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Quark-lepton complementarity revisited

Xinyi Zhang,[1] Ya-juan Zheng,[1,2] and Bo-Qiang Ma[1,3,*]
[1]*School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China*
[2]*Department of Physics, Shandong University, Jinan, Shandong 250100, China*
[3]*Center for High Energy Physics, Peking University, Beijing 100871, China*
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We reexamine the quark-lepton complementarity (QLC) in nine angle-phase parametrizations with the latest result of a large lepton mixing angle ϑ_{13} from the T2K, MINOS, and Double-Chooz experiments. We find that there are still two QLC relations satisfied in P1, P4, and P6 parametrizations, whereas only one QLC relation holds in P2, P3, P5, and P9 parametrizations separately. We also work out the corresponding reparametrization-invariant forms of the QLC relations and check the resulting expressions with the experimental data. The results can be viewed as a check of the validity of the QLC relations, as well as a new perspective into the issue of seeking for the connection between quarks and leptons.

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The mixing of fermions remains mysterious in the flavor physics. In the standard model of the particle physics, the mixing is described by mixing matrices which show up in the charged current interaction. The interaction is described by the following Lagrangian,

$$L = -\frac{g}{\sqrt{2}} U_L^\dagger \gamma^\mu V_{\mathrm{CKM}} D_L W_\mu^+ - \frac{g}{\sqrt{2}} E_L^\dagger \gamma^\mu U_{\mathrm{PMNS}} N_L W_\mu^- + \mathrm{H.c.}, \tag{1}$$

where

$$U_L = (u_L, c_L, t_L)^T, \qquad D_L = (d_L, s_L, b_L)^T,$$
$$E_L = (e_L, \mu_L, \tau_L)^T, \qquad N_L = (\nu_1, \nu_2, \nu_3)^T.$$

In Eq. (1), V_{CKM}, namely, the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1], is the mixing matrix describing the mixing between different generations of quarks. Correspondingly, U_{PMNS}, the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [2], describes the misalignment of the flavor eigenstates with the mass eigenstates of leptons.

This similarity, here we refer to the mixing between generations, combined with the pursuit for unification or symmetry has motivated speculations on connections between quarks and leptons [3]. Of all the attempts, the quark-lepton complementarity (QLC) [4,5] has caught much attention for it provides a tempting way to link quarks and leptons. Both its theoretical base [5] and phenomenological implications [6] have been discussed in the literature.

The original QLC corresponds to two numerical relations between the mixing angles of the CKM matrix and the PMNS matrix, namely,

$$\theta_{12} + \vartheta_{12} = 45°, \qquad \theta_{23} + \vartheta_{23} = 45°, \tag{2}$$

where θ_{ij} and ϑ_{ij} denote the mixing angles of the CKM matrix and the PMNS matrix separately in the standard parametrization [7]. The mixing matrix is a unitary matrix that can be parametrized by three Euler angles and a CP-violating phase. Such kind of parametrization can be referred to as angle-phase parametrization. For neutrinos of the Majorana type, two additional CP-violating phases are needed. Since the Majorana CP-violating phases do not manifest themselves in the oscillation, we discuss the Dirac neutrinos only. The three Euler angles correspond to three rotations in complex planes. There is a freedom in arranging the orders of three rotations and different orders result in different angle-phase parametrizations. There are nine angle-phase parametrizations that are structurally different [8]. Notice that the QLC relations are parametrization dependent [9,10]. Though different parametrizations are equivalent to each other mathematically, there are differences in revealing some phenomenological relations, e.g., the QLC relations. Such differences make them of different significance in analysis and model building.

As a result, it is meaningful to reexamine the QLC relations in nine angle-phase parametrizations especially when several experiments observe a relatively large lepton mixing angle ϑ_{13} [11]. Such a result deviates from the previous thought of a quasivanishing one. Another motivation is that there is still a chance that the QLC relations are purely accidental; evaluating the relations is the first step to take before going further along such a direction.

The starting point is the moduli of mixing matrices, which is

*mabq@pku.edu.cn

$$|V_{\rm CKM}| = \begin{pmatrix} 0.974\,28 \pm 0.000\,15 & 0.2253 \pm 0.0007 & 0.003\,47^{+0.000\,16}_{-0.000\,12} \\ 0.2252 \pm 0.0007 & 0.973\,45^{+0.000\,15}_{-0.000\,16} & 0.0410^{+0.0011}_{-0.0007} \\ 0.008\,62^{+0.000\,26}_{-0.000\,20} & 0.0403^{+0.0011}_{-0.0007} & 0.999\,152^{+0.000\,030}_{-0.000\,045} \end{pmatrix} \quad (3)$$

for quarks [12] and

$$|U_{\rm PMNS}| = \begin{pmatrix} 0.824^{+0.011(+0.032)}_{-0.010(-0.032)} & 0.547^{+0.016(+0.047)}_{-0.014(-0.044)} & 0.145^{+0.022(+0.065)}_{-0.031(-0.113)} \\ 0.500^{+0.027(+0.076)}_{-0.021(-0.071)} & 0.582^{+0.050(+0.139)}_{-0.023(-0.069)} & 0.641^{+0.061(+0.168)}_{-0.023(-0.063)} \\ 0.267^{+0.044(+0.123)}_{-0.027(-0.088)} & 0.601^{+0.048(+0.133)}_{-0.022(-0.069)} & 0.754^{+0.052(+0.143)}_{-0.020(-0.054)} \end{pmatrix} \quad (4)$$

for leptons, which is the latest global fitting results of pre-Daya-Bay experiments ($1\sigma(3\sigma)$) [13,14]. There are novel measurements of the neutrino mixing angle ϑ_{13} by the Daya-Bay Collaboration [15] and the RENO Collaboration [16] recently. The Daya-Bay Collaboration releases $\sin^2 2\vartheta_{13} = 0.092 \pm 0.016(\text{stat}) \pm 0.005(\text{syst})$ of a significance of 5.2σ, and the corresponding angle is $\vartheta_{13} = (8.828 \pm 0.793(\text{stat}) \pm 0.248(\text{syst}))^\circ$. The RENO Collaboration releases $\sin^2 2\vartheta_{13} = 0.113 \pm 0.013(\text{stat}) \pm 0.019(\text{syst})$ of a significance of 4.9σ, and the corresponding angle is $\vartheta_{13} = (9.821 \pm 0.588(\text{stat}) \pm 0.860(\text{syst}))^\circ$. The value of ϑ_{13} in our analysis is based on the global fit in Refs. [13,14], with $\vartheta_{13} = (8.332 \pm 1.399(\pm 4.396))^\circ$, which corresponds to $\sin^2 2\vartheta_{13} = 0.082 \pm 0.027(\pm 0.084)$. Therefore, our analysis is compatible with the new data.

We calculate the mixing angles of the nine angle-phase parametrizations with matrix elements that are independent of the CP-violating phase. For example, from the P1, i.e., the standard parametrization, we have

$$\sin\theta_{13} = |V_{ub}|, \quad \tan\theta_{12} = \frac{|V_{us}|}{|V_{ud}|}, \quad \tan\theta_{23} = \frac{|V_{cb}|}{|V_{tb}|}. \quad (5)$$

Thus, we get the corresponding values of the mixing angles. The results are listed in Table I.

From Table I, we can see that the QLC is satisfied approximately in seven of the nine parametrizations. By "satisfying," we mean 45° being in 2σ error range. Of the seven parametrizations that have at least one QLC relation, the QLC relations for two pairs of mixing angles hold in P1, P4, and P6 parametrizations, of which P1 corresponds to the standard parametrization [7].

Combined with earlier work on the self-complementarity of the lepton mixing angles [17], we find that a parametrization which has the self-complementarity also has at least one of the QLC relations. To be explicit, the P1, P3, P4, P6, and P9 parametrizations hold both the lepton self-complementarity and the quark-lepton complementarity. We see that except for the well-examined P1 (standard) and P3 (Kobayashi-Maskawa) parametrizations, P4, P6, and P9 parametrizations stand out as they have the advantage of satisfying both complementarities. The self-complementarity relation may result in new mixing patterns. The PMNS matrix can be expanded around such patterns in orders of the Wolfenstein parameter λ by using the QLC relation. One such example is given in Ref. [18].

Table I can be viewed as an update of the work Zheng did in Ref. [10]. Compared with the results in Ref. [10], we find that the situation has been changed a lot. Only one QLC relation holds in more parametrizations whereas the original form of the QLC, namely, two complementarity relations, is satisfied in P1, P4, and P6 parametrizations. Additionally, the parametrizations that the QLC relations hold do not have a simple form in their (1,3) entries in common.

As the matrix elements are more relevant to physical observables, we seek relations of the matrix elements which are reparametrization invariant, as Ref. [13] did for the standard parametrization only.

By assuming that the QLC relations are exact, we translate the relations into the form in terms of the matrix elements. For example, in P2 parametrization we have

$$\theta_2 + \vartheta_2 \simeq 45^\circ, \quad (6)$$

since

$$\cos\theta_2 = |V_{tb}|, \qquad \cos\vartheta_2 = |U_{\tau 3}|; \quad (7)$$

by substituting the trigonometric function with the modulus of the matrix elements, we have

$$|U_{\tau 3}| = \frac{1}{\sqrt{2}}\left(\sqrt{1 - |V_{tb}|^2} + |V_{tb}|\right). \quad (8)$$

We list the results in Table II.

From Table II, we see that the relations can be generally divided into two kinds. One is a one-to-one relation as in P2, P3, P4, P5, and P9 parametrizations, while the other kind is the two-to-two relation as in P1, P4, and P6 parametrizations. The same expressions for P1 have been pointed out in Ref. [13]. Notice that for central values, there are two cases with larger deviations, i.e., the second relation in P4 and P6. Such deviations are a natural result of a relatively large deviation from the exact numerical QLC relation, which can be seen in Table I.

The moduli of the CKM matrix elements, i.e., $|V_{ij}|$, are measured to a high precision by various processes. Using the QLC relations in the reparametrization-invariant forms in Table II, we can get information on the relatively

TABLE I. The angle-phase parametrizations and quark-lepton complementarity.

Parametrization	Mixing angles	Quark-lepton complementarity
P1: $V = R_{23}(\theta_{23})R_{31}(\theta_{13}, \phi)R_{12}(\theta_{12})$	$\theta_{12}/\theta_{23}/\theta_{13}\ \ \vartheta_{12}/\vartheta_{23}/\vartheta_{13}$	
$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -c_{12}s_{23}s_{13} - s_{12}c_{23}e^{-i\phi} & -s_{12}s_{23}s_{13} + c_{12}c_{23}e^{-i\phi} & s_{23}c_{13} \\ -c_{12}c_{23}s_{13} + s_{12}s_{23}e^{-i\phi} & -s_{12}c_{23}s_{13} - c_{12}s_{23}e^{-i\phi} & c_{23}c_{13} \end{pmatrix}$	$(13.02 \pm 0.039)^\circ + (33.58^{+0.849}_{-0.748})^\circ = (46.60^{+0.888}_{-0.787})^\circ$	$\theta_{12} + \vartheta_{12} \simeq 45^\circ$ [a]
	$(2.35^{+0.063}_{-0.040})^\circ + (40.37^{+2.880}_{-1.227})^\circ = (42.72^{+2.943}_{-1.267})^\circ$	$\theta_{23} + \vartheta_{23} \simeq 45^\circ$ [b]
	$(0.20 \pm 0.009)^\circ + (8.33 \pm 1.40)^\circ = (8.53 \pm 1.409)^\circ$	
P2: $V = R_{12}(\theta_3)R_{23}(\theta_2, \phi)R_{12}^{-1}(\theta_1)$	$\theta_1/\theta_2/\theta_3\ \ \vartheta_1/\vartheta_2/\vartheta_3$	
$\begin{pmatrix} s_1c_2s_3 + c_1c_3e^{-i\phi} & c_1c_2s_3 - s_1c_3e^{-i\phi} & s_2s_3 \\ s_1c_2c_3 - c_1s_3e^{-i\phi} & c_1c_2c_3 + s_1s_3e^{-i\phi} & s_2c_3 \\ -s_1s_2 & -c_1s_2 & c_2 \end{pmatrix}$	$(12.07^{+0.477}_{-0.340})^\circ + (23.95^{+3.895}_{-2.287})^\circ = (36.02^{+4.372}_{-2.627})^\circ$	
	$(2.36^{+0.042}_{-0.063})^\circ + (41.06^{+4.538}_{-1.745})^\circ = (43.42^{+4.580}_{-1.808})^\circ$	$\theta_2 + \vartheta_2 \simeq 45^\circ$ [b]
	$(4.84^{+0.257}_{-0.186})^\circ + (12.75^{+2.209}_{-2.674})^\circ = (17.59^{+2.466}_{-2.860})^\circ$	
P3: $V = R_{23}(\theta_2)R_{12}(\theta_1, \phi)R_{23}^{-1}(\theta_3)$	$\theta_1/\theta_2/\theta_3\ \ \vartheta_1/\vartheta_2/\vartheta_3$	
$\begin{pmatrix} c_1 & s_1c_3 & -s_1s_3 \\ -s_1c_2 & c_1c_2c_3 + s_2s_3e^{-i\phi} & -c_1c_2s_3 + s_2c_3e^{-i\phi} \\ s_1s_2 & -c_1s_2c_3 + c_2s_3e^{-i\phi} & c_1s_2s_3 + c_2c_3e^{-i\phi} \end{pmatrix}$	$(13.02 \pm 0.038)^\circ + (34.51^{+1.113}_{-1.012})^\circ = (47.53^{+1.151}_{-1.050})^\circ$	$\theta_1 + \vartheta_1 \simeq 45^\circ$ [a]
	$(2.19^{+0.066}_{-0.051})^\circ + (28.10^{+4.131}_{-2.608})^\circ = (30.29^{+4.197}_{-2.659})^\circ$	
	$(0.88^{+0.041}_{-0.031})^\circ + (14.85^{+2.194}_{-3.057})^\circ = (15.73^{+2.235}_{-3.088})^\circ$	
P4: $V = R_{23}(\theta_2)R_{12}(\theta_1, \phi)R_{31}^{-1}(\theta_3)$	$\theta_1/\theta_2/\theta_3\ \ \vartheta_1/\vartheta_2/\vartheta_3$	
$\begin{pmatrix} c_1c_3 & s_1 & -c_1s_3 \\ -s_1c_2c_3 + s_2s_3e^{-i\phi} & c_1c_2 & s_1c_2s_3 + s_2c_3e^{-i\phi} \\ s_1s_2c_3 + c_2s_3e^{-i\phi} & -c_1s_2 & -s_1s_2s_3 + c_2c_3e^{-i\phi} \end{pmatrix}$	$(13.02 \pm 0.041)^\circ + (33.16^{+1.096}_{-0.959})^\circ = (46.18^{+1.137}_{-1.000})^\circ$	$\theta_1 + \vartheta_1 \simeq 45^\circ$ [a]
	$(2.37^{+0.065}_{-0.051})^\circ + (45.92^{+3.360}_{-1.543})^\circ = (48.29^{+3.425}_{-1.584})^\circ$	$\theta_2 + \vartheta_2 \simeq 45^\circ$ [a]
	$(0.20^{+0.009}_{-0.007})^\circ + (9.98^{+1.490}_{-2.095})^\circ = (10.18^{+1.499}_{-2.102})^\circ$	
P5: $V = R_{31}(\theta_3)R_{23}(\theta_2, \phi)R_{12}^{-1}(\theta_1)$	$\theta_1/\theta_2/\theta_3\ \ \vartheta_1/\vartheta_2/\vartheta_3$	
$\begin{pmatrix} -s_1s_2s_3 + c_1c_3e^{-i\phi} & -c_1s_2s_3 - s_1c_3e^{-i\phi} & c_2s_3 \\ s_1c_2 & c_1c_2 & s_2 \\ -s_1s_2c_3 - c_1s_3e^{-i\phi} & -c_1s_2c_3 + s_1s_3e^{-i\phi} & c_2c_3 \end{pmatrix}$	$(13.03 \pm 0.039)^\circ + (40.67^{+2.875}_{-1.634})^\circ = (53.70^{+2.912}_{-1.673})^\circ$	
	$(2.35^{+0.063}_{-0.040})^\circ + (39.87^{+4.556}_{-1.718})^\circ = (42.22^{+4.619}_{-1.758})^\circ$	$\theta_2 + \vartheta_2 \simeq 45^\circ$ [b]
	$(0.20^{+0.009}_{-0.007})^\circ + (10.89^{+1.772}_{-2.290})^\circ = (11.09^{+1.781}_{-2.297})^\circ$	
P6: $V = R_{12}(\theta_1)R_{31}(\theta_3, \phi)R_{23}^{-1}(\theta_2)$	$\theta_1/\theta_2/\theta_3\ \ \vartheta_1/\vartheta_2/\vartheta_3$	
$\begin{pmatrix} c_1c_3 & c_1s_2s_3 + s_1c_2e^{-i\phi} & c_1c_2s_3 - s_1s_2e^{-i\phi} \\ -s_1c_3 & -s_1s_2s_3 + c_1c_2e^{-i\phi} & -s_1c_2s_3 - c_1s_2e^{-i\phi} \\ -s_3 & s_2c_3 & c_2c_3 \end{pmatrix}$	$(13.02 \pm 0.039)^\circ + (31.25^{+1.414}_{-1.111})^\circ = (44.27^{+1.453}_{-1.150})^\circ$	$\theta_1 + \vartheta_1 \simeq 45^\circ$ [b]
	$(2.31^{+0.063}_{-0.040})^\circ + (38.56^{+2.948}_{-1.263})^\circ = (40.87^{+3.011}_{-1.303})^\circ$	$\theta_2 + \vartheta_2 \simeq 45^\circ$ [a]
	$(0.49^{+0.015}_{-0.011})^\circ + (15.49^{+2.617}_{-1.606})^\circ = (15.98^{+2.632}_{-1.617})^\circ$	
P7: $V = R_{31}(\theta_3)R_{12}(\theta_1, \phi)R_{31}^{-1}(\theta_2)$	$\theta_1/\theta_2/\theta_3\ \ \vartheta_1/\vartheta_2/\vartheta_3$	
$\begin{pmatrix} c_1c_3c_2 + s_3s_2e^{-i\phi} & s_1c_3 & -c_1c_3s_2 + s_3c_2e^{-i\phi} \\ -s_1c_2 & c_1 & s_1s_2 \\ -c_1s_3c_2 + c_3s_2e^{-i\phi} & -s_1s_3 & c_1s_3s_2 + c_3c_2e^{-i\phi} \end{pmatrix}$	$(13.23^{+0.038}_{-0.040})^\circ + (54.41^{+3.524}_{-1.621})^\circ = (67.64^{+3.562}_{-1.661})^\circ$	
	$(10.32^{+0.273}_{-0.175})^\circ + (52.04^{+3.042}_{-1.536})^\circ = (62.36^{+3.315}_{-1.711})^\circ$	
	$(10.14^{+0.273}_{-0.175})^\circ + (47.69^{+2.427}_{-1.275})^\circ = (57.83^{+2.700}_{-1.450})^\circ$	
P8: $V = R_{12}(\theta_1)R_{23}(\theta_2, \phi)R_{31}(\theta_3)$	$\theta_1/\theta_2/\theta_3\ \ \vartheta_1/\vartheta_2/\vartheta_3$	
$\begin{pmatrix} -s_1s_2s_3 + c_1c_3e^{-i\phi} & s_1c_2 & s_1s_2c_3 + c_1s_3e^{-i\phi} \\ -c_1s_2s_3 - s_1c_3e^{-i\phi} & c_1c_2 & c_1s_2c_3 - s_1s_3e^{-i\phi} \\ -c_2s_3 & -s_2 & c_2c_3 \end{pmatrix}$	$(13.02 \pm 0.039)^\circ + (43.22^{+2.596}_{-1.347})^\circ = (56.25^{+2.635}_{-1.386})^\circ$	
	$(2.31^{+0.063}_{-0.040})^\circ + (36.94^{+3.443}_{-1.578})^\circ = (39.25^{+3.506}_{-1.618})^\circ$	
	$(0.49^{+0.015}_{-0.011})^\circ + (19.50^{+3.222}_{-1.886})^\circ = (19.99^{+3.237}_{-1.897})^\circ$	
P9: $V = R_{31}(\theta_3)R_{12}(\theta_1, \phi)R_{23}(\theta_2)$	$\theta_1/\theta_2/\theta_3\ \ \vartheta_1/\vartheta_2/\vartheta_3$	
$\begin{pmatrix} c_1c_3 & s_1c_2c_3 - s_2s_3e^{-i\phi} & s_1s_2c_3 + c_2s_3e^{-i\phi} \\ -s_1 & c_1c_2 & c_1s_2 \\ -c_1s_3 & -s_1c_2s_3 - s_2c_3e^{-i\phi} & -s_1s_2s_3 + c_2c_3e^{-i\phi} \end{pmatrix}$	$(13.01 \pm 0.041)^\circ + (30.00^{+1.787}_{-1.390})^\circ = (43.01^{+1.828}_{-1.431})^\circ$	$\theta_1 + \vartheta_1 \simeq 45^\circ$ [a]
	$(2.41^{+0.065}_{-0.041})^\circ + (47.76^{+3.658}_{-1.523})^\circ = (50.17^{+3.723}_{-1.564})^\circ$	
	$(0.51^{+0.015}_{-0.012})^\circ + (17.95^{+2.779}_{-1.712})^\circ = (18.46^{+2.794}_{-1.724})^\circ$	

[a] "$\simeq$" refers to 45° being in 2σ error range.
[b] "$\simeq$" refers to 45° being in 1σ error range.

not-so-well-determined PMNS matrix elements $|U_{ij}|$. All of the relations in the second column of Table II are candidates of reparametrization-invariant QLC relations and their validity could be tested by future experiments. As there is still chance that the QLC relations are accidental, the reparametrization-invariant forms of the QLC relations can also be used as the test for the validity of the QLC relations with the advantage of being more directly related to the observables.

As the parametrizations are independent of each other, the corresponding relations deduced from different parametrizations do not have to be satisfied simultaneously. In fact, when assuming that they are satisfied at the same time, unexpected results may emerge. For example, if we assume that the one-to-one relations of $|U_{\mu 3}|$ and $|U_{\tau 3}|$ hold at the same time, by a usage of the unitarity relation, we will find that $|U_{e3}|$ is purely imaginary, which contradicts the data.

TABLE II. The reparametrization-invariant forms of the QLC relations and their verification.

Parametrization	Reparametrization-invariant form	Verification
P1	$\frac{\lvert U_{e2}\rvert}{\lvert U_{e1}\rvert} = \frac{\lvert V_{ud}\rvert - \lvert V_{us}\rvert}{\lvert V_{ud}\rvert + \lvert V_{us}\rvert}$	$0.664^{+0.021}_{-0.019}$, $0.624\,37 \pm 0.000\,95$
P1	$\frac{\lvert U_{\mu3}\rvert}{\lvert U_{\tau3}\rvert} = \frac{\lvert V_{tb}\rvert - \lvert V_{cb}\rvert}{\lvert V_{tb}\rvert + \lvert V_{cb}\rvert}$	$0.850^{+0.100}_{-0.038}$, $0.921\,17^{+0.002\,09}_{-0.001\,38}$
P2	$\lvert U_{\tau3}\rvert = \frac{1}{\sqrt{2}}(\sqrt{1-\lvert V_{tb}\rvert^2} + \lvert V_{tb}\rvert)$	$0.754^{+0.052}_{-0.020}$, $0.735\,62^{+0.000\,49}_{-0.000\,74}$
P3	$\lvert U_{e1}\rvert = \frac{1}{\sqrt{2}}(\sqrt{1-\lvert V_{ud}\rvert^2} + \lvert V_{ud}\rvert)$	$0.824^{+0.011}_{-0.010}$, $0.848\,26 \pm 0.000\,35$
P4	$\lvert U_{e2}\rvert = \frac{1}{\sqrt{2}}(\sqrt{1-\lvert V_{us}\rvert^2} - \lvert V_{us}\rvert)$	$0.547^{+0.016}_{-0.014}$, $0.529\,62 \pm 0.000\,61$
P4	$\frac{\lvert U_{\tau2}\rvert}{\lvert U_{\mu2}\rvert} = \frac{\lvert V_{cs}\rvert - \lvert V_{ts}\rvert}{\lvert V_{cs}\rvert + \lvert V_{ts}\rvert}$	$1.033^{+0.121}_{-0.056}$, $0.920\,49^{+0.002\,08}_{-0.001\,33}$
P5	$\lvert U_{\mu3}\rvert = \frac{1}{\sqrt{2}}(\sqrt{1-\lvert V_{cb}\rvert^2} - \lvert V_{cb}\rvert)$	$0.641^{+0.061}_{-0.023}$, $0.677\,52^{+0.000\,81}_{-0.000\,52}$
P6	$\frac{\lvert U_{\mu1}\rvert}{\lvert U_{e1}\rvert} = \frac{\lvert V_{ud}\rvert - \lvert V_{cd}\rvert}{\lvert V_{ud}\rvert + \lvert V_{cd}\rvert}$	$0.607^{+0.034}_{-0.027}$, $0.624\,50 \pm 0.000\,95$
P6	$\frac{\lvert U_{\tau2}\rvert}{\lvert U_{\tau3}\rvert} = \frac{\lvert V_{tb}\rvert - \lvert V_{ts}\rvert}{\lvert V_{tb}\rvert + \lvert V_{ts}\rvert}$	$0.797^{+0.084}_{-0.036}$, $0.922\,46^{+0.002\,03}_{-0.001\,29}$
P9	$\lvert U_{\mu1}\rvert = \frac{1}{\sqrt{2}}(\sqrt{1-\lvert V_{cd}\rvert^2} - \lvert V_{cd}\rvert)$	$0.500^{+0.027}_{-0.021}$, $0.529\,70 \pm 0.000\,61$

To sum up, we reexamine the QLC relations in nine angle-phase parametrizations, and work out the corresponding relations of the reparametrization-invariant form. We find that the new experimental data have changed the situation whether a given parametrization has the quark-lepton complementarity or not and whether a given parametrization has one relation or two. The reparametrization-invariant form of the QLC relations may suggest some general connections between quarks and leptons mixing. We look forward to the experimental tests of these relations.

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