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Dark matter, infinite statistics, and quantum gravity

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We elaborate on our proposal regarding a connection between global physics and local galactic dynamics via quantum gravity. This proposal calls for the concept of MONDian dark matter which behaves like cold dark matter at cluster and cosmological scales but emulates modified Newtonian dynamics (MOND) at the galactic scale. In the present paper, we first point out a surprising connection between the MONDian dark matter and an effective gravitational Born-Infeld theory. We then argue that these unconventional quanta of MONDian dark matter must obey infinite statistics, and the theory must be fundamentally nonlocal. Finally, we provide a possible top-down approach to our proposal from the matrix theory point of view.

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I. INTRODUCTION

The fascinating problem of "missing mass", or dark matter [1], has been historically identified on the level of galaxies. But the need for dark matter is in fact even more urgent at larger scales. Dark matter is apparently required to yield: (1) the correct cosmic microwave background spectrum shapes [including the alternating peaks]; (2) the correct large-scale structures; (3) the correct elemental abundances from big bang nucleosynthesis; and (4) the correct gravitational lensing. Naturally dark matter has been accorded a prominent place in the concordant ΛCDM model of cosmology [1] according to which cold dark matter (CDM), dark energy (in the form of cosmological constant), and ordinary matter account for about 23%, 73%, and 4% of the energy and mass of the Universe, respectively.

However, at the galactic scale, dark matter does not fare nearly as well at the larger scales. It can explain the observed asymptotic independence of orbital velocities on the size of the orbit only by fitting data (usually with two parameters) for individual galaxies. It is also not very successful in explaining the observed baryonic Tully-Fisher relation [2,3], i.e., the asymptotic-velocity-mass ($v^4 \propto M$) relation. Another problem with dark matter is that it seems to possess too much power on small scales ($\sim 1 - 1000$ kpc) [4].

On the other hand, there is an alternative paradigm that goes by the name of modified Newtonian dynamics (MOND) [5–7], due to Milgrom. MOND stipulates that the acceleration of a test mass m due to the source M is given by $a = a_N$ for $a \gg a_c$, but $a = \sqrt{a_N a_c}$ for $a \ll a_c$, where $a_N = GM/r^2$ is the magnitude of the usual Newtonian acceleration and the critical acceleration a_c is numerically related to the speed of light c and the Hubble scale H as $a_c \approx cH/(2\pi) \sim 10^{-8}$ cm$/s^2$. With only one parameter MOND can explain rather successfully the observed flat galactic rotation curves and the observed Tully-Fisher relation [8]. Unfortunately there are problems with MOND at the cluster and cosmological scales.

Thus CDM and MOND complement each other well, each being successful where the other is less so. We found it natural to combine their salient successful features into a unified scheme which straddles the fields of astronomy and high energy physics. In our previous work [9], by making use of a novel quantum gravitational interpretation of (dark) matter's inertia, we introduced the new concept of MONDian dark matter which behaves like CDM at cluster and cosmological scales but emulates MOND at the galactic scale.

In this paper, after a short review of our proposal on MONDian dark matter, we first point out a surprising connection between our proposal and an effective gravitational Born-Infeld description of the MOND-like phenomenology of our dark matter quanta. Furthermore, we stress that these unusual quanta of dark matter must obey the crucial property of infinite statistics. We illustrate the properties of an essentially nonlocal theory that describes such dark matter with infinite statistics. We naturally expect that such noncanonical dark matter quanta should have dramatic signatures in high energy particle experiments.

II. FROM ENTROPIC GRAVITY TO MONDIAN DARK MATTER

Our previous proposal [9] makes crucial use of a natural relationship between gravity and thermodynamics [10,11]. The starting point is the recent work of Verlinde [10] in which the canonical Newton's laws are derived from the

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point of view of holography [12–15]. Verlinde applies the first law of thermodynamics to propose the concept of entropic force

$$F_{\text{entropic}} = T\frac{\Delta S}{\Delta x}, \tag{1}$$

where Δx denotes an infinitesimal spatial displacement of a particle with mass m from the heat bath with temperature T. Invoking Bekenstein's original arguments concerning the entropy S of black holes [16] he imposes $\Delta S = 2\pi k_B \frac{mc}{\hbar}\Delta x$. With the help of the famous formula for the Unruh temperature, $k_B T = \frac{\hbar a}{2\pi c}$, associated with a uniformly accelerating (Rindler) observer [17,18], he obtains Newton's second law $F_{\text{entropic}} = T\nabla_x S = ma$.

Next, Verlinde considers an imaginary quasilocal (spherical) holographic screen of area $A = 4\pi r^2$ with temperature T. Assuming the equipartition of energy $E = \frac{1}{2}Nk_B T$ with N being the total number of degrees of freedom (bits) on the screen given by $N = Ac^3/(G\hbar)$, and employing the Unruh temperature formula and the fact that $E = Mc^2$, he obtains $2\pi k_B T = GM/r^2$ and recovers exactly the nonrelativistic Newton's law of gravity, namely $a = GM/r^2$.

But we live in an accelerating Universe (in accordance with the ΛCDM model). Thus we need a generalization [9] of Verlinde's proposal [10] to de Sitter space with a positive cosmological constant (which is related to the Hubble parameter H by $\Lambda \sim 3H^2$ after setting $c = 1$). Since the Unruh-Hawking temperature as measured by a noninertial observer with acceleration a in the de Sitter space is given by $\sqrt{a^2 + a_0^2}/(2\pi k_B)$ [19,20], where $a_0 = \sqrt{\Lambda/3}$ [12], it is natural to define the net temperature measured by the noninertial observer (relative to the inertial observer) to be

$$\tilde{T} = \frac{1}{2\pi k_B}\left(\sqrt{a^2 + a_0^2} - a_0\right). \tag{2}$$

In fact, Milgrom has suggested in [21] that the difference between the Unruh temperatures measured by noninertial and inertial observers in de Sitter space, namely $2\pi k_B \Delta T = \sqrt{a^2 + a_0^2} - a_0$, can give the correct behaviors of the interpolating function between the usual Newtonian acceleration and his suggested MOND for very small accelerations. However, he was not able to justify why the force should be related to the difference between the Unruh temperatures measured by noninertial and inertial observers in de Sitter space. Or, in his own words: "it is not really clear why ΔT should be a measure of inertia". As we will see in the following, adopting Verlinde's entropic force point of view allows us to justify Milgrom's suggestion naturally.

Following Verlinde's approach, the entropic force, acting on the test mass m with acceleration a in de Sitter space, is obtained by replacing the T in Verlinde's argument by $\tilde{T}$ for the Unruh temperature:

$$F_{\text{entropic}} = \tilde{T}\nabla_x S = m(\sqrt{a^2 + a_0^2} - a_0). \tag{3}$$

For $a \gg a_0$, the entropic force is given by $F_{\text{entropic}} \approx ma$, which gives $a = a_N$ for a test mass m due to the source M. But for $a \ll a_0$, we have $F_{\text{entropic}} \approx ma^2/(2a_0)$; and so the terminal velocity v of the test mass m should be determined from $ma^2/(2a_0) = mv^2/r$.

The observed flat galactic rotation curves (i.e., at large r, v is independent of r) and the observed Tully-Fisher relation (the speed of stars being correlated with the galaxies' brightness, i.e., $v^4 \propto M$) now require that $a \approx (2a_N a_0^3/\pi)^{(1/4)}$.[1] But that means

$$F_{\text{entropic}} \approx m\frac{a^2}{2a_0} = F_{\text{Milgrom}} \approx m\sqrt{a_N a_c}, \tag{4}$$

for the small acceleration $a \ll a_0$ regime. Thus we have recovered MOND—provided we identify $a_0 \approx 2\pi a_c$, with the (observed) critical galactic acceleration $a_c \sim \sqrt{\Lambda/3} \sim H \sim 10^{-8}\ \text{cm}/s^2$. Thus, from our perspective, MOND is a phenomenological consequence of quantum gravity. To recapitulate, we have successfully predicted the correct magnitude of the critical galactic acceleration, and furthermore have found that global physics (in the form of a cosmological constant) can affect local galactic motion!

Finally, to see how dark matter can behave like MOND at the galactic scale, we continue to follow Verlinde's holographic approach to write $2\pi k_B \tilde{T} = \frac{G\tilde{M}}{r^2}$, by replacing the T and M in Verlinde's argument by $\tilde{T}$ and $\tilde{M}$ respectively. Here $\tilde{M}$ represents the *total* mass enclosed within the volume $V = 4\pi r^3/3$. Now it is natural to write the entropic force $F_{\text{entropic}} = m[(a^2 + a_0^2)^{1/2} - a_0]$ as $F_{\text{entropic}} = ma_N[1 + (a_0/a)^2/\pi]$ since the latter expression is arguably the simplest interpolating formula for F_{entropic} that satisfies the two requirements: $a \approx (2a_N a_0^3/\pi)^{1/4}$ in the small acceleration $a \ll a_0$ regime, and $a = a_N$ in the $a \gg a_0$ regime. But we can also write F in another, yet equivalent, form: $F_{\text{entropic}} = G\tilde{M}m/r^2 = G(M + M')m/r^2$, where M' is some unknown mass—that is, dark matter. These two forms of F illustrate the idea of CDM-MOND duality [9]. The first form can be interpreted to mean that there is no dark matter, but that the law of gravity is modified, while the second form means that there is dark matter (which, by construction, is consistent with MOND) but that the law of gravity is not modified.

Dark matter of this kind can behave as if there is no dark matter but MOND. Therefore, we call it "MONDian dark matter" [9]. Solving for M' as a function of r in the two acceleration regimes, we obtain $M' \approx 0$ for $a \gg a_0$, and (with $a_0 \sim \sqrt{\Lambda}$)

[1] One can check this by carrying out a simple dimensional analysis and recalling that there are two accelerations in the problem: viz, a_N and a_0.

$$M' \sim (\sqrt{\Lambda}/G)^{1/2} M^{1/2} r, \tag{5}$$

for $a \ll a_0$. This intriguing *dark matter profile* relates, at the galactic scale, dark matter (M'), dark energy (Λ) and ordinary matter (M) to one another. At the moment, it seems prohibitive to check this prediction in astronomical observations. As a remark, this dark matter profile has been derived assuming nonrelativistic sources and so it is only valid within the galactic scale. When we enter the cluster or cosmic scale, we need to take into account of the fully relativistic sources. This may explain why MOND works at the galactic scale, but not at the cluster or cosmic scale. One of reasons is that, for the larger scales, one has to use Einstein's equations with non-negligible contributions from the pressure and explicitly the cosmological constant, which have not been taken into account in the MOND scheme [9].

In the above proposal for the dark matter profile, we have assumed spherical symmetry and so it is solely dependent on r. Since both schemes of AQUAL [22] and QUMOND [23] reduce to the MOND theory in the spherically symmetric limit, our proposal should presumably be consistent with AQUAL and QUMOND in that limit. In principle, we could generalize our derivation to accommodate the general case without spherical symmetry and predict a dark matter disk to compare with AQUAL and QUMOND, but this is certainly beyond the scope of the present paper.

III. GRAVITATIONAL BORN-INFELD THEORY

As we have reviewed in the last section, our proposal combines the MONDian phenomenology with the concept of dark matter. Since the thermodynamic argument we provided is highly constrained (as in the formulae for the effective acceleration and hence the force law), we would like to use the same constraint to likewise elucidate the concept of MONDian dark matter. One way to do this is to look for various reformulations of MONDian phenomenology. Given the specific form for the MONDian force law (3), our choices are limited. One particularly useful reformulation is via an effective gravitational dielectric medium, motivated by the analogy between Coulomb's law in a dielectric medium and Milgrom's law for MOND [7,24]. As we will show below, the form of the Born-Infeld Hamiltonian density for electrodynamics resembles that of the MONDian force law (3). Interestingly, Milgrom has also noted a similar connection between the nonlinear Born-Infeld electrostatics and MOND theory [25]. Thus the effective gravitational medium for our case is precisely that of the Born-Infeld type.

Now, we proceed to construct an effective gravitational Born-Infeld theory and point out its remarkable connection to the MONDian phenomenology. First of all, the original Born-Infeld (BI) theory [26] is defined with the following Lagrangian density (where $\vec{E}$ and $\vec{B}$ are the respective electric and magnetic fields)

$$L_{\rm BI} = b^2\left(1 - \sqrt{1 - \frac{E^2 - B^2}{b^2} - \frac{(\vec{E}\cdot\vec{B})^2}{b^4}}\right), \tag{6}$$

where b is a dimensionful parameter. In fact, if we set $\vec{B} = 0$, it follows that b represents the maximal field strength allowed. The corresponding Hamiltonian density is given by [26]

$$H_{\rm BI} = b^2\left(\sqrt{1 + \frac{D^2 + B^2}{b^2} + \frac{(\vec{D}\times\vec{B})^2}{b^4}} - 1\right). \tag{7}$$

Next, we explore a gravitational analog of the Born-Infeld theory, in which the relevant field strength is of a gravitational type. In particular, we set $\vec{B} = 0$. Then, the corresponding gravitational Lagrangian and Hamiltonian densities read as

$$L_g = b^2\left(1 - \sqrt{1 - \frac{E_g^2}{b^2}}\right), \tag{8}$$

$$H_g = b^2\left(\sqrt{1 + \frac{D_g^2}{b^2}} - 1\right). \tag{9}$$

For our reasoning, the Hamiltonian density is more relevant, and for a normalization purpose (which will become clear in a moment), we start from the following normalized Hamiltonian density which has an extra overall factor of $\frac{1}{4\pi}$:

$$H_g = \frac{b^2}{4\pi}\left(\sqrt{1 + \frac{D_g^2}{b^2}} - 1\right) = \frac{1}{4\pi}\left(\sqrt{b^4 + b^2 D_g^2} - b^2\right). \tag{10}$$

Let $A_0 \equiv b^2$ and $A \equiv b D_g$, then the Hamiltonian density becomes

$$H_g = \frac{1}{4\pi}(\sqrt{A^2 + A_0^2} - A_0). \tag{11}$$

Assuming there exists energy equipartition, then the effective gravitational Hamiltonian density, which correspond to the energy, is equal to

$$H_g = \tfrac{1}{2} k_B T_{\rm eff}, \tag{12}$$

where $T_{\rm eff}$ is an effective temperature associated with the energy through the equipartition of energy.[2] But the Unruh temperature formula implies that

[2]Note that this energy density is energy per unit volume. But we can regard it as energy per degree of freedom by recalling that volume, which usually scales as entropy S, scales as the number of degrees of freedom N in a holographic setting. Interestingly $S \sim N$ is one of the features of infinite statistics [27].

$$T_{\rm eff} = \frac{\hbar}{2\pi k_B} a_{\rm eff}, \quad (13)$$

where $a_{\rm eff}$ is the effective acceleration. As a result, we obtain (after setting $\hbar = 1$)

$$a_{\rm eff} = \sqrt{A^2 + A_0^2} - A_0. \quad (14)$$

For a given test mass m, the Born-Infeld *inspired* force law is then given by

$$F_{\rm BI} = m(\sqrt{A^2 + A_0^2} - A_0). \quad (15)$$

Quite remarkably, $F_{\rm BI}$ is of exactly the same form as the force law (3) derived in our previous paper [9] as reviewed in section II. In what follows, we give a physical interpretation of this somewhat formal result and use it to illuminate the properties of the proposed MONDian dark matter quanta.

IV. MONDIAN DARK MATTER AND INFINITE STATISTICS

In this section, we argue that the surprising connection between an effective gravitational Born-Infeld and the force law (3) points to the concept of infinite statistics for our MONDian dark matter quanta. We argue that this is implied by the equivalence principle. Then we discuss a toy model of a neutral scalar field obeying infinite statistics as a first step towards a phenomenologically realistic model of MONDian dark matter.

First, let us use the equivalence principle within the logic of our argument. In the previous section, the local gravitational fields $\vec{A}$ and $\vec{A}_0$ appeared in the surprising formal connection between an effective gravitational Born-Infeld theory and our MONDian force law (3). The validity of the equivalence principle suggests that we should identify (at least locally) the local accelerations $\vec{a}$ and $\vec{a}_0$ with the local gravitational fields $\vec{A}$ and $\vec{A}_0$ respectively. Namely,

$$\vec{a} \equiv \vec{A}, \qquad \vec{a}_0 \equiv \vec{A}_0. \quad (16)$$

In other words, the validity of the equivalence principle suggests that the temperature $T_{\rm eff}$ should be identified as

$$T_{\rm eff} \equiv \frac{\hbar}{2\pi k_B}\left(\sqrt{a^2 + a_0^2} - a_0\right), \quad (17)$$

which, in turn, implies that the Born-Infeld inspired force law takes the form

$$F_{\rm BI} = m\left(\sqrt{a^2 + a_0^2} - a_0\right), \quad (18)$$

which is precisely the MONDian force law derived in (3). (For consistency, we check that a_0, the counterpart of the constant b in (10), is itself a constant, being proportional to $\sqrt{\Lambda}$.) We thus conclude that the successful phenomenology of MONDian dark matter may actually be described in terms of an effective gravitational Born-Infeld theory.[3]

The gravitational Born-Infeld Hamiltonian H_g is crucially related to the temperature $T_{\rm eff}$ via the energy equipartition $H_g = \frac{1}{2} k_B T_{\rm eff}$. Now, this temperature $T_{\rm eff}$ is obviously very low, because of the factor of $\hbar$ [see Eq. (13)]. As an example, let us consider a typical acceleration of order 10 ms^{-2}. The corresponding effective temperature is of order $T_{\rm eff} \sim 10^{-20}$ K and the effective characteristic energy scale is of order $k_B T_{\rm eff} \sim 10^{-24}$ eV. Obviously, $k_B T_{\rm eff}$ is much smaller than even the tiny neutrino masses of order 10^{-3} eV or the mass of any viable *cold* dark matter candidate which has to be much heavier than 1 eV.

Recall that the equipartition theorem in general states that the average of the Hamiltonian is given by

$$\langle H \rangle = -\frac{\partial \log Z(\beta)}{\partial \beta}, \quad (19)$$

where $\beta^{-1} = k_B T$ and Z denotes the partition function. To obtain $\langle H \rangle = \frac{1}{2} k_B T$ per degree of freedom, we require the partition function to be of the Boltzmann form

$$Z = \exp(-\beta H). \quad (20)$$

To be a viable cold dark matter candidate, the quanta of our MONDian dark matter are expected to be much heavier than $k_B T_{\rm eff}$. One may think that it suffices to use the conventional quantum-mechanical Bose-Einstein or Fermi-Dirac statistics, but they would not lead to $\langle H \rangle = \frac{1}{2} k_B T$ per degree of freedom. As a result, the validity of $H_g = \frac{1}{2} k_B T_{\rm eff}$ for very low temperature $T_{\rm eff}$ somehow requires a unique quantum statistics with a Boltzmann partition function. But this is precisely what is called the infinite statistics [28] as described by the Cuntz algebra (a curious average of the bosonic and fermionic algebras [28])

$$a_i a_j^\dagger = \delta_{ij}. \quad (21)$$

Thus, by invoking infinite statistics, the assumption of energy equipartition $H_g = \frac{1}{2} k_B T_{\rm eff}$, even for very low temperature $T_{\rm eff}$, is justified.

One may reason that the above arguments for infinite statistics also apply to Verlinde's original proposal [10] which invokes energy equipartition, and accordingly he should need introducing infinite statistics as well. This would be true *if* he assumed that the typical mass scale of the quanta of microscopic degrees of freedom (or bits in his terminology) on the holographic screen is much heavier than $k_B T_{\rm eff}$. However, it is *not necessary* for him to make such an assumption, thereby the requirement for infinite

[3]We note that, by using the gravitational Born-Infeld and effective acceleration, we have no need to invoke the gravitational "bits" in Verlinde's scheme. Thus, in some sense, we have bypassed that scheme.

statistics is evaded. It could well be that the typical mass scale of the quanta of his bits is much lighter than $k_B T_{\rm eff}$. In that case, it is in the high temperature limit, and then he can safely use the Boltzmann partition function to obtain the energy equipartition formula. As a result, whether Verlinde requires infinite statistics or not would not change any of his results.[4] On the contrary, to be a viable cold dark matter candidate, the quanta of our MONDian dark matter must be much heavier than $k_B T_{\rm eff}$. This means that infinite statistics is an essential ingredient to our proposal.

Therefore, we have two rather striking observations: (i) the relation between our force law that leads to MONDian phenomenology and an effective gravitational Born-Infeld theory; (ii) the need for infinite statistics of some microscopic quanta which underlie the thermodynamic description of gravity implying such a MONDian force law.

It is natural to ask: How would infinite statistics mesh with an effective gravitational Born-Infeld theory and what would such a connection imply for the physical properties of MONDian dark matter? Here we recall some facts from string theory as a theory of quantum gravity. It is well known that in the open string sector, one naturally induces Born-Infeld theories [30], in general of a non-Abelian kind [31]. Furthermore, in the case of a nonperturbative formulation of string theory via matrix theory [32] (a light-cone version of M theory), it has been argued that infinite statistics arises naturally [27,33]. This matrix theory is non-Abelian, but is of the Yang-Mills and not Born-Infeld kind. However, non-Abelian Born-Infeld like extensions of matrix theory exist in various backgrounds [31], and thus infinite statistics should naturally emerge in that context as well. Thus, from the matrix theory point of view, we should expect that infinite statistics and an effective theory of the gravitational Born-Infeld type are closely related. This may serve as a top-down justification for the assumption of the energy equipartition $H_g = \frac{1}{2} k_B T_{\rm eff}$ which requires the imposition of infinite statistics.

As we have argued earlier, with the validity of energy equipartition and the equivalence principle, the successful phenomenology of MONDian dark matter could be described in terms of an effective gravitational Born-Infeld theory which leads to the correct MONDian force law. But we just showed that the validity of this energy equipartition requires some nonstandard degrees of freedom to obey infinite statistics. It is these nonstandard degrees of freedom in the effective gravitational Born-Infeld theory that generates the gravitational fields and leads to the correct MONDian force law. As is well known, any modifications to general relativity must either introduce new local degrees of freedom or violate the principle of general covariance (and hence the equivalence principle) [34]. Since we keep the equivalence principle intact in our arguments and do not introduce any new local gravitational degrees of freedom, we do not modify general relativity. Thus these nonstandard degrees of freedom in the effective gravitational Born-Infeld theory will essentially manifest as new particle degrees of freedom. Since these new particle degrees of freedom when quantized with infinite statistics leads to the correct MONDian force law, we identify them as our MONDian dark matter quanta quantized with infinite statistics.

In order to discuss the particle phenomenology of MONDian dark matter, we need a relativistic field theory of infinite statistics. It is known that any theory of infinite statistics is fundamentally nonlocal[5] (albeit consistent with Lorentz and CPT invariance) [28]. As far as we know, such a complete field-theoretical description of infinite statistics is not available at present and thus we have to start from scratch. Here we present a toy model of a neutral scalar field obeying infinite statistics (see also [35]) and postpone a full treatment of this problem to the future. We start with the Klein-Gordon equation

$$(\partial^2 + m^2)\phi(x) = 0. \tag{22}$$

Since ϕ is a Hermitian field operator, it can be expanded as

$$\phi(x) = \int d\omega_k (a(\vec{k})e^{-ik\cdot x} + a^\dagger(\vec{k})e^{ik\cdot x}), \tag{23}$$

where $d\omega_k \equiv \frac{d^3k}{(2\pi)^3 2\sqrt{\vec{k}^2+m^2}}$ with $k\cdot x \equiv \sqrt{\vec{k}^2 + m^2}t - \vec{k}\cdot\vec{r}$. The annihilation operator a and creation operator $a^\dagger$ obey the infinite statistics algebra

$$a(\vec{k})a^\dagger(\vec{k}') = 2k^0(2\pi)^3\delta^{(3)}(\vec{k} - \vec{k}'), \tag{24}$$

where $k^0 \equiv \sqrt{\vec{k}^2 + m^2}$, and

$$a(\vec{k})|0\rangle = 0 = \langle 0|a^\dagger(\vec{k}). \tag{25}$$

The Wightman function is given by

[4]Verlinde only invokes energy equipartition for the bits (the unknown microscopic degrees of freedom) on the holographic screen. In his picture, all matter is emergent from these bits and the emergent particles can obey infinite statistics or other statistics. But when ordinary matter particles emerge from these bits, they obey bosonic or fermionic statistics. How this happens is beyond the scope of this paper. In short, it appears that quantum gravitational degrees of freedom obey infinite statistics though this fact is irrelevant in Verlinde's case. Nevertheless, we cannot help but wonder whether quantum gravity is actually the origin of particle statistics and that the underlying statistics is infinite statistics. Is it possible that ordinary particles that obey Bose or Fermi statistics are actually some sort of collective degrees of freedom? For a discussion of constructing bosons and fermions out of particles obeying infinite statistics, see [29].

[5]That is, the fields associated with infinite statistics are not local, neither in the sense that their observables commute at spacelike separation nor in the sense that their observables are pointlike functionals of the fields.

$$\Delta^{(+)}(x-y) = \langle 0|\phi(x)\phi(y)|0\rangle = \int d\omega_k e^{-ik\cdot(x-y)}, \quad (26)$$

where we have used Eqs. (23)–(25). The Feynman propagator

$$\Delta_F(x-y) \equiv -i\langle 0|T(\phi(x)\phi(y))|0\rangle, \quad (27)$$

is given, in terms of the Wightman functions, by

$$\Delta_F(x-y) = -i\theta(x^0-y^0)\Delta^{(+)}(x-y) - i\theta(y^0-x^0)\Delta^{(+)}(y-x) \quad (28)$$

$$= \int \frac{d^4k}{(2\pi)^4}\frac{e^{-ik\cdot(x-y)}}{k^2-m^2+i\epsilon}. \quad (29)$$

Equation (29) can be shown to be equal to Eq. (28) by two different ways. One way is by explicitly performing the integration over k_0 in Eq. (29) to yield Eq. (28). Another way is to show that the Feynman propagator is a Green's function for the Klein-Gordon equation, i.e.,

$$(\partial_x^2 + m^2)\Delta_F(x-y) = -\delta^4(x-y), \quad (30)$$

by applying $(\partial_x^2 + m^2)$ on Eq. (28) and using the fact that the Wightman function solves the Klein-Gordon equation and that, with the aid of Eq. (26), $\Delta^{(+)}(x-y) = \Delta^{(+)}(y-x)$ at equal time $x^0 = y^0$.

From Eq. (29), it is obvious that we get back the conventional result for the scalar propagator and nonlocality is not manifest. Mathematically, this is because only the term $\langle 0|aa^\dagger|0\rangle$ gives a nonzero contribution to the propagator. The commutator of a and $a^\dagger$, which is absent in the infinite statistics case, is not required for the calculation of the propagator. But while nonlocality is not manifest in the propagator, it is not completely lost. The reason is that the equal-time commutator $[\phi(x), \phi(y)]|_{x^0=y^0}$ is nonzero, which is a manifestation of nonlocality.

We thus conclude that this toy model is illuminating to some extent, and indeed it could serve as a preliminary model for MONDian dark matter. However, we will not explore it further. This is because we are more interested in the particle physics phenomenology of the nonlocality associated with MONDian dark matter. Such phenomenological studies will crucially rely on a nonlocal propagator of the infinite statistics quanta, as well as their interactions with the Standard Model particles. In contrast, nonlocality in this toy model is only manifest in the equal-time commutator $[\phi(x), \phi(y)]|_{x^0=y^0}$; but it is not clear that it will lead to any direct and observable phenomenological consequences. Undoubtedly, for the particle physics phenomenology of MONDian dark matter to be relevant, we will need a full description which involves a truly nonlocal field theory of infinite statistics quanta. Investigating the precise nature of such a nonlocal theory is the next step in our research program. However, the proposal that MONDian dark matter quanta should be described by a nonlocal theory of infinite statistics, with ultimate origins in quantum gravity, is already quite remarkable, and this feature of MONDian dark matter uniquely distinguishes our suggestion from other phenomenological models of dark matter.

We end this section with the following observation on the phenomenology of MONDian dark matter. On the one hand, infinite statistics has been associated with the physics of quantum gravitational quanta such as D0-branes, in particular, backgrounds [33] as well as with black hole physics (as in the work of Strominger [33]). On the other hand, there are existing proposals arguing for the relevance of primordial black holes in the physics of dark matter [36], and, what is more important, for experimental searches for such primordial black holes [36]. Naturally we are led to conjecture that the application of the same experimental techniques may be relevant in the observational search of MONDian dark matter with infinite statistics.

V. CONCLUSION: INFINITE STATISTICS AND QUANTUM GRAVITY

In this paper, we have further developed our proposal for MONDian dark matter which unifies the salient features of cold dark matter and the phenomenology of modified Newtonian dynamics. The MONDian dark matter behaves like CDM at cluster and cosmological scales but emulates MOND at the galactic scale. We have pointed out a surprising connection between our proposal and an effective gravitational Born-Infeld description of the MOND-like phenomenology of our dark matter quanta. Furthermore, we have argued that these unusual quanta of dark matter must obey the crucial property of infinite statistics. Thus, MONDian dark matter has to be described as an essentially nonlocal theory for such infinite statistics quanta. Such a theory would be fundamentally quantum gravitational and thus distinguished from the usual phenomenological models of dark matter.

We conclude by presenting a possible top-down approach to our proposal. As already mentioned, it is quite natural to expect that quantum gravity in some form of matrix theory [32], has a non-Abelian Born-Infeld extension. If one concentrates on the $U(1)$ part of that theory, which would correspond to a "center of mass" sector of the full quantum theory of gravity, one will in principle expect to derive an effective gravitational Born-Infeld theory of the kind discussed in this paper. Also matrix theory [33] allows for infinite statistics being a theory of large (infinite size) matrices. Thus it would be possible to envision a gravitational Born-Infeld Hamiltonian which, in conjunction with the equipartition theorem that is true for infinite statistics, would imply the temperature formula and thus the force law derived in our previous paper [9]. Finally, by invoking the equivalence principle in this thermodynamic limit, we would be able to derive the exact formula $[\sqrt{a^2 + a_0^2} - a_0]$ from which we could deduce the MONDian scaling at galactic distances.

This scenario would imply that quantum gravity (in the guise of M theory) is really behind MONDian dark matter and its implications for particle physics as well as astronomy on the galactic and extragalactic scales. In this discussion, we would need to take account of holography (i.e. a matrix model description) in the cosmological asymptotically de Sitter background [37], which will be quite nontrivial. One simple idea would be to envision a matrix model (inspired by matrix theory [32])

$$L = \mathrm{Tr}(\tfrac{1}{2}(\partial\mathbb{M})^2 + m^2\mathbb{M}^2 + gV(\mathbb{M})O_{\mathrm{SM}}), \quad (31)$$

where $\mathbb{M}$ is an infinite dimensional square matrix. The mass term m^2 and the "Yukawa" coupling g are phenomenological parameters. Here $V(\mathbb{M})$ denotes some effective potential (for simplicity, we can envision a quartic term $\mathbb{M}^4$) and O_{SM} is the relevant standard model operator that describes the necessary coupling to the dark matter sector. The mass parameter m could be related to the cosmological SUSY breaking mechanism of Banks [38] if the matrix $\mathbb{M}$ has fundamental origins in matrix theory in a cosmological de Sitter background. However this topic is beyond the scope of our present work and we leave it for further study in the future.

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