Jet quenching via gravitational radiation in thermal AdS space-time

Edward Shuryak, Ho-Ung Yee, and Ismail Zahed

Department of Physics and Astronomy, State University of New York, Stony Brook, New York 11794, USA

We argue that classical bulk gravitational radiation effects in AdS/CFT, previously ignored because of their subleading nature in the $1/N_c$ expansion, are magnified by powers of large Lorentz factors γ for ultrarelativistic jets, thereby dominating other forms of jet energy loss in holography at finite temperature. We make use of the induced gravitational self-force in thermal anti-de Sitter to estimate its effects. In a thermal medium, relativistic jets may lose most of their energy through longitudinal drag caused by the energy accumulated in their nearby field as they zip through the strongly coupled plasma.

I. INTRODUCTION

This paper stems from two different motivations. One is a (century old) quest to understand the radiation reaction force in classical electrodynamics and general relativity, especially in higher space-time dimensions. The second (practical) motivation is the need to estimate the magnitude of such a classical force on a falling gravitational body in thermal anti-de Sitter (AdS$_5$) space-time. This latter problem is related to the empirically important issue of jet quenching in strongly coupled quark-gluon plasma as unravelled by recent collider experiments at the Relativistic Heavy Ion Collider (RHIC) and LHC.

Because of these two different motivations, we make the introduction a bit large to allow for a brief descriptive of the history of both subjects which is perhaps needed for some diverse readers. A more thorough analysis of the radiation reaction force in electrodynamics in higher even-space + time dimensions will be presented in a companion paper [1].

A. Radiation reaction force

One general theoretical problem, with which theorists such as Abraham, Lorentz, Dirac and others struggled from about the beginning of 20th century, deals with the understanding of the radiation reaction force. The concept of nonrelativistic and relativistic classical radiation loss can be found in most textbooks on classical electrodynamics. For instance, the relativistic form of the Larmor formulae for the power radiated by an accelerated charge reads

$$P = -\frac{2e^2}{3}\ddot{x}\cdot\ddot{x}, \tag{1.1}$$

with $x^a(\tau)$ being the particle trajectory parameterized by its proper time τ. The double dots reflect the proper acceleration. It is not difficult to write down a longitudinal force that drags the charged particle matching the energy loss (1.1). This procedure refers to the radiation or large distance derivation of the local force, based essentially on the energy-momentum flux radiated on an asymptotic surface at infinity (large sphere). However, there are many reasons for why a local derivation of the self-force that only makes use of the local fields acting on the accelerated charge is warranted.

There is a large body of research on this approach for both scalar, vector (electromagnetism), and tensor (gravity) radiation both in flat and curved space-time. Once the energy loss is accounted for, the radiation reaction force is fixed by the condition of balancing loss. Putting aside some known paradoxes related to this particular construction, we would like to focus on the following specific questions:

(i) Is the radiation reaction force amenable to a local definition from a regulated self-field at the instantaneous position of the charge?

(ii) In electrodynamics, the radiation reaction force is expressed via the acceleration, but in general relativity particles move along geodesics for which the covariant acceleration is zero. So what makes up the force?

(iii) The Larmor formula involves the acceleration thanks to the dipole approximation in nonrelativistic electrodynamics, but what about higher derivatives in higher-dimensional settings and general relativity?

While we do not address all of these difficult issues here, we aim at exploring the practical importance of some theoretical proposals already in the literature. If (i) is true, it would simplify calculations considerably, especially in the case of general relativity since defining distant surfaces in curved backgrounds is, in general, elusive. More specifically, can the radiation reaction force be the *self-interaction force* following from the charge's own field due to its past trajectory?

The closest to formulating an answer to a local definition of the self-force in four-dimensional gravity was the work of Mino, Sasaki, and Tanaka and also Queen and Wald [2,3], where the gravitational radiation reaction force was obtained in a closed form:

$$m\ddot{x}^a = Gm^2\dot{x}^b\dot{x}^c\int_{-\infty}^{\tau_-}d\tau'\left(\frac{1}{2}\nabla^a\mathbf{G}^-_{bca'b'}\right.$$
$$\left.-\nabla_b\mathbf{G}^{-a}_{c\ a'b'} - \frac{1}{2}\dot{x}^a\dot{x}^d\nabla_d\mathbf{G}^-_{bca'b'}\right)\dot{x}'^a\dot{x}'^b, \tag{1.2}$$

whereby the integral is carried over the past world line of the particle up to a regulated time τ_-. Here, $\mathbf{G}^-$ is the retarded propagator for the Einstein equation with the particle's own stress tensor acting as a source. While more details on this equation will be given below, here, we just note that the bracket in Eq. (1.2) is just the Christoffel's force from a metric perturbation as induced by the past history of the particle. Note that the result (1.2) was derived in harmonic gravitational gauge: in this work, we will ignore all subtleties that may be associated with this particular gauge choice.

Does this force or its simplified analogues really work? Clearly, for synchrotron radiation in flat four-dimensional space-time, a force given by an analogous expression would vanish. The simplest way to see this is to note that the retarded propagator in flat four-dimensional space-time reads

$$\mathbf{G}^-(x, x') = \frac{1}{2\pi} \theta(x^0 - x'^0) \delta((x - x')^2),$$

which is totally localized on the light cone. As a result, there are simply no points on the synchrotron world line that intersect the past light cone. However, this drawback is short-lived if we are to notice that: 1) in curved space-times the retarded propagator gets contributions from light rays reflecting on the curved background thereby causing intersections with the past light cone; 2) in even − space + time dimensions, the retarded propagator has support on the entire interior of past light cone even in flat space, causing contributions from the past history of the particle. Equation (1.2) is a valid local radiative self-force in gravity.

We will use Eq. (1.2) to address the issue of jet quenching in the AdS/CFT correspondence. Our main observation is that for ultrarelativistic jets, Eq. (1.2) is to be used in the regime of very short proper times such that the Taylor expansion of all the factors in the numerator in proper time would provide the first finite term after all infinities are subtracted and renormalized. In our analysis, the magnitude of the proper time ϵ is

$$\epsilon \approx 1/\gamma^2 \ll 1, \tag{1.3}$$

as will be made clear in later sections. In the small ϵ expansion, the radiative self-force in curved and thermal AdS$_5$ follows from the finite term in Eq. (1.2) after using the leading singularity of the retarded propagator $\mathbf{G}^-$ with the appropriate Taylor coefficient.

B. Jet quenching

Jets are the most spectacular aspect of QCD physics in a high-energy setting. Their interaction with the quark-gluon plasma in a heavy-ion collision has long been deemed very interesting, starting from the early work by Bjorken [4]. Indeed, the phenomenon known as jet quenching has been observed in heavy ion collisions at RHIC; see early

summary in Refs. [5,6]. The observed magnitude of jet quenching was significantly stronger than predicted by the perturbative estimates.

A more spectacular jet quenching phenomenon has been reported in the first LHC heavy-ion run in November 2010, events with huge energy (≈ 100 GeV) having been lost and dissipated into the quark-gluon plasma. Many surviving jets retaining a fraction of the initial energy and travelling back-to-back to the trigger jet appear to fragment outside the medium in the standard way. There is no indication of collinear radiation in the jet cone or nearby. The energy loss appears to be due to some longitudinal force applied to the moving charge as opposed to the expected transverse kicks due to perturbative QCD (pQCD).

These observations strongly suggest the notion that the quark-gluon plasma is strongly coupled and that it needs to be treated in the AdS/CFT setting in trying to understand the mechanism of energy loss. Some of the previous works on this line will be briefly reviewed below. What seems particularly important is the dependence of the jet energy loss per unit length dE/dx on the jet energy E. For discussion purposes, let us assume that the energy loss is of some power p of the energy:

$$dE/dx \sim E^p k(x) \tag{1.4}$$

In pQCD, the energy loss through radiation results in the splitting of the leading parton into many partons, each carrying a fraction of the leading energy. The scaling nature of pQCD implies that jet quenching is proportional to the energy itself; $-dE/dx \approx E$ with $p = 1$. This is radically different from e.g. standard synchrotron radiation in accelerators which has a power $p = 4$. As noted by Pomeranchuck in 1939 and more recently by Kharzeev [7], the energy loss with $p > 1$ leads to the following:

$$\frac{1}{(p - 1)E_f^{p-1}} = \frac{1}{(p - 1)E_i^{p-1}} + \int k(x) dx. \tag{1.5}$$

The interesting result is a *finite* final energy E_f, even if the initial jet energy is very large $E_i \rightarrow \infty$. If so, then all companion jets will have nearly same energy, independent of the energy of the original (trigger) jet!

As we will show in this paper, the self-force from classical gravitational radiation of a falling ultrarelativistic object in the holographic bulk does in fact produce a power $p > 1$ of the Lorentz factor. Thus, this interesting behavior with higher energy jets stopping at shorter distances seems to be also a feature of the AdS/CFT correspondence.

C. Jet quenching in AdS/CFT

The idea that strong coupling physics is at work in jet quenching and could be addressed in the context of AdS/CFT was initially put forward by one of us (with Sin) [8]. Since then, the number of applications and discussions along these lines is rather large, see, in particular,

Refs. [9–11]. For completeness, we briefly review these discussions of jets in the AdS/CFT context.

In QCD, the simplest process in which jets have been first seen is $e^+e^- \to$ hadrons. The newborn quark-antiquark pair moves in a random direction and creates two back-to-back jets. Its description started from string-breaking models at low energies (e.g. the so-called Lund model) and later developed into an elaborate pQCD description of the partonic coherent cascade at high energies. The outcome is a good description of the data.

Therefore, it is not surprising that the first papers addressing some of the issues in the AdS/CFT setup have the same setting: two charged particles moving with velocity v back-to-back from each other [12,13]. (The same problem with non back-to-back velocities was addressed earlier in the context of the pomeron problem [14]). As the charged particles recede in thermal AdS, a lightlike string between them falls gravitationally in the bulk. The gravitational hologram on the boundary conformal field theory shows no sign of a falling string, but rather a pointlike explosion. Indeed, the boundary energy-momentum tensor $T^{\mu\nu}$ in the comoving frame yields a spatial (pressure) part which is not isotropic. The falling string has nothing to do with a hydrodynamical explosion, as no black-hole is formed. The strong coupling regime does not tolerate jets, even if the two charges move by straight lines as in $e^+e^- \to$ hadrons. More details about this pointlike explosion, termed as "conformal collider physics," was then discussed by Hofman and Maldacena [15].

Ultrarelativistic jets in holography are well-approximated by lightlike rays travelling near the UV boundary of AdS. Their demise comes at the hands of holographic bulk gravity which causes them to ultimately fall toward the horizon and be absorbed by the standing black hole. This scenario was put forward originally in Ref. [8] and recently revisited and sharpened by many (see Refs. [16–18] and references therein). Although colored objects (gluons or quarks) are the ends of the bulk strings, it was found [9,10,16,17] that their falling path, toward the black hole in thermal AdS, is close to a geodesic. In this work, for simplicity, we approximate the jet by an ultra-relativistic body which falls on near-null geodesics, as initially suggested in Ref. [8]. We do not, of course, suggest that the strings do not exist, but we simply do not focus on their role in this work: obviously the gravitational radiation and the braking force are there without strings.

The current issue on jet physics in the context of the AdS/CFT setup has now shifted to the question of what exactly are the masses of these objects and how the jet process is initiated. One of the main results is that the shape of the geodesic describes "scale evolution" of a jet and is related to the jet-quenching strength versus the length x travelled since its production. The geodesic path consists of two parts: a near horizontal one suggestive of a very

nonlinear rise of energy loss $-dE/dx \approx x^2$, followed by a near-vertical plunge into the horizon, finished by a jet explosion as it splashes on the black hole horizon. Needless to say, both results need to be carefully tested against the phenomenologically accessible information at both RHIC and LHC. We will postpone this until the future.

D. Synchrotron versus gravitational radiation

The first step toward relating the two very different motivations mentioned in the earlier part of the introduction was done by one of us (with Khriplovich) nearly 40 years ago [19], applying the same method to 4 problems: synchrotron electromagnetic and gravitational radiations in flat and curved $3 + 1$-dimensional spaces in the ultrarelativistic regime $\gamma \gg 1$. The results for the radiation intensity are

$$\begin{aligned} \mathbf{I}_{\text{e.m.}}^{\text{flat}} &\approx e^2\gamma^4/R^2, & \mathbf{I}_{\text{grav}}^{\text{flat}} &\approx G_4 m^2\gamma^4/R^2, \\ \mathbf{I}_{\text{e.m.}}^{\text{curv}} &\approx e^2\gamma^2/R^2, & \mathbf{I}_{\text{grav}}^{\text{curv}} &\approx G_4 m^2\gamma^2/R^2, \end{aligned} \quad (1.6)$$

where the Schwarzschild metric was used for the curved space. An ultrarelativistic particle was set to rotate at the (unstable) circular orbit of radius $R \approx (3/2)r_h$. Note the decrease of the powers of γ as we go from synchrotron radiation in flat space to gravitational radiation around a Schwarzchild black hole. Also note that the powers of energy in the formulae consistently exceed one. In the last case of Eq. (1.6), Newton's constant G_4 appears in combination with the particle energy $E = \gamma m$. As a result, if the particle is small and the massless limit $m \to 0$, $\gamma \to \infty$, $m\gamma =$ drag fixed is considered, the limit is nonsingular, and thus the answer is the only possible one which makes sense physically.

For pedagogical reasons, we would like to discuss the problem, i.e. gravitational radiation and self-force in AdS$_5$, in two steps. In this subsection, we will compare the settings of the synchrotron radiation with that in pure or empty AdS$_5$. Then, in a later section, we will introduce thermal AdS$_5$ and study our problem in two different coordinate frames complementarily; the plasma rest frame and the jet comoving frame.

The familiar setting for synchrotron radiation consists of a point charge rotating on a circle of fixed radius R with $\gamma = E/m \gg 1$. The ratio of the proper time τ to the coordinate time t is fixed, and e.g.

$$\dot{x}^a = \frac{dx^a}{d\tau} = \gamma\frac{dx^a}{dt}. \quad (1.7)$$

The 4-velocity is tangent to the world line, while the acceleration (induced by the appropriate magnetic field) is orthogonal to it and is directed radially toward the circle's center. All odd derivatives of coordinates share the same direction with $\dot{x}^a$, while all even ones share the same direction with $\ddot{x}^a$. An extra power of gamma appears

for each derivative over the proper time. While the synchrotron radiation is a textbook example in $3 + 1$-dimensional space-time, it is not so in other dimensions. The difference, in respect to the self-force issue, will be discussed in $2 + 1$ and $4 + 1$ space-times in the companion paper [1]. We note that these space-time dimensions are the relevant ones for holographic models of QCD in $1 + 1$ and $3 + 1$ dimensions.

E. Motion in Schwarzschild versus AdS$_5$ space-times

In the gravity setting, the problem discussed in Ref. [19] is that of an ultrarelativistic particle rotating in (unstable) circular orbit of a $3 + 1$-dimensional Schwarzschild metric in polar coordinates (t, r, θ, ϕ). In contrast, an empty AdS$_5$ space is defined by the metric

$$ds^2 = \frac{1}{z^2}[-dt^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2 + dz^2], \quad (1.8)$$

which is a near-horizon limit of the $9 + 1$-dimensional black three-brane solution. The brane itself is extended in 3 coordinates x^1, x^2, x^3, and the metric is independent of them. These three coordinates are the analogue of the polar coordinates θ, ϕ of the Schwarzschild metric, with z playing the role of the radial coordinate r. Unlike the Schwarzschild black hole, the empty AdS$_5$ has no horizon since it originates from the so-called extremal black hole with maximal possible charge.

In the next section, we will derive an expression for the self-force containing multiple derivatives of the local trajectory over the proper time based on Eq. (1.2). To understand its origin, we recall the geodesic equation for free-falling in general gravity

$$\ddot{x}^a = -\frac{1}{2}g^{ab}(g_{cb,d} + g_{db,a} - g_{cd,b})\dot{x}^c \dot{x}^d, \quad (1.9)$$

where the right-hand side is driven by the Christoffel symbols. Since the Schwarzschild metric depends only on r and the empty AdS$_5$ metric only on z, the acceleration is directed only along these coordinates. The large tangent components of the 4-velocity on the right-hand side are of order γ^2, and so is the acceleration.

For the Schwarzschild case, equatorial geodesics follow by setting $\dot{x}^\theta = u^\theta = 0$. Thus,

$$\ddot{x}^r = -\frac{1}{2}\left(1 - \frac{r_h}{r}\right)\left(-\frac{r_h}{r^2}(u^0)^2 + 2r(u^\phi)^2\right), \quad (1.10)$$

using $\dot{x} = u$. The radius of interest for the ultrarelativistic circular orbit is a textbook result:

$$r_c = \frac{3r_h}{2}\left(1 + \frac{1}{2\gamma^2}\right). \quad (1.11)$$

Repeated differentiations of r_c yields further derivatives over proper time τ if needed. Analogous expressions for AdS$_5$ can readily be obtained using similar arguments after setting the path in the geodesic inside the (t, x^1, z) plane with $\dot{x}^2 = \dot{x}^3 = 0$.

In order to find the geodesic path itself, it is, however, more convenient to use existing integrals of motion rather than solving the second-order differential equations for the acceleration just mentioned. The Schwarzschild metric does not depend on t, θ, ϕ, and, thus, the lower-indexed momenta (per mass) u_0, u_θ, u_ϕ are conserved: those can be called the energy and two components of the angular momentum, respectively. The AdS$_5$ metric is independent of the 4 coordinates of the (boundary Minkowski world) (t, x^1, x^2, x^3). As a result, all 4-momentum components $u_a = P_a$, $a = 1, 2, 3, 4$ are conserved.

The geodesic line follows from the on-shell condition for the squared 4-velocity

$$g_{00}(u^0)^2 + g_{rr}\dot{r}^2 + g_{\phi\phi}(u^\phi)^2 = \text{right-hand side}, \quad (1.12)$$

where right-hand side $= -1$ for a massive particle and 0 for a massless one, after expressing all the velocities in terms of the conserved momenta. For the Schwarzschild case, one should use $u^0 = g^{00}\gamma$, $u^\phi = g^{\phi\phi}L$, and for AdS$_5$,

$$u^a = g^{aa}P_a, \quad a = 1, 2, 3, 4. \quad (1.13)$$

As a result, the respective radial velocities $\dot{r}$ or $\dot{z}$ can be expressed in terms of the coordinates. The result is a first-order equation that is readily integrated. For instance, the trajectory in AdS$_5$ is

$$\frac{dx^5}{dx^1} = \frac{\dot{z}}{\dot{x}} = \frac{1}{P}\sqrt{(E^2 - P^2) - 1/z^2}, \quad (1.14)$$

which integrates to the path

$$x^1 = \int_{z_i}^z dz' \frac{P}{\sqrt{E^2 - P^2 - 1/z'^2}}. \quad (1.15)$$

There is a qualitative difference between falling in a Schwarzschild background and in an AdS$_5$ background. Indeed, while in the former, a trapped circular orbit is possible, it is not in the latter. Particles fall indefinitely in AdS$_5$, irrespective of their initial conditions. A jet in AdS$_5$ assumes a particle being very energetic with $\gamma \gg 1$ along x^1 with 4-momentum conservation. The falling trajectory is composed of two parts: 1) an almost straight initial path with the x momentum much larger than the z momentum with $\dot{x}_1 \gg \dot{z}$; 2) a final diving or almost vertical plunge into the horizon.

Falling in the Schwarzschild and in the AdS$_5$ involves transverse spatial acceleration which is normal to the velocities (at least initially in AdS$_5$). In the Schwarzschild case, the circular fall produces a radiation intensity of order γ^2 as shown in Eq. (1.6). Is the fall in the empty AdS$_5$ also accompanied by a γ^2 radiation? The answer is negative, and one can readily see that there is no dependence of the radiation on γ. Indeed, since the AdS$_5$ metric is Lorentz-invariant, we can switch to a frame where $P_1 = 0$. The trajectory is then confined to the (t, z) plane with no

transverse acceleration. This is not the case for thermal AdS_5, as we discuss in later sections.

II. SELF-FORCE IN GENERAL RELATIVITY

The local self-force in $3 + 1$ gravity with zero cosmological constant was derived originally by Mino, Sasaki, and Tanaka and also Queen and Wald [2,3]. As we noted in the introduction and now repeat for completeness,

$$m\ddot{x}^a = G_5 m^2 \dot{x}^b \dot{x}^c \int_{-\infty}^{\tau_-} d\tau' \Big(\frac{1}{2} \nabla^a \mathbf{G}_{bca'b'}^- - \nabla_b \mathbf{G}_c^{-a}{}_{a'b'}$$
$$- \frac{1}{2} \dot{x}^a \dot{x}^d \nabla_d \mathbf{G}_{bca'b'}^- \Big) \dot{x}'^a \dot{x}'^b, \qquad (2.1)$$

with $\mathbf{G}^-$ being the graviton retarded propagator,

$$\Box \mathbf{G}_{aba'b'}^- - 2\mathbf{R}_{ab}{}^c{}^d \mathbf{G}_{cda'b'}^- = -16\pi \bar{g}_{aa'} \bar{g}_{bb'} \delta_5(x, x'), \qquad (2.2)$$

where $x = x(\tau)$, $x' = x(\tau')$, $\delta_5(x, x') = \delta^5(x - x')\sqrt{-g}$ and $\bar{g}$ is DeWitt's bilocal for parallel displacement along the geodesic [20]. Although the original derivation of Eq. (2.1) was carried in $3 + 1$-dimensional space with zero cosmological constant and matter, its physical interpretation is applicable in any dimensions: the right-hand side is simply a modification of Christoffel symbols due to the retarded metric perturbation of the particle trajectory. Therefore, we assume it to hold in general, especially for thermal AdS_5 in $4 + 1$ dimensions.

For ultrarelativistic jets, the eikonal limit is appropriate:

$$\mathbf{G}_{aba'b'}^-(x, x') \approx 16\pi \bar{g}_{aa'} \bar{g}_{bb'} \mathbf{G}^-(x, x'), \qquad (2.3)$$

with $\Box \mathbf{G}^-(x, x') = -\delta_5(x, x')$. Inserting Eq. (2.3) in Eq. (2.1) yields

$$m\ddot{x}^a \approx 4\pi^2 G_5 m^2 \int_{-\infty}^{\tau_-} d\tau' ((\nabla^a \mathbf{G}^- - \dot{x}^a \dot{x}^d \nabla_d \mathbf{G}^-)(\dot{x} \cdot \dot{x}')^2$$
$$- 2\dot{x} \cdot \dot{x}' \dot{x}'^a \dot{x}^d \nabla_d \mathbf{G}^-). \qquad (2.4)$$

In the above, we have dropped terms of the type $\nabla^a \bar{g}$ as they are subleading in the proper time expansion.

The scalar retarded propagator in a curved background of $4 + 1$ dimensions can be related to the one in $2 + 1$ dimensions. Explicitly,

$$\mathbf{G}^-(x, x') = -\frac{1}{2\pi} \frac{d}{d\sigma} \Big(\Theta(x', x) \frac{\theta(-\sigma)}{2\pi} \frac{\sqrt{\Delta}}{\sqrt{-2\sigma}} \Big), \qquad (2.5)$$

where the expression inside the bracket is the retarded propagator in $2 + 1$ dimensions. $\Theta(x', x)$ is the generalized Heaviside step-function with a spacelike surface through x (the final form (2.5) does not depend on the choice of this surface), and Δ is the Van-Vleck determinant

$$\Delta(x, x') = (g(x)g(x'))^{1/2} \det(\nabla_a \nabla_{a'} \sigma(x, x')), \qquad (2.6)$$

which is a scalar two-point function. σ is another two-point scalar function:

$$\sigma(x, x') = \frac{1}{2}(\tau - \tau') \int_{\tau'}^{\tau} d\tau'' \dot{x}(\tau'') \cdot \dot{x}(\tau''), \qquad (2.7)$$

which is defined by the geodesic between x and x'. It is negative for timelike geodesics

$$\sigma(x, x') = -\frac{1}{2}(\tau - \tau')^2 = \frac{1}{2}d(x, x')^2, \qquad (2.8)$$

where $d(x, x')$ is the chordal distance. While the latter is only defined locally for general curved space-times, it can be defined globally for de Sitter and AdS spaces because of their spherical and hyperbolic nature. Indeed, for AdS_5, the finite distance is

$$\cos(d(x, x')) - 1 = \frac{(\mathbf{x} - \mathbf{x}')^2}{2zz'}, \qquad (2.9)$$

with z being the conformal direction, and $\mathbf{x}^2 = -t^2 + x^2$. This relation is readily derived by embedding AdS_5 in R^6 with a hyperbolic constraint. We further note that the retarded propagator(2.5) was initially derived in Ref. [21]. Our small proper time expansion is consistent with this result.

III. GRAVITY SELF-FORCE IN AdS_5

A. The expansion in ϵ

For ultrarelativistic jets, the trajectory is characterized by small proper times, and one can expand in it. Inserting Eq. (2.5) into Eq. (2.4) and following our arguments for the self-force in $4 + 1$ dimensions shows that the gravitational self-force for ultrarelativistic jets is dominated by the leading singularity in the (covariant) gradient of the propagator,

$$\nabla^a \mathbf{G}^- \approx -\frac{3}{4\pi^2} \frac{\sqrt{\Delta} \sigma^a}{\epsilon^5}, \qquad (3.1)$$

with small $\epsilon = (\tau - \tau') \ll 1$ and $\sigma^a = \epsilon \dot{x}^a$, where $\dot{x}^a$ is tangent to the jet geodesic. The smallness of ϵ will be explained further below. The problem is then reduced to a covariant expansion of $\sqrt{\Delta}$ for x' near x. For that, we follow Refs. [20,22] and expand $\sqrt{\Delta}$, first covariantly in terms of the world function σ and then through a standard Taylor expansion of σ. Specifically,

$$\sqrt{\Delta} = 1 + \frac{1}{12}\mathbf{R}_{ab}\sigma^a\sigma^b - \frac{1}{24}\mathbf{R}_{ab;c}\sigma^a\sigma^b\sigma^c$$
$$+ \Big(\frac{1}{288}\mathbf{R}_{ab}\mathbf{R}_{cd} + \frac{1}{360}\mathbf{R}^m{}_a{}^n{}_b\mathbf{R}_{mcnd} + \frac{1}{60}\mathbf{R}_{ab;cd} \Big)$$
$$\times \sigma^a\sigma^b\sigma^c\sigma^d + \ldots, \qquad (3.2)$$

For both empty and thermal AdS spaces, there are significant simplifications, which come from the fact that the Ricci tensor $\mathbf{R}_{ab}$ is proportional to the metric, since the Einstein equation with cosmological constant tells us

$$\mathbf{R}_{ab} - \frac{1}{2}g_{ab}\mathbf{R} = \frac{\Lambda}{2}g_{ab}, \qquad (3.3)$$

with the scalar curvature $\mathbf{R} = -5\Lambda/3$. As a result, the covariant derivatives of the Ricci tensor simply vanish:

$$\mathbf{R}_{ab;c} = \mathbf{R}_{ab;cd} = 0, \tag{3.4}$$

which yields

$$\sqrt{\Delta} = 1 - \frac{1}{18}\sigma + \frac{1}{648}\sigma^2 + \frac{1}{360}\mathbf{R}^m{}_a{}^n{}_b\mathbf{R}_{mcnd}\sigma^a\sigma^b\sigma^c\sigma^d + \dots, \tag{3.5}$$

after using the identity $\sigma^a\sigma_a = 2\sigma$.

For a bulk object falling in AdS$_5$, a tremendous simplification takes place when rewriting Eq. (3.5) in terms of jet trajectory proper time difference. Indeed, since it follows a geodesic at zeroth order, the bilocal $\sigma(x, x')$ uniquely determines the distance travelled for short or small ϵ. The geodesic definition and length are unique when x' is in the neighborhood of x. Thus, $\sigma = \epsilon^2/2$ and $\sigma^a = \epsilon\dot{x}^a$, where $\dot{x}^a$ is tangent to the jet geodesic.

Another way to say this is to note that the covariant Taylor expansion of σ along the particle trajectory (and not the graviton trajectory) to fourth order is

$$\sigma \approx -\frac{\epsilon^2}{2} - \frac{\epsilon^4}{12}D\dot{x} \cdot D\dot{x}, \tag{3.6}$$

with $D\dot{x}^a \equiv \ddot{x}^a + \Gamma^a_{bc}\dot{x}^b\dot{x}^c$, the covariant or long derivative. The covariant derivative and its higher-order variants vanish along the falling jet geodesic, making the first contribution exact. Thus, along the geodesic jet trajectories in AdS$_5$,

$$\sqrt{\Delta} \approx 1 + \frac{\epsilon^4}{360}\mathbf{R}^m{}_a{}^n{}_b\mathbf{R}_{mcnd}\dot{x}^a\dot{x}^b\dot{x}^c\dot{x}^d. \tag{3.7}$$

In terms of Eq. (3.7), the gravitational self-force (2.4) simplifies:

$$m\ddot{x}^a \approx -\frac{G_5 m^2}{30\pi}\left(\int d\epsilon\right)\mathbf{R}^m{}_e{}^n{}_b\mathbf{R}_{mcnd}\dot{x}^e\dot{x}^b\dot{x}^c\dot{x}^d\dot{x}^a. \tag{3.8}$$

The final integration along the proper time $\int d\epsilon$ will be explained shortly and is of order $1/\gamma^2$. The dragging gravitational covariant self-force on the falling jet is longitudinal and of order γ^3. It is entirely driven by a (squared) Riemann tensor.

B. Final integration and magnitude of ϵ

So far, we have assumed ϵ, the proper time between the emission of the graviton and its action on the charge, to be small. We have not yet specified why and how small it is. A qualitative answer follows from the fact that the more relativistic the jet, the more localized its own trailing field. Indeed, the localization in time is expected to be $t \approx 1/\gamma$ and thus a localization in proper time $\tau \approx \epsilon \approx 1/\gamma^2$.

A way to see this is to note that the final ϵ integration needs to be cutoff at large ϵ. Indeed, Eq. (3.6) in flat space-time reduces to

$$\sigma|_{\text{flat}} \approx -\frac{\epsilon^2}{2} - \frac{\epsilon^4}{12}\ddot{x} \cdot \ddot{x}. \tag{3.9}$$

For ultrarelativistic motion, we can make the substitution

$$\epsilon \rightarrow \epsilon\left(1 + \tfrac{1}{6}\epsilon^2\ddot{x} \cdot \ddot{x}\right)^{1/2}, \tag{3.10}$$

leading to a finite resummed result:

$$\int_0^\infty d\epsilon\left(1 + \frac{1}{6}\epsilon^2\ddot{x} \cdot \ddot{x}\right)^{-5/2} = \frac{4}{\sqrt{6\ddot{x} \cdot \ddot{x}}}, \tag{3.11}$$

with most of the contribution stemming from the range $\epsilon \approx 1/\gamma^2$. As a result, the dropped terms in our quasilocal analysis of the gravitational self-force are all subleading in $1/\gamma$. This argument shows how a schematic resummation of the subleading corrections yields a finite result for $\int d\epsilon$. The qualitative character of this substitution does not fix the overall coefficient exactly. For that, more quantitative work is needed.

The scaling of $\epsilon \approx 1/\gamma^2$ is also in effect in the synchrotron analysis in even space + time dimensions as we discuss in the companion paper [1]. This underlines again the analogy between synchrotron and gravitational radiation as we have noted above.

C. Holographic gravitational force estimate

The previous analysis is purely 5-dimensional, and to apply that to AdS/CFT, we have to translate 5-dimensional (5D) results into 4-dimensional (4D) gauge theory ones. The holographic duality holds in the double limit in which the number of colors is large compared to the 't Hooft coupling $N_c \gg \lambda = g^2 N_c \gg 1$, taken also to be large. In AdS/CFT,

$$G_5 = \frac{\pi L^3}{2N_c^2}. \tag{3.12}$$

The string length l_s and the AdS radius L are tied by $L^4/l_s^4 = \lambda$ in the double holographic limit. For a meaningful gauge theory result, L, which involves l_s, should drop at the end, and we will show this explicitly in our result shortly.

The 5D metric has the form

$$ds^2 = \frac{L^2}{z^2}\left(\frac{dz^2}{f} - f dt^2 + d\vec{x}^2\right), \tag{3.13}$$

where the gauge theory space-time is measured by $(t, \vec{x})$. From this, one can obtain relations between the local 5D energy E_{AdS} and the 4D energy E as follows:

$$E_{\text{AdS}} = \frac{z}{L}E. \tag{3.14}$$

Similarly, the 5D (proper) time and the 4D (proper) time are related as

$$t_{\text{AdS}} = \frac{L}{z} t. \qquad (3.15)$$

As a result, the 4-velocity $\dot{x}$ in the above result, which is defined in terms of the 5D proper time, is roughly

$$\dot{x} \approx \frac{z}{L} \gamma, \qquad (3.16)$$

where γ is the ordinary Lorenz factor in 4 dimensions. Also, since m is the 5D rest energy, it should be rescaled by z/L to tie with the 4D rest energy m_4. We now note that the square of the Riemann tensor in Eq. (3.8) is of dimension but independent of L,

$$\mathbf{R}^2 \approx \frac{\mathbf{P}(z/z_h)}{z^4}, \qquad (3.17)$$

with $\mathbf{P}$ a polynomial of $z/z_h \approx T/Q_s$. Q_s is the jet saturation scale to be fixed below. The final integration $\int d\epsilon$ is of order L/γ^2 times some function of T/Q_s. The $1/\gamma^2$ was explained before, whereas the factor L follows the factor L^2 in ds^2, since the geodesic motion is independent of the overall factor of L^2.

With these considerations, the 5-dimensional *noncovariant force*, which is equal to 5-dimensional gravitational energy loss, takes the form

$$\frac{dE_{\text{AdS}}}{dt_{\text{AdS}}} = \left(\frac{z}{L}\right)^2 \frac{dE}{dt} \sim G_5 m^2 R^2 \left(\int d\epsilon\right)(\dot{x})^4 \ \sim \frac{L^3}{N_c^2}\left(m_4 \frac{z}{L}\right)^2$$
$$\times \frac{1}{z^4} \frac{L}{\gamma^2}\left(\frac{z}{L}\gamma\right)^4 \mathbf{F}(z/z_h)$$
$$= \left(\frac{z}{L}\right)^2 \frac{(m_4\gamma)^2}{N_c^2} \mathbf{F}(z/z_h), \qquad (3.18)$$

with $\mathbf{F}$ a universal function of the ratio of the initial to horizon (final) values of z. With this, one can make a safe massless limit by replacing $m_4\gamma = E$ as it makes sense physically. Thus,

$$\frac{dE}{dt} \sim \frac{E^2}{N_c^2} \mathbf{F}(z/z_h), \qquad (3.19)$$

with $\mathbf{F}(0) = 0$. L drops in the final result for the boundary observables. We set $L \equiv 1$ in the rest of our paper. In thermal AdS$_5$, the universal function $\mathbf{F}$ appears solely dependent on the ratio z/z_h, with the holographic direction playing the role of a renormalization scale evolution. Indeed, along the longitudinal direction, the jet forms at $z_i \approx 1/Q_s$ and ends at $z_h = 1/(\pi T)$, the black hole horizon.

Although Eq. (3.19) is subleading in $1/N_c$, it is magnified by a large factor $\gamma^2 \gg 1$, thereby dwarfing this suppression. We note that the work following from Eq. (3.19) yields a power or intensity

$$\mathbf{I} \approx \frac{E^2}{N_c^2} \int_{z_i}^{z_h} \frac{dz}{\dot{z}} \mathbf{F}(z/z_h), \qquad (3.20)$$

in agreement with Eq. (1.6). We cannot, however, completely trace the origin of this agreement at this time.

Finally, we note that the longitudinal covariant force following from a dragging colored string is of the order of $\gamma\sqrt{\lambda}T^2$ [16,17]. The ratio of the longitudinal drag from gravitational radiation (or self-force) to color is

$$\frac{\text{gravity-radiation-drag}}{\text{color-drag}} \approx \frac{\gamma^2}{N_c^2\sqrt{\lambda}} \approx \frac{10^{2...4}}{10 \times 5}, \qquad (3.21)$$

not small, for typical jets at RHIC and LHC with $\gamma = 10$–100. Of course, the derivation given is perturbative, without backreaction explicitly included. (It means it is only formally valid for N_c, λ exceeding the realistic values of $N_c = 3$ and $\lambda \approx 25$.)

In summary, we argue that the gravitational radiation drag is comparable to the color drag at RHIC and is perhaps the dominant mechanism for jet loss at the LHC.

(Other effects subleading in $1/N_c$ and beyond the Einstein-Hilbert action may also lead to a higher power of γ; we have not yet investigated their effect.)

IV. MATTER VERSUS WIND FRAME FOR THERMAL AdS$_5$

We now proceed to analyze in detail the radiative gravitational force for thermal AdS$_5$ described by a black brane with a horizon located at some value z_h in the 5th coordinate. Holography asserts that this classical setup is dual to thermal $\mathcal{N} = 4$ supersymmetric gauge theory at the boundary with temperature $T = 1/(\pi z_h)$.

A. The geodesics in the (plasma) matter rest frame

To assess the effect of the thermal bath on the longitudinal induced gravitational self-force, we use in this section the standard thermal AdS$_5$ metric with plasma matter at rest. In terms of the coordinates (t, x^1, x^2, x^3, z), the metric is

$$ds^2 = \frac{1}{z^2}\left(-f dt^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2 + \frac{dz^2}{f}\right), \qquad (4.1)$$

with $f = 1 - (z/z_h)^4$. In this frame, the jet is represented by a point particle moving with some 4-velocity $\dot{x}^\mu = u^\mu$. At the start, the jet moves ultrarelativistically in the x^1 direction, so that u^t, $u^x = \gamma^1$, with $\gamma \gg 1$, while u^z is small. As it proceeds, its longitudinal velocity decreases while its velocity along the holographic direction z direction increases until it finally approaches the vicinity of the horizon.

Using conservation of the 4-momentum of the particle, one can readily derive the following expressions for the components of the 4-velocity:

$$\dot{t} = z^2 E/f, \qquad \dot{x} = z^2 P, \qquad (4.2)$$

$$\dot{z}^2 = z^4(E^2 - fP^2) - z^2 f, \qquad (4.3)$$

from which the path

$$x^1 = \int_{z_i}^{z} dz' \frac{P}{\sqrt{E^2 - fP^2 - f/z'^2}}. \qquad (4.4)$$

In pure AdS_5, $f = 1$, leading to the Lorentz-invariant combination $E^2 - P^2$ in the integrand. This is no longer the case in a thermal medium with $f \neq 1$.

An example of the corresponding trajectory in the x-z plane is shown in Fig. 1 by the open circles. For comparison, we also show a geodesic for massless particles (blue asterisks). Note that a jet, here as a particle, starts from the initial value of the holographic coordinate z_i, taken to be 1, and ends at $z = z_h$. In this example $z_h = 5$. After selecting the jet gamma factor $E = 40$ and fixing the initial $z_i = 1$, the value of the jet momentum follows from the on-shell condition as usual. What we see in Fig. 1 is qualitatively similar to the standard trajectory of a stone thrown from a cliff. The trajectory turns down as the increasing z momentum takes over the conserved x momentum.

The choice of z_i is a choice of the particle mass or jet scale (in the renormalization group sense), which is also referred to as the *saturation scale* Q_s. Phenomenology sets it to be 1–2 GeV at RHIC and 2–3 GeV at LHC. With the units thus fixed, the jet energy in this example should correspond to 40–80 GeV at RHIC and 80–120 GeV at LHC. The corresponding matter temperature is, in this example, $T = Q_s/5\pi$, which is lower than the temperature occurring in real hadronic matter. We selected it, however, in order that the stopping distance (about $24/Q_s$) be (for
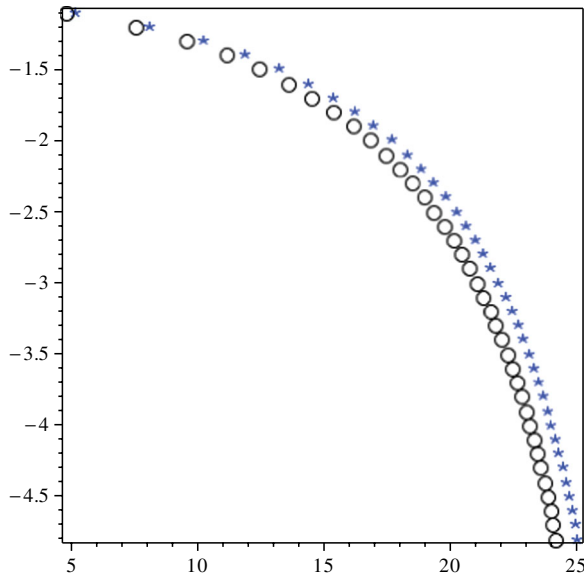
$Q_s = 1$ GeV) about 5 fm, or comparable to the size of heavy nuclei used in the experiments.

The elapsed time (not shown) follows closely the distance except for a divergence as $z \to z_h$. Any falling object freezes in the distant observer frame as it approaches the horizon. In jet experiments, the time is usually limited by the matter freeze-out time; thereby, the falling jet remains close to z_h but never crosses it.

The metric and thus all tensors are independent of gamma, and thus the counting of the power of gamma is straightforward. It is reduced to counting the powers of the 4-velocities. For example,

$$\ddot{x}^a \approx \Gamma_{bc}^a \dot{x}^b \dot{x}^c \approx \gamma^2, \qquad (4.5)$$

so that $\ddot{x} \cdot \ddot{x} \approx \gamma^4$. Of course, one should check that one gets a nonzero result for an ultrarelativistic particle. An important exception involves contractions with the Ricci tensor, which has a special property $R_{ab} \approx g_{ab}$. Therefore, its contraction $\mathbf{R}_{ab}\dot{x}^a\dot{x}^b \approx \gamma^0$ and not γ^2 as naively expected.

Unlike the Ricci terms, the one with the squared Riemann tensor does not have a simplified structure and thus retains its naive order of magnitude $\approx \gamma^4$. More specifically, we found

$$\mathbf{R}\,\mathbf{R}uuuu = \mathbf{R}_{\sigma\gamma\kappa\delta}g^{\sigma\nu}g^{\kappa\mu}\mathbf{R}_{\mu\alpha\nu\beta}u^\alpha u^\beta u^\gamma u^\delta$$

$$= \frac{\gamma^4 \mathbf{P}}{z^4(1 - z^4/z_h^4)^2}, \qquad (4.6)$$

where $\mathbf{P}$ is some lengthy polynomial of degree 4 in the velocities with coefficients depending on γ, z. Instead of presenting it as such, we show in Fig. 2 its dependence on

FIG. 1 (color online). (color online) Example of a geodesic trajectory for a falling particle in the (inverted) 5th coordinate z as a function of x^1. The open (black) circles are for massive particle, (blue) asterisks are for massless geodesic. The parameters are explained in the text.
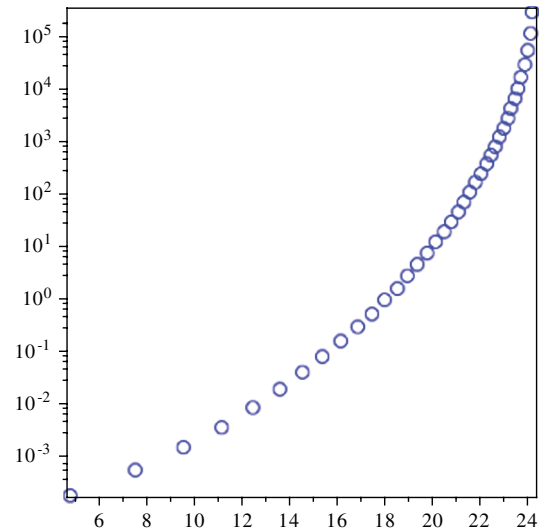
FIG. 2 (color online). Dependence of the combination of two Riemann tensor and 4-velocities normalized to the leading power of the jet Lorentz factor γ, namely $\mathbf{R}\mathbf{R}uuuu/\gamma^4$, versus the longitudinal distance (in the x^1 direction) travelled by the jet, for the numerical example displayed in Fig. 1. The units, as explained in the text, are fixed by the particle mass or Q_s.

the longitudinal distance x^1 travelled rather than z. As we see from the plot, this contribution to the self-force not only depends strongly on the jet energy γ but varies rapidly with the longitudinal distance crossed by the jet. It takes extremely large values near the jet final stopping point. Clearly, the backreaction from this force on the travelling jet needs to be taken into account.

B. The "hot wind" frame

The dependence of all parameters in question on the jet energy (Lorentz factor γ) is also instructive to study in an alternative frame, in which the longitudinal jet momentum P_x can be set to zero. Such boosted frames were used before, e.g. in the study of moving charmonium [23].

First, it is instructive to see how the synchrotron electromagnetic radiation is recovered in this frame. In the original frame, all powers of γ follow from the velocity, while in the boosted frame, the velocities are γ^0. The enhancement in this frame is transferred to the magnetic field which is magnified by γ.

The same happens with the boosted metric. In empty AdS_5, the combination $-dt^2 + dx^{1^2}$ turns to $-dt'^2 + dx'^{1^2}$ after boosting, which is clearly a Lorentz scalar. In the boosted frame, the jet is at rest with all γ's gone. Empty AdS_5 is Lorentz-invariant with free particle motion being uneventful.

At finite temperature, manifest Lorentz invariance in the metric is lost with

$$dt^2 = (\gamma dt' - \sqrt{\gamma^2 - 1}dx^{1'})^2 \qquad (4.7)$$

and $dx^{1^2} = 0$ after the boost. Thus, the metric gets enhanced by γ^2 in this case. This amounts to effectively boosting the energy density. In the rest frame of the jet, the effective temperature is enhanced:

$$T' = T\sqrt{\gamma}. \qquad (4.8)$$

Note that at large γ, the boosted metric in Eq. (4.7) has nonzero g_{12}. There is "wind" in the x direction. This has obvious consequences on the particle motion. While it is still true that there is conserved energy and momentum given by the lower components $u_0 = E$, $u_1 = P$, the upper components are mixed by the nondiaginal metric. As a result, even if $P = 0$, one gets a nonzero velocity in the x direction since

$$u^0 = g^{00}E, \qquad u^1 = g^{10}E. \qquad (4.9)$$

The upper component metric is obtained in the standard way with

$$u^t = \frac{dt}{d\tau} = E\frac{z^2(z_h^4 + z^4\gamma^2 - z^4)}{z^4 - z_h^4},$$

$$u^x = \frac{dx}{d\tau} = E\sqrt{\gamma^2 - 1}\frac{z^6}{z^4 - z_h^4}. \qquad (4.10)$$

The geodesic path is again derived from the on-shell condition $g_{ab}u^au^b = -1$ after solving for the z velocity

$$(u^z)^2 = \left(\frac{dz}{d\tau}\right)^2 = (-z^6E^2 + z^6\gamma^2E^2 + z^4 + E^2z_h^4z^2 - z_h^4)\frac{z^2}{z_h^4}. \qquad (4.11)$$

This is readily integrated. As usual, the ratio $(u^x/u^z)^2$ eliminates the proper time and yields the falling trajectory $x(z)$.

Figure 3 shows the falling jet geodesic in the hot wind frame for the parameters used in the matter rest frame. The motion is a free fall along the z direction, accompanied by a horizontal motion along the wind direction. Initially, the particle is nearly standing and nonrelativistic with $u^t \approx 1$, and all other components are zero. Then, these components get larger with typically $u^a \approx \gamma$.

In the initial time, all the γ dependence is carried by the wind term in the metric $\approx (z/z_h)^4\gamma^2$. Note that for our two figures, the numerical value of the wind term is $\approx 40^2/5^4 = 2.56$, which is not very large compared to 1. It is 10 times more for twice more energetic jets. We now detail how this combination is entering the various contributions to the gravitational self-force in this frame.

First, let us consider the particle acceleration along the geodesic:

$$\ddot{x}^a \approx \Gamma^a_{bc}\dot{x}^b\dot{x}^c. \qquad (4.12)$$

Initially, the velocities are of order γ^0, with all powers of γ carried by the Christoffels. Typically,
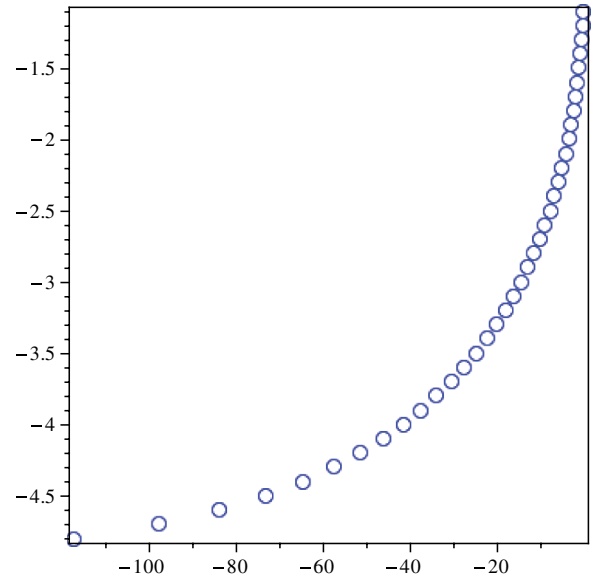

FIG. 3 (color online). (color online) The same example of a geodesic trajectory for a falling particle as in the previous figure, but in the boosted hot wind frame. The points are in the (inverted) 5th coordinate z as a function of x^1, which is now also negative, as the hot wind blows in the negative direction.

$$\Gamma^5_{11} = \frac{(-z_h^4 + z^4)(z_h^4 + z^4\gamma^2)}{z_h^8 z}. \qquad (4.13)$$

Even when the particle is at rest with $\dot{x}^t = 1$ (and all others zero), there is an acceleration of order γ^2 along the z direction. The calculation of the Riemann tensor contribution to the self-force is straightforward in this frame. To leading order, the result is

$$\mathbf{R}_{\sigma\gamma\kappa\delta} g^{\sigma\nu} g^{\kappa\mu} \mathbf{R}_{\mu\alpha\nu\beta} u^\alpha u^\beta u^\gamma u^\delta \approx \gamma^4 4z^4 \mathbf{P}', \qquad (4.14)$$

where $\mathbf{P}'$ is another fourth-order polynomial in the velocities. For $z_h = 1$, it is

$$\begin{aligned}
\mathbf{P}' = &+ 3z^8(u^t)^4 + 3z^8(u^x)^4 - 12z^8(u^t)^3 u^x + 18z^8(u^x u^t)^2 \\
&- 12z^8(u^x)^3 u^t - 8z^4(u^x)^4 - 48z^4(u^x u^t)^2 \\
&+ 32z^4(u^x)^3 u^t - 8z^4(u^t)^4 + 32z^4(u^t)^3 u^x \\
&+ 42(u^x u^t)^2 - 28(u^t)^3 u^x + 7(u^t)^4 \\
&- 28(u^x)^3 u^t + 7(u^x)^4.
\end{aligned} \qquad (4.15)$$

It is nonzero even in the initial part of the trajectory when the particle is nearly at rest ($u^t = 1$, $u^x = u^z = 0$), as it has the $(u^t)^4$ term. However, as the wind accelerates the falling jet, the polynomial is seen to keep the same power of γ. The quantity plotted in Fig. 2 is, of course, a scalar, so it is same in both frames.

V. SUMMARY AND DISCUSSION

Starting with a companion paper [1], we first mention its main result: the longitudinal self-force in flat even space + time is

$$\begin{aligned}
2+1: \ (m\ddot{x}^a)_L &\approx -\frac{2e^2}{\sqrt{3}} \frac{\dddot{x} \cdot \dddot{x}}{\sqrt{\ddot{x} \cdot \ddot{x}}} \dot{x}^a, \\
4+1: \ (m\ddot{x}^a)_L &\approx -\frac{e^2}{10\sqrt{3}} \frac{\dddot{x} \cdot \dddot{x}}{\sqrt{\ddot{x} \cdot \ddot{x}}} \dot{x}^a,
\end{aligned} \qquad (5.1)$$

for an electromagnetically charged particle undergoing circular motion in $2+1$ and $4+1$ dimensions, respectively. Both of these induced forces match the expected radiation asymptotically.

Our main result is the (leading) longitudinal gravitational self-force (3.8) with Eq. (3.11), i.e.

$$m\ddot{x}^a \sim -\left(\frac{G_5 m^2}{\sqrt{\dot{x} \cdot \dot{x}}}\right) \mathbf{R}^m{}_e{}^n{}_b \mathbf{R}_{mcnd} \dot{x}^e \dot{x}^b \dot{x}^c \dot{x}^d \dot{x}^a, \qquad (5.2)$$

derived using similar arguments, starting from Eq. (1.2). The structure itself appears from the leading short distance singularity of the propagator in $4+1$ dimensions. Its part in brackets, coming from a resummation of the higher-order terms, is schematic, and we do not claim exactness of its coefficient, just the power of γ. The convergence of the expansion follows from $\epsilon \approx 1/\gamma^2$, which is expected in the ultrarelativistic limit. Indeed, at large γ, the fields are Lorentz-contracted, causing them to be localized over a

proper time of order $1/\gamma^2$. In Eq. (3.11), we have shown how this cutoff in ϵ is obtained through a qualitative resummation of the higher-order terms. More work is required to fix this resummation quantitatively.

It is interesting to note the fundamentally different nature of the approximations leading to the radiation/self-force in the nonrelativistic versus the relativistic limit. In the former, the radiation proceeds through large wavelengths or small frequencies via a multipole expansion at large distances away from the charge/mass. In the latter, the expansion is dominated by the short wavelengths or frequencies; that is, by the field in the immediate vicinity of the charge/mass. In a way, the two approaches are expected to be tied by the structures of energy-momentum conservation.

To assess the magnitude of the gravitational self-force in AdS$_5$, we have used two different frames: the plasma matter rest frame and the jet rest frame. While scalars and vectors are expected to transform according to the expected lore of relativity, the use of these two distinct frames sheds light on the nature of the contributions to the self-force. The dominant contribution to the covariant longitudinal gravitational drag is of order γ^3.

The longitudinal self-force we have discussed is very different from those discussed so far in the context of holography. In particular, it has nothing to do with the color charge of the jet, but rather with its energy. The massive amount of excitation left behind by the jet produces backward gravitational drag or pull, which we have found to be growing dramatically with the jet energy. This effect is classical, large, and physically well-separated from the variety of other subleading effects one may think of. It is by no means subleading, although in holography, it is formally down by a power of Newton's gravitational constant $G_5 \approx 1/N_c^2$.

The present work is the first indication for the existence of strong self-induced gravitational forces in thermal AdS$_5$ that have a power of the Lorentz factor γ larger than 1. Many of the terms with higher powers do appear for arbitrary paths but are canceled out in the result, mostly because on geodesics, there is no covariant acceleration. The more involved problem of a dropping string with a longitudinal color drag is likely to depart from the geodesic motion and therefore involves higher powers. This issue will be addressed elsewhere.

It is amusing to note that the γ^2 dependence of the noncovariant force, and hence the intensity radiated by energy conservation, is the same as in the $3+1$ Schwarzschild case [19]. We cannot trace the connection except the fact that the combination $(\gamma m)^2 = E^2$ in the radiated intensity is the only one that is finite in the massless limit $m \to 0$ and $\gamma \to \infty$ with E fixed.

This paper is intended as a qualitative indication for the existence and importance of classical gravitational self-forces in studying jet quenching via AdS/CFT

correspondence. We have not tried to assess their effects on jets trajectories in the bulk or their stopping distance on the boundary quantitatively. For realistic values of N_c and λ, Eq. (3.8) appears to be somewhat too strong for ultrarelativistic jets to be treated in perturbation theory. Therefore, more work is needed to ascertain the role of these forces in a reliable calculational framework.

It is very challenging to try to understand the origin of this effect from the gauge theory side without recourse to holography and bulk gravity. As a parting comment, let us speculate that bulk gravitons and gravity are perhaps related to pomerons and effective pomeron field theory, which practitioners in the field have tried to construct for the past many decades.

APPENDIX

The structure of the singular terms in even space + time dimensions can be readily obtained using the same arguments as in odd space + time dimensions. Indeed, the retarded propagator in arbitrary space-time dimensions follows from

$$
\begin{aligned}
G^-(x, x') &= -\Theta(x, x') \, \mathrm{Im} G_F(x, x') \\
&= -\Theta(x, x') \, \mathrm{Im} G_H(x, x')|_{\sigma + i0},
\end{aligned} \tag{A1}
$$

following DeWitt's prescription between the Feynman and Hadamard propagator. In general AdS$_5$, the singular part of the Hadamard propagator reads

$$
G_H(x, x') = \frac{g(\sigma)}{(-2\sigma)^{3/2}} + w(\sigma), \tag{A2}
$$

where σ is the world function biscalar (2.7). Both $g(\sigma)$ and $w(\sigma)$ are regular functions of σ. $g(\sigma)$ is fixed by $\Box G_H = 0$ in leading order:

$$
2\partial^a g \sigma_a + g(\Box \sigma - 5) = 0, \tag{A3}
$$

after using that $\sigma^a \sigma_a = 2\sigma$. Since

$$
\Box \sigma = 5 - \sigma^a \partial_a \ln \Delta, \tag{A4}
$$

it follows that

$$
\partial_a \ln(g/\sqrt{\Delta}) = 0, \tag{A5}
$$

which fixes g up to an overall normalization.

[1] E. Shuryak, H.-U. Yee, and I. Zahed (work in progress).
[2] Y. Mino, M. Sasaki, and T. Tanaka, Phys. Rev. D **55**, 3457 (1997).
[3] T. C. Quinn and R. M. Wald, Phys. Rev. D **56**, 3381 (1997).
[4] J. D. Bjorken, FERMILAB-PUB-82-059-THY, FERMILAB-PUB-82-059-T, 1982.
[5] J. Adams et al. (STAR Collaboration), Nucl. Phys. **A757**, 102 (2005).
[6] K. Adcox et al. (PHENIX Collaboration), Nucl. Phys. **A757**, 184 (2005).
[7] D. E. Kharzeev, Eur. Phys. J. C **61**, 675 (2009).
[8] S.-J. Sin and I. Zahed, Phys. Lett. B **608**, 265 (2005).
[9] C. P. Herzog, A. Karch, P. Kovtun, C. Kozcaz, and L. G. Yaffe, J. High Energy Phys. 07 (2006) 013.
[10] S. S. Gubser, Phys. Rev. D **74**, 126005 (2006).
[11] H. Liu, K. Rajagopal, and U. A. Wiedemann, Phys. Rev. Lett. **97**, 182301 (2006).
[12] S. Lin and E. Shuryak, Phys. Rev. D **77**, 085013 (2008).
[13] S. Lin and E. Shuryak, Phys. Rev. D **76**, 085014 (2007).
[14] M. Rho, S.-J. Sin, and I. Zahed, Phys. Lett. B **466**, 199 (1999).
[15] D. M. Hofman and J. Maldacena, J. High Energy Phys. 05 (2008) 012.
[16] S. S. Gubser, D. R. Gulotta, S. S. Pufu, and F. D. Rocha, J. High Energy Phys. 10 (2008) 052.
[17] P. M. Chesler, K. Jensen, A. Karch, and L. G. Yaffe, Phys. Rev. D **79**, 125015 (2009).
[18] P. Arnold and D. Vaman, J. High Energy Phys. 04 (2011) 027.
[19] I. B. Khriplovich and E. V. Shuryak, Zh. Eksp. Teor. Fiz. **65**, 2137 (1973) [Lett. Nuovo Cimento Soc. Ital. Fis. **7**, 911 (1973)].
[20] B. S. Dewitt, Dynamical Theory of Groups and Fields (Gordon and Breach, New York, 1965).
[21] U. H. Danielsson, E. Keski-Vakkuri, and M. Kruczenski, J. High Energy Phys. 01 (1999) 002.
[22] S. M. Christensen, Phys. Rev. D **14**, 2490 (1976).
[23] Q. J. Ejaz, T. Faulkner, H. Liu, K. Rajagopal, and U. A. Wiedemann, J. High Energy Phys. 04 (2008) 089.