PHYSICAL REVIEW D **85,** 105001 (2012)

Lorentz violation in Hořava-Lifshitz-type theories

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(Received 2 December 2010; revised manuscript received 23 November 2011; published 3 May 2012)

We show that coupling the standard model to a Lorentz symmetry-violating sector may coexist with viable phenomenology provided that the interaction between the two is mediated by higher-dimensional operators. In particular, if the new sector acquires anisotropic-scaling behavior above a ''Hořava-Lifshitz'' energy scale $\Lambda_{\rm HL}$ and couples to the standard model through interactions suppressed by $M_{\rm pl}$, the transmission of the Lorentz violation into the standard model is protected by the ratio $\Lambda_{\rm HL}^2/M_{\rm pl}^2$. A wide-scale separation $\Lambda_{\rm HL} \ll M_{\rm pl}$ can then make Lorentz-violating terms in the standard model sector within experimental bounds without fine-tuning. We first illustrate our point with a toy example of Lifshitz-type neutral fermion coupled to photon via the magnetic moment operator, and then implement similar proposal for the Hořava-Lifshitz gravity coupled to conventional Lorentz-symmetric matter fields. We find that most radiatively induced Lorentz violation can be controlled by a large-scale separation, but the existence of instantaneously propagating non-Lifshitz modes in gravity can cause a certain class of diagrams to remain quadratically divergent above $\Lambda_{\rm HL}$. Such problematic quadratic divergence however can be removed by extending the action with terms of higher Lifshitz-dimension, resulting in a completely consistent setup that can cope with the stringent tests of Lorentz invariance.

DOI: 10.1103/PhysRevD.85.105001 PACS numbers: 11.30.Cp, 04.60.−m

I. INTRODUCTION

Lorentz symmetry and its universality with respect to propagation and interaction of different types of particles is a very well-established symmetry of nature. Stringent constraints are derived on the parameters of effective Lagrangian that encode possible departures from Lorentz symmetry [1,2]. Existing models of Lorentz symmetry-breaking did not go far beyond the effective Lagrangian description, and the idea that either a vector or the gradient of a scalar field condense at intermediate or low-energy while restoring the Lorentz symmetry at high energies [3–5] so far has not found any reasonable ultraviolet (UV) completion. Even more, it is not fully understood whether such completions exist in principle.

It is also conceivable that Lorentz symmetry is somehow broken by the UV physics, and, for example, quantum gravity is often being tauted as being the cause (see e.g. [6]). If Lorentz violation (LV) is indeed a UV-related phenomenon, then there is a significant conceptual hierarchy problem. One would expect that LV should manifest itself in the lowest-dimensional operators. Since the set of such operators starts from dimensions three and four [1,2], one should naively expect that the strength of LV interactions is of the order of $\Lambda_{\rm LV}$ for dimension-three operators, and $O(1)$ for dimension four. Several mechanisms for protecting higher-dimensional LV operators from ''leaking'' into the lower-dimensional ones have been proposed and partially summarized in [7].

The localization of LV to higher-dimensional operators can occur in various ways. For example, Ref. [8] assumed that operators responsible for Lorentz violation are tensors of a higher rank and irreducible, and therefore their appearance in dimension-three and dimension-four operators is prohibited. Refs. [9,10] argue that supersymmetrization of the standard model (SM) leads to automatic elimination of lower-dimensional LV operators. The soft-breaking terms allow this leakage into lower dimensions to happen but in a controllable way: e.g. the coefficients of dimension-four operators are induced by the dimension-six operators

$$c_{\rm LV}^{(4)} \sim m_{\rm soft}^2 c_{\rm LV}^{(6)} \sim \frac{m_{\rm soft}^2}{\Lambda_{\rm LV}^2}. \tag{1}$$

If there is a wide-enough scale separation between the SUSY-breaking mass and the high-energy scale where LV originates, $m_{\rm soft} \ll \Lambda_{\rm LV}$, the existence of Lorentz breaking can be made consistent with the variety of experimental constraints. Dimension four coefficients $c_{\rm LV}^{(4)}$ induce a difference between propagation speed for different particles, limited by the most-stringent constraints to be at the level of 10^{-23} (see e.g. [11]), which is perfectly safe; for example, if $m_{\rm soft}$ is at the weak scale and $\Lambda_{\rm LV}$ is close to Planck scale.

In this paper we examine another generic but very different way of protecting against LV leaking into the SM sector. Consider a LV-sector that couples to the SM via a power-suppressed interaction,

$$\frac{1}{M^{n+k-4}} O_{\rm LV}^{(n)} O_{\rm SM}^{(k)}, \tag{2}$$

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where $O_{\rm LV}^{n}$ and $O_{\rm SM}^{k}$ are some operators from LV- and SM-sectors of dimensions n and k, respectively, $n + k \geq 5$, and M is a very high-energy scale. Being power-suppressed, this operator would typically generate a power-divergent loop integral. For example, when $n = 1$ and $k = 4$, integrating-out fields in the LV-sector is likely to generate a quadratic divergence leading to an LV-term in the SM as

$$\frac{1}{M} O_{\rm LV}^{(1)} O_{\rm SM}^{(4)} \rightarrow \frac{\Lambda_{\rm UV}^2}{M^2} O_{\rm SM,\, LV}^{(4)}. \tag{3}$$

Theories of this kind are usually not considered viable on the phenomenological ground. The induced LV-term is generically of order one since naturally $\Lambda_{\rm UV} \sim M$. However, particularly interesting cases exist when the loops in the LV-sector are stabilized at high energy through certain mechanism so that $\Lambda_{\rm UV}$ gets replaced by a well-defined physical scale that can be separated far from M. In the latter case, the induced LV-terms as in (3) can be made arbitrarily small.

A well-known class of mechanisms of such kind is introducing higher-derivative terms in the interactions or propagators, which improves the convergence of loop integrals. Examples include the noncommutative field theories [12,13], the so-called Lee-Wick theories [14,15] and Hořava-Lifshitz-type theories [16,17]. In the last example, the following modification of a particle propagator is assumed at very large spatial momentum

$$\frac{i}{\omega^2 - \mathbf{k}^2} \rightarrow \frac{i}{\omega^2 - \frac{\mathbf{k}^6}{\Lambda_{\rm HL}^4}}. \tag{4}$$

While such a propagator leads to better convergent-loop integrals, the absence of higher derivatives with respect to time in the Lagrangian and, consequently, the absence of ω^4 etc. terms in the propagator allows one to extend the regime of validity of this theory beyond $\Lambda_{\rm HL}$ without immediately encountering pathological ghostlike features. But, at the same time such a construction leads to the violation of Lorentz symmetry explicitly above the Lifshitz scale. If, however, a theory of this type is coupled to SM sector through power-suppressed interactions only, it is conceivable that the size of induced LV-terms in SM is controlled by the ratio $\Lambda_{\rm HL}^2/M^2$ and can be made small given a sufficiently large separation between $\Lambda_{\rm HL}$ and M. There would be no need for fine-tuning since radiative corrections become stabilized so that $\Lambda_{\rm HL} \ll M$ alone would be sufficient.

We shall illustrate this mechanism in a toy example with a neutral fermion that has a Lifshitz-type propagator. It couples to photon through an anomalous magnetic moment, which is a power-suppressed interaction. In this case, as expected, the LV corrections induced by the fermion to the photon sector is controlled by $\mu^2 \Lambda_{\rm HL}^2$, where μ is the anomalous magnetic moment. Given that this product can be made arbitrarily small, approximate Lorentz symmetry in the photon sector is maintained despite being completely broken for the neutral fermion.

Perhaps the most interesting example of this type would be gravity since its interactions are suppressed by a very large scale. Besides many interesting features of Lifshitz-type field theories that have been intensively studied in the past, they have attracted a lot attention when Hořava proposed that a theory of this type stands as a candidate for a renormalizable theory of gravity [17]. Among different issues that Hořava's theory for gravity is facing at phenomenological level, the question of LV is not the last on the list. Given that the graviton propagators violate Lorentz symmetry in the ultraviolet, is it reasonable to expect that such a theory would respect Lorentz symmetry at low energies without tremendous fine-tuning? The answer to this question is by no means a straightforward one. If Hořava-Lifshitz-type behavior is more than just a cute way of making loops better convergent but indeed a description of nature at short distances, one has to specify how this behavior is consistent with stringent tests of Lorentz symmetry performed with a variety of the SM particles. We have two classes of interaction: (i) those that have dimensionless couplings in the standard model (α_s, α_W, α_{EM}), and (ii) those with gravity whose strength is controlled by the Newton constant $G_N = \frac{1}{8\pi M_{\rm pl}^2}$. Various loop corrections to the propagation of SM particles will have different types of divergences, and all of them must not introduce an overwhelming amount of LV. *A priori*, one has the option as to where to put Lifshitz behavior: in the matter sector, in the gravity sector, or in both. We shall distinguish two generic options:

(i) *Option 1* Both SM- and gravity-sectors flow into the Lifshitz-type behavior above $\Lambda_{\rm HL}$.

(ii) *Option 2* Only gravitational propagators become Lifshitz-type at $\Lambda_{\rm HL}$, while the bare SM action preserves normal Lorentz-symmetric propagators all the way to the Planck scale.

Option 1 leads to fine-tuning issues even in the limit as gravity is decoupled. Indeed, various SM-loop corrections to the dimension-four kinetic operators are not universal for different types of particles; e.g. compared to leptons and photons, quarks and gluons will have extra corrections due to the strong group, etc. *In the absence of additional protective symmetries*, this should lead to a Lorentz nonuniversality of radiative corrections. Even if one assumes an exact universality of the speed of propagation for different species, simple one-loop corrections would introduce a nonuniversality of the order of $\alpha_{\rm SM}/\pi \sim 10^{-3} - 10^{-2}$, which has to be tuned away at 1 part per 10^{20}. This was recently illustrated by the calculation of radiative corrections in the toy model that involved two different scalar fields [18]. Therefore, it seems that this option is troublesome even before the gravity effects are taken into account and regardless of whether one has a large-scale separation between $\Lambda_{\rm HL}$ and Planck scale.

Option 2 seems to be more viable. Indeed all the loop corrections that involve SM fields but not gravity are automatically Lorentz-preserving. The fact that gravity couples to the SM fields only through Planck-mass suppressed interaction leads us to consider the protection mechanism outlined above. Our proposal is that the gravitational loops (which normally would be power-divergent) get stabilized at Hořava-Lifshitz scale, so that possible nonuniversality generated through quantum corrections in the propagation speed of different species will indicate that the induced LV in the SM-sector scales is

$$\Delta c \sim \frac{\Lambda_{\rm HL}^2}{\pi^2 M_{\rm pl}^2}. \quad (5)$$

Similar to the toy example discussed earlier, one could hope to have control over this quantity via the ratio $\Lambda_{\rm HL}^2/M_{\rm pl}^2$, demanding sufficient-scale separation to ensure that it is small.

We perform a detailed one-loop analysis of Hořava-type gravity, calculating corrections to the speed of propagation for vectors and scalars and find that loop corrections produced by the spin-2 and spin-1 graviton do indeed exhibit the behavior described by (5), but some quadratic divergences associated with the vector-graviton loop diagrams remain. We see that these remaining quadratic-divergent corrections are not universal between scalars and vectors, thus potentially reinstating the issue of fine-tuning in the theory.

Our analysis, however, points toward a relatively easy solution to the fine-tuning problem. The inclusion into the action of a single-term that respects all the symmetries of the original model of Hořava-Lifshitz gravity but with a Lifshitz-dimension higher than 6; counted in a naive way is sufficient to suppress all quadratically divergent contributions and render the loop-induced Lorentz violation in the standard model sector completely under control. In such an extended model, the mechanism we conjectured above is fully at work, and the need for fine-tuning to maintain the Lorentz symmetry in consistency with the observations is totally absent. The model, on the other hand, might still harbor additional problems associated with the new terms we introduce, and we will defer extended discussion on this topic to follow-up works.

The status of Hořava's original proposal [17] as well as its various extensions [19–21], both on theoretical and phenomenological ground, is still being actively discussed and debated in the literature [22–39,50]. We make no attempts to delve on these issues in the current study, concentrating instead on the perturbative calculation at one-loop level using linearized-gravity action to illustrate our main points. Furthermore, to make calculations more straightforward we work within the "healthy-extension" framework proposed in [20], and assume that the full nonlinear theory is consistent provided that the parameters are chosen properly. It would become clear that our main conclusion is largely independent of the specific choice of those model-dependent parameters. We mention in passing that Lorentz-violating effects in Hořava's gravity, considered from very different angles, have also been discussed in other works [40,41,43,49].

Gravitational-loop calculations can be cumbersome, not least due to the necessity of introducing explicit gauge-fixing in the gravity sector. At one-loop level, quantum corrections to the effective action for each individual particle is gauge-choice dependent, but fortunately such dependence is always canceled when one compares the same correction for different matter fields. The *actual Lorentz-violation effect* we present in this paper, exhibited by dimension-four operators, i.e. the difference of the propagation speeds of massless particles with different spins or other quantum numbers, is independent of the gauge choice and therefore bares true physical meaning. Of course, radiative corrections to the propagation speed identical for all matters can be absorbed by a simple scaling of space and time coordinates and, therefore, do not lead to real Lorentz symmetry-violating effects.

Our calculations deal with the divergent-loop integrals, and therefore the choice of regularization may affect the answer. Let us further clarify the basic assumptions and the main goals of this paper. We adopt the Wilsonian point of view on radiative corrections and use a hard-cutoff regulator for the 3-momentum implicitly. Introducing a hard-cutoff breaks the diffeomorphism invariance of the general relativity as well as the Hořava-Lifshitz theory. As a consequence, one has to be cautious in calculating the full one-loop-corrected effective action, as coefficients in front of all quadratically divergent terms may not have direct physical meaning. However, the Lorentz symmetry, being a global symmetry, is not broken by the hard cutoff in the 3-momentum space in a perturbative calculation above the flat spacetime background. Therefore, as long as the fate of the Lorentz symmetry is concerned, which is the main topic of this paper, the simplest regularization scheme we use is acceptable. This observation is also related to the fact that one-loop corrections to the kinetic terms for each individual particle we derive below is not gauge-choice independent, but the net effect of the Lorentz-violating observable given the difference among the one-loop corrections for different species turns out to be fully gauge-invariant. There, of course, exist other regularization schemes where any power-law divergences are nullified, such as dimensional regularization. We believe that such regularizations are not useful for answering the question of technical naturalness of LV theories. Such schemes can be adopted as self-consistent methods as long as the theory is believed to be valid up to arbitrarily high scale without any new physics emerging at UV, which in the authors' opinion is a somewhat unrealistic assumption. As long as one holds the viewpoint that the Lagrangian we are dealing with should be understood only within a context of an effective theory below a certain scale, power-law divergent-radiative corrections are real physical effects in the sense that they indicate the low-energy theory is sensitive to the UV physics. Should the quadratic sensitivity to the cutoff appear in the observables such theories are not free from the fine-tuning problem.

In the current context, our working assumption is that the Lorentz symmetry starts emerging in the bare Lagrangian at certain high scale. We wish to calculate how the radiative corrections are to maintain or destroy this symmetry for observations made at a much lower scales within the reach of current experiments. The theory will be free from fine-tuning if power-law divergent-loop diagrams are absent. In fact, in the following discussions, the cancellation or noncancellation for power-law divergent diagrams are presented in terms of the loop integrals themselves without the need of evaluating them explicitly. Different regularization may associate to those integrals different numbers, reflecting the UV sensitivity of the model itself, but if they are canceled identically without the need of evaluating the loop integrals explicitly, it certainly remains so in any regularization scheme, in which case we claim the low-energy theory is robust against radiative corrections. In this sense, a naive UV cutoff serves merely as overly simplified terminology that allows us to refer to the UV-sensitive loop integrals as being ‘‘power-law divergent.’’

On the other hand, any Lorentz-violating effects given by the logarithmically divergent-loop integrals remain the same in any regularization scheme one happens to prefer including, for example, the dimensional regularization with only the $\Lambda_{\rm UV}$ replaced by the renormalization scale.

This paper is organized as follows. In Sec. II we analyze a toy model with a neutral Lifshitz-type fermion interacting with photon via the magnetic moment, and calculate its radiative corrections to the photon action and the induced Lorentz violation. In Sec. III we introduce the Hořava-Lifshitz-type theories for gravity, truncate the action to the quadratic level, and derive the propagators for the gauge-invariant modes. In Sec. IV we calculate the difference of the propagation speed for vectors and scalars, both minimally coupled to gravity, where we will find residual fine-tuning in the standard Hořava-Lifshitz models. Section V presents a simple extension to the same model where such fine-tuning can be eliminated. We include further discussion in Sec. VI. More details regarding the loop calculations are presented in Appendixes A and B. Appendix C includes two toy models for the Lifshitz-type QED, which being gauge theories shares a lot of common properties and issues with the Hořava-Lifshitz gravity.

A few words on the convention we would follow in this paper: we will consider only one-loop diagrams, which either consist of only one propagator and one vertex, or two propagators and two vertices. Each vertex contains a factor of $\frac{1}{i\hbar}$, and it cancels precisely the factor of $i\hbar$ carried by each propagator. Consequently, we can safely ignore these factors altogether. Just to fix the notation, if the action is given by the form $S = -\frac{1}{2}\phi\mathcal{O}\phi + \lambda\phi^2$, we would say the propagator is $\mathcal{O}^{-1}$ and the vertex is λ. We shall also use the convention that $\Box = -\partial_t^2 + \Delta$. Its transformation into the momentum space is given by the rule $\partial_t \to -i\omega$ and $\partial_i \to ik_i$, and therefore $\Box \to \omega^2 - \vec{k}^2$.

II. A TOY MODEL OF A NEUTRAL LIFSHITZ FERMION

Let us first consider a simple toy example. Suppose we have a Lifshitz-type neutral fermion whose action is given by

$$\mathcal{L}_\psi = \bar{\psi}[\gamma^0\partial_t + \Lambda_{\rm HL}^{1-z}(\sqrt{-\Delta})^{z-1}\gamma^i\partial_i]\psi, \tag{6}$$

where we have introduced a Lifshitz scale $\Lambda_{\rm HL}$ and the Lifshitz critical exponent z. When $z > 1$, this action has an anisotropic-scaling behavior. In principle one should include all the lower spatial-derivative-terms but at large $\vec{k}$, which is the limit that we are mainly interested in; the highest spatial-derivative term dominates and we will keep only it.

Let us suppose that this fermion couples to photon through an irrelevant operator given by

$$\mathcal{L}_I = \frac{1}{2M}F^{\mu\nu}\bar{\psi}\sigma_{\mu\nu}\psi, \tag{7}$$

where M is a mass parameter, which gives the fermion an anomalous magnetic moment $\mu = M^{-1}$. The photon-kinetic term takes the usual form $\mathcal{L}_A = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$.

We would like to evaluate the fermion one-loop correction to the photon-kinetic operator. In particular we are looking for Lorentz symmetry-violating effect. It is useful to define

$$(\tilde{k}^0, \tilde{\vec{k}}) \equiv (k^0, |k|^{z-1}\vec{k}/\Lambda_{\rm HL}^{z-1}) \tag{8}$$

and

$$\tilde{k}^2 \equiv -k_0^2 + \frac{|\vec{k}|^{2z}}{\Lambda_{\rm HL}^{2z-2}}. \tag{9}$$

With these notations, the fermion propagator is $1/\tilde{\not{k}}$.

The one-loop integral that contributes to the photon-kinetic operator in the zero external-momentum limit is given by

$$K = -\frac{1}{4i(2\pi)^4M^2}\int d^4k \frac{F^{\mu\nu}F^{\alpha\beta}\mathrm{tr}\,\sigma_{\mu\nu}\tilde{\not{k}}\sigma_{\alpha\beta}\tilde{\not{k}}}{\tilde{k}^4}. \tag{10}$$

Detailed calculation of this integral is presented in Appendix B. It is found that when $z = 1$ and the theory respects the Lorentz symmetry K vanishes identically, leading to no correction to the photon-kinetic term at this level. When $z > 1$ and the Lorentz symmetry is broken

$$K = \frac{\Lambda_{\rm HL}^{3(1-1/z)}(f^t - f^x)}{2M^2}(\mathbf{E}^2 + \mathbf{B}^2), \tag{11}$$

where

$$f^t \equiv -\frac{8}{i(2\pi)^4}\int d^4k \frac{k_0^2}{z|\vec{k}|^{3(1-1/z)}k^4}, \tag{12}$$

and

$$f^x \equiv \frac{8}{3i(2\pi)^4}\int d^4k \frac{|\vec{k}|^2}{z|\vec{k}|^{3(1-1/z)}k^4}. \tag{13}$$

As $z = 3$, both f^t and f^x are logarithmically divergent and it turns out that $f^t = 3f^x$. Consequently, including the one-loop correction, the photon-kinetic term becomes

$$\mathcal{L}_A = \frac{1}{2}\left(1 + \frac{\Lambda_{\rm HL}^2}{3\pi^2M^2}\log\frac{\Lambda_{\rm UV}}{\Lambda_{\rm HL}}\right)\mathbf{E}^2 - \frac{1}{2}\left(1 - \frac{\Lambda_{\rm HL}^2}{3\pi^2M^2}\log\frac{\Lambda_{\rm UV}}{\Lambda_{\rm HL}}\right)\mathbf{B}^2, \tag{14}$$

which leads to an effective “speed of light”

$$c'^2 = \left(1 - \frac{2\Lambda_{\rm HL}^2}{3\pi^2M^2}\log\frac{\Lambda_{\rm UV}}{\Lambda_{\rm HL}}\right)c^2. \tag{15}$$

One can easily see that this correction is under control if there is a wide-scale separation between $\Lambda_{\rm HL}$ and M. Notice also that the Lorentz symmetry of the interaction-term in (7) is not essential for the scaling (15) to hold. We could, for example, “disbalance” $\sigma_{0i}F^{0i}$ and $\sigma_{ij}F^{ij}$ in a LV way, which would affect the numerical coefficient in (15), but not change the ratios of the dimensionful scales.

III. ACTION AND PROPAGATORS FOR THE HORíAVA-TYPE GRAVITY

From this point on, we would like to consider quantum corrections to ordinary matter fields coupled to a Hořava-Lifshitz-type gravity. The main point is that since gravity is coupled to matter through irrelevant couplings, the loop effects are suppressed by $1/M_{\rm pl}^2$ but this suppression is compensated in GR by a quadratic UV divergence. Such divergences have been encountered in previous calculations of LV effects with graviton loops (see e.g. [42,43]). Hořava gravity has the virtue that at least some parts of the loop diagrams are more convergent since the graviton-propagator scales anisotropically at large momentum. For Lifshitz critical exponent $z = 3$, the better convergent loops are logarithmically divergent only, leading to a logarithmic running of the effective speed of light in the matter sector. If all the loop-induced quantum corrections are logarithmically divergent as such, it is conceivable that given a wide separation between the scale $\Lambda_{\rm HL}$ and $M_{\rm pl}$, similar to what we found in the previous section, the induced violation of the Lorentz symmetry might be under control. The main physics question to be addressed is whether the matter actions acquire quantum corrections that lead to the nonuniversality of the propagation speed, and, if so, with what coefficients? In fact, as we show below, such corrections are generically not universal and different c^2 for different species induces observable LV effects.

In this section, we briefly describe the gravity theory of interest. The fact that Hořava theory is a gauge theory, which contains constraints and nondynamical fields makes the problem more involved compared to the simple toy example presented above. We will find that at one-loop level, the theory exhibits mixed properties: while some loops are better convergent as expected, others remain quadratically divergent. Nonlinearity makes any gravitational theory quite difficult to analyze perturbatively without running into various subtleties. The physics is much more transparent in simpler examples, such as a Lifshitz-type quantum electrodynamics, which we present in Appendix C as an analogy to the calculation we perform for the gravity case below.

We define the fields for the metric perturbations above the flat spacetime background as

$$-g_{00} = 1 + n, \tag{16}$$

$$g_{0i} = n_j, \tag{17}$$

$$g_{ij} = \delta_{ij} + h_{ij}. \tag{18}$$

Einstein's theory of general relativity, expressed in ADM formalism, is described by the Lagrangian $\mathcal{L}_{\rm EH} = M_{\rm pl}^2\sqrt{\gamma}N(R + K_{ij}K^{ij} - K^2)$. The action for Hořava gravity is different from it in two aspects, both leading to the violation of Lorentz symmetry. In the low momentum-limit it differs from GR in that the combination $K_{ij}K^{ij} - K^2$ is replaced by a more general expression $K_{ij}K^{ij} - \lambda K^2$, where a model-dependent parameter λ is introduced. In the large momentum-limit, it is proposed that higher spatial-derivative terms dominate the action and are the key ingredients that render the graviton-loop more convergent and the theory renormalizable. For our purpose, the highest dimensional operators are the most important, and they include $R_{ij}\Delta R^{ij}$ and $R\Delta R$. We adopt the so-called “healthy extension” [20] of the original theory in this paper, where additional terms such as

$$R\Delta^2 n = -\frac{2\sigma\Delta^3 n}{M_{\rm pl}^2} \tag{19}$$

and $n\Delta^3 n$ are also needed to completely Lifshitzise the scalar sector. All the fields introduced above are spacetime-dependent functions and it is the so-called “nonprojectable” version that is being considered here. We parameterize the highest spatial-derivative terms by

$$\mathcal{L}_{\rm Hořava} = M_{\rm pl}^2\left(\ldots + \Lambda_{\rm HL}^{-4}R_{ij}\Delta R^{ij} + \frac{a-3}{8}\Lambda_{\rm HL}^{-4}R\Delta R + \frac{b}{2}\Lambda_{\rm HL}^{-4}n\Delta^3 n - \frac{c}{2}\Lambda_{\rm HL}^{-4}R\Delta^2 n\right). \tag{20}$$

Here, a Lifshitz scale $\Lambda_{\rm HL}$ as well as three model-dependent parameters a, b, and c are introduced. We will leave these parameters completely undetermined (other than requiring $b \neq 0$), and simply assume that some reasonable choices of these parameters exist such that the theory is free from instabilities or strong coupling issues.

To derive the propagators, we expand the metric perturbation into different modes that do not mix with each other, and then invert the kinetic-term in each sector individually. It is most natural in this setup to decompose the fields into different spin-sectors with respect to the three-dimensional rotational symmetry. From that point of view, n is a scalar and we define

$$n_i = n_i^{\rm T} + \partial_i \varphi, \tag{21}$$

$$h_{ij} = h_{ij}^{\rm TT} + (\partial_i V_j^{\rm T} + \partial_j V_i^{\rm T}) + \left(\delta_{ij} - \frac{\partial_i \partial_j}{\Delta}\right)\sigma + \frac{\partial_i \partial_j}{\Delta}\tau, \tag{22}$$

where notation TT and T denote traceless-transverse and transverse conditions, respectively. We have altogether one transverse-traceless tensor $h_{ij}^{\rm TT}$, two transverse vectors $V_i^{\rm T}$ and $n_i^{\rm T}$, and four scalars n, φ, σ, and τ.

Expanding the action $\mathcal{L}_{\rm Hořava}$ in terms of these variables to quadratic order, we decompose the result into three independent sectors, which are referred to as the spin-2, spin-1, and spin-0 parts of the action and denoted by $\mathcal{L}_{\mathbf{2,1,0}}$, respectively. Explicitly, they are

$$\begin{aligned}
\mathcal{L}_{\mathbf{2}} &= \frac{1}{4}\dot{h}_{ij}^{\rm TT_2} + \frac{1}{4}h^{{\rm TT}_{ij}}\Delta h_{ij}^{\rm TT} + \frac{1}{4\Lambda_{\rm HL}^4}h_{ij}^{\rm TT}\Delta^3 h^{{\rm TT}_{ij}},\\
\mathcal{L}_{\mathbf{1}} &= -\frac{1}{2}(\dot{V}_i^{\rm T} - n^{{\rm T}_i})\Delta(\dot{V}^{{\rm T}_i} - n_i^{\rm T}),\\
\mathcal{L}_{\mathbf{0}} &= \frac{1-2\lambda}{2}\dot{\sigma}^2 - \frac{1}{2}\sigma\Delta\sigma - (\lambda - 1)\left(\Delta\varphi - \frac{1}{2}\dot{\tau}\right)^2\\
&\quad + \lambda\dot{\sigma}(2\Delta\varphi - \dot{\tau}) - 2n\Delta\sigma + \frac{a}{2\Lambda_{\rm HL}^4}\sigma\Delta^3\sigma\\
&\quad + \frac{b}{2\Lambda_{\rm HL}^4}n\Delta^3 n + \frac{c}{\Lambda_{\rm HL}^4}\sigma\Delta^3 n.
\end{aligned} \tag{23}$$

Since λ appears only in $\mathcal{L}_{\mathbf{0}}$, both $\mathcal{L}_{\mathbf{1}}$ and $\mathcal{L}_{\mathbf{2}}$ are identical for $\mathcal{L}_{\rm EH}$ and $\mathcal{L}_{\rm Hořava}$ if higher-derivative terms are omitted.

In a truncated expansion of the gravity action the full diffeomorphism-symmetry is lost but a "partial gauge-symmetry" remains. It is easily verified that the action given above makes explicit the following gauge symmetry:

$$V_i^{\rm T} \to V_i^{\rm T} + \zeta_i^{\rm T}, \qquad n_i^{\rm T} \to n_i^{\rm T} + \dot{\zeta}_i^{\rm T}, \tag{24}$$

$$\varphi \to \varphi + \dot{\omega}, \qquad \tau \to \tau + 2\Delta\omega, \tag{25}$$

where $\zeta_i^{\rm T}$ and ω are arbitrary infinitesimal functions. When $\lambda = 1$, the linearized Einstein-Hilbert action enjoys an additional gauge-symmetry generated by

$$n \to n - 2\dot{\chi}, \qquad \varphi \to \varphi + \chi. \tag{26}$$

For future purposes, we also define the gauge-invariant combinations

$$v_i^{\rm T} \equiv \dot{V}_i^{\rm T} - n_i^{\rm T}, \qquad \chi \equiv \frac{1}{2}\dot{\tau} - \Delta\varphi, \tag{27}$$

which, instead of $V_i^{\rm T}$, $n_i^{\rm T}$, τ and φ are the real "gauge-independent degrees of freedom." In Hořava-Lifshitz gravity, the "fourth" gauge-symmetry is missing so that n is "physical" by itself (apart from the time-reparameterization symmetry).

The action $\mathcal{L}_{\mathbf{2}}$ leads to the propagators for spin-2 gravitons without any gauge ambiguity. Directly read from the action, it is given by

$$\langle h_{ij}^{\rm TT} h_{kl}^{\rm TT}\rangle = -\frac{\Pi(\vec{k})_{ij,kl}}{\omega^2 - \vec{k}^2 - \Lambda_{\rm HL}^{-4}\vec{k}^6}, \tag{28}$$

where

$$\begin{aligned}
\Pi(\vec{k})_{ij,kl} &= \left(\delta_{ik} - \frac{k_i k_k}{\vec{k}^2}\right)\left(\delta_{jl} - \frac{k_j k_l}{\vec{k}^2}\right)\\
&\quad + \left(\delta_{jk} - \frac{k_j k_k}{\vec{k}^2}\right)\left(\delta_{il} - \frac{k_i k_l}{\vec{k}^2}\right)\\
&\quad - \left(\delta_{ij} - \frac{k_i k_j}{\vec{k}^2}\right)\left(\delta_{kl} - \frac{k_k k_l}{\vec{k}^2}\right).
\end{aligned} \tag{29}$$

The propagators given by $\mathcal{L}_{\mathbf{1,0}}$, to the contrary, cannot be determined without making a gauge choice. The technical details, including the gauge-fixing and the propagators are presented in Appendix A. In what follows, as much as possible, we carry out our calculations without choosing any particular gauge and express the results in terms of the physical quantities consisting of gauge-invariant combinations only. It will be shown that our final conclusion is valid in general and manifestly independent of the gauge conditions.

IV. LOOP-INDUCED LORENTZ VIOLATION IN THE MATTER SECTOR

We will consider in this section one-loop corrections to the matter kinetic-terms due to their coupling to gravity described by a Hořava-type theory. Our goal is to compute the radiative corrections to the effective propagation speed for different species. Any difference $c_{\rm species\,1} - c_{\rm species\,2} \neq 0$, if present, would indicate the violation of Lorentz symmetry at the quantum level.

We briefly mention the strategy for the calculation. Since we are only interested in the one-loop corrections, it is sufficient to expand the action in metric perturbations up to quadratic order. For those terms that are linear in metric perturbations, we "square" them to form a one-loop diagram, using two vertices, each containing one graviton leg. For these diagrams, the loop is formed by one graviton propagator and one matter propagator. For terms quadratic in metric perturbation, we must form a closed graviton-loop with single graviton-propagator. We focus on the leading divergent contributions, and therefore will set the external momentum to zero inside the loop integrals. Moreover, we are interested only in those one-loop radiative corrections to the matter kinetic-term that can actually lead to violation of the Lorentz symmetry, and therefore it suffices to expand $\sqrt{-g}$ to the first-order because at one-loop level the radiative corrections from the quadratic expansion of $\sqrt{-g}$ can only renormalize $M_{\rm pl}$. Since terms that contain metric perturbations at quadratic order contribute only when the metric perturbations are contracted among themselves forming a single graviton closed-loop, we are allowed to replace all the quadratic expression of the metric perturbations in the action by their correlation functions directly, which entails a sequence of simplifications. For example, a term in the action of the form $F_{ij}(h_{kl}, n, n_k)\partial^i\phi\partial^j\phi$, where F_{ij} is a *quadratic* expression of the metric perturbations can be equivalently replaced by its correlation function $\langle F_{ij}\rangle = \frac{1}{3}\langle F_{kl}\delta^{kl}\rangle\delta_{ij}$. In this last step, we have made use of the three-dimensional rotational symmetry that remains valid in Hořava's gravity. Similarly, terms of the form of $F_i(h_{kl}, n, n_k)\partial^i\phi\dot{\phi}$, where F_i is a three-dimensional vector that is also a *quadratic* function of the metric perturbations cannot contribute at one-loop level and will be omitted. Vertices that mix different spin components of gravitons do not contribute at one-loop either and are omitted. We will apply these simplifications implicitly from this point on, and, for brevity, drop the $\langle\cdot\rangle$ sign in the action if any quadratic expression of the metric perturbation fields are replaced by the corresponding correlation function.

We would repeatedly encounter the divergent integrals

$$\mathbb{L} \equiv \frac{1}{i(2\pi)^4}\int\frac{\mathrm{d}\omega\mathrm{d}^3k}{\omega^2 - \Lambda_{\rm HL}^{-4}\vec{k}^6}, \tag{30}$$

and when the fields are canonically normalized and the proper scales are restored, we have $\mathbb{L} = \frac{\Lambda_{\rm HL}^2}{8\pi^2 M_{\rm pl}^2}\log\frac{\Lambda_{\rm UV}^2}{\Lambda_{\rm HL}^2}$. Individual diagrams may also be quadratically divergent and proportional to $\Lambda_{\rm UV}^2/M_{\rm pl}^2$. As explained in the introduction, by referring to those diagrams as quadratically divergent, we have implicitly assumed that the integrals are to be regularized by a hard cutoff $\Lambda_{\rm UV}$ in the 3-momentum space. At $\Lambda_{\rm UV}$-scale the theory for the matter sector is described by the bare Lagrangian that is manifestly Lorentz invariant, such as those given in (31) and (41), or more generally, by the Lagrangian for the standard model of particles and fields. We wish to investigate, for a low-energy observer, to what extent the Lorentz symmetry exhibited by the bare Lagrangian defined at $\Lambda_{\rm UV}$ can be preserved. We emphasize that explicit UV-cutoff breaks the gauge symmetries in the gravity sector and a more involved regulator will be needed if we are looking for the physically meaningful effects coming from the $\Lambda_{\rm UV}$-scale. It is however sufficient in the current discussion since the hard cutoff does not break the Lorentz symmetry itself. We will not need to evaluate those loop integrals explicitly, and it suffices to recognize that the presence of such radiative corrections indicates that the low-energy theory is sensitive to the UV physics, leading to the fine-tuning problem.

Finally, let us fix the convention for the notation of correlation functions. Since all correlation functions considered here are two-point functions of two operators, and we always take the external momentum to zero, we can omit the "," and denote $\langle AB\rangle \equiv \langle A, B\rangle$, which in momentum space should be understood as $\langle A(\omega, \vec{k}), B(-\omega, -\vec{k})\rangle$. For brevity, we also introduce a notation for the correlation functions of two identical operators: we denote $\langle A, \ldots\rangle \equiv \langle A(\omega, \vec{k})A(-\omega, -\vec{k})\rangle$.

Let us first consider a scalar ϕ minimally coupled to gravity described by the Lorentz-symmetric "bare" Lagrangian

$$\mathcal{L} = -\frac{1}{2}\sqrt{-g}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi, \tag{31}$$

whose propagator is of course just $\langle\phi\phi\rangle = -1/(\omega^2 - \vec{k}^2)$. It is meant to represent an elementary SM matter field, such as the Higgs field.

Following the strategy explained above, we expand the action to the quadratic order in terms of the metric perturbations and decompose the interaction terms into each spin sector defined by the relevant metric perturbations involved. Explicitly, we have

$$\mathcal{L}_{\mathbf{2}}^{\rm I} = \left(\frac{1}{2}h^{{\rm TT}_{ij}} - \frac{1}{2\cdot 3}h^{{\rm TT}_{kl}}h^{\rm TT}_{kl}\delta^{ij}\right)\partial_i\phi\partial_j\phi, \tag{32}$$

$$\mathcal{L}_{\mathbf{1}}^{\rm I} = -n^{{\rm T}_i}\partial_i\phi\dot{\phi} + \partial^i V^{{\rm T}_j}\partial_i\phi\partial_j\phi - \frac{1}{3}(\partial_k V_l^{\rm T}\partial^k V^{{\rm T}_l} + n_k^{\rm T}n^{{\rm T}_k})\partial^i\phi\partial_i\phi + \ldots, \tag{33}$$

and

$$\begin{aligned}\mathcal{L}_{\mathbf{0}}^{\rm I} = {}& \frac{1}{2}\left(\sigma + \frac{1}{2}\tau - n\right)\dot{\phi}^2 - \partial^i\varphi\partial_i\phi\dot{\phi} \\ & + \frac{1}{2}\left[\frac{\partial_i\partial_j}{\Delta}(\tau - \sigma) - \frac{1}{2}\tau\delta^{ij} - n\delta^{ij}\right]\partial^i\phi\partial^j\phi \\ & + \frac{1}{2}(2n^2)\dot{\phi}^2 - \frac{1}{2}\cdot\frac{2}{3}\left[n(2\sigma + \tau) - \sigma\tau\right. \\ & \left. + \frac{1}{4}\tau^2 + \partial^k\varphi\partial_k\varphi\right]\partial^i\phi\partial_i\phi + \ldots.\end{aligned} \tag{34}$$

Here, ellipses stands for terms that are manifestly Lorentz invariant, which we drop in the subsequent calculations.

Let us denote the one-loop radiative correction to the kinetic term of ϕ as

$$\delta(\partial_\mu\phi\partial^\mu\phi) = \frac{1}{2}(K^t\dot{\phi}^2 - K^x\partial^i\phi\partial_i\phi), \tag{35}$$

and the contributions from each part of the interaction $\mathcal{L}_{\mathbf{2,1,0}}^{\rm I}$ as $K_{\mathbf{2,1,0}}^t$ and $K_{\mathbf{2,1,0}}^x$, respectively.

It is clear that $K^t_{\mathbf{2}} = 0$ and $K^x_{\mathbf{2}} = -\frac{4}{3}\cdot\mathbb{L}$. The contribution induced by the vector-gravitons is also easy to compute and the result is identical to that in GR. We find $K^t_{\mathbf{1}} = 0$, and[1]

$$K^x_{\mathbf{1}} = \frac{1}{3}\int \mathrm{d}\omega \mathrm{d}^3\vec{k}\frac{\langle \dot{n}^{\mathrm{T}_i} - \Delta V^{\mathrm{T}_i}, \ldots\rangle}{\omega^2 - \vec{k}^2} + \frac{2}{3} \times \int \mathrm{d}\omega \mathrm{d}^3\vec{k}\langle \partial_k V^{\mathrm{T}}_l \partial^k V^{\mathrm{T}_l} + n^{\mathrm{T}}_k n^{\mathrm{T}_k}\rangle. \tag{37}$$

We denote the combination given above as $K^x_{\mathbf{1}\mathrm{scalar}}$. This expression is gauge-choice dependent and therefore cannot be physical. It will be cancelled in the final result by other contributions, as we will show shortly.

Similarly, we can compute the contributions from the spin-0 sector as

$$K^t_{\mathbf{0}} = \int \mathrm{d}\omega \mathrm{d}^3\vec{k}\bigg(-\frac{\langle \dot{\sigma} - \dot{n} + \chi, \ldots\rangle}{\omega^2 - \vec{k}^2} + \langle 2nn\rangle\bigg) \tag{38}$$

and[2]

$$K^x_{\mathbf{0}} = \int \mathrm{d}\omega \mathrm{d}^3\vec{k}\bigg[\frac{1}{3}\cdot\frac{\langle \chi - \dot{\sigma} - \dot{n}, \ldots\rangle}{\omega^2 - \vec{k}^2} + \bigg\langle\frac{1}{12}\tau^2 - \frac{1}{3}(\sigma - n)^2 - \frac{1}{3}\sigma\tau + \tau n + \vec{k}^2\varphi^2\bigg\rangle\bigg]. \tag{39}$$

All contributions combined, we find that the effective change of the propagation speed for a neutral scalar, given by the difference between K^x and K^t is

$$\delta c^2_{\mathrm{scalar}} = -\frac{4}{3}\cdot\mathbb{L} + K^x_{\mathbf{1}\mathrm{scalar}} + \int \mathrm{d}\omega \mathrm{d}^3\vec{k}\frac{1}{\omega^2 - \vec{k}^2} \times \bigg[\frac{1}{3}\langle \chi - \dot{\sigma} - \dot{n}, \ldots\rangle + \langle \dot{\sigma} - \dot{n} + \chi, \ldots\rangle\bigg] + \int \mathrm{d}\omega \mathrm{d}^3\vec{k}\bigg\langle\frac{1}{12}\tau^2 - \frac{1}{3}(\sigma - n)^2 - \frac{1}{3}\sigma\tau + \tau n + \vec{k}^2\varphi^2 - 2n^2\bigg\rangle. \tag{40}$$

[1]It so happens that in the current theory

$$\int d\omega d^3k\langle \partial_i V^{\mathrm{T}}_j \partial^i V^{\mathrm{T}_j} + n^{\mathrm{T}}_i n^{\mathrm{T}_i}\rangle = 0, \tag{36}$$

in any gauge when the symmetry-preserving regularization of the UV divergence is employed. In any event, this term cancels out in the final answer by itself without employing the vanishing of this loop integral.

[2]We attempt to express everything in terms of the gauge-invariant combinations, in this case, χ as defined in (27). To do so, identities as $\frac{\vec{k}^2\omega^2}{\omega^2 - \vec{k}^2} = (\vec{k}^2 + \frac{\vec{k}^4}{\omega^2 - \vec{k}^2})$ and $\frac{\vec{k}^2}{\omega^2 - \vec{k}^2} = (-1 + \frac{\omega^2}{\omega^2 - \vec{k}^2})$ are used so that we can trade time-derivative for spatial-derivatives and vice versa, at the cost of generating extra terms that can be combined with those generated by the single graviton-loop diagrams. We will apply the similar identities while computing the spin-0 graviton-loop corrections to the photon-kinetic term as well.

Let us do the same calculation for a $U(1)$-gauge field coupled to gravity. The action for the minimally coupled photon is given by

$$\mathcal{L} = -\frac{1}{4}\sqrt{-g}g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}. \tag{41}$$

To avoid choosing a gauge for photon, we work with the physical fields $E_i = F_{0i}$ and $B_i = \frac{1}{2}\varepsilon_{ijk}F^{jk}$ and their correlation functions. It is easily verified that in any gauge

$$\langle E_i E_j\rangle = \frac{-\omega^2\delta_{ij} + k_i k_j}{\omega^2 - \vec{k}^2}, \quad \langle B_i B_j\rangle = \frac{-\vec{k}^2\delta_{ij} + k_i k_j}{\omega^2 - \vec{k}^2}, \quad \langle E_i B_j\rangle = \frac{\varepsilon_{ijn}\omega k^n}{\omega^2 - \vec{k}^2}. \tag{42}$$

Following the same procedure as before, we find the relevant part of the graviton-photon interactions, separated into different spin-sectors is given by

$$\mathcal{L}^{\mathrm{I}}_{\mathbf{2}} = -\frac{1}{2}h^{\mathrm{TT}_{ij}}(E_iE_j + B_iB_j) - \frac{1}{2\cdot 6}h^{\mathrm{TT}_{ij}}h^{\mathrm{TT}}_{ij}B^2 + \ldots, \tag{43}$$

$$\mathcal{L}^{\mathrm{I}}_{\mathbf{1}} = -\partial_i V^{\mathrm{T}}_j(E^iE^j + B^iB^j) - \varepsilon_{ijk}n^{\mathrm{T}_i}E^jB^k - \frac{1}{6}(n^{\mathrm{T}_i}n^{\mathrm{T}}_i + \partial_i V^{\mathrm{T}}_j\partial^i V^{\mathrm{T}_j})B^2 + \ldots, \tag{44}$$

$$\mathcal{L}^{\mathrm{I}}_{\mathbf{0}} = -\frac{1}{2}\bigg[\bigg(n - \frac{1}{2}\tau\bigg)\delta^{ij} + \frac{\partial^i\partial^j}{\Delta}(\tau - \sigma)\bigg](E_iE_j + B_iB_j) - \varepsilon_{ijk}\partial^i\varphi E^jB^k + \frac{1}{2}(2n^2)E^2 - \frac{1}{2}\bigg(\frac{1}{3}\sigma^2 + \frac{1}{6}\tau^2 + \frac{1}{3}\partial^i\varphi\partial_i\varphi\bigg)B^2 + \ldots, \tag{45}$$

and, again "…" represents those terms shared by both E^2 and B^2 that do not lead to any Lorentz-symmetry violation.

Let us consider the one-loop correction to the photon-kinetic term, denoted similarly as $\frac{1}{2}(K^tE^2 - K^xB^2)$. Again, the contributions from each spin-sector $\mathcal{L}^{\mathrm{I}}_{\mathbf{2,1,0}}$ are denoted as $K^t_{\mathbf{2,1,0}}$ and $K^x_{\mathbf{2,1,0}}$, respectively. It is most easily checked that

$$K^t_{\mathbf{2}} = \frac{4}{3}\cdot\mathbb{L}, \quad \text{and} \quad K^x_{\mathbf{2}} = -\frac{2}{3}\cdot\mathbb{L}. \tag{46}$$

To evaluate the contributions from the spin-1 sector, the vertex $\varepsilon_{ijk}n^{\mathrm{T}_i}E^jB^k$ is very important. When all the crossing-terms are properly included, one finds that many terms combine into a "complete square" so that $K^t_{\mathbf{1}}$ can be expressed as

$$K^t_{\mathbf{1}} = -\frac{1}{3}\int \mathrm{d}\omega \mathrm{d}^3k\vec{k}^2\langle V^{\mathrm{T}}_i V^{\mathrm{T}_i}\rangle - \frac{1}{3}\int \mathrm{d}\omega \mathrm{d}^3k\frac{\vec{k}^2\langle \upsilon^{\mathrm{T}}_i \upsilon^{\mathrm{T}_i}\rangle}{\omega^2 - \vec{k}^2}, \tag{47}$$

where $\upsilon^{\mathrm{T}}_i = \dot{V}^{\mathrm{T}}_i - n^{\mathrm{T}}_i$ is the gauge-invariant combination defined in Eq. (27). Similarly,

$$K_{\mathbf{1}}^{x} = \frac{1}{3}\int d\omega d^3k\langle n_i^{\mathrm{T}} n^{\mathrm{T}_i}\rangle + \frac{1}{3}\int d\omega d^3k\frac{\langle \dot{n}_i^{\mathrm{T}} - \Delta V_i^{\mathrm{T}}, \ldots\rangle}{\omega^2 - \vec{k}^2} + \frac{1}{3}\int d\omega d^3\vec{k}\langle \partial_k V_l^{\mathrm{T}}\partial^k V^{\mathrm{T}_l} + n_k^{\mathrm{T}} n^{\mathrm{T}_k}\rangle. \tag{48}$$

The contributions from loop diagrams with a scalar-graviton propagator are somewhat more cumbersome but still straightforward to calculate. With all the crossing-terms included and everything expressed in terms of the gauge-invariant combinations whenever possible, one eventually finds that

$$K_{\mathbf{0}}^{t} = \int d\omega d^3k\Big[-\frac{2}{3}\frac{\langle \dot{n} - \chi, \ldots\rangle}{\omega^2 - \vec{k}^2} - \frac{1}{3}\Big\langle n + \frac{1}{2}\tau - \sigma, \ldots\Big\rangle + \langle 2nn\rangle\Big], \tag{49}$$

and

$$K_{\mathbf{0}}^{x} = \int d\omega d^3k\Big[\frac{2}{3}\frac{\langle \dot{n} - \chi, \ldots\rangle}{\omega^2 - \vec{k}^2} + \Big\langle -\frac{2}{3}\Big(n - \frac{1}{2}\tau\Big)^2 + \frac{1}{3}\sigma^2 + \frac{1}{6}\tau^2 + \vec{k}^2\varphi^2\Big\rangle\Big]. \tag{50}$$

Putting these formulas together, we find the effective change of the speed of light for photon is

$$\delta c^2_{\text{photon}} = -2\cdot\mathbb{L} + K^{x}_{\mathbf{1}\,\text{scalar}} + \frac{1}{3}\int d\omega d^3k\frac{\vec{k}^2\langle v_i^{\mathrm{T}} v^{\mathrm{T}_i}\rangle}{\omega^2 - \vec{k}^2} + \frac{4}{3}\int d\omega d^3k\frac{\langle \dot{n} - \chi, \ldots\rangle}{\omega^2 - \vec{k}^2} + \int d\omega d^3k\Big\langle -\frac{1}{3}n^2 + \frac{1}{12}\tau^2 + \frac{2}{3}\sigma^2 + n\tau - \frac{2}{3}n\sigma - \frac{1}{3}\tau\sigma + \vec{k}^2\varphi^2 - 2n^2\Big\rangle, \tag{51}$$

where $K^{x}_{\mathbf{1}\,\text{scalar}}$ is the exact combination given in Eq. (37).

Now, we are ready to examine the real Lorentz-symmetry violating effect given by the difference of the graviton one-loop correction to the propagation speed for different species, e.g. scalar and photon field in the current case. The final answer, being the difference between Eqs. (51) and (40), is rather simple and reads

$$\delta c^2_{\text{photon}} - \delta c^2_{\text{scalar}} = -\frac{2}{3}\cdot\mathbb{L} + \frac{1}{3}\int d\omega d^3k\frac{\vec{k}^2\langle v_i^{\mathrm{T}} v^{\mathrm{T}_i}\rangle}{\omega^2 - \vec{k}^2} + \frac{4}{3}\int d\omega d^3k\frac{\vec{k}^2\langle\sigma n\rangle - \langle\chi\dot{\sigma}\rangle}{\omega^2 - \vec{k}^2} - \frac{1}{3}\int d\omega d^3k\frac{\omega^2 + 3\vec{k}^2}{\omega^2 - \vec{k}^2}\langle\sigma^2\rangle. \tag{52}$$

It is this quantity that measures the *actual violation of the Lorentz symmetry*, which cannot be simply scaled away by field and coordinate redefinitions. In this final result, all gauge-dependent quantities including $K^{x}_{\mathbf{1}\text{scalar}}$ and any correlation functions that explicitly contain τ and φ disappear. Therefore, it is fully physical and independent of the gauge-fixing scheme. The second-term above is quadratically divergent, leading to a residual fine-tuning problem in this model as we discuss further below. This divergence is the direct consequence of the non-Lifshitz behavior of propagators for the spin-1 gravitons. The second line, generated by the spin-0 gravitons, on the other hand, leads only to logarithmic divergence, which can be easily seen from the explicit propagators given in Appendix A. All the model-dependent quadratic divergences contributed by the spin-0 gravitons are completely canceled out in this final answer, so that the only remaining quadratic divergences comes from the vector-graviton loops.

We can furhter simplify this formula slightly if we use the knowledge that all propagators $\langle\chi\dot{\sigma}\rangle$, $\langle\sigma\sigma\rangle$, and $\langle\sigma n\rangle$ are of Lifshitz-type, and loop integrals can be reduced to

$$\int\frac{d\omega d^3k\omega^2}{\omega^2 - \vec{k}^2}\langle\text{Lifshitz}\rangle \approx \int d\omega d^3k\langle\text{Lifshitz}\rangle + \text{ finite terms}, \qquad \int\frac{d\omega d^3k\vec{k}^2}{\omega^2 - \vec{k}^2}\langle\text{Lifshitz}\rangle = \text{finite}. \tag{53}$$

Dropping all finite contributions, we have

$$\delta c^2_{\text{photon}} - \delta c^2_{\text{scalar}} = -\frac{2}{3}\cdot\mathbb{L} + \frac{1}{3}\int d\omega d^3k\Big[\frac{\vec{k}^2\langle v_i^{\mathrm{T}} v^{\mathrm{T}_i}\rangle}{\omega^2 - \vec{k}^2} - \Big(\frac{4}{\omega^2}\langle\chi\dot{\sigma}\rangle + \langle\sigma\sigma\rangle\Big)\Big]. \tag{54}$$

Substituting-in the explicit forms for the propagators given in Appendix A, we reach our final result in the current version of Hořava-Lifshitz gravity,

$$(\delta c^2)_{\text{photon}} - (\delta c^2)_{\text{scalar}} = -\frac{\Lambda^2_{\text{HL}}}{12\pi^2 M^2_{\text{pl}}}\log\frac{\Lambda^2_{\text{UV}}}{\Lambda^2_{\text{HL}}} - \frac{3\lambda + 1}{(3\lambda - 1)}\frac{\Lambda'^2_{\text{HL}}}{24\pi^2 M^2_{\text{pl}}}\log\frac{\Lambda^2_{\text{UV}}}{\Lambda'^2_{\text{HL}}} - \frac{\Lambda^2_{\text{UV}}}{24\pi^2 M^2_{\text{pl}}}. \tag{55}$$

Here Λ'_{HL} is the model-dependent Lifshitz energy-scale defined in Eq. (A5).

Very similar results are found in the case of simple Lifshitz-Abelian gauge theory, which we demonstrate in Appendix C.

We will discuss the implications of this result and propose ways to improve the model in order to eliminate all the quadratic divergence in Sec. V.

V. AN IMPROVED MODEL AND THE ABSENCE OF FINE-TUNING

Our calculations in the previous section show that Hořava-Lifshitz gravity and its extensions discussed in the literature thus far induce Lorentz-violation effects in the standard model sector with quadratic sensitivity to the cutoff. This poses a serious problem since the model has essentially no natural protection against large Lorentz-violation in the matter sector, and therefore a tremendous amount of fine-tunning is required to keep the model consistent with observations. This quadratic divergence in $\delta c^2_{\rm photon} - \delta c^2_{\rm scalar}$ means that our proposal based on a large-scale separation $\Lambda_{\rm HL}/M_{\rm pl} \ll 1$ to protect the Lorentz symmetry in the standard model does not work, and we must modify the theory in order to remove such remaining divergence.

Given our formula (54), the problematic piece is easy to spot. It is the vector-graviton contribution, identical to those in GR that leads to the problem because

$$\langle v_i^{\rm T} v^{{\rm T}_i} \rangle = -\frac{2}{\vec{k}^2}, \qquad (56)$$

and does not go to zero at large $|\vec{k}|$ the same way the Lifshitz propagators do. This part of the calculation entirely parallels its counterpart in the Einstein theory and therefore it is not at all surprising that it remains quadratically divergent.

There are ways to modify the theory to remove the quadratic divergence. Naturally, one thinks of including in the theory a term that contains $v_i^{\rm T}\Delta^2 v^{{\rm T}_i}$ so that at large momenta the propagator receives Lifshitz scaling $v_i^{\rm T} v^{{\rm T}_i} \sim 1/\vec{k}^4$ sufficient to suppress the relevant loop-integral and make it logarithmic. In the three-dimensional covariant notation, such terms may originate either from $K_{ij}\Delta K^{ij}$ or $\nabla^i K_{ij} \nabla^k K^{kj}$. Both possibilities are usually not considered since their Lifshitz dimensions are higher than 6 in the naive counting method. Note, however, such counting is questionable in theories with mixed Lifshitz and non-Lifshitz behavior considered in this paper.

The consequences of $K_{ij}\Delta K^{ij}$ or $\nabla^i K_{ij}\nabla^k K^{kj}$ terms in the action are not explored. One potential worry is the modification to the ordinary kinetic-term for the spin-2 gravitons by $K_{ij}\Delta K^{ij}$-term, and to avoid this we shall consider the addition to the Hořava-Lifshitz Lagrangian given by

$$\mathcal{L}' = \frac{2}{\Lambda^2} \nabla^i K_{ij} \nabla^k K^{kj}, \qquad (57)$$

so that at the linearized-level it only modifies the spin-1 and spin-0 graviton actions, and produces terms

$$\mathcal{L}' = \frac{1}{2\Lambda^2} v_i^{\rm T} \Delta^2 v^{{\rm T}_i} - \frac{2}{\Lambda^2} \chi \Delta \chi. \qquad (58)$$

We can easily repeat our calculation in this new model when such terms are included. The propagators are given in Appendix A, and using them we find

$$(\delta c^2)_{\rm photon} - (\delta c^2)_{\rm scalar} = -\frac{\Lambda_{\rm HL}^2}{12\pi^2 M_{\rm pl}^2} \times \left(1 + \frac{\sqrt{(1-2\lambda)\alpha^{-1}}}{2(2\lambda - 1)}\right) \log\frac{\Lambda_{\rm UV}^2}{\Lambda_{\rm HL}^2} - \frac{\Lambda^2}{12\pi^2 M_{\rm pl}^2} \log\frac{\Lambda_{\rm UV}^2}{\Lambda^2}. \qquad (59)$$

This expression contains logarithmically divergent pieces only, and we note that each of the spin-2, spin-1, and spin-0 sector contributes one term.

In the new theory with the additional term (57) included in the Lagrangian, the mechanism we proposed in the introduction is fully at work. One can safely put both $\Lambda_{\rm HL}$ and Λ well below the Planck scale, and the entire framework, consisting of both a Lifshitz-type gravity and a nearly Lorentz-invariant standard model sector would stay completely consistent with observations.

VI. DISCUSSION

In this paper we argue that a large amount of Lorentz violation in the irrelevantly coupled-sectors (axions, gravity, etc.) can coexist with the Lorentz-symmetric phenomenology of SM particles and fields provided that quantum corrections are stabilized by a Lifshitz-type behavior above $\Lambda_{\rm HL}$, a scale that can be adjusted. This idea is of particular interest if the LV-sector is gravity and is described by a Hořava-type theory. The key to this proposal is the ‘‘self-regulating’’ behavior of Lifshitz-type propagators that participate in the loops. Given that one could entertain a possibility of very large energy-scale separation $\Lambda_{\rm HL} \ll M_{\rm pl}$, the induced differences in the speed of propagation for different SM species can be under control by the ratio $(\Lambda_{\rm HL}/M_{\rm pl})^2$ and no fine-tuned choice of bare parameters to maintain Lorentz symmetry will be needed.

Our explicit calculations for a generalized Hořava-type gravity coupled with conventional matter-fields have confirmed this expectation in the following sense: those fields in the gravitational sector that fully acquire the anisotropic scaling, such as the truly dynamical transverse and traceless gravitons induce Lorentz violation controlled by $(\Lambda_{\rm HL}/M_{\rm pl})^2 \log\Lambda_{\rm UV}$. The quadratic divergence of graviton-loop is explicitly softened to the logarithmic one above the Hořava-Lifshitz scale. However, our result, Eq. (55), shows that in the conventional extensions of Hořava-Lifshitz gravity, loop-induced Lorentz-violating effects do contain a residual quadratic divergence. This divergence is generated by the non-Lifshitz parts of the gravitational action for the vector-gravitons. Therefore, for the choice of $\Lambda_{\rm UV} \sim M_{\rm pl}$, our idea of putting dimension-four LV operators under control of a small ratio of two-dimensionful parameters does not quite work there: LV from the Hořava gravity-sector will be efficiently transmitted to the SM sector with the quadratic sensitivity to the cutoff $(\Lambda_{UV}/M_{pl})^2$. (In some sense, the situation is reminiscent of noncommutative field theories, where certain divergences are self-regulated while others remain.)

Could this problem be resolved? A quick remedy we proposed is to include terms that suppress the vector-graviton propagator in the UV, such as $\nabla^i K_{ij} \nabla_k K^{kj}$. This addition term in the action ensures the Lifshitz-type behavior for the vector modes of the metric perturbations and is consistent with all the symmetries of Hořava gravity. It contains no more than two time-derivatives as required. Typically such terms are not considered because of their higher Lifshitz-dimension, but in the theory with mixed-behavior (Lifshitz for gravity and non-Lifshitz for matter) the naive counting of scaling dimensions can be misleading. Such terms appear to be admissible and they lead to the same softening of the loop integrals for the spin-1 modes as those of the other graviton modes. With these terms included, the one-loop corrections to the propagation speed of different species are fully under control and always proportional to $(\Lambda_{HL}/M_{pl})^2 \log \Lambda_{UV}$. Provided that $\Lambda_{HL} \ll M_{pl}$, the induced Lorentz-violation effect in the standard model can be minimized to the phenomenologically acceptable level. We also emphasize that the introduction of these new operator structures will exorcize quadratic divergence in *any* regulator scheme, while the coefficients in front of the logarithmic-divergent terms will be regularization-scheme independent. We reserve more detailed analysis of the proposed extension for the follow-up works.

We close-up with additional comments on the viability of the whole setup, and various phenomenological options.

(i) *On the choice of scale for* Λ_{HL}. Our answer suggests the *maximum* scale for the transition to Hořava-Lifshitz behavior. Given that various phenomenological constraints on dimension-four LV operators are more stringent than 10^{-20}, one would need to have $\Lambda_{HL} \lesssim 10^{10}$ GeV. This is an intermediate scale often appearing in particle physics, a geometric mean of weak and Planck scales. A much more definitive statement about the limit on Λ_{HL} can be made once we extend our calculations to actual SM fermions (electrons, quarks), which we plan to address in the future. On the other hand, nothing prevents choosing much lower scales for Λ_{HL}, such as a TeV or even meV scales. The latter is the absolute minimum set by precision tests of gravity at sub-mm scales.

(ii) *Graviton propagation speed.* So far we have considered only corrections to the propagation speed of matter, but graviton propagation is also of phenomenological interest. Deep inside the Hořava-Lifshitz regime the gravitons are superluminal and therefore cannot be constrained by e.g. Cerenkov radiation. However, there are also much milder 1%-level accuracy constraints on $c_{graviton}$ coming from the gravitational energy loss of binary pulsars. There are no good arguments in this theory as to why the matter and gravity should propagate with the same speed in the IR and possibly some additional emergent symmetry is required.

(iii) *Higher-dimensional operators and higher-loop corrections.* So far in our considerations we neglected external momenta of particles. This corresponds to explicitly calculating dimension-four LV operators, while neglecting dimension six. It turns out that the highest-energy cosmic rays can also be (barely) sensitive to the Planck scale normalized dimension-six operators [11]. Investigating the actual size of these operators induced by graviton loops is worthly of a separate investigation. Similarly, an important subject to address is the higher-loop order, where normal SM radiative corrections and gravitational corrections are combined.

(iv) *Hořava gravity and supersymmetry.* Supersymmetry of the Hořava-type theories was considered recently in e.g. [44]. It may bring additional benefits of making the speed of light universal, not only among matter fields but also for gravitons. In addition, the nonlinear terms in the gravity action may be used as a way of breaking supersymmetry [45], in which case one should expect the soft-breaking mass in the matter sector to scale as $m_{soft} \sim \Lambda_{HL}^2/M_{pl}$. This is again suggestive of the intermediate scale of 10^{10} GeV as a reasonable choice for Λ_{HL}.

(v) *Regularization dependence.* In the current study, we have simply used a hard cutoff of spatial momentum as a regulator. As we have explained, as long as the Lorentz-violation effects and the associated fine-tuning problem in the standard model is concerned, such regularization scheme is sufficient since it preserves the global symmetry under concern. It is no longer valid if we wish to calculate the complete gauge-invariant one-loop effective action, such as exact value of $\Delta c/c$ associated with quadratic divergence, or nonlogarithmic corrections to $\log(\Lambda_{UV}/\Lambda_{HL})$ coming from the upper-limit of integration. Technically, more challenging regulator procedures may involve a finite splitting between any pair of vertices in loop diagrams in terms of their geodesic distance in the coordinate space, or employ a hard cutoff for some gauge-invariant quantities in the momentum space.

ACKNOWLEDGMENTS

The authors would like to acknowledge useful discussions with N. Afshordi, T. Jacobson, L. Leblond, M. Serone, and M. Trott. M. P. would also like to acknowledge illuminating discussions with S. Groot Nibbelink (many years ago and long before this paper) on the improvement of UV-loop behavior due to resummed LV propagators. Y. S. would like to thank O. Pujolas, S. Sibiryakov, G. Gabadadze, and D. Zwanziger for very useful discussions and communications. In addition, M. P. is grateful to the organizers and participants of the Cambridge workshop on gravity and Lorentz violation, which catalyzed the revision of this paper. This work was supported in part by Natural Sciences and Engineering Research Council Canada. Research at the Perimeter Institute was supported in part by the Government of Canada through Natural Sciences and Engineering Research Council and by the Province of Ontario through Ministry of Economic Development and Trade.

APPENDIX A: PROPAGATORS AND GAUGE-FIXING IN HORíAVA-LIFSHITZ GRAVITY

The action $\mathcal{L}_1$ is identical in both Einstein's and Hořava's theories if operators with Lifshitz dimensions higher than 6 are ignored. Both $\mathcal{L}_1$ and $\mathcal{L}_0$ contain gauge symmetries generated by (24) and (25), and one must choose a gauge-fixing scheme to derive the propagators for vector and scalar gravitons.

The simplest gauge condition[3] to choose that respects the Lifshitz symmetry at large momentum would be $n_i = 0$. In this gauge, we easily derive the propagators

$$\langle V_i^{\mathrm{T}} V_j^{\mathrm{T}}\rangle = -\frac{\delta_{ij} - k_i k_j/\vec{k}^2}{\omega^2 \vec{k}^2}, \qquad \langle n_i^{\mathrm{T}} *\rangle = 0, \tag{A1}$$

for vector-gravitons and at large $\vec{k}$

$$\begin{aligned} \langle\sigma\sigma\rangle &= -\frac{\tilde{\lambda}}{\omega^2 - \alpha\tilde{\lambda}\Lambda_{\mathrm{HL}}^{-4}\vec{k}^6}, \\ \langle\sigma\tau\rangle &= -\frac{\tilde{\lambda} - 1}{\omega^2 - \alpha\tilde{\lambda}\Lambda_{\mathrm{HL}}^{-4}\vec{k}^6}, \\ \langle\sigma n\rangle &= \frac{c}{b}\frac{\tilde{\lambda}}{\omega^2 - \alpha\tilde{\lambda}\Lambda_{\mathrm{HL}}^{-4}\vec{k}^6}, \qquad \langle\varphi *\rangle = 0, \end{aligned} \tag{A2}$$

for spin-0 gravitons. We have defined the parameters

$$\alpha \equiv a - \frac{c^2}{b}, \qquad \tilde{\lambda} \equiv \frac{\lambda - 1}{3\lambda - 1} \tag{A3}$$

in the above expressions, and omitted the correlations functions that are irrelevant to our results. In this gauge, some of the propagators are singular and more involved regularization scheme is proposed [46,47], but those subtleties do not complicate our calculation here since our final answer is manifestly gauge-choice independent and any divergences that may arise in the individual-loop diagram will be canceled out in physical quantities. Or, if any doubts remain, one can also carry out the calculation in the R_ξ gauge where a gauge-fixing term $\mathcal{L} = -\frac{1}{2\xi}(\dot{n}_i - \alpha\Delta V_i)^2$ is introduced. All the propagators in that gauge will be "healthy" and the final answer not only is independent of the choice of ξ and α but also agrees with that found in the $n_i = 0$ gauge. Explicit calculation shows that

$$\langle v_i^{\mathrm{T}} v^{\mathrm{T}_i}\rangle = -\frac{2}{\vec{k}^2}, \qquad \langle\dot{\sigma}\chi\rangle = \frac{\lambda}{3\lambda - 1}\frac{\omega^2}{\omega^2 - \alpha\tilde{\lambda}\Lambda_{\mathrm{HL}}^{-4}\vec{k}^6}. \tag{A4}$$

We would like to make a few more comments on these results. We find a new scale emerging in the spin-0 sector. The propagators do exhibit anisotropic-scaling properties with the Lifshitz critical exponent $z = 3$, but with a new Lifshitz scale related Λ_{HL} as

$$\Lambda'_{\mathrm{HL}} = (\alpha\tilde{\lambda})^{-(1/4)}\Lambda_{\mathrm{HL}}. \tag{A5}$$

Depending on the value of α and $\tilde{\lambda}$, it can be much higher, lower than, or equal to Λ_{HL}. We will refer to this scale as the induced-Lifshitz scale for the spin-0 sector.

Of course the choice of parameters a, b, and c, including their signs, may have direct consequences for the stability and strong coupling problems in the gravity sector. It is well-known by analyzing the case for pure gravity that $\tilde{\lambda} > 0$ is a necessary condition to avoid ghosts. This cannot be immediately seen from the above propagators, as we have not fully diagonalized the action. We do not dwell on this issue further and simply assume that there exists reasonable choices of parameters so that the theory is well-defined.

To remove all quadratic divergence in the loop-induced Lorentz-violation effect observed in Sec. IV, we introduce the additional term in the theory

$$\mathcal{L}' = \frac{2}{\Lambda^2}(\nabla_i K^{ij})(\nabla^k K_{kj}) = \frac{1}{2\Lambda^2} v_i^{\mathrm{T}}\Delta^2 v^{\mathrm{T}_i} - \frac{2}{\Lambda^2}\chi\Delta\chi. \tag{A6}$$

It is easily checked that the propagators become

$$\langle V_i^{\mathrm{T}} V_j^{\mathrm{T}}\rangle = -\frac{\delta_{ij} - k_i k_j/\vec{k}^2}{\omega^2(\vec{k}^2 + \Lambda^{-2}\vec{k}^4)}, \qquad \langle n_i^{\mathrm{T}} *\rangle = 0, \tag{A7}$$

for vector-gravitons and at large $\vec{k}$

$$\begin{aligned} \langle\sigma\sigma\rangle &= \frac{1}{(2\lambda - 1)}\frac{1}{\omega^2 - \alpha(1 - 2\lambda)^{-1}\Lambda_{\mathrm{HL}}^{-4}\vec{k}^6}, \\ \langle\sigma\chi\rangle &= \frac{i\lambda}{(4\lambda - 2)}\frac{\Lambda^2\omega}{\vec{k}^2(\omega^2 - \alpha(1 - 2\lambda)^{-1}\Lambda_{\mathrm{HL}}^{-4}\vec{k}^6)}, \\ \langle\sigma n\rangle &= -\frac{c}{b}\frac{1}{(2\lambda - 1)}\frac{1}{\omega^2 - \alpha(1 - 2\lambda)^{-1}\Lambda_{\mathrm{HL}}^{-4}\vec{k}^6}, \end{aligned} \tag{A8}$$

for spin-0 gravitons. Because of the additional $\vec{k}^2$ suppression in the σ-χ correlator, only the very last term in

[3] As a matter of additional check, we have also performed calculations in the generalized R_ξ gauge for the spin-1 gravitons, when $\mathcal{L} = -\frac{1}{2\xi}(\dot{n}_i - \alpha\Delta V_i)^2$-term is added to the action. Explicit calculations of c_{scalar} and c_{photon} can be carried out, and the result for their difference shows complete independence on the choice of ξ and α parameters.

Eq. (54) contributes to the logarithmic divergence produced by the scalar-graviton loops.

APPENDIX B: ADDITIONAL DETAILS ABOUT LIFSHITZ-TYPE LOOP INTEGRALS

When the internal propagators all share the same Liftshitz-type behavior with the same exponent z, the loop integral can be easily cast into a normal Feynman integral so that standard textbook formulas are directly applicable. Take, as an example, the fermion one-loop correction discussed in Sec. II,

$$K = -\frac{1}{4i(2\pi)^4 M^2}\int d^4k \frac{F^{\mu\nu}F^{\alpha\beta}\mathrm{tr}\,\sigma_{\mu\nu}\tilde{k}\sigma_{\alpha\beta}\tilde{k}}{\tilde{k}^4}, \quad (B1)$$

where

$$(\tilde{k}^0, \tilde{\vec{k}}) \equiv (k^0, |k|^{z-1}\vec{k}/\Lambda_{\rm HL}^{z-1}). \quad (B2)$$

It is easy to verify that

$$d^4\tilde{k} = \frac{z|\vec{k}|^{3(z-1)}}{\Lambda_{\rm HL}^{3(z-1)}}d^4k = \frac{z|\tilde{\vec{k}}|^{3(1-1/z)}}{\Lambda_{\rm HL}^{3(1-1/z)}}d^4k. \quad (B3)$$

Changing the loop-integral variable from d^4k to $d^4\tilde{k}$, and dropping the tilde for brevity, we have

$$K = -\frac{\Lambda_{\rm HL}^{3(1-1/z)}}{4i(2\pi)^4M^2}\int d^4k\, \frac{F^{\mu\nu}F^{\alpha\beta}\mathrm{tr}\,\sigma_{\mu\nu}k\sigma_{\alpha\beta}k}{z|\vec{k}|^{3(1-1/z)}k^4}. \quad (B4)$$

The following identity is also easily checked

$$\mathrm{tr}\ \sigma_{\mu\nu}\not{k}\sigma_{\alpha\beta}\not{k} = 4k^2(g_{\mu\beta}g_{\nu\alpha} - g_{\mu\alpha}g_{\nu\beta}) + 8(g_{\mu\alpha}k_\nu k_\beta - g_{\mu\beta}k_\nu k_\alpha + g_{\nu\beta}k_\mu k_\alpha - g_{\nu\alpha}k_\mu k_\beta). \quad (B5)$$

Full Lorentz symmetry emerges as $z = 1$, in which case each pair of $k_\alpha k_\beta$ in the above expressions can be replaced by $\frac{1}{4}k^2 g_{\alpha\beta}$ and, consequently, K vanishes identically by simple symmetry considerations.

While $z > 1$ and the Lorentz symmetry is broken, using parity and spatial-rotational symmetry, one can still replace each pair of $k_\alpha k_\beta$ in the integral by $g_{00}f^t + \sum_i g_{ii}f^x$. Here

$$f^t \equiv -\frac{8}{i(2\pi)^4}\int d^4k \frac{k_0^2}{z|\vec{k}|^{3(1-1/z)}k^4}, \quad (B6)$$

and

$$f^x \equiv \frac{8}{3i(2\pi)^4}\int d^4k \frac{|\vec{k}|^2}{z|\vec{k}|^{3(1-1/z)}k^4}. \quad (B7)$$

Therefore,

$$K = -\frac{\Lambda_{\rm HL}^{3(1-1/z)}}{M^2}\Big(-\frac{f^t + 3f^x}{4}F_{\mu\nu}F^{\mu\nu} + f^tF_{\mu 0}F^{\mu 0} + \sum_i f^x F_{\mu i}F^{\mu i}\Big) = \frac{\Lambda_{\rm HL}^{3(1-1/z)}(f^t - f^x)}{2M^2}(\mathbf{E}^2 + \mathbf{B}^2). \quad (B8)$$

Generally in the Euclidean signature, we have the integral [48]

$$I_{r,s}^A \equiv \int\frac{d^{\hat{D}}\hat{p}}{(2\pi)^{\hat{D}}}\int\frac{d^{\bar{D}}\bar{p}}{(2\pi)^{\bar{D}}}\frac{(\hat{p}^2)^r(\bar{p}^2)^s}{(\hat{p}^2+\bar{p}^2+m^2)^A} = m^{2(r+s)+\hat{D}+\bar{D}-2A}\frac{\Gamma(s+\frac{\bar{D}}{2})\Gamma(r+\frac{\hat{D}}{2})\Gamma(A-r-s-\frac{\hat{D}+\bar{D}}{2})}{(4\pi)^{(\hat{D}+\bar{D})/2}\Gamma(\hat{D}/2)\Gamma(\bar{D}/2)\Gamma(A)}. \quad (B9)$$

Choosing $\hat{D} = 1$, $\bar{D} = 3$, and a Wick rotation leads to $f^t = \frac{8}{z}I^2_{1,3/(2z)-3/2}$ and $f^x = \frac{8}{3z}I^2_{0,3/(2z)-1/2}$. For both f^t and f^x

$$r + s = \frac{3}{2z} - \frac{1}{2}.$$

Therefore, the ratio of the two is immediately given by

$$\frac{f^t}{f^x} = \frac{3\Gamma(\frac{3}{2})\Gamma(\frac{3}{2z})}{\Gamma(\frac{1}{2})\Gamma(\frac{3}{2z}+1)}. \quad (B10)$$

As $z = 1$, $f^t = f^x$ as expected. $z = 3$ is a particular interesting case where we find

$$f^t = 3f^x = \frac{1}{2\pi^2}\Gamma(0). \quad (B11)$$

$\Gamma(0)$ encodes the UV divergence in this formula, which gives rise to a logarithmically divergent-terms for both f^t and f^x, if we use dimensional regularizations. The fact that they are different by a factor of 3 implies violation of the Lorentz symmetry.

APPENDIX C: TWO TOY MODELS OF LIFSHITZ SCALAR-QED

To achieve better understanding of the physics in Lifshitz-type gauge theories, we intend to work out two different toy models in which an ordinary, complex scalar is coupled to a Lifshitz-type photon. In order to retain analogy to the graviton-radiative corrections to the kinetic terms of non-Lifshitz matter fields, we evaluate the mass renormalization of the complex scalar generated by the photon loops. These loop integrals are also quadratically divergent in ordinary QED and expected to become better convergent if the photon is Lifshtiz-like.

There are two way of "Lifshitzizing" the photon. One can do so by breaking all the gauge symmetries as in the following theory:

$$\mathcal{L} = -(\partial_\mu \phi - iA_\mu \phi)(\partial^\mu \phi + iA^\mu \phi^\dagger) + \frac{1}{2}A^\mu\{[\Box - (-\Delta)^z \Lambda_{\rm HL}^{-2(z-1)}]g_{\mu\nu} - \partial_\mu \partial_\nu\}A^\nu, \quad \text{(C1)}$$

where $z \gtrsim 2$. We used the combination $(-\Delta) = \vec{k}^2$ since it is a positive-definite operator. This theory appears like the standard scalar-QED if $\Lambda_{\rm HL} \to \infty$ but breaks gauge symmetry explicitly as long as $\Lambda_{\rm HL}$ is finite. In this theory there is no need of gauge fixing and the propagators of A_μ is given by

$$\langle A_\mu A_\nu\rangle = -\frac{g_{\mu\nu} + \frac{k_\mu k_\nu}{\Lambda_{\rm HL}^{-2(z-1)}\vec{k}^{2z}}}{\omega^2 - \vec{k}^2 - \Lambda_{\rm HL}^{-2(z-1)}\vec{k}^{2z}}. \quad \text{(C2)}$$

There are two relevant diagrams to evaluate for the mass renormalization of ϕ. The single-vertex diagram corresponds to the following loop integral:

$$I_1 = \frac{1}{(2\pi)^4}\int \frac{{\rm d}\omega {\rm d}^3\vec{k}[4 + \Lambda_{\rm HL}^{2(z-1)}(-\omega^2 + \vec{k}^2)/\vec{k}^{2z}]}{\omega^2 - \vec{k}^2 - \Lambda_{\rm HL}^{-2(z-1)}\vec{k}^{2z}} \approx \frac{3}{(2\pi)^4}\int \frac{{\rm d}\omega {\rm d}^3\vec{k}}{\omega^2 - \Lambda_{\rm HL}^{-2(z-1)}\vec{k}^{2z}}. \quad \text{(C3)}$$

Here we have used the residue theorem and assumed that the dominant part of the integral is contributed by the pole at $\omega = \pm|\vec{k}|^z/\Lambda_{\rm HL}^{z-1}$. This integral is logarithmically divergent if $z \gtrsim 3$.

The double-vertex diagram consists of one scalar-propagator and one photon-propagator, and in the limit of zero-external momentum is given by the integral

$$I_2 = \frac{1}{(2\pi)^4}\int \frac{{\rm d}\omega {\rm d}^3\vec{k}k^2[1 + k^2\Lambda_{\rm HL}^{2(z-1)}/\vec{k}^{2z}]}{(\omega^2 - \vec{k}^2)[\omega^2 - \vec{k}^2 - \lambda_L^{-2(z-1)}\vec{k}^{2z}]}. \quad \text{(C4)}$$

This integral is finite as long as $z \gtrsim 2$.

Therefore, in this toy model the mass renormalization of ϕ is only linearly divergent if $z = 2$, logarithmically if $z = 3$, and finite if $z > 3$.

We will now examine a different toy model, which is much closer in spirit to Hořava's theory of gravity. We would Lifshitzize photon without breaking the gauge symmetry. Consider the Lagrangian

$$\mathcal{L} = -\frac{1}{2}F_{0i}F^{0i} - \frac{1}{4\Lambda_{\rm HL}^{2(z-1)}}F_{ij}(-\Delta)^{z-1}F^{ij}. \quad \text{(C5)}$$

Similar to Arnowitt-Deser-Misner formalism in Lifshitz gravity, we separate the variables A_0 and $A_i \equiv A_i^{\rm T} + \partial_i\varphi$ and rewrite the action as

$$\mathcal{L} = -\frac{1}{2}A_i^{\rm T}[\partial_t^2 + \Lambda_{\rm HL}^{-2(z-1)}(-\Delta)^z]A^{{\rm T}_i} - \frac{1}{2}(A^0 + \dot{\varphi})\Delta(A^0 + \dot{\varphi}). \quad \text{(C6)}$$

This expression makes explicit the gauge symmetry

$$A^0 \to A^0 - \dot{\omega}, \qquad \varphi \to \varphi + \omega, \quad \text{(C7)}$$

which is nothing but the original gauge symmetry $A_\mu \to A_\mu + \partial_\mu\omega$. We would like to compute the mass renormalization for the complex scalar in this model as well. Clearly,

$$I_2 = \frac{1}{(2\pi)^4}\int d^4k k^\mu k^\nu \langle A_\mu A_\nu\rangle\langle\phi\phi^\dagger\rangle, \quad \text{(C8)}$$

and

$$I_1 = -\frac{1}{(2\pi)^4}\int {\rm d}^4k g^{\mu\nu}\langle A_\mu A_\nu\rangle. \quad \text{(C9)}$$

Therefore, the sum

$$I_1 + I_2 = -\frac{1}{(2\pi)^4}\int {\rm d}^4k\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right)\langle A_\mu A_\nu\rangle \quad \text{(C10)}$$

pickup only the gauge-independent part of the photon propagator automatically. We can choose any gauge that we like to evaluate these integrals. For example, in the $A^0 = 0$ gauge, analogous to the $n_i = 0$ gauge in gravity, the photon propagators are

$$\langle A_i A_j\rangle = -\frac{1}{\omega^2 - \Lambda_{\rm HL}^{-2(z-1)}\vec{k}^{2z}}\left(\delta_{ij} - \frac{k_i k_j}{\vec{k}^2}\right) - \frac{k_i k_j}{\omega^2\vec{k}^2}, \quad \langle A^0 A^0\rangle = \langle A^0 A_i\rangle = 0. \quad \text{(C11)}$$

Therefore,

$$I_1 + I_2 = \frac{2}{(2\pi)^4}\int \frac{{\rm d}\omega {\rm d}^3\vec{k}}{\omega^2 - \Lambda_{\rm HL}^{-2(z-1)}\vec{k}^{2z}} + \frac{1}{(2\pi)^4}\int \frac{{\rm d}\omega {\rm d}^3\vec{k}}{\omega^2 - \vec{k}^2}. \quad \text{(C12)}$$

Just as we have observed in the case of Hořava-type gravity, the result for $z = 3$ contains both logarithmic and quadratic divergences. The difference is that it is manifestly gauge-independent in this simple toy model. When $z = 1$, $I_1 + I_2 = \frac{3}{(2\pi)^4}\int {\rm d}\omega {\rm d}^3k(\omega^2 - \vec{k}^2)^{-1}$, recovering the standard scalar-QED result.

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