

Vector interaction strength in Polyakov–Nambu–Jona-Lasinio models from hadron-quark phase diagrams

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We estimate the vector interaction strength of the Polyakov–Nambu–Jona-Lasinio (PNJL) parametrizations, assuming that its transition curves should be as close as possible to the recently studied RMF-PNJL hadron-quark phase diagrams. Such diagrams are obtained matching relativistic mean-field hadronic models and the PNJL quark ones. By using this method we found for the magnitude of the vector interaction, often treated as a free parameter, a range of $7.66 \text{ GeV}^{-2} \lesssim G_V \lesssim 16.13 \text{ GeV}^{-2}$, or equivalently, $1.52 \lesssim G_V/G_s \lesssim 3.2$, with G_s being the scalar coupling constant of the model. These values are compatible but restrict the range of $4 \text{ GeV}^{-2} \lesssim G_V \lesssim 19 \text{ GeV}^{-2}$, recently obtained from lattice QCD data through a different mean-field model approach.

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The hadron-quark phase transition is still a challenging task for both theoretical and experimental fields. From the theoretical point of view, the strongly interacting matter is treated by QCD. However, in the regime of low energies, QCD is nonperturbative and still difficult to solve for intermediate temperatures and chemical potentials, although lattice methods have faced a huge progress in the last years [1]. For such a regime, it is useful to use effective models in the description of the quark matter that share the same features of QCD, for example, the MIT bag model [2], the NJL (Nambu–Jona-Lasinio) one [3,4], and its version coupled with the Polyakov loop, named the PNJL model [5].

In this context such models are used to construct the QCD phase diagram [6], where the different regions are identified as those in which the chiral symmetry is broken or restored. Studies in this direction were performed, for instance, for the linear σ model [7] and the NJL one [8]. The additional information about the confined/deconfined phases can also be taken into account when the PNJL model is used to construct the quark phase transitions [9–11], or even when the Polyakov loop is linked with the linear σ model [12].

In a very recent study [13], a comparison was made among the hadron-quark phase diagrams generated by PNJL models and those constructed by matching a large class of relativistic mean-field (RMF) hadronic models with four different parametrizations of the PNJL quark model (other studies based on this treatment can be found in Refs. [14–18]). The results shown pointed out to a difference between the phase transition curves due to the repulsive interaction of the RMF models.

Based on these results, we propose the construction of a PNJL model that minimizes this difference. This will be done by including a vector interaction in the original PNJL structure. We name hereafter the resulting model as the PNJLv model, and the strength of its vector interaction will

then be estimated in order to approximate the PNJLv transition curves to the RMF-PNJL ones as much as possible.

Actually, the inclusion of vector interactions in effective quark models was already discussed in the literature; see Ref. [4] for a study in the context of the NJL model. It is known that the effect of the increase of the repulsive interaction strength in the quark matter phase diagram is to shrink the first-order transition region as the chemical potential μ increases and also that the critical end point of the transition moves to larger μ and lower temperature T . Such effects and further studies are reported for both NJL model [19,20] and the PNJLv one [10,19,21–23], where the vector interaction strength is often used as a free parameter. Therefore, taking into account the aforementioned effects, we furnish here a method to estimate this interaction strength, based on the RMF-PNJL hadron-quark phase diagrams. We also compare our results with the recent ones, based on a mean-field calculation related to QCD lattice data, proposed in Ref. [24].

Our starting point is the Lagrangian density of the two-flavor PNJLv model, that reads

$$\begin{aligned} \mathcal{L} = & \bar{\psi}(i\gamma_\mu D^\mu - m)\psi + G_s[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\vec{\tau}\psi)^2] \\ & - G_V(\bar{\psi}\gamma_\mu\psi)^2 - \mathcal{U}(T, \mu, \Phi, \Phi^*), \end{aligned} \quad (1)$$

with $D^\mu = \partial^\mu - iA^\mu$ being the covariant derivative, A^μ the gluon field, and m the current quark mass. The vector interaction strength is regulated by the parameter G_V , and the Polyakov loop potential is given by $\mathcal{U}(T, \mu, \Phi, \Phi^*)$. As in Ref. [13], we use four different versions of this potential, namely, RRW06 [9,25], RTW05 [26], FUKU08 [10], and DS10 [27].

From this Lagrangian density one obtains, following the procedure used in Ref. [26] and taking into account the new vector term, the grand thermodynamic potential,

$$\Omega = \mathcal{U}(T, \mu, \Phi, \Phi^*) + G_s \rho_s^2 - \frac{\gamma}{2\pi^2} \int_0^\Lambda E(k, M) k^2 dk - \frac{\gamma}{6\pi^2} \int_0^\infty \mathcal{F}_+ \frac{k^4 dk}{E(k, M)} - G_V \rho^2, \quad (2)$$

in the isospin symmetric system, in which $E(k, M) = (k^2 + M^2)^{1/2}$, and with the degeneracy factor given by $\gamma = 12$. The constituent quark mass is $M = m - 2G_s \rho_s$, and the quark density is obtained from $\rho = -\partial\Omega/\partial\tilde{\mu}$, where $\tilde{\mu}$ is related with G_V and ρ through $\tilde{\mu} = \mu - 2G_V \rho$. Note that the quark density can be also found by requiring that $\partial\Omega/\partial\rho = 0$. The functions $\mathcal{F}_\pm = \mathcal{F}_\pm(k, T, \tilde{\mu}, \Phi, \Phi^*)$ are defined by $\mathcal{F}_\pm = F(k, T, \tilde{\mu}, \Phi, \Phi^*) \pm \bar{F}(k, T, \tilde{\mu}, \Phi, \Phi^*)$.

The strength of the scalar interaction (G_s), the vacuum integral cutoff (Λ), and m are also parameters of the model. Here they are given by $G_s = 5.04 \text{ GeV}^{-2}$, $\Lambda = 651 \text{ MeV}$, and $m = 5.5 \text{ MeV}$ [26].

The inclusion of the confinement information, via the Polyakov loop, affects the statistical distributions of the PNJLv model in such way that the new Fermi-Dirac functions are now given by

$$F(k, T, \tilde{\mu}, \Phi, \Phi^*) = \frac{\Phi e^{2x} + 2\Phi^* e^x + 1}{3\Phi e^{2x} + 3\Phi^* e^x + e^{3x} + 1} \quad (3)$$

and $\bar{F}(k, T, \tilde{\mu}, \Phi, \Phi^*) = F(k, T, -\tilde{\mu}, \Phi^*, \Phi)$, with $x = (E - \tilde{\mu})/T$.

The quark condensate $\langle \bar{\psi} \psi \rangle = \rho_s$, and the Polyakov loop Φ are found by requiring that $\partial\Omega/\partial\rho_s = \partial\Omega/\partial\Phi = 0$, in the lowest order approximation [9,28], that leads to $\Phi = \Phi^*$. The explicit form of the equations of motion (EOM) of the PNJLv model, found by minimizing Ω in respect to ρ , ρ_s and Φ , are

$$\rho = \frac{\gamma}{2\pi^2} \int_0^\infty \mathcal{F}_- k^2 dk - \frac{\partial\mathcal{U}}{\partial\mu}, \quad (4)$$

$$\rho_s = \frac{\gamma}{2\pi^2} \int_0^\infty \mathcal{F}_+ \frac{M}{E} k^2 dk - \frac{\gamma}{2\pi^2} \int_0^\Lambda \frac{M}{E} k^2 dk, \quad (5)$$

and

$$\frac{\partial\mathcal{U}}{\partial\Phi} - \frac{T\gamma}{2\pi^2} \int_0^\infty (g_1 + g_2) k^2 dk = 0, \quad (6)$$

where

$$g_1 = g_1(k, T, \tilde{\mu}, \Phi) = \frac{1 + e^{-x}}{3\Phi(1 + e^{-x}) + e^x + e^{-2x}}, \quad (7)$$

and $g_2 = g_2(k, T, -\tilde{\mu}, \Phi)$. Note that these EOM are the same PNJL ones, with the chemical potential μ shifted by the vector interaction, as given by $\tilde{\mu}$. In the RMF case, such a shift also occurs in the value of the baryonic chemical potential due to the contribution of the mean-field value of the isoscalar vector meson field ω .

It is important to remark that we are dealing with the simplest PNJLv version, in which color quark condensates are not being taken into account. Here, we are not consid-

ering any possibility of emergence of two-flavor superconducting color, or color flavor locked phases [29,30].

To construct the PNJLv transition curves varying the G_V parameter, we follow the procedure adopted by Fukushima [10] that uses the magnitude of the order parameters ρ_s and Φ to define the transition temperature for each fixed μ . In that work, the author uses the condition of $\rho_s/\rho_s^{\text{vac}} = 1/2$ to construct the phase diagrams. The transition temperature at $\mu = 0$ found in that case is around $T_c(\mu = 0) = 200 \text{ MeV}$. It is noteworthy to observe that in the construction of the quark phase diagrams, the approach using fixed values for ρ_s is qualitatively equivalent, at least at moderated μ values and for $G_V \neq 0$, to those based on the local maximum of $\partial\rho_s/\partial T$ and $\partial\Phi/\partial T$, used in Ref. [26]. In the $G_V = 0$ case, the agreement between the crossover transition curves obtained with the two criteria is still better.

Here, we use different values of $\rho_s/\rho_s^{\text{vac}}$ for each Polyakov potential in order to obtain a better agreement of $T_c(\mu = 0)$ with the lattice QCD results, $T_c(\mu = 0) = 173 \pm 8 \text{ MeV}$ [31]. The adopted values are $\rho_s/\rho_s^{\text{vac}} = 0.73, 0.70, 0.72, 0.71$, respectively, for the RRW06, RTW05, FUKU08, and DS10 parametrizations. Furthermore, we still maintain the rescaling of the original parameters T_0 and b of $\mathcal{U}(T, \mu, \Phi, \Phi^*)$ to $T_0 = 190 \text{ MeV}$ (RRW06, RTW05, and DS10), and $b = 0.007\Lambda^3$ (FUKU08) also used in Ref. [13].

The PNJLv diagrams constructed from the adopted method are displayed in Figs. 1(a)–1(d).

From Fig. 2, one can see that there is a range of values for G_V , at least in a certain temperature region, that makes the transitions constructed from the PNJLv model very close to those obtained via the RMF-PNJL matchings, represented by the gray bands. For these matchings, we have used a large class of RMF hadronic models coupled to the PNJL ones in which $G_V = 0$ [13].

The overlap between the hadron-quark phase transitions provided by the PNJLv and RMF-PNJL models is found for $G_V^{\text{min}} \leq G_V \leq G_V^{\text{max}}$, with $G_V^{\text{min}}/G_s = 1.60, 1.52, 1.60, 1.54$, and $G_V^{\text{max}}/G_s = 3.20, 2.74, 2.86, 2.98$, respectively, for the RRW06, RTW05, FUKU08, and DS10 Polyakov potentials. This give us a total range of $7.66 \text{ GeV}^{-2} \leq G_V \leq 16.13 \text{ GeV}^{-2}$. We can also define a temperature region of better overlap between the PNJLv results and the RMF-PNJL bands given by $T \leq T^{\text{max}}$. The maximum temperatures are $T^{\text{max}} \approx 90, 80, 50, 80 \text{ MeV}$, for the same aforementioned PNJLv models. Therefore, our findings can be useful, for instance, in the study of proton-neutron stars that are described at $T \lesssim 50 \text{ MeV}$. Applications of the RMF-PNJL models to compact stars have been done recently for the proton-neutron stars evolution [32], for quark [33], and for hybrid stars [34].

In terms of the ratio G_V/G_s , the total range obtained here is $1.52 \leq G_V/G_s \leq 3.2$. This result suggests that the G_V values calculated from hadron-quark phase transition curves are in fact greater than some used in the literature, namely,

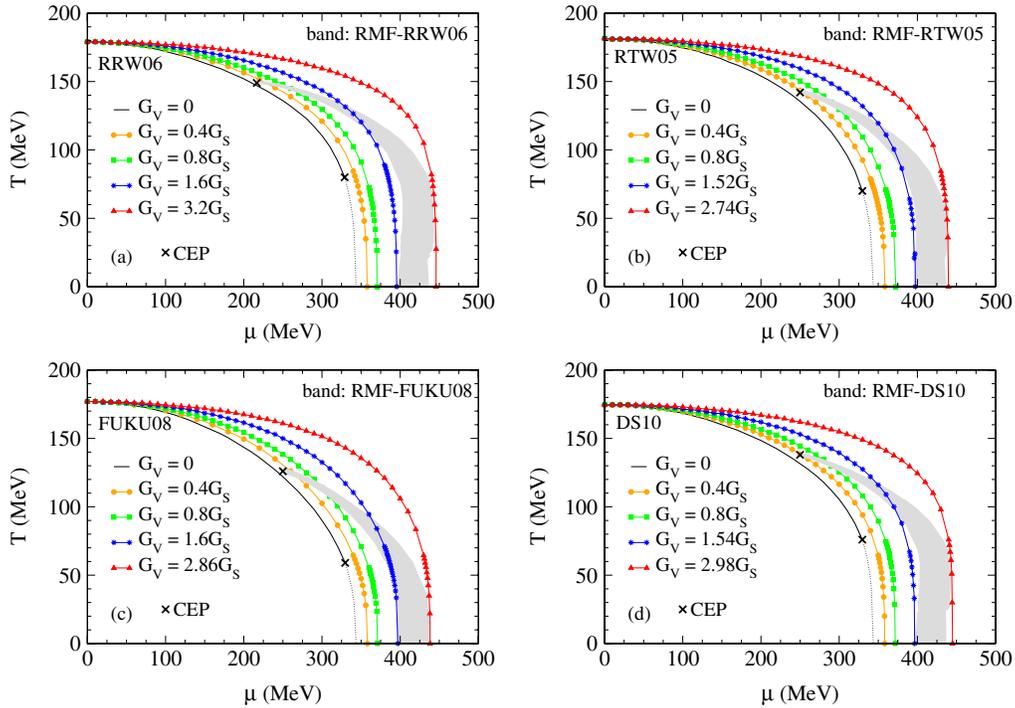


FIG. 1 (color online). PNJLv phase diagrams results for different G_V values compared to RMF-PNJL models. The crosses indicate the critical end points.

$G_V/G_s = 0.25$ [35] and $G_V/G_s = 0.5$ [36]. In Ref. [22] it was argued that the range of acceptable values in the non-local PNJLv model are $0.25 < G_V/G_s < 0.5$. This range is substantiated by a Fierz transformation of an effective one-

gluon exchange interaction, with G_V depending on the strength of the $U_A(1)$ anomaly in the two-flavor model. Our values are considerably above 0.5, which should not be unexpected, as the vector term in the PNJLv model, in

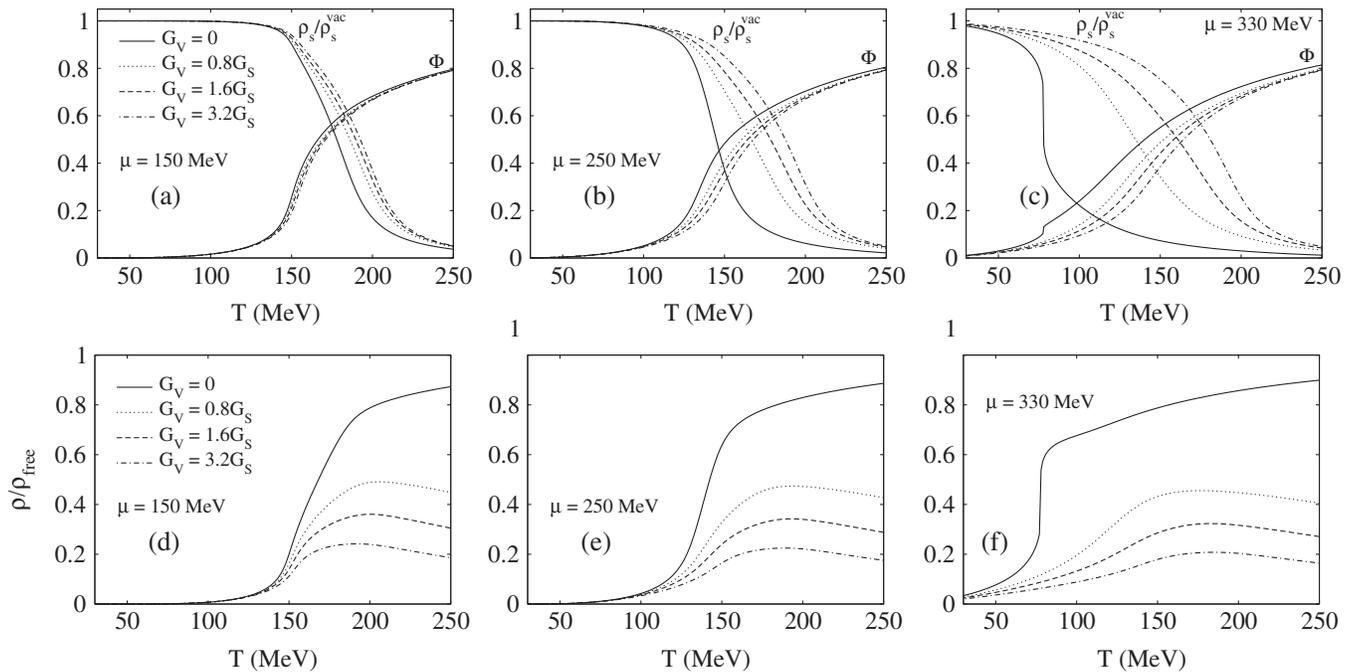


FIG. 2. (a)–(c): Order parameters versus temperature. (d)–(f): Quark density normalized by the free massless quark density, $\rho_{\text{free}} = 2\mu^3/\pi^2 + 2T^2\mu$, versus temperature. In all figures the RRW06 parametrization were used.

our fitting procedure, mimics the repulsive short-range part of the nucleon-nucleon interaction present in the RMF model. The nature of the repulsive nuclear force is beyond one-gluon exchange and acts between colorless hadron degrees of freedom, that essentially are of nonperturbative origin, explaining the difference in the values of G_V . Consistently, our values are larger than, for instance, $G_V/G_s = 1$ used for the PNJL model of Ref. [23].

It is important to stress that our estimated range for the vector coupling strength is compatible with those found recently in Ref. [24], given by $4 \text{ GeV}^{-2} \leq G_V \leq 19 \text{ GeV}^{-2}$. Actually that range encompasses ours. In that work, the authors estimated such range through a completely different way. They used the lattice QCD data of the diagonal and off-diagonal quark susceptibilities, as input in an effective QCD mean-field model approach, described by a Lagrangian density of a two-flavor quark system interacting only via massive vector fields. By doing so, they assume that the vector part of the QCD interaction can be isolated and treated in that approximation. Actually, the authors considered a temperature-dependent vector coupling, $G_V = G_V(T)$. They obtained the range $4 \text{ GeV}^{-2} \leq G_V(T_c) \leq 19 \text{ GeV}^{-2}$ by assuming a transition temperature of $T_c(\mu = 0) = 170 \text{ MeV}$. In our calculations we consider a fixed vector interaction strength.

For the sake of completeness we also present in Fig. 2 the order parameters ρ_s and Φ , and the quark density, obtained self-consistently from the PNJL EOM given by Eqs. (4)–(6). All the curves were constructed for the RRW06 model and for some values of μ . Notice that the dependence of ρ_s and Φ with temperature becomes smoother when G_V increases. This is consistent with our findings that the vector interaction in PNJL models favors the crossover in the quark phase diagram (see Fig. 2). Note also that, the increase of G_V decreases the quark density for fixed μ and T values, as expected from the relation $\rho = (\mu - \tilde{\mu})/2G_V$.

The approaches presented here for the construction of the hadron-quark phase transitions can be understood from a qualitative analysis of the structure of the PNJL model. It is known that the differences between the NJL and the PNJL models are the modification in the Fermi-Dirac distributions and the inclusion of a Polyakov potential, $\mathcal{U}(\Phi, \Phi^*, T)$, in the equations of state. Regarding the statistical distributions, the extreme case in which $\Phi = \Phi^* = 0$ leads, as one can verify from Eq. (3) for $G_V = 0$, to $F(k, T, \mu, \Phi = \Phi^* = 0) = [e^{3(E-\mu)/T} + 1]^{-1}$. The same result is obtained for the antiquarks distributions by replacing μ by $-\mu$.

Notice that in the case of total confinement, i.e. $\Phi = \Phi^* = 0$, the PNJL model gives rise to the Fermi-Dirac statistics of a three-quark cluster, which can be seen as a prototype of hadrons. The PNJL model embodies the

formation of hadronic degrees of freedom. From this point of view, the interaction has to recognize the hadronic formation, and should account for a more realistic description of the case in which the quarks are totally confined. Therefore, the interaction between the clusters should also contain the characteristic repulsion present in the force between hadrons. This feature is qualitatively built in PNJL models, by including the term $G_V(\bar{\psi}\gamma_\mu\psi)^2$ in the Lagrangian density (PNJL model).

In the RMF-PNJL approach of the hadron-quark phase transition with no vector interaction in the PNJL model, all the repulsion is restricted to the hadronic sector. What our results reveal is that the effect of such repulsion in the hadron-quark transition can be reproduced in PNJL models by adjusting the vector interaction strength. However, we should point out that at high μ and low T , in the region where the baryonic degrees of freedom dominate, models formulated only with quark degrees of freedom are not strictly valid (see e.g. [37]). The PNJL model should also be limited in the same sense, while it still incorporates the nuclear repulsion close to the hadron-quark phase transition. As it is known [10], by increasing the vector interaction strength in the PNJL model the critical end point disappears, while in the RMF-PNJL model in all cases it still remains. We stress that the vector interaction strength, in our model, was fitted in a region of $T \times \mu$ far from the RMF-PNJL critical end point (see Fig. 2). The present version of the PNJL model should be improved to account for a possible critical end point, for instance, by including density and T dependence on the model parameters in order to incorporate more effects from QCD. In particular, the recent calculation of the Polyakov quark-meson model with the functional renormalization group method [38] has already shown that the μ dependence of T_0 affects the QCD phase diagram at finite μ .

As a last remark, we point out that the construction of the hadron-quark phase transition is still an open question, from the experimental point of view. Experiments planned to occur in new facilities, such as the Facility for Antiproton and Ion Research (FAIR) [39] at GSI, and the Nuclotron-based Ion Collider Facility (NICA) [40] at the Joint Institute for Nuclear Research (JINR), will be performed to reach the highly compressed matter covering thus, as far is possible, the strongly interacting matter phase diagram. For this reason, it is always important to present and discuss the different predictions of effective models in the intermediate T and μ values of the hadron-quark phase diagram, where the lattice QCD calculations still cannot reach.

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