String amplitudes and framelike formalism for higher spins

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We analyze open string vertex operators describing connection gauge fields for spin 3 in Vasiliev's framelike formalism and perform their extended Becchi-Rouet-Stora-Tyutin analysis. Gauge symmetry transformations, generalized zero torsion constraints relating extra fields to the dynamical framelike field, and the relation between the dynamical framelike field and fully symmetric Fronsdal's field for spin 3 are all realized in terms of Becchi-Rouet-Stora-Tyutin constraints on these vertex operators in string theory. Using the construction, we analyze the 3-point correlator for the spin 3 field and calculate Chern-Simons type cubic interactions described by the 3-derivative Berends-Burgers-Van Dam type vertex in the framelike formalism.

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I. INTRODUCTION

Constructing consistent gauge theories of interacting higher spin fields is a long-standing, fascinating, and difficult problem (for an incomplete and very subjective list of references see [1–58]).

Despite significant progress in describing the dynamics of higher spin field theories, achieved over the past few decades, our understanding of the general structure of the higher spin interactions is still very far from complete. String theory appears to be a particularly efficient and natural framework to construct and analyze consistent gauge-invariant interactions of higher spins [4–6,21,22,34,36,37,59–68].

Within string theory, there are several approaches to this problem. The first approach is based on the observation that excitations with higher spins appear naturally in the massive spectrum of open and closed strings with the masses of the states on the leading Regge trajectory given by $m \sim (\frac{s}{\alpha'})^{(1/2)}$, so in the tensionless limit $\alpha' \to \infty$ the corresponding operators technically become massless. There are several difficulties within this approach; e.g., it is generally not easy to combine the vertex operators so as to recover the explicit set of the Stueckelberg symmetries of the corresponding states. The known examples of such operators typically mix the excitations with different spin values [68]. In addition, since the tensionless limit is opposite to the low-energy one, field theoretic interpretation of the correlation functions of these vertex operators is not easy. This formalism is also hard to extend to the anti–de Sitter (AdS) case since the world sheet correlators of string theory in AdS backgrounds are difficult to analyze beyond the semiclassical limit. Another string-theoretic approach to higher spins, based on the formalism of ghost cohomologies, is independent on the tension arguments and in principle allows circumventing some of the difficulties related to the tensionless limit. This approach is based on new physical [Becchi-Rouet-Stora-Tyutin (BRST) invariant and nontrivial] vertex operators that we analyzed in previous works (see, e.g., [66]) that are essentially coupled to the β-γ system of superconformal ghosts in Ramond-Neveu-Schwarz formulation of superstring theory (RNS) formalism. This ghost coupling cannot be removed by picture-changing transformation and can be classified in terms of ghost cohomologies [66,67]. This class of vertex operators is ghost picture-dependent, distinguishing them from standard operators such as a photon or a graviton, which exist at any picture. In the open string sector, there is a subclass of these operators corresponding to massless higher spin excitations. BRST-invariance conditions lead to Pauli-Fierz on-shell conditions for higher spin fields in Fronsdal's metriclike formalism, while BRST nontriviality constraints lead to gauge transformations for these operators. Their world sheet amplitudes are thus gauge-invariant by construction and describe polynomial interactions of massless higher spin fields in the low-energy effective limit. In our previous works we calculated some examples of such interations—cubic interaction of $s = 3 - 3 - 4$, the disc amplitude of spin 3 operators with the graviton (reproducing the coupling of spin 3 to gravity through the linearized Weyl tensor), and the quartic interaction of spin 3 and spin 1 gauge fields [66,67]. In practice, however, explicit calculations involving these operators are in most cases complicated, as their explicit structure is generally quite cumbersome. More significantly, due to the picture dependence, in many physically important cases the options to manipulate with the picture changing are limited and it is often hard to find the appropriate picture combination of the higher spin vertex operator satisfying the correct ghost number balance in correlation

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functions to cancel the background charges of the ghosts (e.g., on the sphere all the appropriate correlators must carry total ϕ-ghost number -2, χ-ghost number $+1$, and $b - c$ ghost number $+3$). One important example when such a complication appears is the cubic interaction of spin $s = 3$ corresponding to cubic amplitude of spin 3 vertex operators in open string theory. Straightforward calculation of this amplitude using vertex operators for Fronsdal-type fields requires 4 picture-changing transformations which, given the cumbersome structure of the operators, makes the computations practically insurmountable. In this paper we approach this problem by developing vertex operator formalism for auxiliary (extra) fields in Vasiliev's framelike approach. We construct vertex operators for connection gauge fields in this formalism. As in the Fronsdal's case the on-shell conditions on the operators lead to standard trace and symmetry constraints on the fiber indices of the connection gauge fields, along with gauge fixing conditions for diffeomorphism symmetries. Gauge transformations of the connection fields lead, in turn, to shifting the vertex operators by BRST-exact terms that do not affect the correlators that determine the structure of the interaction terms in the low-energy limit. The generalized zero torsion constraints follow from ghost cohomology conditions on the vertex operators that will be derived in the next section.

The rest of the paper is organized as follows. In Sec. II we review the basic ideas of the framelike description of higher spin fields and construct vertex operators for the dynamical and auxiliary connection gauge fields. In Sec. III we analyze the 3-point correlation function of these operators for spin 3, limiting ourselves to terms with 3 derivatives. The result is given by the Berends-Burgers-Van Dam type 3-derivative vertex in a certain gauge, modulo total derivative terms. In the conclusion and discussion section we outline generalizations of the developed formalism for the AdS case and discuss the relation between the vertex operators, constructed in this paper, and generators of higher spin algebra in AdS.

II. FRAMELIKE FORMALISM AND VERTEX OPERATORS FOR CONNECTION GAUGE FIELDS

Framelike formalism in higher spin field theories, originally proposed by Vasiliev and later developed in a number of works (e.g., see [2,26,43,44,69–72]), is a powerful tool to describe gauge-invariant interactions of higher spin fields in various backgrounds including AdS geometry. Unlike the approach used by Fronsdal that considers higher spin tensor fields as metric-type objects, the framelike formalism describes the higher spin dynamics in terms of higher spin connection gauge fields that generalize objects such as vielbeins and spin connections in gravity (in standard Cartan-Weyl formulation or MacDowell-Mansouri-Stelle-West in case of nonzero cosmological constant). The higher spin connections for a given spin s are described by the collection of two-row gauge fields

$$\omega^{s-1|t} \equiv \omega_m^{a_1\ldots a_{s-1}|b_1\ldots b_t}(x), \qquad 0 \le t \le s-1, \\ 1 \le a, b, m \le d \tag{1}$$

traceless in the fiber indices, where m is the curved d-dimensional space index while a, b label the tangent space with ω satisfying

$$\omega_m^{(a_1\ldots a_{s-1}|b_1)\ldots b_t} = 0. \tag{2}$$

The gauge transformations for ω are given by

$$\omega_m^{a_1\ldots a_{s-1}|b_1\ldots b_t} \to \omega_m^{a_1\ldots a_{s-1}|b_1\ldots b_t} + D_m\rho^{a_1\ldots a_{s-1}|b_1\ldots b_t}, \tag{3}$$

while the diffeomorphism symmetries are

$$\omega_m^{a_1\ldots a_{s-1}|b_1\ldots b_t}(x) \to \omega_m^{a_1\ldots a_{s-1}|b_1\ldots b_t}(x) + \partial_m\epsilon^n(x)\omega_n^{a_1\ldots a_{s-1}|b_1\ldots b_t}(x) + \epsilon^n(x)\partial_n\omega_m^{a_1\ldots a_{s-1}|b_1\ldots b_t}(x). \tag{4}$$

The $\omega^{s-1|t}$ gauge fields with $t \ge 0$ are auxiliary fields related to the dynamical field $\omega^{s-1|0}$ by generalized zero torsion constraints:

$$\omega_m^{a_1\ldots a_{s-1}|b_1\ldots b_t} \sim \partial^{b_1}\ldots\partial^{b_t}\omega_m^{a_1\ldots a_{s-1}} \tag{5}$$

skipping pure gauge terms (for convenience of the notations, we set the cosmological constant to 1, anywhere the AdS backgrounds are concerned).

It is also convenient to introduce the $d+1$-dimensional index $A = (a, \hat{d})$ (where $\hat{d}$ labels the extra dimension) and to combine $\omega^{s|t}$ into a single two-row field $\omega^{A_1\ldots A_{s-1}|B_1\ldots B_{s-1}}(x)$ identifying

$$\omega^{s-1|t} = \omega^{a_1\ldots a_{s-1}|b_1\ldots b_t\hat{d}\ldots\hat{d}}, \\ \omega^{A_1\ldots A_{s-1}|B_1\ldots B_{s-1}}V_{A_{t+1}}\ldots V_{A_{s-1}} = \omega^{A_1\ldots A_{s-1}|B_1\ldots B_t}, \tag{6}$$

where V_A is the compensator field satisfying $V_AV^A = 1$. The Fronsdal field $H^{a_1\ldots a_s}$ is then obtained by symmetrizing $\omega^{(a_1\ldots a_s)} = e^{m(a_s}\omega_m^{a_1\ldots a_{s-1})}$. We now turn to the question of constructing vertex operators for $\omega^{s-1|t}$. The operators for spins greater than 2 constructed in our previous works [66] were in fact limited to the Fronsdal-type objects only. In particular, in RNS superstring theory the operators for $s = 3$ are given by

$$V^{(-3)} = H_{abm}(p)ce^{-3\phi}\partial X^a\partial X^b\psi^m e^{ipX} \tag{7}$$

at an unintegrated minimal negative picture and

$$V^{(+1)} = K \circ H_{abm}(p)\oint dze^{\phi}\partial X^a\partial X^b\psi^m e^{ipX} \tag{8}$$

at integrated minimal positive picture $+1$ where $a, b, m = 0, \ldots, d-1$ are Minkowski space-time indices, $X^a(z)$ are space-time coordinates, ψ^a are their world sheet superpartners, b, c are reparametrizational fermionic ghosts, and β, γ are bosonic superconformal ghosts. The homotopy transformation $K \circ T$ of an integrated operator

$T = \oint dz V(z)$ [with $V(z)$ being a primary field of dimension 1] is defined according to

$$K \circ T = T + \frac{(-1)^N}{N!}\oint \frac{dz}{2i\pi}(z-w)^N :K\partial^N W:(z) + \frac{1}{N!}\oint \frac{dz}{2i\pi}\partial_z^{N+1}[(z-w)^N K(z)]K\{Q_{\rm brst}, U\}, \tag{9}$$

where

$$Q = \oint dz\left\{cT - bc\partial c - \frac{1}{2}\gamma\psi_m\partial X^m - \frac{1}{4}b\gamma^2\right\} \tag{10}$$

is the BRST operator, $K = -4ce^{2\chi-2\phi}$ is the homotopy operator satisfying $\{Q, K\} = 1$, U and W are the operators appearing in the commutator $[Q, V(z)] = \partial U(z) + W(z)$, and the ghost fields are bosonized as usual according to

$$c = e^{\sigma}, \qquad b = e^{-\sigma}, \qquad \gamma = e^{\phi-\chi} \equiv e^{\phi}\eta, \qquad \beta = e^{\chi-\phi}\partial\chi \equiv e^{-\phi}\partial\xi. \tag{11}$$

The operators (7) and (8) are the elements of negative and positive ghost cohomologies H_{-3} and H_1, respectively (see [66] for definitions and review). They are related according to $V^{(+1)} =: Z\Gamma^2 Z\Gamma^2 : V^{(-3)}$ by a combination of BRST-invariant transformations by picture-changing operators for b-c and β-γ systems: $Z =: b\delta(T):$ and $\Gamma =: \delta(\beta)G$: (T is the full stress tensor and G is the supercurrent). Therefore, the on-shell conditions and gauge transformations for H_{abm} at positive and negative pictures are identical. The manifest expression for $V^{(+1)}$ is given by

$$V_{s=3}(p;w) = \oint dz(z-w)^2 U(z) \equiv A_0 + A_1 + A_2 + A_3 + A_4 + A_5 + A_6 + A_7 + A_8, \tag{12}$$

where

$$A_0(p;w) = \frac{1}{2}H_{abm}(p)\oint dz(z-w)^2 \times P^{(2)}_{2\phi-2\chi-\sigma}e^{\phi}\partial X^a\partial X^b\psi^m e^{i\vec{p}\vec{X}}(z) \tag{13}$$

and

$$A_8(w) = H_{abm}(p)\oint dz(z-w)^2 \times \partial cc\partial\xi\xi e^{-\phi}\partial X^a\partial X^b\psi^m\} e^{i\vec{p}\vec{X}}(z) \tag{14}$$

have ghost factors proportional to e^{ϕ} and $\partial cc\partial\xi\xi e^{-\phi}$, respectively, and the rest of the terms carry a ghost factor proportional to $c\xi$:

$$\begin{aligned}
A_1(p;w) &= -2H_{abm}(p)\oint dz(z-w)^2 c\xi(\vec{\psi}\partial\vec{X})\partial X^a\partial X^b\psi^m e^{i\vec{p}\vec{X}}(z),\\
A_2(p;w) &= -H_{abm}(p)\oint dz(z-w)^2 c\xi\partial X^a\partial X^b\partial X^m P^{(1)}_{\phi-\chi}e^{i\vec{p}\vec{X}}(z),\\
A_3(p;w) &= H_{abm}(p)\oint dz(z-w)^2 c\xi\partial X^a\partial X^b\partial^2 X^m e^{i\vec{p}\vec{X}}(z),\\
A_4(p;w) &= 2H_{abm}(p)\oint dz(z-w)^2 c\xi\partial\psi^a P^{(1)}_{\phi-\chi}\partial X^b\psi^m e^{i\vec{p}\vec{X}}(z),\\
A_5(p;w) &= 2H_{abm}(p)\oint dz(z-w)^2 c\xi\partial^2\psi^a\partial X^b\psi^m e^{i\vec{p}\vec{X}}(z),\\
A_5(p;w) &= -2H_{abm}(p)\oint dz(z-w)^2 c\xi\partial X^a\partial X^b(\partial^2 X^m + \partial X^{a_3}P^{(1)}_{\phi-\chi})e^{i\vec{p}\vec{X}}(z),\\
A_6(p;w) &= 2iH_{abm}(p)\oint dz(z-w)^2 c\xi(\vec{p}\,\vec{\psi})P^{(1)}_{\phi-\chi}\partial X^a\partial X^b\psi^m e^{i\vec{p}\vec{X}}(z),\\
A_7(p;w) &= 2iH_{abm}(p)\oint dz(z-w)^2 c\xi(\vec{p}\partial\vec{\psi})\partial X^a\partial X^b\psi^m e^{i\vec{p}\vec{X}}(z).
\end{aligned} \tag{15}$$

Here w is an arbitrary point on the world sheet; since all the w derivatives of $s = 3$ operators are BRST-exact in a small Hilbert space [66], all the correlation functions involving higher spin operators $V_{s=3}(p, w)$ are w-independent and the choice of w is arbitrary. Conformal dimension n polynomials $P^{(n)}_{A\phi+B\chi+C\sigma}$ (where A, B, C are some numbers) are defined according to

$$e^{-A\phi(z)-B\chi(z)-C\sigma(z)}\frac{d^n}{dz^n}e^{A\phi(z)+B\chi(z)+C\sigma(z)} \tag{16}$$

[where the product is understood in the algebraic rather than operator-product expansion (OPE) sense].

As it is straightforward to check, the BRST-invariance constraints on the operators (7) and (8) lead to Pauli-Fierz type conditions

$$p^2 H_{abm} = p^a H_{abm} = \eta^{ab} H_{abm} = 0. \tag{17}$$

However, in general

$$\eta^{am} H_{abm} \neq 0 \tag{18}$$

as the tracelessness in a and m or b and m indices is not required for $V^{(-3)}$ to be a primary field. In what follows below we shall interpret H_{abm} with the dynamical spin 3 connection form $\omega^{2|0}$, identifying m with the manifold index and a, b with the fiber indices. So the tracelessness condition is generally imposed by BRST-invariance constraint on any pair of fiber indices only (but not on a pair of manifold and fiber indices). The same is actually true also for the vertex operators for framelike gauge fields of spins higher than 3. Altogether, this corresponds precisely to the double tracelessness constraints for corresponding metric-like Fronsdal's fields for higher spins (although the zero double trace condition does not of course appear in the case of $s = 3$). As it is clear from the manifest expressions (7) and (8) the tensor H_{abm} is by definition symmetric in indices a and b and therefore can be represented as a sum of two Young diagrams. However, only the fully symmetric diagram is the physical state, since the second one (with two rows) can be represented as the BRST commutator in the small Hilbert space:

$$\begin{aligned} &V^{(-3)} \sim \{Q, W\},\\ &W = H_{abm}(p) c\partial\xi e^{-4\phi + ipX} \partial X^a \left(\psi^{[m} \partial^2 \psi^{b]} - 2\psi^{[m} \partial \psi^{b]} \partial\phi + \psi^m \psi^b \left(\frac{5}{13} \partial^2 \phi + \frac{9}{13} (\partial\phi)^2 \right) \right) + a \leftrightarrow b. \end{aligned} \tag{19}$$

If Ω_{abm} is two-row, the $V^{(-3)}$ operator is obtained as the commutator of W with the matter supercurrent term of Q given by $\sim \oint \gamma \psi_m \partial X^m$. As W commutes with the $\oint(-\frac{1}{4} b\gamma^2 - bc\partial c)$ term in Q, $V^{(-3)}$ is BRST-exact if and only if it commutes the stress energy part of Q given by $\oint cT$. This is the case if the integrand of W is a primary field. It is, however, easy to check that the integrand is primary only when the last term in its expression is present. Since this term is proportional to $\sim \partial\xi e^{-4\phi + ipX} \partial X^a \psi^m \psi^b (\frac{5}{13} \partial^2 \phi + \frac{9}{13} (\partial\phi)^2)$ it is automatically antisymmetric in m and b and is absent when multiplied by fully symmetric H_{abm}. In the latter case this term is not a primary since its OPE with T contains cubic singularities and therefore the commutator of Q with W does not give $V^{(3)}$. Similarly, shifting H_{abm} by symmetrized derivative $H_{abm} \to H_{abm} + p_{(m}\Lambda_{ab)}$ is equivalent to shifting the vertex operator (7) by BRST-exact terms given by

$$\begin{aligned} &V^{(-3)} \to V^{(-3)} + \{Q, U\},\\ &U = \Lambda_{ab} c\partial\xi e^{-4\phi + ipX} \Big\{ \partial X^a \left((p\psi)\partial^2 \psi^b - 2(p\psi)\partial\psi^b \partial\phi + (p\psi)\psi^b \left(\frac{5}{13} \partial^2 \phi + \frac{9}{13} (\partial\phi)^2 \right) \right)\\ &\qquad + \partial X^a \partial X^b ((p\partial^2 X) - \partial\phi (p\partial X)) \Big\}. \end{aligned} \tag{20}$$

Of course everything described above also applies to the vertex operator (8) at positive picture, with appropriate Z, Γ transformations. This altogether already sends a strong hint to relate (7) and (8) to vertex operators for the dynamical framelike field $\omega^{2|0}$ describing spin 3. However, to make the relation between string theory and framelike formalism, we still need the vertex operators for the remaining extra fields $\omega^{2|1}$ and $\omega^{2|2}$. The expressions that we propose are given by

$$V^{2|1}(p) = 2\omega_m^{ab|c}(p) c e^{-4\phi} (-2\partial\psi^m \psi_c \partial X_{(a} \partial^2 X_{b)} - 2\partial\psi^m \partial\psi_c \partial X_a \partial X_b + \psi^m \partial^2 \psi_c \partial X_a \partial X_b) e^{ipX} \tag{21}$$

for $\omega^{2|1}$ and

$$\begin{aligned} V^{2|2}(p) = &-3\omega_m^{ab|cd}(p) c e^{-5\phi} \bigg(\psi^m \partial^2 \psi_c \partial^3 \psi_d \partial X^a \partial X_b - 2\psi^m \partial\psi_c \partial^3 \psi_d \partial X_a \partial^2 X_b + \frac{5}{8} \psi^m \partial\psi_c \partial^2 \psi_d \partial X_a \partial^3 X_b\\ &+ \frac{57}{16} \psi^m \partial\psi_c \partial^2 \psi_d \partial^2 X_a \partial^2 X_b \bigg) e^{ipX} \end{aligned} \tag{22}$$

for $\omega^{2|2}$. We start with analyzing the operator for $\omega^{2|1}$. Straightforward application of Γ to this operator gives

$$:\!\Gamma V^{2|1}\!:(p) = V^{(-3)}(p), \qquad H_m^{ab}(p) = i p_c \omega_m^{ab|c}(p); \tag{23}$$

i.e., the picture changing of $V^{2|1}$ gives the vertex operator for $\omega^{2|0}$ with the 3-tensor given by the divergence of $\omega^{2|1}$; i.e., for $p_c \omega_m^{ab|c}(p) \neq 0$ $V^{2|1}$ is the element of H_{-3}. If, however, the divergence vanishes, the cohomology rank changes and $V^{2|1}$ shifts to H_{-4}. This is precisely the case we are interested in. Namely, consider the H_{-4} cohomology condition

$$p_c \omega_m^{ab|c}(p) = 0. \tag{24}$$

The general solution of this constraint is

$$\omega_m^{ab|c} = 2p^c \omega_m^{ab} - p^a \omega_m^{bc} - p^b \omega_m^{ac} + p_d \omega_m^{acd;b}, \tag{25}$$

where ω_m^{ab} is traceless and divergence free in a and b and satisfies the same on-shell constraints as H_m^{ab}, while $\omega_m^{acd;b}$ is some three-row field, antisymmetric in a, c, d and symmetric in a and b. It is, however, straightforward to check that the operator $V^{2|1}$ with the polarization given by $\omega^{ab|c} = p_d \omega_m^{acd;b}$ can be cast as the BRST commutator:

$$p_d \omega_m^{acd;b}(p) V^m_{ac|b}(p) = \Big\{ Q, \omega_m^{acd;b}(p) \oint dz e^{\chi - 5\phi + ipX} \partial\chi (-2\partial\psi^m \psi_c \partial X_a \partial^2 X_b - 2\partial\psi^m \partial\psi_c \partial X_a \partial X_b + \psi^m \partial^2 \psi_c \partial X_a \partial X_b) \times \Big(\partial^2 \psi_d - \frac{4}{3} \partial\psi_d \partial\phi + \frac{1}{141} \psi_d (41(\partial\phi)^2 - 29\partial^2\phi) \Big) \Big\}. \tag{26}$$

Therefore, modulo pure gauge terms the cohomology condition (24) is the zero torsion condition relating the extra field $\omega^{2|1}$ to the dynamical $\omega^{2|0}$ connection. Similarly, constraining $V^{2|2}$ to be the element of H_{-5} cohomology results in the second generalized zero torsion condition

$$\omega_m^{ab|cd} = 2p^d \omega^{ab|c} - p^a \omega^{bd|c} - p^b \omega^{ad|c} + 2p^c \omega^{ab|d} - p^a \omega^{bc|d} - p^b \omega^{ac|d} \tag{27}$$

relating $\omega^{2|2}$ to $\omega^{2|1}$ modulo BRST-exact terms $\sim \{Q, W^{2|2}(p)\}$ where

$$W^{2|2}(p) = \omega^{ab;cdf}(p) \oint dz e^{ipX} \Big[\Big(\psi^m \partial^2 \psi_c \partial^3 \psi_d \partial X^a \partial X_b - 2\psi^m \partial\psi_c \partial^3 \psi_d \partial X_a \partial^2 X_b + \frac{5}{8} \psi^{(m} \partial\psi_c \partial^2 \psi_{d)} \partial X_a \partial^3 X_b + \frac{57}{16} \psi^m \partial\psi_c \partial^2 \psi_d \partial^2 X_a \partial^2 X_b \Big) \Big] \Big(-\frac{5}{2} L_f \partial^2 \xi + \partial L_f \partial \xi \Big), \tag{28}$$

where, as previously, $\xi = e^\chi$ and

$$L_f = e^{-6\phi} \Big(\partial^2 \psi_f - \partial\psi_f \partial\phi + \frac{3}{25} \psi_f ((\partial\phi)^2 - 4\partial^2\phi) \Big). \tag{29}$$

The gauge transformations for $\omega^{2|1}$ and $\omega^{2|2}$, $\delta\omega_m^{ab|c}(p) = p_m \Lambda^{ab|c}$ and $\delta\omega_m^{ab|cd}(p) = p_m \Lambda^{ab|cd}$, with Λ's having the same symmetries in the fiber indices as ω's shift the operators (21) and (22) by terms that are BRST-exact in the small Hilbert space; the explicit expressions for the appropriate BRST commutators are given in the Appendix B. Similarly to the $\omega^{2|0}$ case, for $\omega^{2|1}$ and $\omega^{2|2}$ with the manifold m index antisymmetric with any of the fiber indices a or b the operators (21) and (22) become BRST-exact in the small Hilbert space. Given the cohomology (“zero torsion”) conditions, (25) and (27) ensure that the fully symmetric $s = 3$ Fronsdal field is related to the dynamical field $\omega^{2|0}$ by the gauge transformation removing the two-row diagram. The expressions for the appropriate BRST commutators are given in the Appendix B.

This concludes the construction of the vertex operators for framelike gauge fields for spin 3. In the next section we shall use this construction to analyze the 3-point open string amplitude for spin 3.

III. THREE-POINT AMPLITUDE AND 3-DERIVATIVE VERTEX

In this section we use the vertex operator formalism, developed in the previous section, to compute the cubic coupling of massless spin 3 fields. In this paper we limit ourselves to the 3-derivative contributions corresponding to the Berends, Burgers, and Van Dam [20,73] type vertex in the field theory limit. The first step is to choose the ghost pictures of the operators to ensure the correct ghost number balance, i.e., so that the correlator has total ϕ-ghost number -2, $b-c$ ghost number $+3$, and χ-ghost number $+1$. This requires two out of three operators to be taken unintegrated at negative pictures and the third one at positive picture (note that higher spin operators at positive pictures are always integrated). It is convenient to take unintegrated operators at the minimal ghost picture -3; i.e., we shall use the $V^{(-3)}$ operator for $\omega^{2|0}$. Then the remaining integrated operator must be taken at picture $+5$, and only the terms proportional to the ghost factor $\sim ce^{\chi+4\phi}$ will contribute, while the terms proportional to $\sim \partial cce^{2\chi+3\phi}$ and to $\sim e^{5\phi}$ will drop out as they do not satisfy the balance of ghosts. It is therefore appropriate to choose the operator for $\omega^{2|2}$ for the third operator (for which the minimal positive picture is $+3$) and to apply the picture-changing transformation twice to bring it to the picture $+5$. The result is given by

$$\left\{Q,\xi\left\{Q,\xi K\circ\omega_m^{ab|cd}(p)\oint de^{3\phi}F_{abcd}^{m(17/2)}\right\}\right\}$$
$$=V_1(p)+V_2(p)$$
$$\equiv \omega_m^{ab|cd}(p)\oint du(u_0-u)^8 ce^{\chi+4\phi+ipX}R^m_{ab|cd}(u), \quad (30)$$

where

$$V_1(p)$$
$$=\frac{3}{64}\omega_m^{ab|cd}(p)\oint du(u_0-u)^8 ce^{\chi+4\phi+ipX}$$
$$\times L_{abcd}^{m(9)}\left\{\frac{24}{11!}P^{(11)}_{2\phi-2\chi-\sigma}\left(\frac{1}{8}P^{(2)}_{\chi}+\frac{1}{8}P^{(2)}_{2\phi-2\chi-\sigma}\right.\right.$$
$$\left.-\frac{1}{4}P^{(1)}_{\chi}P^{(1)}_{2\phi-2\chi-\sigma}-12P^{(11)}_{\phi-\chi}P^{(1)}_{2\phi-2\chi-\sigma}-12(P^{(1)}_{\phi-\chi})^2\right)$$
$$\left.+\frac{1}{11!}P^{(12)}_{2\phi-2\chi-\sigma}P^{(1)}_{-(3/2)\phi+(55/4)\chi+(11/4)\sigma}-\frac{102}{13!}P^{(13)}_{2\phi-2\chi-\sigma}\right\} \quad (31)$$

and

$$V_2(p)=\frac{1}{9!-8!}\sum_{n=0}^{7}2^{n-7}\sum_{\{l,m,p,q\geq 0;l+m+p+q=8-n\}}\sum_{r=0}^{p}\sum_{a=0}^{l}\sum_{b=0}^{q}\sum_{N=0}^{a+b+r+5}\left\{\frac{(-1)^{a+b+p+q}N!}{m!l!(p-r)!r!(N-r)!(5+a+b+r-N)!}\omega_m^{ab|cd}(p)\right.$$
$$\left.\times\oint du(u_0-u)^8 ce^{\chi+4\phi}\partial^{(p-r)}L_{abcd}^{m(N+9)}\partial^{(m)}P^{n|8}_{2\phi-2\chi-\sigma|\chi}P^{l-a|l}_{\chi|\phi-\chi}P^{q-b|q}_{3\phi+\chi|\phi-\chi}P^{(5+a+b+r-N)}_{\phi-\chi}(u)\right\}. \quad (32)$$

Here u_0 is an arbitrary point on the boundary (will be fixed later) and

$$F_{abcd}^{m(17/2)}=\left(\psi^m\partial^2\psi_c\partial^3\psi_d\partial X^a\partial X_b-2\psi^m\partial\psi_c\partial^3\psi_d\partial X_a\partial^2X_b+\frac{5}{8}\psi^m\partial\psi_c\partial^2\psi_d\partial X_a\partial^3X_b+\frac{57}{16}\psi^m\partial\psi_c\partial^2\psi_d\partial^2X_a\partial^2X_b\right)e^{ipX} \quad (33)$$

is the dimension $\frac{17}{2}$ primary field (given the on-shell conditions on ω); conformal dimension $N+9$ fields are defined as the OPE terms in the product of $F_{abcd}^{m(17/2)}$ with the matter supercurrent $G=-\frac{1}{2}\psi_n\partial X^n$ on the world sheet:

$$G_m(z)F_{abcd}^{m(17/2)}(w)=\sum_{N=0}^{\infty}(z-w)^{N-1}L_{abcd}^{m(N+9)}(w) \quad (34)$$

or manifestly

$$L_{abcd}^{m(N+9)}=\frac{e^{ipX}}{N!}\left\{\frac{15}{8}\left(\partial X_a\partial^3X_b+\frac{171}{16}\partial^2X_a\partial^2X_b\right)\left(\partial^{(N+1)}X^m\partial\psi_c\partial^2\psi_d-\frac{1}{N+1}\partial^{(N+2)}X_c\psi^m\partial^2\psi_d\right.\right.$$
$$\left.+\frac{2}{(N+1)(N+2)}\partial^{(N+3)}X_d\psi^m\partial\psi_c\right)+\psi^m\partial\psi_c\partial^2\psi_d\left(-\frac{15}{8(N+1)}\partial^{N+1}\psi_a\partial^3X_b\right.$$
$$\left.-\frac{45}{4(N+1)(N+2)(N+3)}\partial^{N+3}\psi_b\partial X_a-\frac{171}{8(N+1)(N+2)}\partial^{N+2}\psi_a\partial^2X_b\right)+3\partial X_a\partial X_b\left(\partial^{(N+1)}X^m\partial^2\psi_c\partial^3\psi_d\right.$$
$$-\frac{2}{(N+1)(N+2)}\partial^{(N+3)}X_c\psi^m\partial^3\psi_d-\frac{6}{(N+1)(N+2)(N+3)}\partial^{(N+4)}X_d\partial^2\psi_c\psi^m$$
$$\left.-\frac{3}{N+1}\psi^m\partial^2\psi_c\partial^3\psi_d\partial^{(N+1)}\psi_{(a}\partial X_{b)}\right)-6\partial X_a\partial^2X_b\left(\partial^{(N+1)}X^m\partial\psi_c\partial^3\psi_d-\frac{1}{N+1}\partial^{(N+2)}X_c\psi^m\partial^3\psi_d\right.$$
$$\left.-\frac{6}{(N+1)(N+2)(N+3)}\partial^{(N+4)}X_d\partial\psi_c\psi^m\right)+6\psi^m\partial\psi_c\partial^3X_d\left(\frac{1}{N+1}\partial^{(N+1)}\psi_a\partial^2X_b\right.$$
$$\left.\left.+\frac{2}{(N+1)(N+2)}\partial^{(N+2)}\psi_b\partial X_a\right)+N{:}\partial^{N-1}G_mF_{abcd}^{m(17/2)}{:}(1-\delta_{0;N})-i{:}(p^n\partial^N\psi_n)F_{abcd}^{m(17/2)}{:}\right\}. \quad (35)$$

The associate ghost polynomials $P^{n|N}_{A_1\phi+B_1\chi+C_1\sigma|A_2\phi+B_2\chi+C_2\sigma}$ are conformal dimension n polynomials in derivatives of ϕ,χ, and σ defined as the terms in the operator product

$$P^{(N)}_{A_1\phi+B_1\chi+C_1\sigma}(z)e^{A_2\phi+B_2\chi+C_2\sigma}(w) = \sum_{n=0}^{N}(z-w)^{n-N}P^{n|N}_{A_1\phi+B_1\chi+C_1\sigma|A_2\phi+B_2\chi+C_2\sigma}(z)e^{A_2\phi+B_2\chi+C_2\sigma}(w) \tag{36}$$

(see Appendix A for some of the techniques related to these polynomials). Note that, for example,

$$P^{N|N}_{A_1\phi+B_1\chi+C_1\sigma|A_2\phi+B_2\chi+C_2\sigma} \equiv P^{(N)}_{A_1\phi+B_1\chi+C_1\sigma}, \tag{37}$$

while

$$P^{0|N}_{A_1\phi+B_1\chi+C_1\sigma|A_2\phi+B_2\chi+C_2\sigma} = \prod_{k=0}^{n-1}(C_1C_2+B_1B_2-A_1A_2-k). \tag{38}$$

We are now prepared to analyze the 3-point function given by

$$A(p,k,q) = \omega_n^{s_1s_2}(p)\omega_p^{t_1t_2}(k)\omega_m^{ab|cd}(q)\oint du(u-u_0)^8\langle c\partial X_{s_1}\partial X_{s_2}\psi^n e^{ipX}(z)c\partial X_{t_1}\partial X_{t_2}\psi^n e^{ikX}(w)ce^{\chi+4\phi+iqX}R^m_{ab|cd}(u)\rangle. \tag{39}$$

Using the $SL(2,R)$ symmetry, it is convenient to set $z\to\infty$, $u_0=w=0$ (see [67] where the details related to this choice were discussed). For the notation purposes, however, it is convenient to retain z and w in our notations for the time being. We start with computing the "static" exponential ghost part of the correlator. Simple calculation gives

$$\langle ce^{-3\phi}(z)ce^{-3\phi}(w)ce^{4\phi+\chi(u)}\rangle = (z-w)^{-8}(z-u)^{13}(w-u)^{13}\to z^5(w-u)^{13}, \tag{40}$$

where we substituted the $z\to\infty$ limit. Next, consider the ψ part of the correlator. The expression for $R^m_{ab|cd}$ contains two types of terms: those that are quadratic in ψ and those that are quartic ψ. Since the remaining two spin 3 operators are linear in ψ, only the quadratic terms contribute to the correlator. Note that all the terms quadratic in ψ are also cubic in ∂X. So the pattern for the ψ correlators is

$$\langle\psi^n(z)\psi^p(w){:}\partial^{(P_1)}\psi_c\partial^{(P_2)}\psi_d{:}(u)\rangle = P_1!P_2!\left(\frac{\eta^n_d\eta^p_c}{z^{P_2+1}(w-u)^{P_1+1}}-\frac{\eta^n_c\eta^p_d}{z^{P_1+1}(w-u)^{P_2+1}}\right), \tag{41}$$

where, according to the manifest expression (35) for $R^m_{ab|cd}$, the numbers P_1 and P_2 can vary from 0 to $N+3$ (and $N_{\max}=8$). Next, consider the X part. As in this paper we limit ourselves to just three-derivative terms, it is sufficient to compute the terms linear in momentum (since the $\omega^{2|2}$ field already contains 2 derivatives out of 3). According to (31), (32), and (35) the X factor is a combination of the 3-point correlators of the type $\sim(\omega^{2|0})^2\langle\omega^{2|2}(\partial X)^2e^{ipX}(z)\times(\partial X)^2e^{ikX}(w)\partial^{(M_1)}X\partial^{(M_2)}X\partial^{(M_3)}Xe^{iqX}\rangle$ with different values of M_1, M_2, and M_3. Straightforward computation gives

$$\begin{aligned}
&\lim_{z\to\infty}\omega_n^{s_1s_2}(p)\omega_p^{t_1t_2}(k)\omega_m^{ab|cd}(q)\langle\partial X_{s_1}\partial X_{s_2}e^{ipX}(z)\partial X_{t_1}\partial X_{t_2}e^{ikX}(w)\partial^{(M_1)}X_a\partial^{(M_2)}X_b\partial^{(M_1)}X^m e^{iqX}(u)\rangle\\
&= M_1!M_2!M_3!\omega_n^{s_1s_2}(p)\omega_p^{t_1t_2}(k)\omega_m^{ab|cd}(q)\bigg\{\frac{2iq_{t_2}\eta_{s_1a}\eta_{s_2b}\eta^m_{t_1}}{z^{2+M_1+M_2}(w-u)^{2+M_3}}+iq_{t_2}\eta_{s_1a}\eta^m_{s_2}\eta_{t_1b}\bigg(\frac{1}{z^{2+M_1+M_3}(w-u)^{2+M_2}}\\
&\quad+\frac{1}{z^{2+M_2+M_3}(w-u)^{2+M_1}}\bigg)-\frac{2ik_{s_2}\eta_{t_1a}\eta_{t_2b}\eta^m_{s_1}}{z^{3+M_3}(w-u)^{1+M_1+M_2}}-ik_{s_2}\eta_{t_1a}\eta^m_{t_2}\eta_{s_1b}\bigg(\frac{1}{z^{3+M_1}(w-u)^{1+M_2+M_3}}\\
&\quad+\frac{1}{z^{3+M_2}(w-u)^{1+M_1+M_3}}\bigg)-\frac{1}{M_3}\eta_{s_1t_1}\eta_{s_2a}\eta_{t_2b}\bigg(ik^m\bigg(\frac{1}{z^{3+M_1}(w-u)^{1+M_2+M_3}}+\frac{1}{z^{3+M_2}(w-u)^{1+M_1+M_3}}\bigg)\\
&\quad+ip^m\bigg(\frac{1}{z^{3+M_1+M_3}(w-u)^{1+M_2}}+\frac{1}{z^{3+M_2+M_3}(w-u)^{1+M_1}}\bigg)\bigg)+ip_b\eta_{s_1t_1}\eta_{s_2a}\eta^m_{t_2}\bigg(\frac{1}{M_2}\frac{1}{z^{3+M_1}(w-u)^{1+M_2+M_3}}\\
&\quad+\frac{1}{M_1}\frac{1}{z^{3+M_2}(w-u)^{1+M_1+M_3}}\bigg)+\frac{ip_b\eta_{s_1t_1}\eta^m_{s_2}\eta_{t_2a}}{z^{3+M_3}(w-u)^{1+M_1+M_2}}\bigg(\frac{1}{M_1}+\frac{1}{M_2}\bigg).
\end{aligned} \tag{42}$$

Comparing this with the explicit expression (30)–(35) for the $\omega^{2|2}$ vertex operator, it is easy to notice that, while the static ghost factor (40) is proportional to $\sim z^5$, the z asymptotics of the ψ correlator (41) is $\sim\frac{1}{z}+O(\frac{1}{z^2})$ and the asymptotics for the X correlator is $\sim\frac{1}{z^4}+O(\frac{1}{z^5})$. This means that only the terms proportional to $\sim z^0$ contribute to the interaction vertex. Terms proportional to negative powers of z disappear in the limit $z\to\infty$ and correspond to pure gauge contributions. There are no terms proportional to positive powers of z (their presence would be a signal of problems with the gauge invariance). Moreover the z asymptotics further simplifies the analysis of the ghost polynomials in the expressions (30)–(32) for the $\omega^{2|2}$ operator; namely, all the polynomials have to couple to the $ce^{-3\phi}$ ghost exponent of the $\omega^{2|0}$ operator sitting at w as any couplings of these polynomials with the operator sitting at z produce contributions vanishing in the $z \to \infty$ limit.

Combining (30), (32), (41), and (42) we arrive to the following expression for the main matter building block for the matter part of the correlator:

$$\lim_{z\to\infty} \omega_n^{s_1 s_2}(p)\omega_p^{t_1 t_2}(k)\omega_m^{ab|cd}(q)\langle \psi^n \partial X_{s_1}\partial X_{s_2} e^{ipX}(z)\psi^p \partial X_{t_1}\partial X_{t_2} e^{ikX}(w) L_{abcd}^{m(N+9)}(u;q)\rangle = z^{-5}(w-u)^{-9-N} A_N(p,k,q),$$

$$\begin{aligned} A_N(k,p,q) = {}& \omega_n^{s_1 s_2}(p)\omega_p^{t_1 t_2}(k)\omega_m^{ab|cd}(q) \times \Big\{ \eta^{nm}\eta_{pd}\Big(-72(N+5)\eta^{s_1 a}\eta^{s_2 b}\eta^{t_1 c} q^{t_2} \\ & + \Big(72(N+5) - \frac{45}{4}(N+1)^2(N+2) - 144\Big)\eta^{t_1 a}\eta^{s_1 b}\eta^{t_2 c} k^{s_2} + \Big(144 - \frac{45}{4}N(N+1)\Big)\eta^{s_1 t_1}\eta^{s_2 a}\eta^{t_2 b} k^c \\ & + \Big(\frac{45}{4}N(N+1)^2 - 72(N+4)\Big)\eta^{s_1 t_1}\eta^{s_2 a}\eta^{t_2 c} p^b\Big) + \mathrm{Symm}(m,a,b)\Big\}. \end{aligned} \quad (43)$$

Using the manifest expression (35) for the $V_{2|2}$ vertex operator in terms of L_{9+N} and their derivatives, it is now straightforward to calculate the cubic coupling. First of all, it is immediately clear that only the V_2 part of $V_{2|2}$ contributes to the overall correlator. No terms from V_1 contribute since, as it was pointed out above, all the ghost polynomials entering $V_{2|2}$ must be completely absorbed by the ghost exponent $\sim ce^{-3\phi}$ located at w (no couplings to the exponent at z are allowed as they would result in contributions vanishing at $z \to \infty$). At the same time all the terms in V_1 carry the factors of $P^{(n)}_{2\phi-2\chi-\sigma}$ ($n = 11, 12, 13$) which cannot be absorbed by $ce^{-3\phi}$ [i.e.. their OPEs with $ce^{-3\phi}$) are less singular than $(z-w)^{-n}$]. Indeed, since $e^{2\phi-2\chi}b(z)ce^{-3\phi}(w) \sim (z-w)^5 e^{-\phi-2\chi}(w) + O(z-w)^6$, clearly for $n \geq 5$

$$\partial^{(n)}(e^{2\phi-2\chi}b)(z) \equiv ce^{-3\phi}(w) \equiv :P^{(n)}_{2\phi-2\chi-\sigma}e^{2\phi-2\chi}b:ce^{-3\phi}(w) \sim \frac{n!}{(n-5)!}:P^{(n-5)}_{2\phi-2\chi-\sigma}e^{-\phi-2\chi}:(w) + O(z-w) \quad (44)$$

implying that

$$P^{(n)}_{2\phi-2\chi-\sigma}(z)ce^{-3\phi}(w) \sim O\Big(\frac{1}{z^5}\Big), \quad (45)$$

i.e., no complete contractions for $n \geq 6$. Next, combining (42) with the expression (35) for $V_2(q)$ we obtain the following result for the overall correlator:

$$\begin{aligned} & \frac{6}{9!-8!}\sum_{n=0}^{7} 2^{n-7} \sum_{\{l,m,p,q\geq 0; l+m+p+q=8-n\}} \sum_{r=0}^{p}\sum_{b=0}^{q}\sum_{N=l+b+r+2}^{l+b+r+5} \\ & \times \Big\{ \frac{(-1)^{1+b+m+q+r+N} N!(N+8+p-r)! \prod_{j=0}^{m-1}(n+j)!}{m!l!(p-r)!r!(N-r)!(5+l+b+r-N)!(N-l-b-r-2)!(N+8)!} \\ & \times \alpha^{-3;0;1}_{3;1;0|1;-1;0}(q-b|q)\alpha^{-3;0;1}_{2;-2;-1|0;1;0}(n|8)A_N(p,k,q)\Big\}, \end{aligned} \quad (46)$$

where we used the fact that the only nonzero contributions from the summation over a are the terms with $a = l$ for which $P^{l-a|l}_{\chi|\phi-\chi} = P^{0|l}_{\chi|\phi-\chi} = (-1)^l l!$; while for $a \neq l$ $P^{l-a|l}_{\chi|\phi-\chi}$ are the polynomials in χ of dimension $l-a$ which are not contractible with $ce^{-3\phi}$. The numbers $\alpha^{A_3;B_3;C_3}_{A_1;B_1;C_1|A_2;B_2;C_2}(n|N)$ appearing in (46) are the coefficients in front of the leading order terms in the operator products

$$P^{n|N}_{A_1\phi+B_1\chi+C_1\sigma|A_2\phi+B_2\chi+C_2\sigma}(z)e^{A_3\phi+B_3\chi+C_3\sigma}(w) \sim \frac{\alpha^{A_3;B_3;C_3}_{A_1;B_1;C_1|A_2;B_2;C_2}(n|N)}{(z-w)^n}e^{A_3\phi+B_3\chi+C_3\sigma}(w). \quad (47)$$

The calculation of these coefficients is explained and the values are given in the Appendix A [see (58) and (60)]. Finally, substituting for $A_N(p,k,q)$ and evaluating the series (46) we obtain the following answer for the cubic coupling:

$$A(p,k,q) = \frac{691\,072\,283\,467i}{720}\omega_n^{s_1s_2}(p)\omega_p^{t_1t_2}(k)\omega_m^{ab|cd}(q)\times\Big\{\eta^{nm}\eta_{pd}\Big(\frac{1}{36}\eta^{s_1a}\eta^{s_2b}\eta^{t_1c}q^{t_2}+\frac{4}{3}\eta^{t_1a}\eta^{s_1b}\eta^{t_2c}k^{s_2} + \frac{1}{12}\eta^{s_1t_1}\eta^{s_2a}\eta^{t_2b}k^c - \eta^{s_1t_1}\eta^{s_2a}\eta^{t_2c}p^b\Big) + \mathrm{Symm}(m,a,b)\Big\}. \tag{48}$$

This concludes the calculation of the 3-derivative part of the cubic vertex. Inclusion of the appropriate Chan-Paton's indices is straightforward and leads to vertices of the type considered in [20,73].

IV. CONCLUSION AND DISCUSSION

In this paper we performed the analysis of open string vertex operators describing generalized connection gauge fields in Vasiliev's framelike formalism for higher spin fields. We have shown that generalized zero curvature conditions relating auxiliary connections $\omega^{s-1|t}$ to the dynamical $\omega^{s|0}$ fields are realized (up to BRST-exact terms) through ghost cohomology conditions on vertex operators that ensure that the fields with higher values of t belong to cohomologies of higher orders. We have also given precise BRST arguments relating $\omega^{2|0}$ to the symmetric Fronsdal field for spin 3, presenting the BRST commutator for the nonsymmetric spin 3 diagram (an important point which has been somewhat obscure before). We also demonstrated how the 3-derivative cubic vertex of spin 3 fields appears from the string-theoretic 3-point amplitude computed in this work. Obvious directions for future research include the computation of the 5-derivative vertex in the flat space (which technically appears to be significantly more tedious than the 3-derivative one) and generalizing the construction proposed in this work to framelike gauge fields with spins greater than 3. We hope to present these results soon in our future papers. The cubic vertex computed in this work is the one for the flat space and an important next step would be to generalize it to AdS. For that, one has to generalize the computation, analyzing of the 3-point function of operators for framelike spin 3 fields in the sigma-model background studied in [74]. That is, one has to perturb the flat background with the vertex operators for spin 2 vielbeins and connections in AdS space constructed in [74,75]. These operators carry negative cosmological constant and the vacuum solution of the low-energy equations of motion is described by AdS geometry. To calculate the cubic coupling of spin 3 framelike fields in the AdS space one has to consider their disc amplitude with insertions of closed string operators for spin 2 connections. As the insertions carry the dependence on the cosmological constant parameter, the important question to explore is the relation of this amplitude to AdS deformations of flat vertices considered by Vasiliev by methods of vertex complex analysis [26]. It is particularly interesting to clarify how the insertions of the closed string operators give rise to terms with a lower number of derivatives, as observed in [26] for the AdS deformations of vertices in Minkowski space. Another issue is to explore the relevance of the zero momentum parts of the vertex operators for framelike fields to space-time symmetry generators and higher spin algebra in AdS space. Typically, physical vertex operators in string theory are related to generators of global space-time symmetries in the zero momentum limit. For example, a photon operator at $p = 0$ is the generator of translations. A similar question can be asked about the vertex operators of framelike gauge fields for higher spins at zero momentum. While in general these operators at $p = 0$ do not generate global symmetries for RNS string theory in flat space, it is possible that they realize the symmetries of the sigma model perturbed by operators for spin 2 connections and vielbeins, provided that AdS vacuum constraints are imposed on spin 2. So far we have been able to show this for spin 3 only [76] and this conjecture needs to be generalized for higher spins. If the vertex operators for the framelike gauge fields are indeed related to the symmetries of the sigma model, their operator algebras may provide nice realizations of higher spin algebras in various AdS backgrounds.

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APPENDIX A: ASSOCIATE GHOST POLYNOMIALS AND $\alpha^{A_3;B_3;C_3}_{A_1;B_1;C_1|A_2;B_2;C_2}(n|N)$ COEFFICIENTS

In this appendix we explain some of the techniques to calculate the α coefficients that appear in the series (46) for the spin 3 cubic coupling. As was explained above, the associate ghost polynomials $P^{n|N}_{A_1\phi+B_1\chi+C_1\sigma|A_2\phi+B_2\chi+C_2\sigma}$ $(0 \le n \le N)$ are the conformal dimension n polynomials in bosonized ghost fields ϕ, χ, and σ, defined as the OPE terms in the product

$$P^{(N)}_{A_1\phi+B_1\chi+C_1\sigma}(z)e^{A_2\phi+B_2\chi+C_2\sigma}(w) = \sum_{n=0}^{N}(z-w)^{n-N}P^{n|N}_{A_1\phi+B_1\chi+C_1\sigma|A_2\phi+B_2\chi+C_2\sigma}(z)e^{A_2\phi+B_2\chi+C_2\sigma}(w), \tag{49}$$

where the conformal dimension N polynomials $P^{(N)}_{A_1\phi+B_1\chi+C_1\sigma}$ are defined according to (16). Then the α coefficients $\alpha^{A_3;B_3;C_3}_{A_1;B_1;C_1|A_2;B_2;C_2}(n|N)$ are defined as coefficients in front of the leading order n terms in the OPE

$$P^{n|N}_{A_1\phi+B_1\chi+C_1\sigma|A_2\phi+B_2\chi+C_2\sigma}(z)e^{A_3\phi+B_3\chi+C_3\sigma}(w) \sim \frac{\alpha^{A_3;B_3;C_3}_{A_1;B_1;C_1|A_2;B_2;C_2}(n|N)}{(z-w)^n}e^{A_3\phi+B_3\chi+C_3\sigma}(w) \tag{50}$$

(if the actual leading order of a given OPE is less than n, the appropriate coefficient is zero). Although the manifest form of the associate ghost polynomials is generally complicated, there is an algorithm significantly simplifying the computations of both associate ghost polynomials and related α coefficients. The algorithm is based on a comparison of two similar operator products. Below we shall explain the algorithm and present the results for $\alpha^{-3;0;1}_{3;1;0|1;-1;0}(q-b|q)$ (where $0 \leq b \leq q \leq 8$) and $\alpha^{-3;0;1}_{2;-2;-1|0;1;0}(n|8)$ entering the series (46). Consider the operator product

$$e^{A_1\phi+B_1\chi+C_1\sigma}(z)e^{A_2\phi+B_2\chi+C_2\sigma}(w) = \sum_{m=0}^{\infty}\frac{(z-w)^{-A_1A_2+B_1B_2+C_1C_2+m}}{m!}:e^{(A_1+A_2)\phi+(B_1+B_2)\chi+(C_1+C_2)\sigma}(z)P^{(m)}_{A_1\phi+B_1\chi+C_1\sigma}:(w) \tag{51}$$

around the point w. Differentiating N times over z we get

$$\partial^{(N)}e^{A_1\phi+B_1\chi+C_1\sigma}(z)e^{A_2\phi+B_2\chi+C_2\sigma}(w) = \sum_{m=0}^{\infty}\prod_{l=0}^{N-1}(-A_1A_2+B_1B_2+C_1C_2+m-l) \times \frac{(z-w)^{-A_1A_2+B_1B_2+C_1C_2+m-N}}{m!}:e^{(A_1+A_2)\phi+(B_1+B_2)\chi+(C_1+C_2)\sigma}P^{(m)}_{A_1\phi+B_1\chi+C_1\sigma}:(w). \tag{52}$$

On the other hand this product by definition coincides with the OPE:

$$\begin{aligned}&:P^{(N)}_{A_1\phi+B_1\chi+C_1\sigma}e^{A_1\phi+B_1\chi+C_1\sigma}:(z)e^{A_2\phi+B_2\chi+C_2\sigma}(w)\\ &=\sum_{j,k=0}^{\infty}\sum_{n=0}^{N}\frac{(z-w)^{-A_1A_2+B_1B_2+C_1C_2+n-N+j+k}}{j!k!}:\partial^{(j)}P^{n|N}_{A_1\phi+B_1\chi+C_1\sigma|A_2\phi+B_2\chi+C_2\sigma}P^{(k)}_{A_1\phi+B_1\chi+C_1\sigma}e^{(A_1+A_2)\phi+(B_1+B_2)\chi+(C_1+C_2)\sigma}:(w)\end{aligned} \tag{53}$$

[the derivative of the associate polynomials appears since in the definition (49) the polynomials are located at z while the OPE (53) is around w]. This gives the characteristic equation on $P^{n|N}_{A_1\phi+B_1\chi+C_1\sigma|A_2\phi+B_2\chi+C_2\sigma}$:

$$\begin{aligned}&\sum_{j,k=0}^{\infty}\sum_{n=0}^{N}\frac{(z-w)^{-A_1A_2+B_1B_2+C_1C_2+n-N+j+k}}{j!k!}:\partial^{(j)}P^{n|N}_{A_1\phi+B_1\chi+C_1\sigma|A_2\phi+B_2\chi+C_2\sigma}P^{(k)}_{A_1\phi+B_1\chi+C_1\sigma}:\\ &=\sum_{m=0}^{\infty}\prod_{l=0}^{N-1}(-A_1A_2+B_1B_2+C_1C_2+m-l)\frac{(z-w)^{-A_1A_2+B_1B_2+C_1C_2+m-N}}{m!}P^{(m)}_{A_1\phi+B_1\chi+C_1\sigma}.\end{aligned} \tag{54}$$

Matching the coefficients in front of each power of $(z-w)$ (starting from the most singular term) then gives recurrence relations on $:P^{n|N}_{A_1\phi+B_1\chi+C_1\sigma|A_2\phi+B_2\chi+C_2\sigma}:$, expressing them in terms of conformal dimension n combinations of $P^{(l)}_{A_1\phi+B_1\chi+C_1\sigma}$ and their derivatives (with $1 \leq l \leq n$). The coefficients $\alpha^{A_3;B_3;C_3}_{A_1;B_1;C_1|A_2;B_2;C_2}(n|N)$ are then obtained by replacing each of $\partial^{(k)}P^{(l)}$ according to

$$P^{(l)}_{A_1\phi+B_1\chi+C_1\sigma} \to (-1)^k\prod_{i=0}^{k-1}\prod_{j=0}^{l-1}(l+i)(-A_1A_3+B_1B_3+C_1C_3-j) \tag{55}$$

in each of these combinations since

$$P^{(l)}_{A_1\phi+B_1\chi+C_1\sigma}(z)e^{A_3\phi+B_3\chi+C_3\sigma}(w) \sim (z-w)^{-l}\prod_{j=0}^{l-1}(-A_1A_3+B_1B_3+C_1C_3-j)e^{A_3\phi+B_3\chi+C_3\sigma}(w) + O(z-w)^{1-l}. \tag{56}$$

Applied to $P^{n|8}_{2\phi-2\chi-\sigma|\chi}$ this procedure gives the recurrence relations

$$P^{n|8}_{2\phi-2\chi-\sigma|\chi} = 9!P^{(n)}_{2\phi-2\chi-\sigma}\delta_{1;n} - \sum_{k=1}^{n-1}\sum_{l=0}^{k}\frac{\partial^{(l)}P^{k|8}_{2\phi-2\chi-\sigma|\chi}P^{(n-l-k)}_{2\phi-2\chi-\sigma}}{l!(k-l)!} \tag{57}$$

and the corresponding α coefficients $\alpha^{-3,0,1}_{2,-2,-1|0,1,0}(n|8)$ are given by

$$\begin{aligned}
&\alpha^{-3,0,1}_{2,-2,-1|0,1,0}(0|8) = 9!, \quad \alpha^{-3,0,1}_{2,-2,-1|0,1,0}(1|8) = -8!\times 40, \quad \alpha^{-3,0,1}_{2,-2,-1|0,1,0}(2|8) = 8!\times 150,\\
&\alpha^{-3,0,1}_{2,-2,-1|0,1,0}(3|8) = -8!\times 300, \quad \alpha^{-3,0,1}_{2,-2,-1|0,1,0}(4|8) = 8!\times 275, \quad \alpha^{-3,0,1}_{2,-2,-1|0,1,0}(5|8) = -8!\times 94,\\
&\alpha^{-3,0,1}_{2,-2,-1|0,1,0}(6|8) = 0, \quad \alpha^{-3,0,1}_{2,-2,-1|0,1,0}(7|8) = 0, \quad \alpha^{-3,0,1}_{2,-2,-1|0,1,0}(8|8) = 0.
\end{aligned} \tag{58}$$

Similarly, when applied to $P^{q-b|q}_{3\phi+\chi|\phi-\chi}$ $(0\leq b\leq q\leq 8)$ this procedure gives the recurrence relations

$$P^{q-b|q}_{3\phi+\chi|\phi-\chi} = \frac{(q+3)!}{3!}P^{(q)}_{3\phi+\chi}(\delta_{1;q}+\delta_{2;q}+\delta_{3;q}) - \sum_{k=1}^{q-1}\sum_{l=0}^{k}\frac{\partial^{(l)}P^{k|8}_{3\phi+\chi|\phi-\chi}P^{(n-l-k)}_{3\phi+\chi}}{l!(k-l)!} \tag{59}$$

and the corresponding α coefficients are $\alpha^{-3,0,1}_{3,-1,0|1,-1,0}(q-b|q)(0\leq b\leq q\leq 8)$ are

$$\begin{aligned}
&\alpha^{-3,0,1}_{3,-1,0|1,-1,0}(0|q) = \frac{(q+3)!}{6}, \qquad (\alpha^{-3,0,1}_{3,-1,0|1,-1,0}(1|q) = -\frac{3}{2}(q+2)!q)\sum_{p=1}^{8}\delta_{p;q},\\
&\alpha^{-3,0,1}_{3,-1,0|1,-1,0}(2|q) = (-6(q+3)! + 12(q+2)!q + 72(q+1)!)\sum_{p=2}^{8}\delta_{p;q},\\
&\alpha^{-3,0,1}_{3,-1,0|1,-1,0}(3|q) = (28(q+3)! - 42(q+2)!q - 504q!q)\sum_{p=3}^{8}\delta_{p;q},\\
&\alpha^{-3,0,1}_{3,-1,0|1,-1,0}(4|q) = (84(q+3)! + 126(q+2)!q - 1512(q+1)! + 3024q!q)\sum_{p=4}^{8}\delta_{p;q},\\
&\alpha^{-3,0,1}_{3,-1,0|1,-1,0}(5|q) = (-639(q+3)! - 339(q+2)!q + 4896(q+1)! - 7560q!q)\sum_{p=5}^{8}\delta_{p;q},\\
&\alpha^{-3,0,1}_{3,-1,0|1,-1,0}(6|q) = (1352(q+3)! + 600(q+2)!q - 6984(q+1)! + 10080q!q)\sum_{p=6}^{8}\delta_{p;q},\\
&\alpha^{-3,0,1}_{3,-1,0|1,-1,0}(7|q) = (1158(q+3)! - 3918(q+2)!q + 7992(q+1)! - 52920q!q)\sum_{p=7}^{8}\delta_{p;q},\\
&\alpha^{-3,0,1}_{3,-1,0|1,-1,0}(8|8) = 9!.
\end{aligned} \tag{60}$$

This fully determines the α coefficients in the series (46).

APPENDIX B: BRST RELATIONS AND GAUGE TRANSFORMATIONS FOR $\omega^{2|1}$ AND $\omega^{2|2}$

In this appendix we present explicit expressions for BRST commutators leading to the gauge transformations for the framelike fields and relating framelike and Fronsdal fields for spin 3. The gauge transformation for the $\omega^{2|1}$ field

$$\omega^{ab|c}_m(p) \to \omega^{ab|c}_m(p) + p_m\Lambda^{ab|c}(p) \tag{61}$$

leads to shifting the $V^{2|1}$ vertex operator (21) by BRST-exact terms: $V^{2|1}(p)\to V^{2|1}(p) + \{Q, W^{2|1}_1(p)\}$ where, up to overall numerical factor,

$$W^{2|1}_1(p) \sim \Lambda^{ab|c}(p)\oint dz c e^{-5\phi+ipX}((p\partial\psi)(\psi_c\partial^2X_b - 2\partial\psi_c\partial X_b) + (p\psi)\partial^2\psi_c\partial X_b)\left(\frac{2}{5}\partial L_a\partial\xi - L_a\partial^2\xi\right), \tag{62}$$

where

$$L_a = \partial^2\psi_a - 2\partial\psi_a\partial\phi + \frac{1}{13}\psi_a(5\partial^2\phi + 9(\partial\phi)^2) \tag{63}$$

and Λ has the same symmetry in the fiber indices as $\omega^{2|1}$. This operator is BRST-exact if ω is transverse in the a, b fiber indices (which, in turn, is the invariance condition). Next, if $\omega_m^{ab|c}(p)$ is antisymmetric in m and a (so that the corresponding $\omega^{2|0}$ is the two-row field), $V^{2|1}$ is again the BRST commutator in the small Hilbert space:

$$V^{2|1}(p) = \{Q, W_2^{2|1}(p)\} \tag{64}$$

with

$$W_2^{2|1}(p) \sim \omega_m^{ab|c}(p)\oint dz c e^{-5\phi+ipX}(\psi_c\partial^2 X_b - \partial\psi_c\partial X_b)\left(\frac{2}{5}\partial\psi^{[m}\partial L_{a]}\partial\xi - \partial\psi^{[m}L_{a]}\partial^2\xi\right) + \partial^2\psi_c\partial X_b\left(\frac{2}{5}\psi^{[m}\partial L_{a]}\partial\xi - \psi^{[m}L_{a]}\partial^2\xi\right). \tag{65}$$

Next, we analyze $\omega^{2|2}$ and its vertex operator (22). The gauge transformation for the $\omega^{2|2}$ field

$$\omega_m^{ab|cd}(p) \to \omega_m^{ab|cd}(p) + p_m\Lambda^{ab|cd}(p) \tag{66}$$

leads to shifting the $V^{2|2}$ vertex operator (21) by BRST-exact terms: $V^{2|2}(p) \to V^{2|2}(p) + \{Q, W_1^{2|2}(p)\}$ with

$$W_2^{2|2}(p) \sim \Lambda^{ab|cd}(p)\oint dz c e^{-6\phi+ipX}\left\{\left(\frac{1}{4}(p_n\partial N^n)\partial\xi - (p_nN^n)\partial^2\xi\right)\left(\partial^2\psi_c\partial^3\psi_d\partial X^a\partial X_b - 2\partial\psi_c\partial^3\psi_d\partial X_a\partial^2 X_b + \frac{5}{8}\partial\psi_c\partial^2\psi_d\partial X_a\partial^3 X_b + \frac{57}{16}\partial\psi_c\partial^2\psi_d\partial^2 X_a\partial^2 X_b\right)\right\}, \tag{67}$$

where

$$N_n = \partial^3 X_n - \frac{3}{2}\partial^2 X_n - \frac{1}{3}\partial X_n\left((\partial\phi)^2 - \frac{17}{6}\partial^2\phi\right). \tag{68}$$

As before, this operator is BRST-exact if ω is transverse in the a, b fiber indices. Finally, if $\omega_m^{ab|cd}(p)$ is antisymmetric in m and a or b (so that the corresponding $\omega^{2|0}$ is the two-row field), $V^{2|2}$ is again the BRST commutator in the small Hilbert space:

$$V^{2|2}(p) = \{Q, W_2^{2|2}(p)\} \tag{69}$$

with

$$W_2^{2|2}(p) \sim \omega_m^{ab|cd}(p)\oint dz c e^{-6\phi+ipX}\left\{\left(\frac{1}{4}N^m\partial\xi - (N^m)\partial^2\xi\right)\left(\partial^2\psi_c\partial^3\psi_d\partial X^a\partial X_b - 2\partial\psi_c\partial^3\psi_d\partial X_a\partial^2 X_b + \frac{5}{8}\partial\psi_c\partial^2\psi_d\partial X_a\partial^3 X_b + \frac{57}{16}\partial\psi_c\partial^2\psi_d\partial^2 X_a\partial^2 X_b\right) - (a \leftrightarrow m)\right\}. \tag{70}$$

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