Constraining the dark energy and smoothness parameter with type Ia supernovae and gamma-ray bursts

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The existence of inhomogeneities in the observed Universe modifies the distance-redshift relations thereby affecting the results of cosmological tests in comparison to the ones derived assuming spatially uniform models. By modeling the inhomogeneities through a Zeldovich-Kantowski-Dyer-Roeder approach which is phenomenologically characterized by a smoothness parameter α, we rediscuss the constraints on the cosmic parameters based on type Ia supernovae (SNe Ia) and gamma-ray bursts (GRBs) data. The present analysis is restricted to a flat ΛCDM model with the reasonable assumption that Λ does not clump. A χ^2 analysis using 557 SNe Ia data from the Union2 compilation data (R. Amanullah *et al.*, Astrophys. J. **716**, 712 (2010).) constrains the pair of parameters (Ω_m, α) to $\Omega_m = 0.27^{+0.08}_{-0.03}$ (2σ) and $\alpha \geq 0.25$. A similar analysis based only on 59 Hymnium GRBs (H. Wei, J. Cosmol. Astropart. Phys. 08 (2010) 020.) constrains the matter density parameter to be $\Omega_m = 0.35^{+0.62}_{-0.24}$ (2σ) while all values for the smoothness parameter are allowed. By performing a joint analysis, it is found that $\Omega_m = 0.27^{+0.06}_{-0.03}$ and $\alpha \geq 0.52$. As a general result, although considering that current GRB data alone cannot constrain the smoothness α parameter, our analysis provides an interesting cosmological probe for dark energy even in the presence of inhomogeneities.

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I. INTRODUCTION

It is widely known that the Friedmann-Lemaitre-Robertson-Walker (FLRW) uniform geometry provides a very successful description of the Universe at large scales ($\ell \geq 100$ Mpc). However, due to the structure formation process, the inhomogeneities present at small and moderate scales influence the trajectories of light beams thereby producing observable phenomena like the ones associated with gravitational lensing. Since lensing effects must cause either brightening or dimming of cosmic sources, a basic consequence of the inhomogeneities is to alter the cosmic distances in comparison to the standard homogeneous description. In other words, any cosmic distance calculated along the line of sight of the local observers must be somehow corrected by taking into account the presence of inhomogeneities.

At present, the solution of the problem related to the light propagation in the framework of a late time clumpy Universe is far from a consensus [1,2]. One possibility to deal with the inhomogeneities is to consider them in randomly distributed compact clumps with higher density compensated by a lower density of the smoothly distributed matter. The distance obtained in such an approach is called Dyer-Roeder distance [3,4], although its necessity was already discussed by Zeldovich [5] and Kantowski [6]. Then we refer to it here as the Zeldovich-Kantowski-Dyer-Roeder (ZKDR) distance (for an overview on cosmic distances taking into account the presence of inhomogeneities, see the paper by Kantowski [7]). In this model, the effects experienced by the light beam are phenomenologically quantified by the smoothness α parameter. There are two limiting cases, namely: $\alpha = 1$ (filled beam), where the FLRW uniform distances are fully recovered and $\alpha = 0$ (empty beam) which represents the limit of a totally clumped universe. Therefore, for a partial clumpiness, the smoothness parameter lies on the interval $0 < \alpha < 1$. Notice that in this model only demagnification happens. This is physically expected by the fact that light travels preferentially in voids, with light in denser environments being absorbed or scattered.

There is a rich literature concerning the ZKDR approach and its applications to cosmology. Investigations involving many different physical aspects and phenomenologies were performed, among them: analytical expressions [8–10]; critical redshift for the angular diameter distance (d_A) [11,12], i.e., the redshift where d_A attains its maximum value; time delays [12,13]; gravitational lensing [14,15]; and accelerating universe models driven by particle creation [16]. More recently, some quantitative analyses by using ultracompact radio sources as standard rulers [17,18] and type Ia supernovae as standard candles [19] were also performed.

In a previous analysis, Santos *et al.* [19] applied the ZKDR approach for a flat ΛCDM model by considering two different samples of SNe Ia, namely, the Astier

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et al. (2006) sample [20] and the gold sample of Riess *et al.* (2007) [21]. The first sample, composed by low redshift supernovae, provided no constraints to the smoothness parameter, while the latter, which is composed of higher redshift supernovae, restricted it over the interval $0.42 \leq \alpha \leq 1.0$. In principle, such a result is strongly suggesting that objects at redshifts higher than those probed by SNe Ia could constrain the smoothness parameter. Since gamma-ray bursts (GRBs) have been detected up to redshifts $z \sim 8$, they are the natural candidates to test such a conjecture.

So far, GRBs are the most luminous explosions observed in the Universe. In the last few years, some theoretical and observational developments have shown that the presence of afterglows and that the best candidates to progenitors of GRBs are core-collapse supernovae (for comprehensive reviews on GRB physics, see Refs. [22,23]). Still more important, the possibility of applying them as standard candles has also been discussed by several authors [24,25]. Recently, some studies employing GRBs have shown that they may provide a complementary test to constrain cosmological parameters [25–28]. Indeed, GRBs are also very promising tools for cosmology from many different viewpoints. In particular, the association of long GRBs with peculiar type Ib/c SNe or hypernovae, and thus the death of very massive stars, is supported both by theories and observations [29]. Thus, given their huge luminosity and redshift distribution extending up to at least $z \approx 8$, GRBs may be considered powerful and unique tracers for the evolution of the star formation rate up to the reionization epoch [30,31].

In this paper, by assuming a flat ΛCDM model, we derive new constraints to the smoothness parameter α and the matter density parameter Ω_m. The ZKDR inhomogeneous distance approach will be adopted here, however, different from [19], the present statistical analysis will be based on the 557 SNe Ia from Union2 compilation data [32] plus 59 Hymnium GRBs [33]. As we shall see, the current SNe Ia and GRBs samples, separately used, do not provide tight constraints to the α parameter. Nevertheless, our joint analysis restricts the pair of parameters (Ω_m, α) on the intervals $0.24 \leq \Omega_m \leq 0.33$ and $0.52 \leq \alpha \leq 1.0$ within 95.4% confidence level (2σ). As an extra bonus, it is also found that the Einstein–de Sitter model is excluded with high statistical confidence level, and, as such, our analysis provides an interesting cosmological probe for dark energy even in the presence of inhomogeneities.

The paper is organized as follows. In Sec. II, we present the basic equations and the distance description by taking into account the inhomogeneities as described by the ZKDR equation. In Sec. III, we determine the constraints on the cosmic parameters from the SNe Ia and GRBs samples separately and also through a joint analysis involving both samples. Finally, we summarize the main conclusions in Sec. IV.

II. ZKDR EQUATION FOR LUMINOSITY DISTANCE

In order to describe the degree of inhomogeneity in the local distribution of matter, we also adopt the phenomenological description based on the so-called smoothness parameter, α. This parameter was originally introduced by Dyer and Roeder [3], when writing a differential equation for the angular diameter distance in locally clumpy cosmological models. To obtain the ZKDR equation, let us consider the light propagation in the geometric optics approximation [4,34],

$$\frac{d^2\sqrt{A}}{d\lambda^2} + \frac{1}{2} R_{\mu\nu} k^\mu k^\nu \sqrt{A} = 0, \tag{1}$$

where λ is an affine parameter, A is the cross-sectional area of the light beam, $R_{\mu\nu}$ the Ricci tensor, and k^μ the photon four-momentum. In this form, it is implicit that the influence of the Weyl tensor (shear) can be neglected. This means that the light rays are propagating far from the mass inhomogeneities so that the large-scale homogeneity implies that their shear contribution are canceled [35]. Further, the Ricci tensor $R_{\mu\nu}$ is related to the energy-momentum tensor $T_{\mu\nu}$ through the Einstein field equations,

$$R_{\mu\nu} - \tfrac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{2}$$

where R is the scalar curvature, $g_{\mu\nu}$ is the metric described by a FLRW geometry, and G is Newton's constant (in our units $c = 1$). The clustering phenomenon is introduced by considering the following energy-momentum tensor (ΛCDM model):

$$T_{\mu\nu} = T^m_{\mu\nu} + T^\Lambda_{\mu\nu} = \alpha \rho_m u_\mu u_\nu + \rho_\Lambda g_{\mu\nu}, \tag{3}$$

where u_μ is the four-velocity of the comoving volume elements, ρ_m is the matter energy density, $\rho_\Lambda = \Lambda/8\pi G$ is the vacuum energy density associated with the cosmological constant, and $\alpha = 1 - \frac{\rho_{\rm cl}}{\langle \rho_m \rangle}$ is the smoothness parameter introduced by Dyer and Roeder [3]. Such a parameter quantifies the portion of matter in clumps ($\rho_{\rm cl}$) relative to the amount of background matter which is uniformly distributed (ρ_m). In general, due to the structure formation process, it should be dependent on the redshift, as well as on the direction along the line of sight (see, for instance, [18,36] and references therein). However, in the majority of the works α is assumed to be a constant parameter.

Now, we assume a flat ΛCDM cosmology, as well as the validity of the standard duality relation between the angular diameter and luminosity distances, $d_L = (1+z)^2 d_A$, sometimes called the Etherington principle [37]. Since the cross-sectional length $A^{1/2}$ is proportional to the angular distance d_A, it is readily seen that Eq. (1) can be rewritten for the dimensionless luminosity distance ($D_L = H_0 d_L$) as [8–11,13,19]

$$(1+z)^2\mathcal{F}\frac{d^2D_L}{dz^2} - (1+z)\mathcal{G}\frac{dD_L}{dz} + \mathcal{H}D_L = 0, \quad (4)$$

which satisfies the boundary conditions

$$D_L(0) = 0, \qquad \frac{dD_L}{dz}\bigg|_0 = 1. \quad (5)$$

This is the ZKDR equation, where $\mathcal{F}$, $\mathcal{G}$, and $\mathcal{H}$ are functions of the cosmological parameters, expressed in terms of the redshift by

$$\mathcal{F} = \Omega_m + (1-\Omega_m)(1+z)^{-3},$$
$$\mathcal{G} = \frac{\Omega_m}{2} + 2(1-\Omega_m)(1+z)^{-3}, \quad (6)$$
$$\mathcal{H} = \left(\frac{3\alpha-2}{2}\right)\Omega_m + 2(1-\Omega_m)(1+z)^{-3}.$$

As remarked before, the α parameter appearing in the $\mathcal{H}$ expression quantifies the clustered fraction of the pressureless matter, and would be a redshift dependent quantity. Here we follow the standard treatment so that α is also assumed to be a constant (see, for instance, [19] and references therein).

III. SAMPLES AND RESULTS

We know that the Universe is homogeneous only at large scales. Then a more realistic description is to consider that at moderate and small scales matter is clumped, being homogeneous only on average. As light is affected by local quantities, not global, expressions like the distance modulus $\mu(H_0, \Omega_m, \Lambda, z)$ must be altered when the clumpiness phenomenon is taken into account through the ZKDR equation.

In Fig. 1, we display the effects of the inhomogeneities in the reduced Hubble-Sandage diagram for the Union2 [32] and Hymnium [33] samples for some selected values of the smoothness parameter. The plots correspond to several values of Ω_m and α as indicated in the panels. The difference between the data and models from an empty universe case prediction is also displayed there. For the sake of comparison, we also show the Einstein-de Sitter model, i.e. $\Omega_m = 1$ and $\alpha = 1$, as well as the present cosmic concordance ($\Omega_m = 0.26$, $\Omega_\Lambda = 0.74$, $\alpha = 1$). Note that cosmologies with only matter and inhomogeneities can show a behavior resembling to some degree the cosmic concordance model.

In order to constrain the Ω_m and α parameters, a χ^2 minimization will be applied for the sets of SNe Ia and GRB data. Following standard lines, we maximize the posterior probability $\propto \mathcal{L} \times$ prior, where

$$\mathcal{L} \propto \exp\left(-\frac{\chi^2}{2}\right). \quad (7)$$

We adopt a Gaussian prior for the nuisance parameter H_0 centered at $74.2 \pm 3.6\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ [38], which will be marginalized, and χ^2 is given by

$$\chi^2(z|\mathbf{p}) = \sum_i \frac{(\mu(z_i;\mathbf{p}) - \mu_{0,i})^2}{\sigma^2_{\mu_{0,i}}}. \quad (8)$$

In the above expression, μ is the theoretical distance modulus for the set of parameters $\mathbf{p} \equiv (H_0, \alpha, \Omega_m)$, $\mu_{0,i}$ is the measured distance modulus, and $\sigma_{\mu_{0,i}}$ its respective uncertainty. For a joint analysis, we just add the χ^2 of each

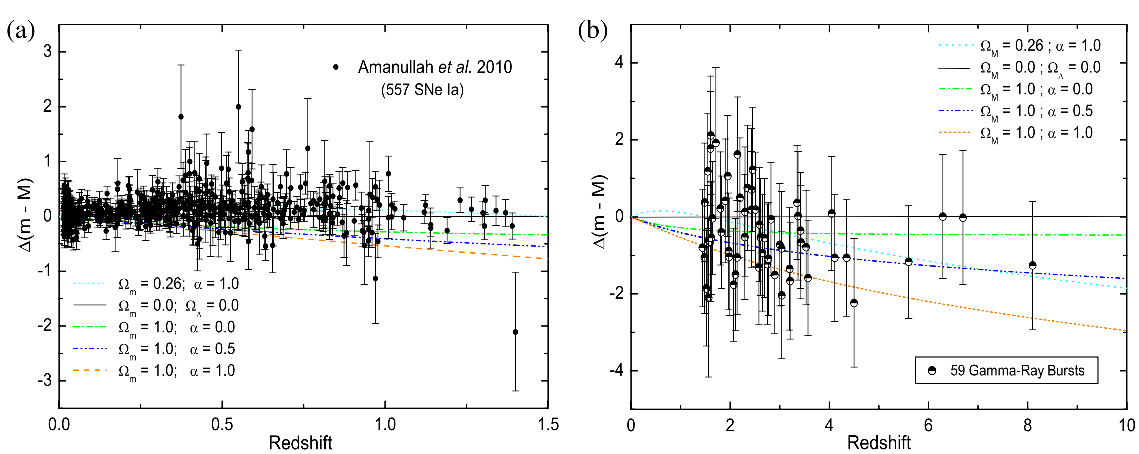

FIG. 1 (color online). The α effect on the residual magnitudes. (a) The 557 supernovae data from the Union2 compilation data [32] and the predictions of the ZKDR luminosity distance for several values of α relative to an empty model of the Universe ($\Omega_m = 0$ and $\Omega_\Lambda = 0$). (b) The same graph but now for the 59 Hymnium GRBs [33]. For comparison, in both panels we see the predictions (light blue dotted curves) of the cosmic concordance model ($\Omega_m = 0.26$, $\Omega_\Lambda = 0.74$, $\alpha = 1$).

sample. We consider the parameters α and Ω_{m} restricted on the interval [0,1] in steps of 0.01 for all numerical computations.

A. SNe Ia

Let us now discuss the bounds arising from SNe Ia observations on the pair of parameters $(\Omega_{\mathrm{m}}, \alpha)$ defining the ZKDR luminosity distance.

The Union2 compilation data [32] are the largest SNe Ia sample and consist of 557 objects, where SALT2 light-curve fitter [39] was used to calibrate the supernovae events. We have applied a χ^2 minimization using this sample and the results are displayed in Figs. 2(a)–2(c). We see from them that the smoothness α parameter is poorly constrained, being restricted on the interval $0.25 \leq \alpha \leq 1.0$ within 2σ confidence level. However, good constraints were obtained for the matter density parameter, which is restricted on the interval $0.24 \leq \Omega_m \leq 0.35 (2\sigma)$. Notice that a universe composed only by inhomogeneously distributed matter ($\Omega_\Lambda = 0$) is also strongly disfavored by these data.

At this point, it is convenient to compare the results derived here with a previous analysis performed by Santos *et al.* [19] using 182 SNe Ia from the *gold* sample observed by Riess *et al.* [21]. Within 2σ C.L., the following intervals were achieved: $0.42 \leq \alpha \leq 1.0$ and $0.25 \leq \Omega_m \leq 0.44$. It is interesting that the increase in the number of supernovae data provides a better constraint to Ω_m, but a greater range for the smoothness parameter is now allowed. We can understand this behavior by noting that low redshift data are compatible with a more inhomogeneous set of data. Indeed, this fact is in agreement with the conclusion that the structure formation process leads to a more locally inhomogeneous universe, and, therefore, with a greater sample, the effects caused by the inhomogeneities will be more likely detected. Further, since the smoothness parameter appears only in the third order in the $D_L(z)$ expansion [1], it is interesting to investigate the parameter space $(\Omega_{\mathrm{m}}, \alpha)$ using higher redshifts data. This will be examined in the next section.

B. Gamma-ray bursts

Gamma-ray bursts now offer a possible route to probe the expansion history of the Universe up to redshifts $z \sim 8$. However, it is widely known that before using GRBs to constrain cosmological models, their correlations must be first calibrated. Here we consider a relation between the isotropic-equivalent radiated energy in gamma rays (E_{iso}) and the photon energy at which the νF_ν is brightest (E_{peak}), known as the Amati relation [40]. This relation is a power law: $E_{p,i} = a \times E_{\mathrm{iso}}^b$, where $E_{p,i} = E_{\mathrm{peak}} \times (1+z)$ is the cosmological rest-frame spectral peak energy. The quantity E_{iso} is defined by

$$E_{\mathrm{iso}} = 4\pi d_L^2 S_{\mathrm{bolo}} (1+z)^{-1}, \tag{9}$$

where S_{bolo} is the bolometric fluence of gamma rays in a given GRB and d_L is its luminosity distance.

The general procedure to calibrate the relation for cosmological purposes is to use a low redshift sample, where the distance does not depend on the cosmological parameters. That is not the case for GRBs, since the observed nearby GRBs may be intrinsically different as GRB 980425 and GRB 031203 [41]. So, the cosmological parameters one would like to constrain enter into the determination of the parameters of the correlation. This is called the circularity problem. Some attempts to overcome the problem have been studied in the literature [42–44]. In this work, we use the method proposed independently by Kodama *et al.* [43] and Liang *et al.* [44], which was recently updated by Wei [33].

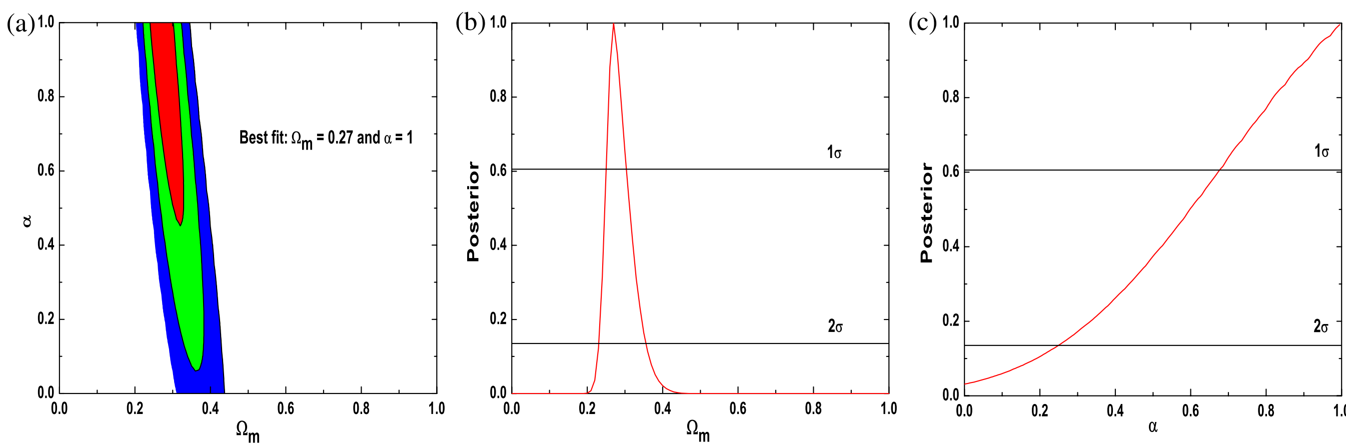

FIG. 2 (color online). (a) The $\Omega_m - \alpha$ plane for flat ΛCDM models obtained from 557 SNe Ia from the Union2 compilation data [32]. Contours stand for 68.3%, 95.4%, and 99.7% confidence levels. Note that the α parameter is not well constrained by the data. (b) Posterior probability for the matter density parameter. We see that $0.24 \leq \Omega_m \leq 0.35$, with 2σ confidence level. (c) Posterior probability for the α smoothness parameter. We see that at 2σ the smoothness parameter is restricted on the interval $(0.25 \leq \alpha \leq 1.0)$.

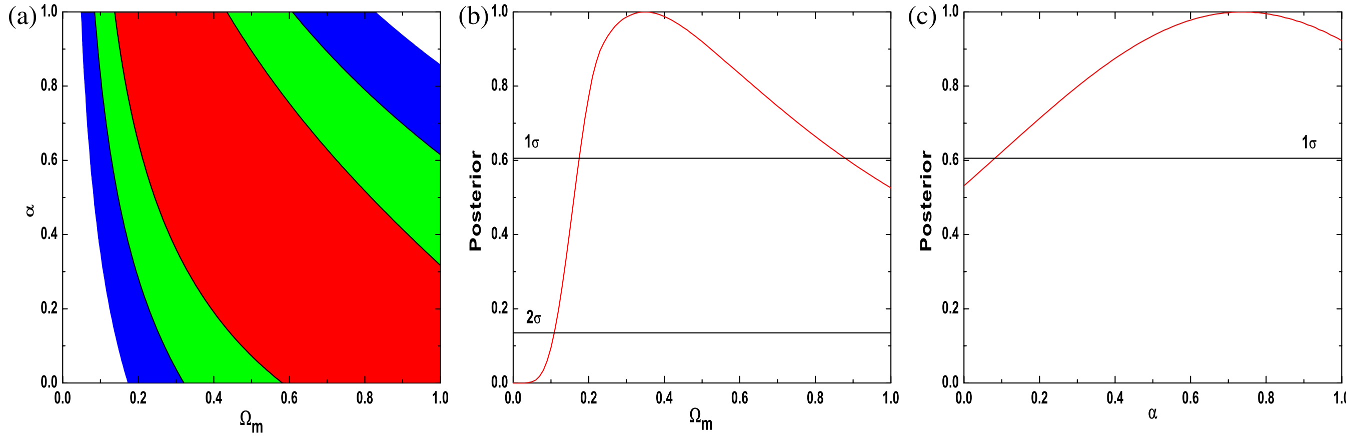

FIG. 3 (color online). (a) Contours of 68.3%, 95.4%, and 99.7% confidence on the (Ω_m, α) plane for flat ΛCDM models as inferred from 59 Hymnium GRBs [33]. (b) Posterior probability for the matter density parameter. In this case, almost all values are allowed within (2σ) confidence level ($0.11 \le \Omega_m \le 1.0$). (c) Posterior probability for the α smoothness parameter. We see that at 2σ the smoothness parameter is not constrained by the data.

The method consists of using SNe Ia as a distance ladder to calibrate the GRBs. Since the distance moduli for the SNe Ia are known, a cubic interpolation is performed to determine the parameters a and b in the Amati relation for the low redshift GRBs ($z < 1.4$). Then, the distance moduli for the highest GRBs are obtained and they can be used as standard candles without the circularity problem. In this connection, it is worth mentioning that the calibration of GRBs is still a quite controversial subject. Indeed, even the Amati relation has been contested by some authors (see, for instance, Ref. [45]).

Wei [33] used the 557 Union2 SNe Ia [32] to calibrate 109 GRBs compiled in [46]. By applying a cubic interpolation with 50 low redshift GRBs ($z < 1.4$), the parameters of the Amati relation were determined and the distance moduli for the other 59 GRBs were derived. This sample is called the Hymnium sample and can be used to derive cosmological parameters without the circularity problem.

In Fig. 3 we display the results of our statistical analysis using the GRB Hymnium sample. From Fig. 3(a) we see that both parameters are poorly constrained by the data. The likelihoods appearing in Figs. 3(b) and 3(c) allow us to get the following constraints within 2σ: $\Omega_m = 0.35^{+0.65}_{-0.24}$ while all values for the smoothness parameter are allowed within 2σ. These data are also compatible with a model composed by inhomogeneously distributed matter ($\Omega_\Lambda = 0$, $\Omega_m = 1$). In principle, such a fact can be understood by noticing that the considered sample is dominated by data with high redshifts, and, therefore, just in a moment where the dark energy component does not play a prominent role for the cosmic evolution. Naturally, the low restriction over α may also reflect the poor quality of the current GRB data as

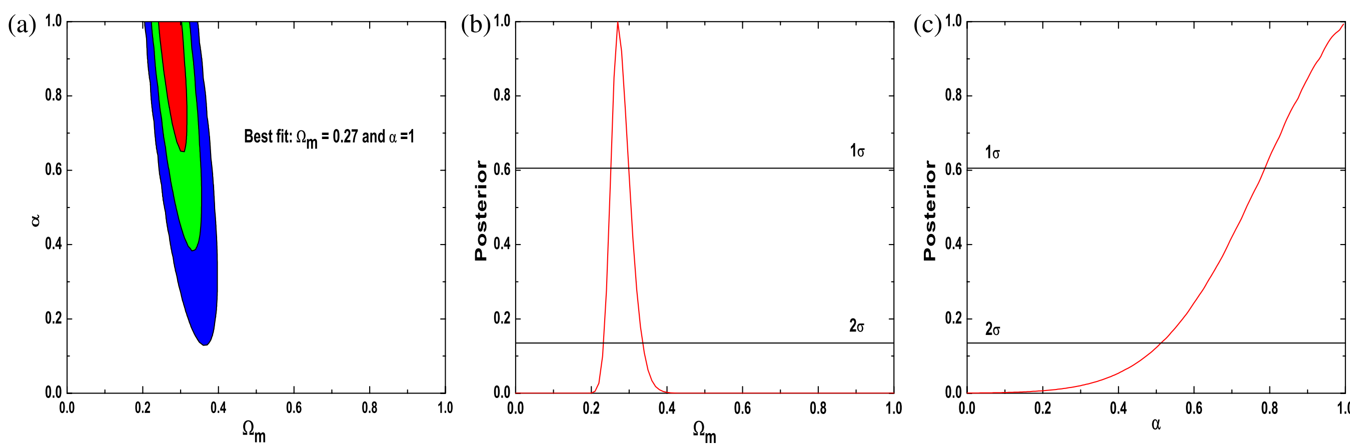

FIG. 4 (color online). (a) Contours of 68.3%, 95.4%, and 99.7% on the (Ω_m, α) plane for flat ΛCDM models as inferred from 557 SNe Ia from the Union2 compilation data [32] and 59 Hymnium GRBs [33]. (b) Posterior probability for the matter density parameter. In this case a comparatively small region is permitted, $0.24 \le \Omega_m \le 0.33$, with ($2\sigma$) confidence level. (c) Posterior probability for the α smoothness parameter. We see that at 2σ the smoothness parameter is restricted on the interval ($0.52 \le \alpha \le 1.0$).

TABLE I. Limits to α and Ω_m.

Sample	Ω_m (2σ)	α (2σ)	$\chi^2_{\rm min}$
SNe Ia	$0.24 \leq \Omega_m \leq 0.35$	$0.25 \leq \alpha \leq 1.0$	545
GRBs	$0.11 \leq \Omega_m \leq 1.0$	unconstrained	23
Joint	$0.24 \leq \Omega_m \leq 0.33$	$0.52 \leq \alpha \leq 1.0$	568

seen by the intrinsic scatter in the Amati relation. In this concern, although out of the scope of this work, it would be interesting to analyze how different phenomenological relations used to calibrate the GRBs can alter the current constraints.

C. SNe Ia and gamma-ray bursts

It is widely recognized that joint analyses in cosmology usually provide a powerful tool to improve constraints in the basic cosmological parameters. Therefore, it is interesting to perform a statistical analysis by combining the 557 SNe Ia from the Union2 compilation data [32] with the 59 Hymnium GRBs [33].

In Figs. 4(a)–4(c) we display the main results of our joint analysis. As can be seen from Fig. 4(a), the constraints on both parameters are significantly improved. The best fit obtained is $\Omega_m = 0.27$ and $\alpha = 1$, with a $\chi^2_{\rm min} = 568.36$. The confidence interval within 2σ for the matter density parameter was slightly changed ($0.24 \leq \Omega_m \leq 0.33$) compared to the SNe Ia sample while for the smoothness parameter a great improvement was achieved ($0.52 \leq \alpha \leq 1.0$) as compared to the limits individually obtained from each sample. Again, the Einstein–de Sitter model is excluded with high confidence. The better restriction over α can be understood as follows: the high redshift GRB data prefer a homogeneous universe, and, as such, they should contribute to diminishing the corresponding space parameter [see Fig. 4(a)]. In other words, since the high redshift Universe is more homogeneous, higher values of α are favored, exactly as happened.

In Table I, we have summarized the main results of our joint analysis.

IV. COMMENTS AND CONCLUSIONS

In the era of precision cosmology, it is expected that standard rulers and candles of ever-increasing accuracy will provide powerful constraints on dark energy and other cosmic parameters. However, in order to proceed with such a program it is also necessary to analyze carefully the physical hypotheses underlying the basic probes. It should also be recalled that even the large-scale homogeneity (Copernican principle) has been challenged in the last few years [47]. Besides, we know that the Universe is effectively inhomogeneous at least in the small-scale domain. In this concern, the approach based on the ZKDR equation is a simple alternative (together with weak lensing) for assessing quantitatively the effects of the clumpiness phenomenon on the light propagation. As discussed here, this approach also provides a simple extension of the Hubble-Sandage diagram, thereby altering the standard cosmological parameter estimation.

In this article, by using SNe Ia and GRBs samples, we have adopted the ZKDR approach to constrain the influence of inhomogeneities in the context of a flat ΛCDM model. Our results are summarized in Table I.

We have shown that the SNe Ia sample [32] was unable to constrain the smoothness parameter, while the matter density parameter was well constrained, being restricted on the interval $0.24 \leq \Omega_m \leq 0.35(2\sigma)$. Comparatively to a previous result [19], the smoothness parameter was less constrained even with an increase of 375 supernovae. In principle, such a result may be justified based on the fact that the Union2 sample has many low redshift supernovae, and this may suggest a redshift dependent smoothness parameter already discussed by some authors [18,36]. In addition, the GRBs sample also provided poor constraints for the pair of parameters. In this case, the data are compatible with the present Universe dominated only by inhomogeneously distributed matter ($\Omega_\Lambda = 0$). This is also expected since at high redshifts dark energy plays only a secondary role. The joint analysis provided good constraints for both parameters. The intervals within 2σ were: $0.52 \leq \alpha \leq 1.0$ and $0.24 \leq \Omega_m \leq 0.33$, where the best fit was $\alpha = 1.0$ and $\Omega_m = 0.27$.

It is important to point out that a smoothness parameter very different from unity is allowed by the current data, which may imply a cosmic concordance model with cosmological parameters shifted by several percent from the standard analysis. In particular, this means that the influence of the late time inhomogeneities can be important to decide which is the best candidate to dark energy. For the near future, we believe that new and more precise GRB data together the ZKDR approach (or some plausible extension of it) will play an important role in determining the real contribution of dark energy.

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