 + 1.24i$	$-0.12 - 0.06i$
	$A(P_N)$	-0.15	$0.26 - 0.15i$	$-0.05 - 0.02i$	$-0.08 + 0.04i$
	$A(P_T)$	-0.06	$0.50 - 0.1i$	$-0.08 + 0.00i$	$-0.34 - 0.14i$

TABLE III. Polarization amplitudes of different diagrams for the decays $\bar{B}^0 \rightarrow a_1^+ \rho^-$, $a_1^0 \rho^0 (\times 10^{-2} \text{ GeV}^3).$

diagram. So one expects that their branching ratios are also very small.

From Table [II](#page-5-0), one can find that our predictions are well consistent with the results calculated by the QCDF approach for most decays. Certainly, there also exist large differences for some decays, which are needed to clarify by the present LHCb experiments. At present, BABAR has given the upper limits of the branching ratios for the decays $B \rightarrow b_1 \rho$, ranging from 1.4–5.2 \times 10⁻⁶ at the 90% confidence level [[8](#page-11-5)], which are not far from our predictions for the decays $\bar{B}^0 \rightarrow b_1^0 \rho^0$ and $B^- \rightarrow b_1^- \rho^0$, but much smaller than those of $\bar{B}^0 \rightarrow b_1^+ \rho^-$ and $B^- \rightarrow b_1^0 \rho^-$. In Ref. [[7\]](#page-11-4), the *BABAR* Collaboration searched the decay $\bar{B}^0 \to a^{\pm}_{1} \rho^{\mp}$ and obtained an upper limit of 61 \times 10⁻⁶ by assuming that a^{\pm} obtained an upper limit of 61×10^{-6} by assuming that a_1^{\pm} decays exclusively to $\rho^0 \pi^{\pm}$. Our prediction for the branching ratio of $\bar{B}^0 \rightarrow a_1^{\pm} \rho^{\mp}$ is about 60×10^{-6} , which agrees with the experiment.

In Table [IV,](#page-6-0) we list the polarization fractions of the $B \rightarrow$ $a_1\rho(\omega)$, $b_1\rho(\omega)$ decays and find that the longitudinal polarizations are dominant in most of these decays, which occupy more than 80%. For the tree-dominated decays, the main contributions come from the factorizable emission diagrams, where the two kinds of transverse polarization amplitudes are highly suppressed by the aforementioned factor $r_{a_1(b_1)} \cdot r_{\rho(\omega)}$. From Table [IV,](#page-6-0) one can find that f_{\parallel} and f_{\perp} have near values and both about a few percent in general. Certainly, for the decays $\bar{B}^0 \rightarrow a_1^0 \rho^0$ and $B^- \rightarrow$ $b_1^- \rho^0(\omega)$, their polarization fractions are very different

TABLE IV. Longitudinal polarization fraction (f_L) and two transverse polarization fractions $(f_{\parallel}, f_{\perp})$ for the decays $B \rightarrow$ $a_1(1260)\rho(\omega)$ and $B \to b_1(1235)\rho(\omega)$. In our results, the uncertainties of f_L come from ω_B and threshold resummation parameter c . The results of f_L predicted by the QCDF approach are also displayed in parentheses for comparison.

	$f_I(\%)$	$f_{\parallel}(\%)$	$f_+(\%)$
$\bar{B}^0 \rightarrow a_1^+ \rho^-$	$90.7^{+0.2+1.3}_{-0.2-1.3}(82^{+5}_{-13})$	3.9	5.4
$\bar{B}^0 \rightarrow a_1^- \rho^+$	$90.4^{+0.0+0.1}_{-0.1-0.1}$ (84^{+2}_{-6})	5.2	4.4
$\bar{B}^0 \rightarrow a_1^0 \rho^0$	$43.3^{+1.2+2.9}_{-1.3-2.9}(82^{+6}_{-68})$	29.7	27.0
$B^{-} \to a_1^0 \rho^{-}$	$93.6^{+0.2+0.1}_{-0.2-0.1} (91^{+3}_{-10})$	2.8	3.6
$B^{-} \to a_{1}^{-} \rho^{0}$	$82.3^{+0.1+2.0}_{-0.3-2.0}(89^{+11}_{-18})$	9.3	8.4
$\bar{B}^0 \rightarrow a_1^0 \omega$	$80.7^{+0.3+3.4}_{-0.1-3.4}(75^{+11}_{-65})$	9.9	9.4
$B^{-} \rightarrow a_{1}^{-} \omega$	$79.5^{+0.6+2.2}_{-0.6-2.2}(88^{+10}_{-14})$	8.9	11.6
$\bar{B}^0 \rightarrow b_1^+ \rho^-$	$95.4^{+0.2+0.1}_{-0.1-0.1} (96^{+1}_{-2})$	2.2	2.4
$\bar{B}^0 \rightarrow b_1^- \rho^+$	$95.8^{+0.5+1.1}_{-0.5-1.1} (98^{+0.9}_{-3.3})$	1.7	2.5
$\bar{B}^0 \rightarrow b_1^0 \rho^0$	$95.3^{+0.2+0.4}_{-0.4-0.4}(99^{+0}_{-18})$	2.8	1.9
$B^{-} \to b_1^0 \rho^{-}$	$92.5^{+0.9+0.6}_{-1.1-0.6} (96^{+1}_{-6})$	0.8	6.7
$B^{-} \to b_{1}^{-} \rho^{0}$	$44.9^{+1.8+5.6}_{-2.0-5.6}(90^{+5}_{-38})$	1.1	54.0
$\bar{B}^0 \rightarrow b_1^0 \omega$	$93.5^{+0.2+0.3}_{-0.1-0.3}(4^{+96}_{-0})$	4.3	2.2
$B^{-} \rightarrow b_{1}^{-} \omega$	73.1 ^{+0.5+1.0} (91 ⁺⁷ ₋₃₃)	25.5	1.4

from those of other decays. In the decay $\bar{B}^0 \rightarrow a_1^0 \rho^0$, the contributions from the two transverse polarization components become prominent and are larger than that from the longitudinal component. It is because the decay is suppressed by the cancellation of the Wilson coefficients C_1 + $C_2/3$ for the color-suppressed amplitude. So the contribution from the factorizable emission diagrams become very small. The left dominant contributions are the nonfactorizable amplitudes from tree operators, where neither of the transverse polarizations is suppressed compared with the longitudinal polarization. Therefore, numerically we get a small longitudinal polarization fraction of about 43%. In Table [V,](#page-6-1) if we ignore the contribution from the nonfactorizable amplitudes of $\bar{B}^0 \rightarrow a_1^0 \rho^0$ and find that the longitudinal polarization becomes dominant, but the branching ratio becomes very small. If we ignore the contributions from its penguin operators or annihilation diagrams, the results have small changes. As to the other charged decays $B^- \to b_1^- \rho^0(\omega)$, either of their transverse polarizations is very sensitive to the contributions listed in lines (2)–(4) in Table [V.](#page-6-1)

Now we turn to the evaluations of the CP-violating asymmetries in the pQCD approach. The CP asymmetries of $B^0/\overline{B}^0 \rightarrow a_1^{\pm} (b_1^{\pm}) \rho^{\mp}$ are very complicated and left for future study. Here, we only research the decays $B^- \rightarrow$ $a_1^0(b_1^0)\rho^-$ and $\bar{B}^0 \to b_1^0 \rho^0(\omega)$, where the transverse polarization fractions are very small and range from 4.7 to 7.5%. Using Eqs. [\(29](#page-4-0)) and ([31](#page-4-1)), one can get the expression for the direct CP-violating asymmetry:

$$
\mathcal{A}_{CP}^{\text{dir}} = \frac{|\bar{\mathcal{M}}|^2 - |\mathcal{M}|^2}{|\mathcal{M}|^2 + |\bar{\mathcal{M}}|^2} = \frac{2z_L \sin \alpha \sin \delta_L}{(1 + 2z_L \cos \alpha \cos \delta_L + z_L^2)}.
$$
 (33)

Here for the four decays we consider, the contributions from the transverse polarizations are very small, so we neglected them in our calculations. Using the input parameters and the wave functions as specified in this section

TABLE V. Contributions from different parts in the decays $\bar{B}^0 \to a_1^0 \rho^0$ and $B^- \to b_1^- \omega$: line (1) is for full contribution, line (2), (3) and (4) are the contributions after ignoring annihilation diagrams, penguin operators and nonfactorization diagrams, respectively.

$\rightarrow a_1^0 \rho^0$ \bar{B}^0	$Br(10^{-7})$	$f_L(\%)$	$f_{\parallel}(\%)$	$f_+(\%)$
(1)	6.4	43.3	29.7	27.0
(2)	5.1	28.4	40.4	31.2
(3)	6.3	42.5	30.1	27.4
(4)	0.2	86.1	9.4	4.5
$B^{-} \rightarrow b_{1}^{-} \omega$	$Br(10^{-6})$	$f_L(\%)$	$f_{\parallel}(\%)$	$f_+(\%)$
(1)	2.1	73.1	25.5	1.4
(2)	0.9	63.5	18.5	18.0
(3)	0.7	67.9	0.1	32.0
(4)	1.8	83.2	11.1	5.7

and Sec. [II](#page-1-0), one can find the pQCD predictions (in units of 10^{-2}) for the direct CP-violating asymmetries of the decays considered:

$$
\mathcal{A}_{CP}^{\text{dir}}(B^{-} \to a_{1}^{0} \rho^{-}) = 11.8^{+1.6+0.0}_{-1.4-0.0},\tag{34}
$$

$$
\mathcal{A}_{CP}^{\text{dir}}(B^- \to b_1^0 \rho^-) = -3.7^{+0.4+1.2}_{-0.3-1.2},\tag{35}
$$

$$
\mathcal{A}_{CP}^{\text{dir}}(\bar{B}^0 \to b_1^0 \rho^0) = 23.8^{+4.3+1.9}_{-4.2-1.9},\tag{36}
$$

$$
\mathcal{A}_{CP}^{\text{dir}}(\bar{B}^0 \to b_1^0 \omega) = 80.3^{+3.8+3.2}_{-4.8-3.2},\tag{37}
$$

where the errors are induced by the uncertainties of the B meson shape parameter $\omega_b = 0.4 \pm 0.04$ and the threshold resummation parameter c , vary from 0.3 to 0.4.

V. NUMERICAL RESULTS AND DISCUSSIONS FOR B_s DECAYS

A. CP-averaged branching ratios

The decays $\bar{B}_s^0 \to a_1^- K^+ (K^{*+})$ have contributions from the factorization emission diagrams with a large Wilson coefficient $C_2 + C_1/3$ (order of 1), so they have the largest branching ratios and arrive at the order 10^{-5} shown in Table [VI](#page-8-0). While for the decays $\bar{B}_s^0 \to a_1^0 K^0(K^{*0})$, the Wilson coefficient is $C_1 + C_2/3$ in tree level and color suppressed, so their branching ratios are small and fall in the order of $10^{-7} \sim 10^{-8}$. Although the decay $\bar{B}_s^0 \to a_1^0 K^0$ is tree dominated, the contributions from tree operators between the factorization and nonfactorization emission diagrams cancel each other mostly, which induces its tree amplitudes to have a very small real part. It does not happen in the channel $\bar{B}_s^0 \rightarrow a_1^0 K^{*0}$. At the same time, there exist three polarization states for the final mesons and the transverse polarizations are about 30%. So the decay mode $a_1^0 K^{*0}$ has a larger branching ratio compared with the mode $a_1^0 K^0$. For the decay $\bar{B}_s^0 \to b_1^0 K^0$, the amplitude of the nonfactorization emission diagrams M_{ek}^T (T denotes the contribution from tree operators), including the large Wilson coefficient C_2 receives a larger value, which is about 5 times the decay $a_1^0 K^0$. Furthermore, because of the vanishing decay constant f_{b_1} , the amplitude F_{eK} becomes zero for the decay $b_1^0 K^0$, while it has a large value but the opposite sign with amplitude M_{ek} in the decay $a_1^0 K^0$. So one can find that there is a much larger contribution from the tree operator for the decay $b_{\parallel}^0 K^0$ than that for the decay $a_1^0 K^0$. The decay $\bar{B}_s^0 \to b_1^0 K^{*0}$ has a large branching ratio, which is also because of the large contribution from the nonfactorizable emission diagrams.

The decays $\bar{B}_s^0 \to a_1(b_1)\pi(\rho, \omega)$ belong to the annihilation type decays, contributed by the W-annihilation and W-exchange diagrams. The decays $\bar{B}_s^0 \to a_1(b_1)\pi(\rho)$ are sensitive to the wave functions of the final states. If the final mesons are π and a_1 , the branching ratios can arrive at order 10^{-6} , while for the π and b_1 final states, the branching ratios become of order 10^{-7} even smaller. In a word, $\mathcal{B}(\bar{B}_s^0 \to a_1\pi) > \mathcal{B}(\bar{B}_s^0 \to b_1\pi)$. The condition is contrary for the decay modes $a_1(b_1)\rho$. The branching ratios of the decays $\bar{B}_s^0 \rightarrow \rho^+ a_1^-$, $\rho^0 a_1^0$, $\rho^- a_1^+$ are very near each other. A similar case exists with the decays $\bar{B}_s^0 \to \rho^+ \pi^-$, $\rho^0 \pi^0$, $\rho^-\pi^+$, whose branching ratios are predicted as $(2.2, 2.3, 2.4) \times 10^{-7}$ [\[3\]](#page-11-27), respectively. As for the other two annihilation type decays $\bar{B}_s^0 \rightarrow a_1^0 \omega$, $b_1^0 \omega$, whose branching ratios are of the order of $10^{-8} \sim 10^{-9}$. It is easy to see that this kind decay is sensitive to the quark structure of the final mesons. Compared with the decays $\bar{B}_{\underline{s}}^0 \rightarrow a_1^0(b_1^0)\rho^0$, the difference is mainly from the signs of $d\bar{d}$ component in the mesons ω and ρ^0 , which induces different interference effects between the amplitudes from the penguin operators: constructive for the decays $a_1^0(b_1^0)\rho^0$, destructive for the decays $a_1^0(b_1^0)\omega$. From our calculations, we find that the penguin amplitude for the decay $a_1^0(b_1^0)\rho^0$ is about 20.4 (48.2) times of that for the decay $a_1^0(b_1^0)\omega$.

The main contributions to the decays $\bar{B}_s^0 \rightarrow a_1(b_1)\eta^{(i)}$ are from the electroweak (EW) penguin operators. Although the contributions from the tree operators have a prominent increase for the decays $\bar{B}_s^0 \rightarrow b_1^0 \eta^{(i)}$ compared with those for the decays $\bar{B}_s^0 \to a_1^0 \eta^{(i)}$ [the former are about 5 (7) times larger than the latter], the increased tree operator contributions for the decays $\bar{B}_s^0 \to b_1^0 \eta^{(i)}$ bring a slight increase to the branching ratios, for the tree operator contributions are Cabibbo-Kobayashi-Maskawa suppressed.

We also checked the sensitivity to the values on the Gegenbauer moments for the decays considered. If one takes smaller Gegenbauer moments, such as $a_1^K = 0.05 \pm 1$ 0.02 [[34](#page-11-28)], 0.10 ± 0.12 [\[35\]](#page-11-29), $a_2^{\pi,K} = 0.115$ [\[36\]](#page-11-30), the branching ratios have a few percent change for most of the decays $\bar{B}_s^0 \rightarrow a_1(b_1)\pi(K)$, more than a 10% change for only very few channels. So we considered that the uncertainties caused by the Gegenbauer moments are small and can be neglected. But it is not the case for the decays $\bar{B}_s^0 \rightarrow$ $a_1(b_1)\eta^{(l)}$. If one takes the newer Gegenbauer moments as given in Ref. [\[36\]](#page-11-30):

$$
a_2^{\pi} = 0.115, \qquad a_4^{\pi} = -0.015, \tag{38}
$$

the branching ratios will have a prominent change,

$$
\mathcal{B}(\bar{B}_s^0 \to a_1^0 \eta) = (0.97^{+0.33}_{-0.34} \times 0.01^{+0.01}_{-0.17} \times 2.3) \times 10^{-7}, \quad (39)
$$

$$
\mathcal{B}(\bar{B}_s^0 \to a_1^0 \eta') = (2.1^{+0.7+0.0+0.0+0.8}_{-0.5-0.0-0.2-0.8}) \times 10^{-7},\qquad(40)
$$

$$
\mathcal{B}(\bar{B}_s^0 \to b_1^0 \eta) = (0.21^{+0.00+0.03+0.05+0.02}_{-0.05-0.05-0.05-0.11}) \times 10^{-7}, \quad (41)
$$

$$
\mathcal{B}(\bar{B}_s^0 \to b_1^0 \eta') = (0.75^{+0.00+0.00+0.16+0.06}_{-0.17-0.16-0.16-0.35}) \times 10^{-7}, \quad (42)
$$

where the errors come from the B_s meson wave function shape parameter $\omega_B = 0.5 \pm 0.05$ GeV, the B_s meson

TABLE VI. Branching ratios (in units of 10^{-6}) for the decays $\bar{B}_s^0 \to a_1(b_1)K(\pi, \eta, \eta')$ and $\bar{B}_s^0 \to a_1(b_1) K^*(\rho, \omega, \phi)$. In our results, the errors for these entries correspond to the uncertainties from the B_s meson wave function shape parameter ω_B , the B_s meson decay constant f_{B_s} , the QCD scale $\Lambda_{\text{QCD}}^{(5)}$ and the threshold resummation parameter c, respectively.

Decay mode	$Br(\times 10^{-6})$	Decay mode	$Br(\times 10^{-6})$
$\bar{B}^0_s \rightarrow a_1^0 K^0$	$0.081^{+0.016+0.005+0.013+0.029}_{-0.010-0.005-0.011-0.029}$	$\bar{B}^0_s \to a^0_1 K^{*0}$	$0.69_{-0.13 - 0.04 - 0.12 - 0.12}^{+0.19 + 0.03 + 0.10 + 0.12}$
$\bar{B}_s^0 \rightarrow a_1^- K^+$	$21.4 + 8.1 + 0.1 + 0.9 + 7.0$ $-5.5 - 0.0 - 1.5 - 7.0$	$\bar{B}^0_s \to a_1^- K^{*+}$	$29.4^{+10.3+0.1+0.6+9.8}_{-7.2-0.1-1.8-9.8}$
$\bar{B}_s^0 \rightarrow a_1^- \pi^+$	$2.7^{+0.7+0.2+0.3+0.0}_{-0.5-0.1-0.4-0.0}$	$\bar{B}_s^0 \rightarrow a_1^- \rho^+$	$0.38^{+0.3+0.1+0.5+0.8}_{-0.3-0.1-0.7-0.8}$
$\bar{B}^0_s \rightarrow a_1^+ \pi^-$	$1.8 + 0.5 + 0.0 + 0.2 + 0.0$ $-0.4 - 0.1 - 0.3 - 0.0$	$\bar{B}_s^0 \rightarrow a_1^+ \rho^-$	$0.37^{+0.2+0.1+0.3+0.4}_{-0.5-0.1-0.7-0.4}$
$\bar{B}^0_s \rightarrow a_1^0 \pi^0$	$2.2^{+0.7+0.1+0.4+0.0}_{-0.4-0.0-0.2-0.0}$	$B_s^0 \rightarrow a_1^0 \rho^0$	$0.38^{+0.3+0.1+0.5+0.6}_{-0.2-0.1-0.6-0.6}$
$\bar{B}^0_s \rightarrow a_1^0 \eta$	$2+0.04+0.00+0.00+0.03$ $-0.04-0.00-0.02-0.03$	$\bar{B}_s^0 \rightarrow a_1^0 \omega$	$0.0049^{+0.0003+0.0003+0.0005+0.0004}_{-0.0003-0.0004-0.0002-0.0004}$
$\bar{B}^0_s \rightarrow a_1^0 \eta'$	$0.30^{+0.09+0.02+0.00+0.10}_{-0.08-0.01-0.03-0.10}$	$\bar{B}_s^0 \rightarrow a_1^0 \phi$	$0.33_{-0.08 - 0.00 - 0.03 - 0.12}^{+0.13 + 0.00 + 0.02 + 0.12}$
$\bar{B}^0_s \rightarrow b^0_1 K^0$	$2.8 + 0.5 + 0.1 + 0.4 + 0.1$ $-0.4 - 0.0 - 0.3 - 0.1$	$\bar{B}^0_s \to b^0_1 K^{*0}$	$3.5^{+0.6+0.1+0.6+0.1}_{-0.5-0.2-0.6-0.1}$
$\bar{B}^0_s \rightarrow b_1^- K^+$	$1.3_{-0.2-0.1-0.3-0.0}^{+0.1+0.0+0.1+0.0}$	$\bar{B}^0_s \to b_1^- K^{*+}$	$2.0^{+0.2+0.1+0.2+0.3}_{-0.2-0.1-0.3-0.3}$
$\bar{B}^0_s \rightarrow b_1^- \pi^+$	$0.079^{+0.013+0.001+0.006+0.000}_{-0.013-0.000-0.004-0.000}$	$\bar{B}_s^0 \rightarrow b_1^- \rho^+$	$0.88^{+0.06+0.01+0.19+0.05}_{-0.08-0.02-0.18-0.05}$
$\bar{B}_s^0 \rightarrow b_1^+ \pi^-$	$0.17^{+0.02+0.00+0.02+0.00}_{-0.02-0.00-0.01-0.00}$	$\bar{B}_s^0 \rightarrow b_1^+ \rho^-$	$1.1_{-0.1-0.0-0.3-0.0}^{+0.1+0.0+0.2+0.0}$
$\bar B^0_s\to b_1^0\pi^0$	$0.085_{-0.017 - 0.000 - 0.013 - 0.000}^{+0.025 + 0.000 + 0.002 + 0.000}$	$\bar{B}^0_s \rightarrow b^0_1 \rho^0$	$0.95^{+0.04+0.01+0.25+0.03}_{-0.06-0.01-0.24-0.03}$
$\bar B^0_s\to b^0_1\eta$	$0.13^{+0.05+0.01+0.01+0.01}_{-0.01-0.01-0.00-0.01}$	$\bar{B}_s^0 \rightarrow b_1^0 \omega$	$0.011_{-0.001}^{+0.001}$ + 0.000 + 0.001 + 0.002 -0.001 - 0.000 - 0.000 - 0.002
$\bar B^0_s\to b^0_1\eta^\prime$	$0.32^{+0.09+0.02+0.00+0.02}_{-0.04-0.00-0.01-0.02}$	$\bar{B}_s^0 \rightarrow b_1^0 \phi$	$0.21_{ -0.03 -0.00 -0.04 -0.00}^{ +0.04 +0.00 +0.03 +0.00}$

decay constant $f_{B_s} = 0.23 \pm 0.02$ GeV, the QCD scale $\Lambda_{\text{QCD}}^{(5)} = 0.25 \pm 0.05 \text{ GeV}$ and threshold resummation parameter c varying from 0.3 to 0.4, respectively. Especially for the decays $\bar{B}_s^0 \rightarrow a_1^0 \eta^{(i)}$, their branching ratios are sensitive to Gegenbauer moments and increase to 7–8 times by using the newer Gegenbauer moments. Certainly, the increases of the branching ratios for the decays $\bar{B}_s^0 \rightarrow b_1^0 \eta^{(i)}$ are not so large. It is needs to clarify which Gegenbauer moments are more reasonable.

The decays $\bar{B}_s^0 \rightarrow a_1(b_1)\phi$ are dominated by the EW penguin operators. Though their branching ratios are small, these two decays are interesting to invest the effect from the electroweak penguins, where there might exist new physics [\[37\]](#page-11-31). The presence of a new physics contribution from EW can enhance the branching ratios of the decays $\bar{B}_s^0 \to \pi(\rho)\phi$, which are used to improve the $B \to \pi K$ "puzzle" [\[38\]](#page-11-32). Whether the two decays considered here have such an effect merits further research.

B. Polarization in the decays $B_s \to a_1(b_1)V$

The formalism of the wave function has great influence on the polarization fractions for some decays. In Ref. [[39\]](#page-11-33), the author suggested that taking the asymptotic models for the K^* meson distribution amplitudes instead of its traditional formalism leads to a smaller $B \to K^*$ form factor $(A_0 \sim 0.3)$. The smaller form factor responds to the smaller longitudinal polarization fraction. Another result is that the strengthened penguin annihilation and nonfactorizable

contributions further bring it down. In the decays $\bar{B}_s^0 \rightarrow$ $a_1(b_1)K^*$, we also take the asymptotic models for the K^* meson wave functions and only find the decay mode $a_1^0 K^{*0}$ with a smaller longitudinal polarization fraction of about 70%. If we neglect the penguin annihilation contribution in the decay $\bar{B}_s^0 \to a_1^0 K^{*0}$, and find that the branching ratio changes from 6.9×10^{-7} to 5.5×10^{-7} , the longitudinal polarization receives a larger increase and arrives at 93.1%. If we neglect the nonfactorizable contribution, both the branching ratio and the polarization fractions will become much smaller. Compared with the $\bar{B}_s^0 \to a_1^0 K^{*0}$ and $\bar{B}_s^0 \to a_2^0 K^{*0}$ $b_1^0 K^{*0}$ decays, we argue that the polarization fractions are also connected with the symmetric properties of the a_1 and b_1 distribution amplitudes, which might have a sensitive effect in the penguin annihilation contribution. If one neglects the penguin annihilation contribution in the decay $\bar{B}_s^0 \rightarrow b_1^0 \omega$, the longitude fraction can amount to 95.4% and the branching ratio decreases by 30%. In a word, the contributions from the penguin annihilation diagrams are very sensitive to the final polarization fractions for some decays.

In Table [VII,](#page-9-0) we list the longitudinal polarization fraction (f_L) and the transverse polarization fractions $(f_{\parallel}, f_{\perp})$ for the decays $\bar{B}_s^0 \rightarrow a_1(b_1)V$, where the errors come from the B_s meson wave function shape parameter $\omega_b = 0.5 \pm 0.05$ GeV, the B_s meson decay constant $f_{B_s} = 0.23 \pm 0.02$ GeV, the QCD scale $\Lambda_{QCD}^{(5)} = 0.25 \pm 0.25$ 0.05 GeV and the threshold resummation parameter c,

TABLE VII. Longitudinal polarization fraction (f_L) and two transverse polarization fractions $(f_{\parallel}, f_{\perp})$ for the decays $\bar{B}_s^0 \to a_1(b_1)V$. In our results, the uncertainties of f_L , f_{\parallel} , f_{\perp} come from the B_s meson wave function shape parameter ω_b , the B_s meson decay constant f_{B_s} , the QCD scale $\Lambda_{\text{QCD}}^{(5)}$ and threshold resummation parameter c, respectively.

	$f_I(\%)$	$f_{\parallel}(\%)$	$f_{\perp}(\%)$
$\bar{B}^0_s \rightarrow a_1^0 K^{*0}$	$68.9_{-6.4-2.8-2.4-3.9}^{+6.1+2.7+1.5+3.9}$	$15.1_{-3.0-1.2-0.9-2.0}^{+3.1+1.3+0.9+2.0}$	$16.0^{+3.4+1.5+1.6+1.9}_{-3.1-1.4-0.8-1.9}$
$\bar{B}_s^0 \rightarrow a_1^- K^{*+}$	$90.6^{+0.2+0.3+0.2+0.1}_{-0.3-0.2-0.3-0.1}$	$4.9^{+0.1+0.0+0.2+0.1}_{-0.1-0.1-0.0-0.1}$	$4.5^{+0.1+0.1+0.2+0.1}_{-0.2-0.1-0.1-0.1}$
$\bar{B}_s^0 \rightarrow a_1^- \rho^+$	$97.7_{-0.3-0.4-1.2-0.9}^{+0.1+0.5+0.6+0.9}$	$2.2^{+0.2+0.3+1.1+0.8}_{-0.1-0.2-0.6-0.8}$	$0.1_{-0.0}^{+0.0+0.0+0.1+0.1}_{-0.0-0.0-0.0-0.1}$
$\bar{B}_s^0 \rightarrow a_1^+ \rho^-$	$97.8^{+0.2+0.4+0.6+1.0}_{-0.2-0.3-1.1-1.0}$	$2.1_{-0.2-0.5-0.2-1.0}^{+0.2+1.1+0.3+1.0}$	$0.1_{-0.0}^{+0.0+0.0+0.0+0.1}_{-0.0-0.0-0.0-0.1}$
$\bar{B}^0_s \rightarrow a_1^0 \rho^0$	$97.8^{+0.2+0.3+0.6+1.0}_{-0.1-0.3-1.0-1.0}$	$2.1_{-0.1-0.3-0.6-0.9}^{+0.1+0.3+1.0+0.9}$	$0.1_{-0.1}^{+0.0+0.0+0.1+0.1}_{-0.1-0.0-0.1-0.1}$
$B_s^0 \rightarrow a_1^0 \omega$	$83.4^{+1.0+2.4+3.5+6.1}_{-0.9-2.2-2.5-6.1}$	$9.8^{+0.5+1.4+1.0+2.8}_{-0.4-1.3-2.2-2.8}$	$6.8^{+0.3+0.8+1.5+2.3}_{-0.4-0.9-1.4-2.3}$
$\bar{B}^0_s \rightarrow a_1^0 \phi$	$94.8^{+0.0+0.1+0.0+0.2}_{-0.0-0.1-0.0-0.2}$	$2.8^{+0.0+0.0+0.0+0.1}_{-0.0-0.0-0.0-0.1}$	$2.4^{+0.0+0.0+0.0+0.1}_{-0.0-0.0-0.1-0.1}$
$\bar{B}^0_s \rightarrow b^0_1 K^{*0}$	$98.2_{-0.4-0.2-0.4-0.3}^{+0.2+0.2+0.2+0.2+0.3}$	$0.9^{+0.1+0.1+0.1+0.1}_{-0.1-0.1-0.1-0.1}$	$0.9_{-0.1-0.1-0.1-0.1}^{+0.1+0.2+0.3+0.1}$
$\bar{B}_s^0 \to b_1^- K^{*+}$	$94.1_{-0.7-0.6-1.2-1.7}^{+0.7+0.6+0.8+1.7}$	$2.8^{+0.3+0.3+0.6+0.8}_{-0.3-0.2-0.4-0.8}$	$3.1_{-0.4-0.4-0.4-0.9}^{+0.3+0.3+0.7+0.9}$
$\bar{B}_s^0 \rightarrow b_1^+ \rho^-$	$96.9_{-0.3-0.6-2.3-2.7}^{+0.3+0.5+0.9+2.7}$	$2.3^{+0.2+0.4+1.8+2.0}_{-0.2-0.4-1.0-2.0}$	$0.8^{+0.1+0.1+0.5+0.7}_{-0.1-0.1-0.6-0.7}$
$\bar{B}_s^0 \rightarrow b_1^- \rho^+$	$91.6^{+0.4+1.5+3.1+4.5}_{-0.6-1.4-5.7-4.5}$	$8.1_{-0.4-1.2-3.0-5.5}^{+0.5+1.4+5.5+5.5}$	$0.3^{+0.1+0.0+0.2+0.1}_{-0.0-0.1-0.1}$
$\bar{B}^0_s \rightarrow b^0_1 \rho^0$	$95.0^{+0.2+0.8+1.9+3.8}_{-0.4-0.9-4.1-3.8}$	$4.7^{+0.3+1.1+3.8+3.6}_{-0.2-1.2-1.8-3.6}$	$0.3^{+0.0+0.1+0.3+0.3}_{-0.0-0.1-0.1-0.3}$
$\bar{B}^0_s \rightarrow b^0_1 \omega$	$63.4_{-2.6-3.8-12.5-12.2}^{+3.3+3.7+12.7+12.2}$	$21.7^{+1.7+2.4+7.6+8.2}_{-2.0-2.2-7.5-8.2}$	$14.8 + 1.0 + 1.5 + 5.0 + 4.2$ $-1.2 - 1.4 - 5.1 - 4.2$
$\bar{B}^0_s \rightarrow b^0_1 \phi$	$99.5^{+0.0+0.0+0.0+0.0}_{-0.0-0.0-0.0-0.0}$	$0.25^{+0.01+0.00+0.01+0.00}_{-0.03-0.00-0.03-0.00}$	$0.25_{-0.01}^{+0.01}$ + 0.00 + 0.03 + 0.00 0.25 - 0.01 - 0.00 - 0.01 - 0.00

varying from 0.3 to 0.4, respectively. With the exception of the decays $\bar{B}_s^0 \to a_1^0 K^{*0}$, $a_1^0 \omega$, $b_1^0 \omega$, the longitudinal polarization fractions of other $\bar{B}_s^0 \rightarrow a_1(b_1)V$ decays are very large and more than 90%.

C. Direct CP-violating asymmetries

Considering the smallness (only about a few percent) of the transverse polarization fractions of most decays $\bar{B}_s^0 \rightarrow$ $a_1(b_1)V$, we can neglect them in our calculations and the expression for the direct CP-violating asymmetries of the decays $\bar{B}_s^0 \to a_1(b_1)V$ (with the exception of $\bar{B}_s^0 \to a_1^0 K^{*0}$, $a_1^0\omega$, $b_1^0\omega$) become simple, which can be calculated by Eq. ([33](#page-6-2)). Certainly, for the $b \rightarrow s$ transition, one only needs to replace α with $\gamma = \arg[-\frac{V_{tb}V_{ts}^*}{V_{ub}V_{us}^*}]$ in Eq. ([33](#page-6-2)). The direct *CP*-violating asymmetries for the decays $\bar{B}_s^0 \rightarrow a_1(b_1)P$ have a similar expression. We calculate the pQCD predictions (in units of 10^{-2}) for the direct CP-violating asymmetries of the decays considered, which are listed in Table [VIII](#page-10-1), where the errors induced by the uncertainties of $\omega_b = 0.5 \pm 0.05$ GeV, $f_{B_s} = 0.23 \pm 0.02$ GeV, $\Lambda_{QCD}^{(5)} =$ 0.25 ± 0.05 GeV and the threshold resummation parameter c varying from 0.3 to 0.4, respectively. We find the following points:

(i) Like the decay $\bar{B}_s^0 \to \pi^0 K^0$, whose direct CP asymmetry is predicted at more than 40% by several methods [\[3,](#page-11-27)[40,](#page-11-34)[41\]](#page-11-35), the decays $\bar{B}_s^0 \rightarrow a_1^0(b_1^0)K^0$ also have large direct CP asymmetries. Unlike the channel $\bar{B}_s^0 \to b_1^- K^+$, the decay $\bar{B}_s^0 \to a_1^- K^+$ has a smaller direct CP asymmetry. It is because that though there are near penguin amplitudes in these two decays, the tree amplitude of the latter is about 3 times as large as that of the former; furthermore, the sine values of their strong phases are close to each other, so the direct CP-asymmetry value for the decay mode $a_1^- K^+$ calculated by Eq. [\(33\)](#page-6-2) is small.

- (ii) The direct CP asymmetries of the decays $\bar{B}_s^0 \rightarrow$ $b_1^0 \eta^{(i)}$ are sensitive to take different Gegenbauer moments for $\eta^{(l)}$. If we take the newer Gegenbauer moments given in Eq. ([38](#page-7-1)), their direct CP asymmetries will change not only in magnitudes but also in signs.
- (iii) The decays $\bar{B}_{s}^0 \rightarrow a_1(b_1)\rho$, with the exception of the channel $\bar{B}_s^0 \rightarrow b_1^- \rho^+$, have smaller direct CP-violating asymmetries compared with the decays $\bar{B}_{s}^{0} \rightarrow a_{1}(b_{1})\pi$. The direct CP-violating asymmetry for the decay $\bar{B}_s^0 \rightarrow b_1^- \rho^+$ is very sensitive to the tree operator contribution from the nonfactorization annihilation diagrams: if we neglect such contribution, its branching ratio can increase 14%, while the direct CP-violating asymmetry becomes only 1.3%.
- (iv) There only exist factorization and nonfactorization emission diagrams for the decays $\bar{B}_s^0 \to a_1(b_1)\phi$. The direct CP-violating asymmetries in these two decays are small, because the interactions between the tree and penguin contributions are very small. From our calculations, we find the ratios of penguin

TABLE VIII. Direct CP-violating asymmetries (in units of %) for the decays $\bar{B}_{s}^{0} \rightarrow$ $a_1(b_1)K(\pi, \eta, \eta')$ and $\bar{B}_s^0 \to a_1(b_1)K^*(\rho, \omega, \phi)$ (with the exception of $\bar{B}_s^0 \to a_1^0K^{*0}$, $a_1^0\omega$, $b_1^0\omega$). In our results, the errors for these entries correspond to the uncertainties from ω_b , f_{B_s} , the QCD scale $\Lambda_{\text{QCD}}^{(5)}$ and the threshold resummation parameter c, respectively.

Decay mode	Direct CP	Decay mode	Direct CP
$\bar{B}^0_s \rightarrow a_1^0 K^0$	$-66.1_{-6.5-3.3-5.1-37.3}^{+10.4+3.3+4.2+37.3}$	$\bar{B}^0_s \rightarrow b^0_1 K^0$	$41.4 + 5.3 + 3.0 + 2.1 + 0.3$ $-5.0 - 3.1 - 0.8 - 0.3$
$\bar{B}^0_s \rightarrow a_1^- K^+$	$-9.7^{+1.4+0.8+0.4+0.7}_{-1.6-0.9-0.2-0.7}$	$\bar{B}_s^0 \rightarrow b_1^- K^+$	$-74.7^{+8.1+3.3+0.4+2.5}_{-7.3-2.6-3.6-2.5}$
$\bar{B}_s^0 \rightarrow a_1^- \pi^+$	$20.5 + 1.3 + 0.3 + 0.4 + 0.2$ $-1.3 - 0.0 - 0.6 - 0.2$	$\bar{B}^0_s \rightarrow b_1^- \pi^+$	$12.7^{+2.3+0.0+1.4+0.0}_{-3.3-0.1-2.6-0.0}$
$\bar{B}_s^0 \rightarrow a_1^+ \pi^-$	$3.2_{-0.3-0.1-0.8-0.2}^{+0.3+0.0+0.6+0.2}$	$\bar{B}_s^0 \rightarrow b_1^+ \pi^-$	$24.5^{+0.7+0.0+2.5+0.1}_{-3.4-0.0-5.3-0.1}$
$\bar{B}_s^0 \rightarrow a_1^0 \pi^0$	$14.0^{+1.1+0.1+0.2+0.0}_{-1.2-0.0-0.0}$	$\bar{B}_s^0 \rightarrow b_1^0 \pi^0$	$-23.3_{-1.3-0.1-2.8-0.2}^{+2.8+0.1+5.0+0.2}$
$\bar{B}_s^0 \rightarrow a_1^0 \eta$	$-31.3_{-2.8-0.2-5.2-4.1}^{+0.0+0.3+0.2+4.1}$	$\bar{B}_s^0 \rightarrow b_1^0 \eta$	$25.0^{+0.0+0.0+0.5+3.4}_{-4.0-2.8-4.8-3.4}$
$\bar{B}_s^0 \rightarrow a_1^0 \eta'$	$-10.2^{+1.4+1.2+2.3+2.1}_{-0.0-1.3-0.4-2.1}$	$\bar{B}_s^0 \rightarrow b_1^0 \eta'$	$22.7^{+0.0+0.0+0.9+2.4}_{-3.6-2.5-7.2-2.4}$
		$\bar{B}^0_s \to b^0_1 K^{*0}$	$2.7^{+4.2+0.3+5.8+3.2}_{-3.7-0.2-5.2-3.2}$
$\bar{B}^0_s \to a_1^- K^{*+}$	$-11.1^{+1.5+1.0+0.7+1.5}_{-1.7-0.9-0.5-1.5}$	$\bar{B}^0_s \to b_1^- K^{*+}$	$0.80^{+7.4+0.3+7.5+3.9}_{-7.4-0.2-6.6-3.9}$
$\bar{B}_s^0 \rightarrow a_1^- \rho^+$	$4.3^{+0.6+0.7+1.6+1.5}_{-0.4-0.5-3.3-1.5}$	$\bar{B}_s^0 \rightarrow b_1^- \rho^+$	$31.6^{+0.2+0.0+3.6+0.2}_{-0.2-0.1-2.8-0.2}$
$\bar{B}_s^0 \rightarrow a_1^+ \rho^-$	$6.0^{+2.1+1.1+2.4+3.1}_{-0.8-1.0-3.5-3.1}$	$\bar{B}_s^0 \rightarrow b_1^+ \rho^-$	$-9.3_{-0.6}^{+0.2+0.2+0.5+0.0}_{-0.6-0.3-0.4-0.0}$
$\bar{B}^0_s \rightarrow a_1^0 \rho^0$	$4.6^{+1.3+0.7+1.6+2.1}_{-1.5-0.8-2.9-2.1}$	$\bar{B}_s^0 \rightarrow b_1^0 \rho^0$	$8.3^{+0.2+0.1+0.8+0.0}_{-0.0-0.2-0.2-0.0}$
$\bar{B}_s^0 \rightarrow a_1^0 \phi$	$-6.2_{-1.4-0.0-1.9-0.9}^{+1.4+0.0+1.4+0.9}$	$\bar{B}_s^0 \rightarrow b_1^0 \phi$	$-0.81_{-0.15-0.00-0.12-0.00}^{+0.32+0.00+0.11+0.00}$

to tree amplitudes for the decays $\bar{B}_s^0 \rightarrow a_1 \phi$ and $\bar{B}_s^0 \rightarrow b_1 \phi$ are about 0.06 and 0.004, respectively. The strong phases penguin and tree amplitudes are only 0.15 and 0.026 rad, respectively.

(v) Compared with the decays $\bar{B}_s^0 \rightarrow a_1(b_1)P$, most of the $\bar{B}_s^0 \rightarrow a_1(b_1)V$ decays have smaller direct CP-violating asymmetries.

VI. CONCLUSION

In this paper, by using the decay constants and the lightcone distribution amplitudes derived from the QCD sumrule method, we researched the $B_{(s)} \to a_1(b_1)\rho(\omega, \phi)$ and $B_s \to a_1(b_1)K^*(K, \pi, \eta^{(l)})$ decays in the pQCD factorization approach and found that

- (i) with the exception of the decays $\bar{B}^0 \to a_1^0 \rho^0(\omega)$, other tree-dominated $B \to a_1 \rho(\omega)$ decays have larger branching ratios, at order 10^{-5} . With the exception of the decays $\bar{B} \to b_1^+ \rho^-$ and $B^- \to b_1^0 \rho^-$, other $B \to b_1 \rho(\omega)$ decays have smaller branching ratios, at order 10⁻⁶. The decays $B \to a_1(b_1)\phi$ are highly suppressed and have very small branching ratios, at order 10^{-9} .
- (ii) The decays $\bar{B}_s^0 \to a_1^- K^+ (K^{*+})$ have contributions from the factorization emission diagrams with a large Wilson coefficient $C_2 + C_1/3$ (order of 1), so they have the largest branching ratios and arrive at order 10^{-5} . While for the decays $\bar{B}_s^0 \rightarrow$ $a_1^0 K^0(K^{*0})$, the Wilson coefficient is $C_1 + C_2/3$ in tree level and color suppressed, so their branching ratios are small and fall in the order of $10^{-7} \sim 10^{-8}$.

For the decays $\bar{B}_s^0 \to b_1 K(K^*)$, all of their branching ratios are of order a few times 10^{-6} . For the pure annihilation type decays $\bar{B}^0_s \rightarrow a_1(b_1)\rho$, with the exception of the decays $\bar{B}_s^0 \rightarrow a_1 \pi$ having large branching ratios of order a few times 10^{-6} , most of them have branching ratios of order 10^{-7} . The branching ratios of the decays $\bar{B}_s^0 \rightarrow a_1^0(b_1^0)\omega$ are the smallest and fall in the order of $10^{-8} \sim 10^{-9}$.

- (iii) For the decays $\bar{B}^0 \rightarrow a_1^0 \rho^0$ and $B^- \rightarrow b_1^- \rho^0$, their two transverse polarizations are larger than their longitudinal polarizations, which are about 43.3% and 44.9%, respectively. The two transverse polarization fractions have near values in the decays $B \to a_1 \rho(\omega)$, while have large differences in some of the $B \to b_1 \rho(\omega)$ decays. With the exception of the decays $\overline{B}_s^0 \to a_1^0 K^{*0}$, $a_1^0 \omega$, $b_1^0 \omega$, the longitudinal polarization fractions of other $\bar{B}_s^0 \rightarrow$ $a_1(b_1)V$ decays are very large and more than 90%.
- (iv) For the decays $B^- \to a_1^0 \rho^-, b_1^0 \rho^-$ and $\bar{B}^0 \to b_1^0 \rho^0$, $b_1^0\omega$, where the transverse polarization fractions range from 4.7 to 7.5%, we calculate their direct CP-violating asymmetries, neglecting the transverse polarizations and find that those two charged decays have smaller values, which are about 11.8% and -3.7% , respectively. The branching ratios and the direct CP asymmetries of the decays $\bar{B}_s^0 \rightarrow$ $a_1^0(b_1^0)\eta^{(l)}$ are very sensitive to take different Gegenbauer moments for $\eta^{(i)}$. Compared with the decays $\bar{B}_s^0 \to a_1(b_1)P$, most of the $\bar{B}_s^0 \to a_1(b_1)V$ decays have smaller direct CP-violating asymmetries.

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