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Photon polarization tensor in the light front field theory at zero and finite temperatures

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In this work, we consider the light front quantum electrodynamics in (3 + 1) dimensions and evaluate the photon polarization tensor at one loop for both zero and finite temperatures. In the first case, we apply the dimensional regularization method to extract the finite contribution and find the transverse structure for the amplitude in terms of the light front coordinates. The result agrees with one-loop covariant calculation. For the thermal corrections, we generalize the hard thermal loop approximation to the light front and calculate the dominant temperature contribution to the polarization tensor, consistent with the Ward identity. In both zero as well as finite temperature calculations, we use the oblique light front coordinates.

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I. INTRODUCTION

In recent years, light front (LF) quantized field theories have been successfully generalized to finite temperature. The light front frame was introduced by Dirac [1], and the quantization of field theories on the null-plane has found applications in many branches of physics [2,3]; see also Ref. [4] for a review and a guide to the extensive literature.

The proper thermal description of LF quantized field theories was pointed out in a number of publications, including Refs. [5–10]. It was shown that the thermal contributions to the self-energy in scalar theories at one loop [11] coincide with the results from conventional calculations. Furthermore, the anomaly term and fermion condensate at zero and finite temperature in the LF Schwinger model match their conventional counterparts [7]. Thermodynamical properties were computed nonperturbatively using discrete light cone quantization [12] in the massive Schwinger model [13], two-dimensional supersymmetric theories [14,15], and, in four dimensions, for SU(N_c) pure gauge theory in the large N_c approximation using the transverse lattice approach [16]. Moreover, the formalism was applied investigating the in-medium properties of quark bound states [17,18] and the nontrivial vacuum structure of the Unruh effect [19].

Particularly in Refs. [8,11], it has been shown that there is a convenient coordinate system: the oblique one, in which the study of thermal effects is straightforward. One can collect both the usual light front coordinates as well as the oblique one in the general light cone coordinate frame,

$$\bar{t} = t + z, \qquad \bar{z} = At + Bz, \qquad \bar{x} = x, \qquad \bar{y} = y, \tag{1}$$

where A and B are arbitrary real constants with the restriction that $A \pm B \neq 0$ and $x^\mu = (t, x, y, z)$ are the usual Minkowski coordinates. In particular, for $A = 0$, $B = 1$, Eq. (1) represents the oblique light front coordinates (OLFC) proposed in Ref. [5] and used in Refs. [7,11,13] to carry out the discussions of statistical mechanics within the LF.

One of the distinct features of LF dynamics is the energy-momentum dispersion relation which is linear in the LF energy. This property is also present in the OLFC coordinates. Accordingly, the propagators in the OLFC momentum space behave differently at $\bar{k}_0 \to \infty$. For instance, the propagator of a scalar particle reads

$$\begin{aligned} iG(k) &= \frac{i}{k^2 - m^2 + i\epsilon} \\ &= \frac{i}{-2k_0k_3 - k_1^2 - k_2^2 - k_3^2 - m^2 + i\epsilon} \\ &= \frac{i}{-2k_0k_3 - k_i^2 - m^2 + i\epsilon}, \qquad i = 1, 2, 3. \end{aligned} \tag{2}$$

Because of the $1/k_0$ dependence of the propagator, the computation of loop integrals is more demanding. One has to properly take into account contributions from the arc contours, used to close the complex integration at infinity, and singular point contributions from moving poles to recover the correct covariant result [20,21]. Particularly, in Ref. [21], various techniques were used to demonstrate the equivalence between equal time and light front dynamics for certain one-loop computations at zero temperature.

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In this work, we use OLFC to analyze the polarization tensor for the quantum electrodynamics, both at zero and finite temperature. At zero temperature, working in dimensional regularization, we find the instant form textbook result for the transverse photon self-energy.

In order to obtain the thermal contribution, we consider the hard thermal loop (HTL) approximation. This technique was developed by Braaten and Pisarski [22] for the thermal quantum field theory at equal times and is particularly useful to extract the leading thermal contributions to the amplitudes in perturbative quantum field theories. HTL theory has found its way into textbooks, e.g. see Ref. [23] for an introduction. At equal times, the vertex functions in HTL approximation respect the Ward identities. We demonstrate that the Ward identity also holds at finite temperature in OLFC and use this fact to determine the Π_{00} component of the polarization tensor whose direct LF computation is challenging due to the k_0 dependence of the LF propagator, as explained above.

Since HTL is a high-energy approximation to the thermal amplitudes and LF dynamics is especially suitable for high-energy, highly relativistic systems, we think it is rewarding to bring these two formalisms together. This contribution presents a first step in this direction.

The paper is organized as follows. In Sec. II, we briefly review the finite temperature formalism for light front quantum field theories. The imaginary time propagators for scalar and the fermionic fields are given. For the latter, we choose to work with the full Dirac space without eliminating the constrained variables. This proposal was made in Refs. [6,7] where the authors showed that using the full Dirac propagator, calculations are usually simplified, as one does not having to worry about additional noncovariant contact interactions in the propagator and the vertex.

In Sec. III A, we present the HTL approximation on the light cone, considering as a first example the scalar ϕ^3 theory and evaluating the two-point function. Sec. III B is concerned with the photon self-energy at one loop, both at zero and finite temperature in HTL approximation. We discuss the Ward identity in both cases.

Our conclusions are given in Sec. IV.

II. LIGHT FRONT QUANTUM FIELD THEORY AT FINITE TEMPERATURE

Light front field theories do not admit a naive generalization to finite temperature, basically because it is not possible to have a heat bath at rest on the light front [5,10]. One way of introducing thermal effects into the LF quantization is to consider the generalized light front coordinates [8]. A particular choice of these coordinates is given by

$$x^\mu \to \bar{x}^\mu, \qquad \mu = 0, 1, \ldots, 3, \tag{3}$$

such that

$$\bar{x}^0 = x^0 + x^3, \qquad \bar{x}^i = x^i, \qquad i = 1, \ldots, 3. \tag{4}$$

Through the dynamical equations, the system evolves in the lightlike $\bar{x}^0$ direction, while $\bar{x}^i = x^i$ coordinates are kept constant. The method is different from usual light front quantization, where x^1, x^2 and $x^- = x^0 - x^3$ are kept constant during LF time evolution.

The components of the momentum vector transform as

$$\bar{p}_0 = p_0, \quad \bar{p}_3 = -p_0 + p_3, \quad \bar{p}_\alpha = p_\alpha; \quad \alpha = 1, 2. \tag{5}$$

Therefore, the dispersion relation of a massive particle takes the form

$$\bar{p}^2 - m^2 = -2\bar{p}_0\bar{p}_3 - \bar{p}_i\bar{p}_i - m^2 = 0, \tag{6}$$

from which one finds the light front energy

$$\bar{p}_0 = -\frac{\bar{p}_i^2 + m^2}{2\bar{p}_3} = -\frac{\omega_{\bar{p}}^2}{2\bar{p}_3}. \tag{7}$$

Finally, the density matrix for a system, interacting with a heat bath at rest, takes the form

$$\rho = e^{-\beta\bar{p}_0}, \tag{8}$$

where β is the inverse equilibrium temperature (we set $k_B = 1$).

The tree-level propagator for the scalar field at zero temperature reads

$$iG(\bar{p}) = \frac{i}{\bar{p}^2 - m^2 + i\epsilon} = \frac{i}{-2\bar{p}_0\bar{p}_3 - \omega_{\bar{p}}^2 + i\epsilon}. \tag{9}$$

For fermionic fields, there are two suggestions for a LF propagator. Traditionally, the constrained spinor degrees of freedom are explicitly eliminated and expressed in terms of the dynamical components. Thereby, noncovariant contact interactions terms are added to the fermion propagator and fermion photon vertex in QED. In standard LF coordinates [(i.e. $A = 1$ and $B = -1$ in (1)], one finds the following propagator [24]:

$$\begin{aligned} iS_C(p) &= i\left(\frac{1}{\gamma_\mu p^\mu - m + i\varepsilon} + \frac{\gamma^+}{2p^+}\right) \\ &= i\frac{\gamma_\mu p_{\rm on}^\mu + m}{p^2 - m^2 + i\varepsilon}, \end{aligned} \tag{10}$$

where $p_{\rm on}$ denotes the on-shell momentum. Furthermore, one has to include a noncovariant piece $-i\gamma^+/2p^+$ to the tree-level fermion photon vertex. As the noncovariant terms cancel to all orders in perturbation theory, the equivalence between LF and covariant formulation on the level of the scattering matrix was established [24].

We follow an alternative proposal [7] and work in the full Dirac space without integrating the constrained spinor components out. The advantage is that noncovariant contact terms are avoided, and no nonlocal terms are introduced to the theory. The details of the Dirac constrained quantization procedure in this case for scalar, fermionic, and gauge fields were clarified in Ref. [6].

The fermion propagator and bare vertex of the theory are then given by

$$iS(\bar{p}) = \frac{i(\bar{p}_\mu \bar{\gamma}^\mu + m)(\bar{\gamma}^0 - \bar{\gamma}^3)}{-2\bar{p}_0\bar{p}_3 - \omega_{\bar{p}}^2 + i\epsilon}, \tag{11}$$

and

$$\Gamma_{ee\gamma} = e(\bar{\gamma}^0 - \bar{\gamma}^3)\bar{\gamma}^\mu, \tag{12}$$

where the transformed γ matrices are

$$\bar{\gamma}^0 = \gamma^0 + \gamma^3, \qquad \bar{\gamma}^i = \gamma^i. \tag{13}$$

In the imaginary time formalism of thermal field theory, one puts $\bar{x}^0$ imaginary. Note that going to imaginary time does not commute with the coordinate transformation (4). If one demands $\bar{x}^0$ to be purely imaginary, then some of the Cartesian coordinates become complex, namely

$$\bar{x}^0 = -i(x^0 + x^3). \tag{14}$$

The linear transformation from Minkowski coordinates to imaginary time OLCF is given by

$$\bar{x}^\mu = L^\mu{}_\alpha x^\alpha, \tag{15}$$

where $L^\mu{}_\alpha$ is the matrix

$$L^\mu{}_\alpha = \begin{pmatrix} -i & 0 & 0 & -i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

We use here the same notation for the OLFC coordinates (4) and their imaginary time counterparts (15). It is clear from the context which coordinates are used.

Under the linear transformation (15), the metric tensor transforms as

$$\bar{g}^{\mu\nu} = L^\mu{}_\alpha L^\nu{}_\beta \eta^{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & i \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ i & 0 & 0 & -1 \end{pmatrix}, \tag{16}$$

where $\eta^{\alpha\beta}$ is the usual Minkowski metric. Furthermore, the energy-momentum four-vector is written as

$$\bar{p}_\mu = \begin{pmatrix} ip_0 \\ p_1 \\ p_2 \\ p_3 - p_0 \end{pmatrix},$$

which leads to

$$\bar{p}^2 = 2ip_0^n\bar{p}_3 - \bar{p}_i^2 = 4\pi nTi\bar{p}_3 - \bar{p}_i^2, \tag{17}$$

where the Matsubara frequencies $p_0^n = 2\pi nT$ for bosons were explicitly inserted.

The gamma matrices change under Eq. (15) as

$$\bar{\gamma}^0 = -i(\gamma^0 + \gamma^3), \tag{18}$$

such that

$$\{\bar{\gamma}^\mu, \bar{\gamma}^\nu\} = 2\bar{g}^{\mu\nu}, \tag{19}$$

where $\bar{g}^{\mu\nu}$ is the complex metric (16).

Finally, the propagators in the imaginary time formalism are given by

$$G^{(T)}(\bar{p}) = \frac{1}{\bar{p}^2 - m^2} = \frac{1}{(2\pi nT)i\bar{p}_3 - \bar{p}_i^2 - m^2} \tag{20}$$

and

$$S^{(T)}(\bar{p}) = \frac{(\bar{p}_\mu \bar{\gamma}^\mu + m)(i\bar{\gamma}^0 - \bar{\gamma}^3)}{2\pi(2n+1)Ti\bar{p}_3 - \bar{p}_i^2 - m^2}, \tag{21}$$

for the scalar and fermionic fields, respectively.

III. POLARIZATION TENSOR

A. Scalar theory

We start with the ϕ^3 theory and evaluate the one-loop self-energy in $(3+1)$ dimensions in the HTL approximation.

In the imaginary time formalism, the self-energy is

$$\begin{aligned}\Pi(\bar{p}) &= \int \frac{d^3\bar{k}}{(2\pi)^3} T\sum_n \frac{1}{\bar{k}^2}\frac{1}{(\bar{k}+\bar{p})^2} \\ &= \int \frac{d^3\bar{k}}{(2\pi)^3} T\sum_n \frac{1}{2(2\pi nT)i\bar{k}_3 - \bar{k}_i^2} \\ &\quad \times \frac{1}{2(2\pi nT + \bar{p}_0)i(\bar{k}_3 + \bar{p}_3) - (\bar{k}+\bar{p})_i^2}. \end{aligned} \tag{22}$$

Here, we are already considering the HTL approximation, i.e. we can neglect the masses. In general, we apply the following two rules to obtain the HTL approximation for any diagram:

(1) neglect masses and soft momentum p dependence in the numerator in favor of the hard loop momentum k,

(2) expand distribution functions and denominators in loop integrals to first order in p/k.

The Matsubara sum in Eq. (22) can be solved using [25]

$$T\sum_n \frac{1}{n+ix}\frac{1}{n+iy} = \frac{\pi}{x-y}(\coth(\pi x) - \coth(\pi y)). \tag{23}$$

This immediately leads to

$$\Pi(\bar{p}) = \int \frac{d^3\bar{k}}{(2\pi)^3}\left[\frac{-1}{8\bar{k}_3(\bar{k}_3+\bar{p}_3)}\frac{\coth\left(\frac{\beta E_1}{2}\right) - \coth\left(\frac{\beta E_2}{2}\right)}{E_1 - E_2 + i\bar{p}_0}\right],$$

where we have defined

$$E_1 = \frac{\bar{k}_i^2}{2\bar{k}_3}, \qquad E_2 = \frac{(\bar{k}_i + \bar{p}_i)^2}{2(\bar{k}_3 + \bar{p}_3)}. \tag{24}$$

Finally, applying the HTL rule (2), we find the following finite temperature contribution:

$$\Pi^\beta(\bar{p}) = -\frac{1}{4}\int \frac{d^3\bar{k}}{(2\pi)^3}\frac{1}{\bar{k}_3^2}\bigg[1 - \frac{i\bar{p}_0\bar{k}_3}{\bar{\eta}\cdot\bar{p}}\bigg]\frac{dn_B(E)}{dE}, \tag{25}$$

where we have introduced the four-vector

$$\bar{\eta}_\mu = (-iE\epsilon(\bar{k}_3), \bar{k}_\alpha, \bar{k}_3), \tag{26}$$

with

$$E = \frac{\bar{k}_i^2}{2|\bar{k}_3|} > 0 \tag{27}$$

and the sign function $\epsilon(\bar{k}_3)$.

This new lightlike four-vector $\bar{\eta}$ is the LF generalization of the four-vector $\eta = (-i, \hat{k})$ which appears in the equal time calculations. Furthermore, the structure of the integral (25) is consistent with the results found in the literature [23]. In the next section, we encounter integrals similar to Eq. (25) in order to obtain the transverse polarization tensor in QED.

B. QED

After the generalization of the HTL approximation to the LF in the scalar ϕ^3 theory, we are now in position to use this method to obtain the HTL polarization tensor $\Pi^{(T)}_{\mu\nu}$ in QED at one loop. In the standard LF frame, the perturbative computation of $\Pi_{\mu\nu}$ at zero temperature is challenging, and the outcoming results are strongly affected by the choice of the fermion propagator [26]. We therefore discuss first the calculation of the polarization tensor at zero temperature in the OLFC.

Using Eqs. (11) and (12), the zero-temperature contribution is given by

$$i\Pi_{\mu\nu}(\bar{p}) = -e^2\int\frac{d^4\bar{k}}{(2\pi)^4}\,\mathrm{Tr}(\bar{\gamma}^0 - \bar{\gamma}^3)\bar{\gamma}_\mu iS(\bar{k}) \times (\bar{\gamma}^0 - \bar{\gamma}^3)\bar{\gamma}_\nu iS(\bar{k} + \bar{p}). \tag{28}$$

Choosing dimensional regularization, we rewrite the previous equation in $d = 4 - \epsilon$ dimensions as

$$i\Pi^\epsilon_{\mu\nu}(\bar{p}) = -e^2\mu^\epsilon\int\frac{d^d\bar{k}}{(2\pi)^d}\,\mathrm{Tr}(\bar{\gamma}^0 - \bar{\gamma}^3)\bar{\gamma}_\mu iS(\bar{k}) \times (\bar{\gamma}^0 - \bar{\gamma}^3)\bar{\gamma}_\nu iS(\bar{k} + \bar{p}). \tag{29}$$

The usual properties of the gamma matrices in d dimensions, i.e.

$$\mathrm{Tr}\,\bar{\gamma}^\alpha\bar{\gamma}^\beta\bar{\gamma}^\sigma\bar{\gamma}^\rho = d(\bar{g}^{\alpha\beta}\bar{g}^{\sigma\rho} - \bar{g}^{\alpha\sigma}\bar{g}^{\beta\rho} + \bar{g}^{\alpha\rho}\bar{g}^{\beta\sigma}), \tag{30}$$

lead to

$$i\Pi^\epsilon_{\mu\nu}(\bar{p}) = -e^2\mu^\epsilon\int\frac{d^d\bar{k}}{(2\pi)^d}d\frac{\bar{k}_\mu(\bar{k}+\bar{p})_\nu + \bar{k}_\nu(\bar{k}+\bar{p})_\mu - \bar{g}_{\mu\nu}[\bar{k}\cdot(\bar{k}+\bar{p}) - m^2]}{(\bar{k}^2 - m^2)((\bar{k}+\bar{p})^2 - m^2)}. \tag{31}$$

We follow next the analysis in Ref. [7] and change variables in Eq. (31),

$$\bar{k}_0 = k_0, \quad \bar{k}_3 = -k_0 + k_3, \quad \bar{k}_\alpha = k_i, \quad (\alpha = 1, 2, \ldots, d-2), \tag{32}$$

such that

$$\bar{k}^2 = -2\bar{k}_0\bar{k}_3 - \bar{k}_i^2 = k^2 = k_0^2 - k_i^2, \qquad i = 1, \ldots, d-1. \tag{33}$$

One observes that the denominators, after substitution, are the usual Minkowski coordinate ones. By the standard argumentation, we then find

$$i\Pi^\epsilon_{\mu\nu}(\bar{p}) = \frac{-ie^2}{2\pi^2}(\bar{g}_{\mu\nu}\bar{p}^2 - \bar{p}_\mu\bar{p}_\nu)\int_0^1 dx x(1-x) \times\bigg(\frac{2}{\epsilon} - \ln\frac{q^2}{4\pi\mu^2} - \gamma - \frac{1}{2}\bigg), \tag{34}$$

with $q^2 = m^2 - x(1-x)p^2$ and γ as the Euler Mascheroni constant. Note that the numerical factor is exactly the same as the equal times one, and the polarization tensor is transverse in the barred variables.

As a next step, we determine the finite temperature contribution to the photon self-energy in the HTL approximation. Using the imaginary time formalism, one has

$$-\Pi^{(T)}_{\mu\nu}(\bar{p}) = -e^2T\sum_n\int\frac{d^3\bar{k}}{(2\pi)^3}\,\mathrm{Tr}(i\bar{\gamma}^0 - \bar{\gamma}^3)\bar{\gamma}_\mu S^{(T)}(\bar{k}) \times (i\bar{\gamma}^0 - \bar{\gamma}^3)\bar{\gamma}_\nu S^{(T)}(\bar{k}+\bar{p}), \tag{35}$$

where the LF fermion propagator in the oblique coordinates is given in Eq. (21). Evaluating the trace of the gamma matrices using Eq. (30) and (16) leads to

$$\Pi^{(T)}_{\mu\nu} = e^2\int\frac{d^3\bar{k}}{(2\pi)^3}T\sum_n\bigg[\frac{8\bar{k}_\mu\bar{k}_\nu}{\bar{k}^2(\bar{k}+\bar{p})^2} - \frac{4\bar{g}_{\mu\nu}}{(\bar{k}+\bar{p})^2}\bigg] \equiv I_{\mu\nu} - 4e^2\bar{g}_{\mu\nu}I. \tag{36}$$

Even before calculating explicitly the components, one can easily see that the Ward identity is satisfied. In fact,

$$\begin{aligned}\bar{p}_\mu \Pi^{\mu\nu(T)} &= 8e^2 \int \frac{d^3\bar{k}}{(2\pi)^3} T \sum_n \frac{(\bar{k}\cdot\bar{p})\bar{k}^\nu}{\bar{k}^2(\bar{k}+\bar{p})^2} - 4e^2 \delta^\nu_\mu \bar{p}^\mu I \\ &\cong 8e^2 \int \frac{d^3\bar{k}}{(2\pi)^3} T \sum_n \frac{[(\bar{k}+\bar{p})^2 - \bar{k}^2]\bar{k}^\nu}{2\bar{k}^2(\bar{k}+\bar{p})^2} \\ &\quad - 4e^2 \delta^\nu_\mu \bar{p}^\mu I \\ &= 4e^2 \int \frac{d^3\bar{k}}{(2\pi)^3} T \sum_n \left[\frac{\bar{k}^\nu}{\bar{k}^2} - \frac{(\bar{k}+\bar{p})^\nu}{(\bar{k}+\bar{p})^2}\right], \end{aligned} \tag{37}$$

where in the second line, we have neglected the $\bar{p}^2$ term in the numerator according to the HTL rules. The Ward identity is now obtained by first evaluating the Matsubara sums and then shifting $(\bar{k}_i + \bar{p}_i) \longrightarrow \bar{k}_i$. For example, for $\nu = \alpha$, we find

$$\begin{aligned}\bar{p}_\mu \Pi^{\mu\alpha(T)} &= 4e^2 \int \frac{d^3\bar{k}}{(2\pi)^3} T \sum_n \left[\frac{\bar{k}^\alpha}{\bar{k}^2} - \frac{(\bar{k}+\bar{p})^\alpha}{(\bar{k}+\bar{p})^2}\right] \\ &= -4e^2 \int \frac{d^3\bar{k}}{(2\pi)^3} T \sum_n \frac{(\bar{k}+\bar{p})^\alpha}{(\bar{k}+\bar{p})^2}, \end{aligned}$$

by antisymmetry. Evaluating the Matsubara sum leads to

$$\bar{p}_\mu \Pi^{\mu\alpha(T)} = e^2 \int \frac{d^3\bar{k}}{(2\pi)^3} (\bar{k}+\bar{p})^\alpha \frac{\tanh(\beta|E_2|/2)}{(\bar{k}_3+\bar{p}_3)^2}.$$

Considering only the finite temperature contribution, we are left with

$$\bar{p}_\mu \Pi^{\mu\alpha(\beta)} = -2e^2 \int \frac{d^3\bar{k}}{(2\pi)^3} (\bar{k}+\bar{p})^\alpha \frac{n_F(|E_2|)}{(\bar{k}_3+\bar{p}_3)^2}.$$

Once the finite temperature result is finite, we consider the shift $(\bar{k}+\bar{p})^i \longrightarrow \bar{k}^i$, and the Ward Identity follows for this component by antisymmetry. For the other two components, the calculations are similar, such that one can prove that, in the HTL approximation, the identity is valid,

$$\bar{p}_\mu \Pi^{\mu\nu(\beta)} = 0, \qquad \nu = 0, 1, 2, 3. \tag{38}$$

The rest of this section is concerned with the explicit evaluation of the polarization tensor. We will use Eq. (36) to explicitly evaluate each one of its components. With only one of them, the zero-zero component, the usual method of evaluating the Matsubara frequencies will fail because of the degree of divergence of the series involved. We will use the Ward identity to indirectly obtain it.

Starting with the scalar part I, one finds

$$\begin{aligned} I &= \int \frac{d^3\bar{k}}{(2\pi)^3} T \sum_n \frac{1}{(\bar{k}+\bar{p})^2} = \int \frac{d^3\bar{k}}{(2\pi)^3} T \sum_n \frac{1}{\bar{k}^2} \\ &\equiv \int \frac{d^3\bar{k}}{(2\pi)^3} S. \end{aligned} \tag{39}$$

Evaluating the Matsubara sum S, we use the relation

$$\sum_n f(n+1/2) = \pi \sum_{\text{Res f(z)}} \tan(\pi z) f(z). \tag{40}$$

Subsequently, one finds

$$\begin{aligned} S &= T \sum_n \frac{1}{2i\bar{k}_0\bar{k}_3 - \bar{k}_i^2} = T \sum_n \frac{1}{2(n+1)\pi T i \bar{k}_3 - \bar{k}_i^2} \\ &= -\frac{\tanh(\beta E/2)}{4|\bar{k}_3|}. \end{aligned} \tag{41}$$

Thus, the finite temperature contribution to I is given by

$$I^\beta = \frac{1}{2} \int \frac{d^3\bar{k}}{(2\pi)^3} \frac{n_F(E)}{|\bar{k}_3|}. \tag{42}$$

To solve this integral, we utilize the following identity:

$$n_F(x) = n_B(x) - 2n_B(2x), \tag{43}$$

such that

$$\begin{aligned} I^\beta &= \frac{1}{2} \int \frac{d^3\bar{k}}{(2\pi)^3} \frac{1}{|\bar{k}_3|} \left(\frac{1}{e^{\beta E} - 1} - \frac{2}{e^{2\beta E} - 1}\right) \\ &= \left(\frac{T}{2\pi}\right)^2 \int_0^\infty r dr \int \frac{dz}{z} \left[\frac{1}{e^{1/2(r^2/z+z)} - 1} - \frac{2}{e^{r^2/z+z} - 1}\right] \\ &= \frac{2T^2}{(2\pi)^2} \int_0^\infty \frac{dz}{z} \sum_{n=0}^\infty \int_0^\infty r dr e^{-n(r^2/z+z)} \\ &= \frac{T^2}{4\pi^2} \zeta(2) \\ &= +\frac{T^2}{24}, \end{aligned} \tag{44}$$

where we use cylindrical coordinates in the second line, and the $\zeta(x)$ denotes the ζ function.

The integral for the spatial components I_{ij} reads

$$I_{ij} = 8e^2 \int \frac{d^3\bar{k}}{(2\pi)^3} \bar{k}_i \bar{k}_j S_1, \tag{45}$$

with

$$\begin{aligned} S_1 &= T \sum_n \frac{1}{\bar{k}^2(\bar{k}+\bar{p})^2} \\ &= -\frac{1}{8} \frac{1}{\bar{k}_3(\bar{k}_3+\bar{p}_3)} \frac{\tanh(\beta E_1/2) - \tanh(\beta E_2/2)}{E_1 - E_2 + i\bar{p}_0}. \end{aligned} \tag{46}$$

The HTL approximation gives rise to the following simplifications:

$$E_1 - E_2 \simeq \frac{\bar{\eta}\cdot\bar{p}}{\bar{k}_3} - i\bar{p}_0, \tag{47}$$

with $\bar{\eta}_\mu$ defined in Eq. (26). Hence, one has the finite temperature contribution to I_{ij} as

$$I^\beta_{ij} \sim \frac{2e^2}{(2\pi)^3} \int d^3\bar{k} \frac{\bar{k}_i\bar{k}_j}{\bar{k}_3^2} \left(1 - \frac{i\bar{p}_0\bar{k}_3}{\bar{\eta}\cdot\bar{p}}\right) \frac{dn_F(E)}{dE}. \tag{48}$$

The integrals containing only the derivative of the distribution function can be evaluated analytically. For example, the first term of I^β_{33} is given by

$$\begin{aligned}\int \frac{d^3\bar{k}}{(2\pi)^3}\frac{dn_F(E)}{dE} &= \beta\frac{d}{d\beta}\int\frac{d^3\bar{k}}{(2\pi)^3}\frac{n_F(E)}{E}\\ &= -\frac{2T^2}{\pi^2}\int_0^\infty zdz\sum_{n=1}^{\infty}\int_{z^2}^{\infty}\frac{du}{u}e^{-(n/z)u}\\ &= -\frac{2T^2}{\pi^2}\int_0^\infty zdz\sum_{n=1}^{\infty}\int_1^\infty\frac{dv}{v}e^{-nzv}\\ &= +\frac{2T^2}{\pi^2}\sum_{n=1}^{\infty}\int_0^\infty zdzEi(-nz)\\ &= -\frac{T^2}{\pi^2}\zeta(2),\end{aligned} \tag{49}$$

where we have utilized the definition

$$Ei(-\mu) = -\int_1^\infty \frac{e^{-\mu x}dx}{x} \quad \text{for } \Re\mu > 0 \tag{50}$$

and the relation

$$\int_0^\infty Ei(-\beta x)x^{\mu-1}dx = -\frac{\Gamma(\mu)}{\mu\beta^\mu} \text{ with } \Re\beta, \quad \Re\mu > 0. \tag{51}$$

Inserting this result, I_{33}^β can be expressed as

$$I_{33}^\beta = -\frac{e^2T^2}{3} - 2e^2\int\frac{d^3\bar{k}}{(2\pi)^3}\frac{i\bar{p}_0\bar{k}_3}{\bar{\eta}\cdot\bar{p}}\frac{dn_F(E)}{dE}. \tag{52}$$

Analogously, the other components of I_{ij} are

$$I_{11}^\beta = I_{22}^\beta = -\frac{e^2T^2}{6} - 2e^2\int\frac{d^3\bar{k}}{(2\pi)^3}\frac{1}{\bar{k}_3}F(\bar{p}_0, \bar{\eta}\cdot\bar{p}, E), \tag{53}$$

$$I_{12}^\beta = -2e^2\int\frac{d^3\bar{k}}{(2\pi)^3}\frac{\bar{k}_1\bar{k}_2}{\bar{k}_3}F(\bar{p}_0, \bar{\eta}\cdot\bar{p}, E), \tag{54}$$

$$I_{\alpha 3}^\beta = -2e^2\int\frac{d^3\bar{k}}{(2\pi)^3}\bar{k}_\alpha F(\bar{p}_0, \bar{\eta}\cdot\bar{p}, E), \tag{55}$$

$$I_{0j}^\beta = \frac{ie^2T^2}{6}\delta_{j3} + 2ie^2\int\frac{d^3\bar{k}}{(2\pi)^3}\frac{\bar{k}_jE}{|\bar{k}_3|}F(\bar{p}_0, \bar{\eta}\cdot\bar{p}, E), \tag{56}$$

with

$$F(\bar{p}_0, \bar{\eta}\cdot\bar{p}, E) = \frac{i\bar{p}_0}{\bar{\eta}\cdot\bar{p}}\frac{dn_F(E)}{dE}. \tag{57}$$

At this point, one can check the consistency of the explicit calculations given above with the Ward identity (38). For example, using Eq. (36) and considering $\alpha = 1, 2$, one finds

$$\bar{p}_\mu\Pi^{\mu\alpha} = \frac{2e^2}{(2\pi)^3}\int d^3\bar{k}\frac{dn_F}{dE}\frac{i\bar{p}_0}{\bar{k}_3}\bar{k}_\alpha = 0 \tag{58}$$

by antisymmetry. A similar computation shows that $\bar{p}_\mu\Pi^{\mu 0} = 0$.

On the other hand, the explicit computation of I_{00} is much more intricate; basically the assumptions of Eq. (40) do not hold. Actually, proving Eq. (40), one needs the residue theorem and the assumption that $|f(z)| \le \frac{M}{|z|^k}$, with $k > 1$ and M being constants. If this is not the case, then one has to include the contributions coming from the arc and poles into the formula. At equal time field theories, such contributions always vanish because the dispersion relation is quadratic in k_0. In LF dynamics, this is not the case anymore. We think that these technical problems can be cured by following a strategy analogous to the zero-temperature discussions carried out in Ref. [20,21].

However, there is another way to obtain I_{00}. Using the $\nu = 3$ component of the Ward identity [Eq. (38)], one has

$$\begin{aligned}0 = \bar{p}_\mu\Pi^{\mu 3} &= 8e^2\int\frac{d^3k}{(2\pi)^3}T\sum_n\frac{(\bar{k}\cdot\bar{p})\bar{k}^3}{\bar{k}^2(\bar{k}+\bar{p})^2} - 4e^2\bar{p}^3I\\ &= -\bar{p}_3I_{00} - (2i\bar{p}_3 + \bar{p}_0)I_{03} - i\bar{p}_\alpha I_{0\alpha}\\ &\quad - (i\bar{p}_0 - \bar{p}_3)I_{33} + \bar{p}_\alpha I_{\alpha 3} - 4e^2(i\bar{p}_0 - \bar{p}_3)I.\end{aligned} \tag{59}$$

Solving for I_{00}, one finds

$$I_{00} = \frac{e^2T^2}{6} + 2e^2\int\frac{d^3\bar{k}}{(2\pi)^3}\frac{E^2}{\bar{k}_3}\frac{i\bar{p}_0}{\bar{\eta}\cdot\bar{p}}\frac{dn_F}{dE}. \tag{60}$$

Collecting all the results, the polarization tensor can be written as

$$\begin{aligned}\Pi_{\mu\nu}(p) &= 2m^2(\delta_{\mu 0}\delta_{\nu 0} - \delta_{\mu 3}\delta_{\nu 3} + i\delta_{\mu 0}\delta_{\nu 3} + i\delta_{\mu 3}\delta_{\nu 0})\\ &\quad - 2e^2\int d^3\bar{k}F(\bar{p}_0, \bar{\eta}\cdot\bar{p}, E)\bar{\eta}_\mu\bar{\eta}_\nu,\end{aligned} \tag{61}$$

where $m^2 = e^2T^2/6$ is the thermal mass.

IV. CONCLUSIONS

In this work, we have studied the QED polarization tensor at one loop in the oblique light front coordinates both at zero and at finite temperature in the HTL approximation.

At zero temperature, we have shown how using the fermion propagator (11) and dimensional regularization, one is able to make close contact to the equal time result. We have recovered the transverse structure in terms of the oblique coordinates and the same numerical factor as in covariant equal time calculations.

At finite temperature, we have considered the HTL approximation to obtain the dominant temperature dependent contributions. The finite temperature part of the polarization tensor obeys the Ward identity. All components of $\Pi_{\mu\nu}$, except Π_{00}, were computed directly. For Π_{00}, the usual analytic techniques, i.e. closing complex contours in the evaluation of Matsubara sums, fail due to the analytic structure of the integrand. This is one of the typical peculiarities of light cone coordinates. It is our expectation that these problems can be solved by resorting to more elaborated LF integration methods, as indicated in Ref. [21]. However, the result for the 00 component as given here will stay, as the Ward identity constitutes an independent constraint.

It is worth mentioning that even though the finite temperature results of Sec. (III B) were given in the imaginary time formalism, we have also considered the closed time path formalism. Using the real-time fermion propagator, as given in Ref. [7], we have evaluated Π_{11}. The finite temperature contribution to the retarded polarization tensor in the closed time path formalism is

$$\begin{aligned}\Pi^{\beta}_{R} &= \Pi^{\beta}_{++} + \Pi^{\beta}_{+-}\\ &\propto \int d^3k \frac{2\bar{k}_1^2 n_F(E)}{|\bar{k}_3|(\bar{k}_3 + \bar{p}_3)} \frac{1}{E_1 - E_2 - \bar{p}_0 + \frac{i\epsilon(\bar{p}_0 - E_1)}{2(\bar{k}_3 + \bar{p}_3)}}\\ &\quad - \frac{2\bar{k}_1^2 n_F(|E_2|)}{|\bar{k}_3 + \bar{p}_3|\bar{k}_3} \frac{1}{E_1 - E_2 - \bar{p}_0 - \frac{i\epsilon(\bar{p}_0 + E_2)}{2\bar{k}_3}}\\ &\quad + 2\frac{n_F(E)}{|\bar{k}_3|}. \end{aligned} \tag{62}$$

On the other hand, the finite temperature contribution to Π_{11} in imaginary time is

$$\begin{aligned}\Pi^{\beta} &= I^{\beta}_{11} + 4e^2 I^{\beta}\\ &\propto \int d^3k \frac{2\bar{k}_1^2 n_F(E)}{|\bar{k}_3|(\bar{k}_3 + \bar{p}_3)} \frac{1}{E_1 - E_2 - \bar{p}_0 + \frac{i\epsilon\bar{p}_0}{2\bar{p}_3}}\\ &\quad - \frac{2\bar{k}_1^2 n_F(|E_2|)}{|\bar{k}_3 + \bar{p}_3|\bar{k}_3} \frac{1}{E_1 - E_2 - \bar{p}_0 + \frac{i\epsilon\bar{p}_0}{2\bar{p}_3}}\\ &\quad + 2\frac{n_F(E)}{|\bar{k}_3|}. \end{aligned} \tag{63}$$

Equations (62) and (63) indicate that the two results are consistent as long as the right analytic continuation in $\bar{p}_0$ is made. Any analytic continuation must handle the pole structure in the complex plane. At the LF, even at zero temperature, the poles inside loop integrals depend on the integration variables [21], and it is not clear *a priori* which is the correct analytic continuation method. This raises very interesting questions as to how to compare the two formalisms in general light cone field theory.

Another perspective of the presented work is the discussion of the three vanishing momentum limits of the LF polarization tensor, namely:

(1) $\bar{k}_0 = 0$, $\bar{k}_3 = 0$ and $|\bar{k}_\alpha| \to 0$,
(2) $|\bar{k}_\alpha| = 0$, $\bar{k}_3 = 0$ and $\bar{k}_0 \to 0$,
(3) $|\bar{k}_\alpha| = 0$, $\bar{k}_0 = 0$ and $\bar{k}_3 \to 0$.

These different ways to approach the origin $\bar{k}_\mu = 0$ are connected to collective excitations in the plasma. At equal times, there are only two possible limits $k_0 = 0$, $|\vec{k}| \to 0$ and $|\vec{k}| = 0$, $k_0 \to 0$ because of rotational invariance. Taking these different limits at equal times, one finds screening and plasmon masses which should be independent from the chosen quantization plane. For the self-energy in scalar field theory, it was shown in Ref. [11] how to restore rotational symmetry of the spacelike directions by a transformation in order to reduce the three LF limits to two. Such a transformation should also exist for the QED polarization tensor (61). We hope to show in the future that this is indeed the case.

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