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Dimension-five effective operators in electroweak $SU(4)_L \otimes U(1)_X$ gauge models

Adrian Palcu

Faculty of Exact Sciences, "Aurel Vlaicu" University Arad, Street Elena Drăgoi 2, Arad-310330, Romania

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We prove in this paper that the electroweak $SU(4)_L \otimes U(1)_X$ gauge models with spontaneous symmetry breaking can offer a natural framework for generating neutrino masses by simply exploiting the tree level realization of dimension-five effective operators. The novelty of our approach resides in the fact that the scalar sector needs not to be enlarged, since these operators are constructed as direct products among scalar multiplets already existing in the model. There is a unique generic matrix for Yukawa couplings in the neutrino sector. The charged leptons are already in their diagonal basis. This framework can lead to a suitable fit of the established phenomenology for the left-handed neutrinos, while the right-handed neutrino masses come out in the sub-keV region, independently of the cutoff Λ. The latter introduces in the theory an intermediate scale (however, more close to GUT than to SM) at about 10^{12} GeV which is a crucial ingredient for the left-handed neutrino phenomenology.

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I. INTRODUCTION

One of the main challenges in particle physics [1] today is the neutrino mass issue. Both its origin and order of magnitude are still awaiting a compelling theoretical explanation. Observational collaborations [2] such as SuperKamiokande, K2K, SNO, KamLAND, LSND, and others have stated the phenomenon of neutrino oscillation as an indisputable evidence. This state of affairs claims for tiny but nonzero masses for neutrinos, regardless they will prove themselves as Dirac or Majorana particles. Consequently, the theory is called to supply a convenient framework and suitable mechanisms for generating these tiny masses, assuming that the standard model (SM) does not necessarily include right-handed neutrinos (otherwise unavoidable ingredients in accomplishing a nonzero mass term). The experimental side of the neutrino mass issue enforces certain restrictions, namely the observed mass splitting ratio $r_\Delta = \Delta m_\odot^2/\Delta m_{\mathrm{atm}}^2 \simeq 0.033$ and particular patterns for the mixing angles $\theta_\odot \simeq 34°$ and $\theta_\odot \simeq 45°$, along with a likely $\theta_{13} \simeq 0$. The absolute mass hierarchy remains still undetermined on theoretical grounds. What we only know at present is that it lies in the sub-eV region [1].

We mention the seesaw mechanism [3] (with its variants) and higher-dimension operators in effective theories [4] among the most appealing theoretical devices designed to accommodate neutrino phenomenology and predict viable consequences of it at low energies. These approaches generally require a larger framework than the one offered by the SM. For instance, the canonical seesaw mechanism essentially relies on a new higher scale (not subject in the gauge symmetry of the SM, but violating *B-L* symmetry) to generate Majorana right-handed masses. In a self-explanatory notation, the seesaw 6×6 matrix can be put as

$$M^{M+D} = \begin{pmatrix} 0 & m_D^T \\ m_D & M_R \end{pmatrix}. \qquad (1)$$

By diagonalizing it one can get both the left-handed and right-handed neutrino masses as

$$M(\nu_L) = -m_D^T M_R^{-1} m_D, \qquad M(\nu_R) = M_R. \qquad (2)$$

Therefore it seems worthwhile to investigate some extensions of the SM that include in a natural way right-handed neutrinos. During the past two decades, gauge models such as $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ [5,6] and $SU(3)_C \otimes SU(4)_L \otimes U(1)_X$ [7–9] have been intensely investigated. They offer promising results when addressing the neutrino issue. In most of the 3-3-1 models [6], a scalar sextet must be added to the Higgs sector in order to give rise to some seesaw mechanisms. In 3-4-1 models the same strategy was considered in Ref. [8] where a new Higgs decaplet is introduced in the scalar sector of the model in order to set up proper seesaw terms. Yet, this can affect the boson masses previously calculated, while some *ad hoc* hypothesis on the breaking scales of the decaplet must be speculated.

Our approach here avoids such new ingredients and proves itself able to set up the canonical seesaw mechanism via dimension-five effective operators constructed out of the existing ingredients in the model. It deals with the class of 3-4-1 models without exotic electric charges, so that even the exotic new quarks exhibit the $\pm\frac{2}{3}$ and $\pm\frac{1}{3}$ electric charges and mix with the traditional quarks of the SM. It also does not involve any enlargement of the scalar sector, but just makes use of the cutoff Λ up to which the model works as a safe renormalizable effective theory. The latter not only determines the highest bound for the validity of the theory but also plays a crucial role in predictions regarding the order of magnitude of the active neutrinos' masses.

Our paper is conceived as follows: after a brief overview of the gauge model and its main phenomenological features in Sec. II, we focus on its lepton content in Sec. III presenting the mass generating procedure based on dimension-5 effective operators in the neutrino sector. Section IV is reserved for some numerical estimates and predictions in a particular scenario taken into consideration, while in Sec. V we sketch our conclusions.

II. ELECTROWEAK $SU(4)_L \otimes U(1)_X$ GAUGE MODEL

We start here by briefly presenting the particular 3-4-1 gauge model under consideration here. However, the reader can find its phenomenological details treated *in extenso* in Ref. [8]. Evidently, our focus will go to its lepton sector, as we intend to exploit the realization of dimension-five effective operators responsible for giving rise (at the tree level) to the well-known seesaw terms.

In the electroweak gauge group $(SU(3)_C \otimes SU(4)_L \otimes U(1)_X$ the electric charge operator is a linear combination of diagonal Hermitian generators from Cartan subalgebra. It is realized in the manner: $Q = T_{3L} + \frac{1}{\sqrt{3}}T_{8L} + \frac{1}{\sqrt{6}}T_{15L} + \frac{1}{2}XI$, where $T_a = \frac{1}{2}\lambda_a$ is normalized as $\mathrm{Tr}(T_aT_b) = \frac{1}{2}\delta_{ab}$.

The lepton representations in this model are

$$f_L = \begin{pmatrix} e \\ \nu_e \\ N_e \\ N'_e \end{pmatrix}_L, \quad \begin{pmatrix} \mu \\ \nu_\mu \\ N_\mu \\ N'_\mu \end{pmatrix}_L, \quad \begin{pmatrix} \tau \\ \nu_\tau \\ N_\tau \\ N'_\tau \end{pmatrix}_L \sim (\mathbf{1}, \mathbf{4}^*, -1/2) \tag{3}$$

$$e_R, \mu_R, \tau_R \sim (\mathbf{1}, \mathbf{1}, -2). \tag{4}$$

For the sake of completeness, we present, in addition, the quark representations of the 3-4-1 model of interest here. In order to cancel all the chiral anomalies, two left-handed quark families must transform as

$$Q_\alpha = (\, u_\alpha \quad d_\alpha \quad D_\alpha \quad D'_\alpha\,)^T_L \sim (\mathbf{3}, \mathbf{4}, -1/6),$$

differently from a third generation which does it in the manner

$$Q_3 = (\, d_3 \quad u_3 \quad U \quad U'\,)^T_L \sim (\mathbf{3}, \mathbf{4}^*, 5/6).$$

Their right-handed partners are singlets with respect to the electroweak gauge group, namely d_{3R}, $d_{\alpha R}$, $D_{\alpha R}$, $D'_{\alpha R} \sim (\mathbf{3}, \mathbf{1}, -2/3)$, u_{3R}, $u_{\alpha R}$, U_R, $U'_R \sim (\mathbf{3}, \mathbf{1}, 4/3)$, with $\alpha = 1, 2$. Capital letters denote exotic quarks, yet their electric charges exhibit (in the particular class of models under consideration here) the same pattern as ordinary quarks. Note that other classes of 3-4-1 models [7] allow for new quarks with some exotic electric charges such as $\pm 4/3$ or $\pm 5/3$. However, the exact colored symmetry $SU(3)_C$ of the QCD remains to describe the strong interaction as a vector-like theory, and its predictions are not affected at low energies by this enlargement of content.

The gauge bosons of the electroweak sector occur in connection with the standard generators T_{aL} of the $su(4)$ algebra. In this basis, the gauge fields are A^0_μ of $U(1)_X$ and $A_\mu \in su(4)$, that is

$$A_\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} D_{1\mu} & W^+_\mu & K^+_\mu & X^+_\mu \\ W^-_\mu & D_{2\mu} & K^0_\mu & X^0_\mu \\ K^-_\mu & K^{0*}_\mu & D_{3\mu} & Y^0_\mu \\ X^-_\mu & X^{0*}_\mu & Y^{0*}_\mu & D_{4\mu} \end{pmatrix}, \tag{5}$$

with $D_{1\mu} = A^3_\mu/\sqrt{2} + A^8_\mu/\sqrt{6} + A^{15}_\mu/\sqrt{12}$, $D_{2\mu} = -A^3_\mu/\sqrt{2} + A^8_\mu/\sqrt{6} + A^{15}_\mu/\sqrt{12}$, $D_{3\mu} = -2A^8_\mu/\sqrt{6} + A^{15}_\mu/\sqrt{12}$, $D_{4\mu} = -3A^{15}_\mu/\sqrt{12}$ as diagonal Hermitian bosons. By inspecting Eq. (5), one notes that—apart from the charged Weinberg bosons ($W^\pm$)—there are several heavy degrees of freedom, namely two new charged bosons $K^\pm$ and $X^\pm$ along with X^0, K^0, and Y^0 (and their complex conjugated). The diagonal entries provide us with the neutral physical bosons: the massless photon A^{em}_μ for the electromagnetic interaction and massive Z_μ, along with two heavy neutral bosons Z'_μ and Z''_μ involved in the neutral currents of the model.

The scalar quadruplets in order to break the symmetry of the model stand in the following representations:

$$\phi^{(1)} = \begin{pmatrix} \chi^0 \\ \chi^-_1 \\ \chi^-_2 \\ \chi^-_3 \end{pmatrix} \sim (\mathbf{1}, \mathbf{4}, -3/2) \tag{6}$$

$$\phi^{(i)} = \begin{pmatrix} \rho^+ \\ \rho^0_1 \\ \rho^0_2 \\ \rho^0_3 \end{pmatrix}, \quad \begin{pmatrix} \eta^+ \\ \eta^0_1 \\ \eta^0_2 \\ \eta^0_3 \end{pmatrix}, \quad \begin{pmatrix} \xi^+ \\ \xi^0_1 \\ \xi^0_2 \\ \xi^0_3 \end{pmatrix} \sim (\mathbf{1}, \mathbf{4}, 1/2). \tag{7}$$

The superscripts denote their electric charge in units of e and $i = 2, 3, 4$.

The electroweak sector of the model provides us with two distinct couplings: g for $SU(4)_L$ and g_X for $U(1)_X$ respectively. Hence, the covariant derivatives read: $D_\mu = \partial_\mu - i(gA_\mu + g_X\frac{X}{2}A^0_\mu)$. Evidently, g is the SM coupling of the $SU(2)_L$ and g' of the $U(1)_Y$, since $SU(2)_L \otimes U(1)_Y$ must be a subgroup of $SU(4)_L \otimes U(1)_X$. With respect to this subgroup, a Higgs doublet occurs from $\phi^{(2)}$; namely,

$$\rho = \begin{pmatrix} \rho^+ \\ \rho^0_1 \end{pmatrix} \sim (\mathbf{1}, \mathbf{2}, 1/2),$$

and two Higgs singlets $\rho^0_2, \rho^0_3 \sim (\mathbf{1}, \mathbf{1}, -1/2)$. Consequently, there is also

$$\chi = \begin{pmatrix} \chi^0 \\ \chi^-_1 \end{pmatrix} \sim (\mathbf{1}, \mathbf{2}, 1/2)$$

and corresponding singlets $\chi^-_2, \chi^-_3 \sim (\mathbf{1}, \mathbf{1}, -1/2)$. When the symmetry is spontaneously broken up to the SM electroweak group, these two scalar doublets can be seen as the traditional

$$\phi = \begin{pmatrix} \rho^+ \\ \rho_1^0 \end{pmatrix} \sim (\mathbf{1}, \mathbf{2}, 1)$$

of the SM—with the completely decoupled $\rho_2^0, \rho_3^0 \sim (\mathbf{1}, \mathbf{2}, 0)$ at this level—and

$$\tilde{\phi} = i\sigma\phi^* = \begin{pmatrix} \chi^0 \\ \chi_1^- \end{pmatrix} \sim (\mathbf{1}, \mathbf{2}, -1),$$

due to the equivalence $\mathbf{2} \sim \mathbf{2}^*$ specific to $SU(2)$ only.

Now, the symmetry breaking pattern becomes quite obvious. The four scalar multiplets in Eqs. (6) and (7) break the symmetry of the model in three steps to the residual one, namely to the electromagnetic $U(1)_{\rm em}$:

$$\begin{aligned} SU(3)_C \otimes SU(4)_L \otimes U(1)_X &\xrightarrow{V'} SU(3)_C \otimes SU(3)_L \otimes U(1)_{X'} \\ &\xrightarrow{V} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \\ &\xrightarrow{v+v'} SU(3)_C \otimes U(1)_{\rm em} \end{aligned}$$

by developing the vacuum expectation values (vev): $\langle\phi^{(1)}\rangle = (v' \quad 0 \quad 0 \quad 0)^T$, $\langle\phi^{(2)}\rangle = (0 \quad v \quad 0 \quad 0)^T$, $\langle\phi^{(3)}\rangle = (0 \quad 0 \quad V \quad 0)^T$, $\langle\phi^{(4)}\rangle = (0 \quad 0 \quad 0 \quad V')^T$. A reasonable alignment is assumed here $v \cong v' \ll V \cong V'$ in order to get rapid and simpler estimations. Evidently, v, $v' = 174$ GeV is the SM electroweak breaking scale, while the new scales V, V' are specific to this 3-4-1 model.

In the symmetry breaking limit, one can easily obtain the couplings match, namely,

$$\frac{1}{g'^2} = \frac{1}{2g^2} + \frac{1}{g_X^2}.$$

This relation leads straightforwardly to the

$$\frac{g_X^2}{g^2} = \frac{\sin^2\theta_W(m_X)}{1 - \frac{3}{2}\sin^2\theta_W(m_X)}. \tag{8}$$

When applying the renormalization group procedure to Eq. (8) in the limit $\sin^2\theta_W = \sin^2\theta_W(m_Z)$ it results that the unification scale m_X is not sensitive to $\alpha_X(m_Z)$ which remains $\alpha_X(m_Z) \sim 10^{-2}$ even for m_X up to 10^{16} GeV [while $\alpha(m_Z) \sim 1/128$].

Of special interest is the boson mass spectrum. It can be computed as

$$m_{W^\pm}^2 = \tfrac{1}{2}g^2(v^2 + v'^2), \qquad m_{Y^0(Y^{0*})}^2 = \tfrac{1}{2}g^2(V^2 + V'^2), \tag{9}$$

$$m_{K^\pm}^2 = \tfrac{1}{2}g^2(V^2 + v'^2), \qquad m_{K^0(K^{0*})}^2 = \tfrac{1}{2}g^2(V^2 + v^2), \tag{10}$$

$$m_{X^\pm}^2 = \tfrac{1}{2}g^2(V'^2 + v'^2), \qquad m_{X^0(X^{0*})}^2 = \tfrac{1}{2}g^2(V'^2 + v^2), \tag{11}$$

$$m_Z^2 = \frac{m_{W^\pm}^2}{\cos^2\theta_W}, \qquad M_{Z',Z''}^2 = \frac{1}{2}g^2\begin{pmatrix} \frac{V^2+V'^2}{4} & \frac{V^2-V'^2}{2} \\ \frac{V^2-V'^2}{2} & V^2 + V'^2 \end{pmatrix}. \tag{12}$$

In our approximation $V \cong V'$, one can easily diagonalize the above matrix and get: $m_{Z'}^2 = \frac{1}{4}g^2V^2$ and $m_{Z''}^2 = g^2V^2$. Z''_μ couples only with exotic fermions having no interaction with ordinary particles and thus totally decoupling from the low energy phenomenology, while Z' mixes with the SM Z and exhibits a mass [7,8] lower bounded by 2 TeV (or greater), in close dependence on which one of the three generations of quarks transforms differently from the other two. The restrictions on the Z' neutral boson come from plugging into this model the data supplied by atomic parity violation experiments and certain processes in the meson systems $D^0 - \bar{D}^0$, $B_s^0 - \bar{B}_s^0$, $B_d^0 - \bar{B}_d^0$, $K^0 - \bar{K}^0$ [with their corresponding mixings in the flavor-changing neutral-current interactions]. However, these theoretical bounds [7,8] are consistent with recent experimental observations [1] that suggest a lower bound around 1 TeV for Z'.

In order to ensure the consistency with the SM phenomenology, the heavy particles of the 3-4-1 symmetry must not compromise the precision tests of the SM, particularly the oblique corrections must remain unaffected. That is, the new heavy quarks, new heavy bosons, and the extra scalar fields must give negligible contributions to the oblique parameters U, S, T, namely to the vacuum polarization amplitudes of the W and Z bosons. Being completely decoupled from low energy physics, Z''_μ does not contribute to these oblique corrections. Also, $Y^0(Y^{0*})$ does not contribute as long as it is a singlet under $SU(2)$. The $SU(2)$ doublets (K^+, K^0) and (X^+, X^0) (their complex conjugates) do not alter the one-loop calculations due to their degenerate masses [see Eqs. (10) and (11)]. There remains to be investigated only the contribution of some scalar doublets such as

$$\begin{pmatrix} \eta^+ \\ \eta_1^0 \end{pmatrix}$$

and

$$\begin{pmatrix} \xi^+ \\ \xi_1^0 \end{pmatrix}$$

whose couplings are subject to a proper tuning. However, these detailed calculations exceed the aim of this paper.

All the fermion masses are generated through the Higgs mechanism by scalar particles interacting with fermion fields. In this connection, the large split of breaking scales works efficiently in preserving the SM phenomenology. Ordinary quarks and leptons acquire their masses at the SM scale v while exotic quarks at the new V, V' scales.

A vast amount of theoretical research has been accomplished in this field due to several striking features that make such SM extensions quite appealing and much valuable despite the fact that they claim for a plethora of new particles. We count some of their assets. (i) First of all, the generation number in the fermion sector seems to get its explanation (absent in the SM, where simply an *ad hoc* triplication of the first generation is performed). In order to cancel the chiral anomalies—this time by an interplay among families—the number of generations must be divisible by the number of colors $N_C = 3$. This leads to exactly 3 generations if one assumes the asymptotic freedom condition from QCD that limits them to no more than 5. (ii) Contrary to SM, these models supply a natural framework for charge quantization (Doff and Pisano [7]). (iii) The strong CP problem can be elegantly solved due to a natural existence of Peccei-Quinn symmetry (Pal, Montero, Sanchez-Vega [5]). (iv) It offers a suitable framework for implementing the little Higgs mechanism (Kong [7]). (v) All SM neutral currents and masses are identically recovered. (vi) All new particles acquire their masses from the high scales V, V' so they do not interfere with the SM phenomenology at low energies supplied by present facilities. (vii) If the third generation of quarks is the one transforming differently, that accounts naturally for the unbalancing heaviness of the top quark.

We mention that our work focuses on a particular model from the 3-4-1 class of models, namely the one corresponding in the systematic classification accomplished by Ponce and Sanchez [9] to $b = c = 1$, Model A. The rich phenomenology of such models can be found and compared in Refs. [7,8].

III. LEPTON MASSES

The Yukawa sector of any gauge model is set up to supply fermion masses (consequently the SSB). There are introduced certain Yukawa coefficients (complex matrices h) that couple left-handed and right-handed fermion fields. We write down the most general combinations allowed by the gauge symmetry:

$$\mathcal{L}_Y^{\rm lept} = h^l_{ii}\bar{f}_{iL}\phi^{(1)+}l_{iR} + \frac{1}{\Lambda}\bar{f}_{iL}(h^M_{ij}S^+_R f^c_{jL} + h^D_{ij}S^+_D f^c_{jL} + h^D_{ij}S'^+_D f^c_{jL}) + \text{H.c.}, \tag{13}$$

where $l_L = e_L, \mu_L, \tau_L$. S matrices are defined as follows $S_R = (\phi^{(3)} \otimes \phi^{(4)} + \phi^{(4)} \otimes \phi^{(3)}) \sim (\mathbf{1}, \mathbf{10}, 1)$, $S_D = (\phi^{(2)} \otimes \phi^{(3)} + \phi^{(3)} \otimes \phi^{(2)}) \sim (\mathbf{1}, \mathbf{10}, 1)$, $S'_D = (\phi^{(2)} \otimes \phi^{(4)} + \phi^{(4)} \otimes \phi^{(2)}) \sim (\mathbf{1}, \mathbf{10}, 1)$ with h^M_{ij}, h^D_{ij} as the generic Yukawa matrices for Dirac and Majorana terms. Evidently, this is the basis where the charged leptons are already diagonal, so $h^l_{ij} = 0$. Up to this point we proved that the electroweak gauge symmetry $SU(4)_L \otimes U(1)_X$ allows for a natural implementation of neutrino masses via dimension-five effective operators plus the Yukawa couplings, usually subject to certain extra assumptions to overcome their arbitrariness.

In order to restrict ourselves to a simpler version, the entries in the generic neutrino matrix are the same regardless of their nature, namely $h^M_{ij} = h^D_{ij}$. Therefore, the Yukawa Lagrangian becomes in the case at hand here:

$$\mathcal{L}_Y^{\rm lept} = h^l_{ii}\bar{f}_{iL}\phi^{(1)+}l_{iR} + h_{ij}\Big[\frac{1}{\Lambda}\bar{f}_{iL}(S^+_R f^c_{jL} + S^+_D f^c_{jL} + S'^+_D f^c_{jL})\Big] + \text{H.c.} \tag{14}$$

In concrete expressions below the Yukawa coefficients will be denoted in order $A = h_{ee}$, $B = h_{\mu\mu}$, $C = h_{\tau\tau}$, $D = h_{e\mu}$, $D' = h_{\mu e}$ $E = h_{e\tau}$, $E' = h_{\tau e}$, $F = h_{\mu\tau}$, $F' = h_{\tau\mu}$.

It is natural to consider that the positions 3 and 4 in each lepton quadruplet are precisely $N = \nu_R$ and $N' = \nu^c_R$, as long as they are sterile with respect to Z and exhibit indistinguishable couplings to the new Z' and Z'' bosons (see for instance Ref. [8]). This assumption leads straightforwardly to the following identification:

$$\mathcal{L}_Y^{\nu} = \mathcal{L}_Y^R + \mathcal{L}_Y^D + \mathcal{L}_Y^{D'}. \tag{15}$$

By inspecting Eq. (13) one can easily identify

$$m(e) = h^l_e v, \qquad m(\mu) = h^l_\mu v, \qquad m(\tau) = h^l_\tau v, \tag{16}$$

since we work in a basis where the charged lepton sector is diagonal.

Taking into consideration the field theory mass formulas for, respectively, Dirac ($\mathcal{L}^D_Y = -m_D\bar{\psi}^c\psi + \text{H.c.}$) and Majorana ($\mathcal{L}^M_Y = -\frac{1}{2}m_M\bar{\psi}^c\psi + \text{H.c.}$) terms, along with the above vev alignment and the normal seesaw (1), one gets the mass matrices:

$$M_D = h\frac{vV}{\Lambda}, \tag{17}$$

$$M_R = (h + h^T)\frac{V^2}{\Lambda}. \tag{18}$$

From Eq. (2), in the flavor basis the Majorana terms for left-handed and right-handed neutrinos can be read

$$M(\nu_L) = [h^T(h + h^T)^{-1}h]\frac{v^2}{\Lambda}, \tag{19}$$

$$M(\nu_R) = (h + h^T)\frac{V^2}{\Lambda}. \tag{20}$$

By diagonalizing Eq. (18) with a proper U_R one obtains those matrices in the mass basis. The first step is

$$(\widehat{h + h^T}) = \text{Diag}(r_1, r_2, r_3) = U^T_R(h + h^T)U_R. \tag{21}$$

Now, inserting this result in Eq. (19) one computes

$$M(\nu_L) = [h^T U_R(\widehat{h + h^T})^{-1}U^T_R h]\frac{v^2}{\Lambda} \tag{22}$$

which can be diagonalized as $\hat{M}_L = U^T M(\nu_L)U$; namely,

$$\hat{M}_L = [U^T h^T U_R(\widehat{h + h^T})^{-1}U^T_R h U]\frac{v^2}{\Lambda}. \tag{23}$$

The physical neutrino masses can be computed via Eq. (23) if we consider their mixing (for details, see the reviews in Ref. [10]). The unitary mixing matrix U ($U^+U = 1$) links the gauge-flavor basis to the physical basis of massive neutrinos:

$$\nu_{\alpha L}(x) = \sum_{i=1}^{3} U_{\alpha i}\nu_{iL}(x), \tag{24}$$

where $\alpha = e, \mu, \nu$ (corresponding to neutrino gauge eigenstates), and $i = 1, 2, 3$ (corresponding to massive physical neutrinos with masses m_i). The mixing matrix $U_{\rm PMNS}$ (Pontecorvo-Maki-Nakagawa-Sakata) has in the standard parametrization the form

$$U_{\rm PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix}, \tag{25}$$

where the notations $\sin\theta_{23} = s_{23}$, $\sin\theta_{12} = s_{12}$, $\sin\theta_{13} = s_{13}$, $\cos\theta_{23} = c_{23}$, $\cos\theta_{12} = c_{12}$, $\cos\theta_{13} = c_{13}$ stand for the mixing angles and δ is the Dirac CP phase (with phenomenological meaning). Usually, to this one a diagonal Majorana phase matrix $P = {\rm Diag}(1, e^{i\alpha}, e^{i\beta})$ is sticked, though it can be absorbed by redefining fields. The standard identification leads to solar angle—θ_{12}, atmospheric angle—θ_{23}, reactor angle—θ_{13}. Several different patterns for matrix (25) have been considered in the literature. The most appealing approach stemming from the seminal work of Harrison Perkins, Scott [11], the so-called "tri-bi-maximal" ansatz is largely invoked when the PMNS matrix is analyzed and its phenomenological consequences are worked out in different models. A possible alternative to it followed the "bi-maximal" line and was developed in Ref. [12]. These particular textures—in good agreement with data—are often unfolded by enforcing certain discrete flavor symmetries on M.

The global data [1] regarding neutrino oscillations impose certain restrictions, namely $\sin^2\theta_{12} \simeq 0.3$, $\sin^2\theta_{23} \simeq 0.5$, and a small reactor angle θ_{13} (probably near zero), along with the mass splittings $\Delta m_{12}^2 \simeq 7.6 \times 10^{-5}$ eV^2 and $\Delta m_{23}^2 \simeq 2.4 \times 10^{-3}$ eV^2.

IV. PLAUSIBLE SCENARIO AND NUMERICAL ESTIMATES

In this section we prove that our construction is not a mere theoretical device. On the contrary, it can work very well when it comes to confronting the experimental data. We do not claim to make a general analysis here and get general predictions, but just apply the above procedure to a very particular scenario to get a quick fit with the data. The results make it obvious that our approach can be further developed and employed in investigating the neutrino sector.

A. Left-handed physical neutrinos

Since U_R is not restricted on observational ground, one can assume for the sake of simplicity a suitable scenario in which $h_{ji} = -h_{ij}$—namely $D' = -D$, $E' = -E$, $F' = -F$—that leads to $U_R = I$, and hence,

$$\begin{aligned}(h + h^T) &= {\rm Diag}(2A, 2B, 2C),\\ (h + h^T)^{-1} &= {\rm Diag}\left(\frac{1}{2A}, \frac{1}{2B}, \frac{1}{2C}\right)\end{aligned} \tag{26}$$

and consequently,

$$M(\nu_L) \simeq \frac{1}{2}\begin{pmatrix} A + \frac{D^2}{B} + \frac{E^2}{C} & \frac{EF}{C} & -\frac{DF}{B} \\ \frac{EF}{C} & B + \frac{D^2}{A} + \frac{F^2}{C} & \frac{DE}{A} \\ -\frac{DF}{B} & \frac{DE}{A} & C + \frac{E^2}{A} + \frac{F^2}{B} \end{pmatrix} \times \frac{\upsilon^2}{\Lambda}. \tag{27}$$

Bearing in mind that *Trace* is independent of the basis we work in, so that ${\rm Tr}M(\nu_L) = \sum_i m_{iL}$, one obtains

$${\rm Tr}M(\nu_L) = \frac{\upsilon^2}{2\Lambda}\left(A + \frac{D^2}{B} + \frac{E^2}{C} + B + \frac{D^2}{A} + \frac{F^2}{C} + C + \frac{E^2}{A} + \frac{F^2}{B}\right). \tag{28}$$

If all the Yukawa couplings are in the same range with the coupling of the charged τ lepton (at most comparable but no greater than it), then one obtains an upper bound for the sum of the individual neutrino masses. This is

$$\sum_i m_{iL} \leq \frac{9}{2} m(\tau)\frac{\upsilon}{\Lambda}. \tag{29}$$

As long as the experimental evidence imposes an upper bound on the range of left-handed neutrino masses (a few eV), one can estimate the cutoff energies up to which this model is valid. If $\upsilon = 174$ GeV and $m(\tau) = 1777$ MeV, then $\Lambda \geq 1.4 \times 10^{12}$ GeV which evidently is an intermediate level between SM and GUT energies.

B. Mass hierarchy and mixing angles

In order to get some rough estimates of the mass spectrum and fit properly the mixings between the three species of left-handed neutrinos, let us consider a simple setting: $A \simeq -0.371 \times 10^{-4}$, $B \simeq C \simeq -0.5383 \times 10^{-3}$, $D \simeq E \simeq 0.9034i \times 10^{-3}$, $F \simeq -0.172\,66i \times 10^{-2}$, where $i^2 = -1$ the complex unity.

Under these circumstances, the left-handed neutrino mass matrix (27) becomes

$$M(\nu_L) \simeq \begin{pmatrix} 0.150 & 0.145 & -0.145 \\ 0.145 & 1.350 & 1.100 \\ -0.145 & 1.100 & 1.350 \end{pmatrix} \times 10^{-2} \frac{v^2}{\Lambda} \quad (30)$$

so that it can be roughly diagonalized by the unitary matrix

$$U = \begin{pmatrix} 0.831 & 0.556 & 0 \\ -0.393 & 0.587 & 0.707 \\ 0.393 & -0.587 & 0.707 \end{pmatrix} \quad (31)$$

corresponding to the experimentally observed values $\sin^2\theta_{12} \simeq 0.31$, $\sin^2\theta_{23} \simeq 0.5$, and $\sin^2\theta_{13} \simeq 0$.

The mass spectrum comes out in this particular case in a normal hierarchy, namely,

$$|m_1| \simeq 0.002\,16 \text{ eV}, \qquad m_2 \simeq 0.088\,66 \text{ eV}, \qquad m_3 \simeq 0.529\,83 \text{ eV}. \quad (32)$$

The mass splitting ratio yields $r_\Delta \simeq 0.03$ in good agreement with experimental data and so the mass squared splittings are.

Of course, a further work could consider a nonzero *CP* phase violation and a small but nonzero reactor angle (as it is suggested by a recent series of papers [14]) so that from this stage one can perform a more accurate calculus and take into consideration a plethora of scenarios once the solar (θ_{12}) and atmospheric angles (θ_{23}) are firmly established. All these scenarios, of course, can lead to many different $M(\nu_L)$ matrices and, finally, even to a different mass hierarchy. Our results here are nothing but the proof that our method can work and is not claimed to be a general analytical analysis (which will be performed in a future work).

C. Right-handed sterile neutrinos

Now, a few words about the right-handed neutrinos in our approach framework. They acquire [Eq. (26)] the following masses in the scenario at hand:

$$\sum_i m_{iR} = \frac{2V^2}{\Lambda}(A + B + C). \quad (33)$$

According to the assumption in Sec. IVA, this can be approximated as

$$\sum_i m_{iR} \simeq 6m(\tau)\frac{V^2}{v\Lambda} \quad (34)$$

or equivalently

$$\sum_i m_{iR} \simeq \frac{4}{3}\left(\frac{V}{v}\right)^2 \sum_i m_{iL} \quad (35)$$

which, as expected, is not affected by the cutoff Λ. If V is not very high—say around 1–10 TeV—so that its new physics is testable at LHC, the so-called sterile neutrinos develop masses in the sub-keV range. The stability of these three species of right-handed neutrinos could recommend them as good candidates for the warm component of the dark matter [13]. Indeed, their only interactions with left-handed neutrinos are mediated by heavy bosons K and X [see Eq. (5)] via such couplings as $\frac{g}{\sqrt{2}}\overline{(\nu_R)^c}\gamma^\mu \nu_L X^0_\mu$ and $\frac{g}{\sqrt{2}}\overline{(\nu_R)}\gamma^\mu \nu_L K^0_\mu$. When calculating the width for a kinematically allowed decay $\nu_R \rightarrow \nu'_R \nu_L \nu'_L$, the expression $\Gamma = G_F^2 m_{\nu_R}^5 m_W^4 / 192\pi^3 m_{X,K}^4$ satisfactorily supplies the order of magnitude. The lifetimes $\tau = \Gamma^{-1} \sim 10^{27}$ s, provided the fact that both bosons are in TeV mass region. This result is by far greater than the estimated age of the Universe $t_0 \simeq 4.3 \times 10^{17}$ s according to [15], so the right-handed neutrinos as WIMP particles have a safe behavior from the stability viewpoint. A deeper investigation of the cosmological and astrophysical implications exceeds the scope of this paper and will be performed in a future work.

V. SUMMARY

In this paper we worked out the canonical seesaw mechanism in the framework of electroweak $SU(4)_L \otimes U(1)_Y$ extension of the SM, by simply constructing dimension-five effective operators as suitable direct products among scalar quadruplets existing in the model. Then we made use of the same generic Yukawa matrix for Dirac and Majorana couplings in order to obtain the plausible mass spectrum for the left-handed neutrinos. It came out in the range 10^{-3}–10^{-1} eV. For this purpose, an intermediate scale between SM and GUT was set by the cutoff $\Lambda \simeq 1.4 \times 10^{12}$ GeV. The right-handed partners develop masses in dependence on the high breaking scale V of the model. If the latter one ranges in the 1–10 TeV domain, then the right-handed neutrinos can be seen as plausible warm dark matter candidates with masses around 10^{-1} keV.

The advantage of our method over other approaches is that it exploits in a natural way all the ingredients supplied by the 3-4-1 model itself, without enlarging the scalar sector of the model, without invoking any new particle in the lepton sector or a fine-tuning procedure. Of course, a more detailed analysis can further be performed based on different hypotheses regarding the Yukawa couplings that can be subject to a new and appropriate flavor symmetry.

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