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$B^0 - \bar{B}^0$ mixing in gauge-Higgs unification

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We discuss flavor mixing and resulting flavorchanging neutral current in a five-dimensional $SU(3)_{\mathrm{color}} \otimes SU(3) \otimes U'(1)$ gauge-Higgs unification. Flavor mixing is realized by the fact that the bulk and brane-localized mass terms are not diagonalized simultaneously. As the concrete flavor-changing neutral current processes, we calculate the rate of $B_d^0 - \bar{B}_d^0$ mixing and $B_s^0 - \bar{B}_s^0$ mixing due to the exchange of nonzero Kaluza-Klein gluons at the tree level. We obtain a lower bound on the compactification scale of order $\mathcal{O}$(TeV) by comparing our prediction on the mass difference of neutral B meson with the recent experimental data.

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I. INTRODUCTION

In spite of the great success of the standard model (SM), the origin of electroweak gauge symmetry breaking is still unknown in particle physics. Though in the SM, the Higgs boson is assumed to play a role for the symmetry breaking, it seems to have various theoretical problems such as the hierarchy problem and the presence of many theoretically unpredicted arbitrary coupling constants in its interactions.

Gauge-Higgs unification (GHU) [1] is one of the fascinating scenarios beyond the SM. It provides a possible solution to the hierarchy problem without supersymmetry. In this scenario, the Higgs boson in the SM is identified with the extra spatial components of the higher-dimensional gauge fields. A remarkable fact is that the quantum correction to Higgs mass is UV-finite and calculable due to the higher-dimensional gauge symmetry, regardless of the nonrenormalizability of the theory. This has opened up a new avenue to solve the hierarchy problem [2]. The finiteness of the Higgs mass has been studied and verified in various models and types of compactification at one-loop level[1] [4] and even at the two-loop level [5]. The fact that the Higgs boson is a part of gauge fields implies that Higgs interactions are restricted by gauge principle and may provide a possibility to solve the arbitrariness problem of Higgs interactions as well.

From such point of view, it seems that the following issues are particularly important for the GHU to be phenomenologically viable. The first one is whether there is any characteristic prediction on the observables subject to precision tests. The second one is how *CP* violation is achieved, since the Higgs interactions are given by gauge interactions with real couplings. The last one is how flavor mixing is generated, since Yukawa coupling in GHU is given by gauge interactions, which are universal for all flavors.

As for the first issue, it will be desirable to find finite (UV-insensitive) and calculable observables, in spite of the fact that the theory is nonrenormalizable and observables are very UV-sensitive in general. Works on the oblique electroweak parameters and fermion-anomalous magnetic moment from such a viewpoint have been already done in the literature [6–8]. The second issue has been addressed in our previous papers [9,10], where *CP* violation is claimed to be achieved spontaneously either by the vacuum expectation value of the Higgs field or by the complex structure of the compactified extra space.

In this paper, we focus on the remaining issue concerning the flavor physics in the GHU scenario. It is highly nontrivial problem to explain the variety of fermion masses and flavor mixings in this scenario, since the gauge interactions should be universal for all matter fields, while the flavor symmetry has to be broken eventually in order to distinguish each flavor and to realize their mixings. In our previous papers [11,12], we addressed this issue and have clarified the mechanism to generate the flavor mixings by the interplay between bulk masses and the brane-localized masses.

An important point is that such introduced two types of mass terms generically may be flavor nondiagonal without contradicting with gauge invariance, which leads to the flavor mixing in the up- and down-types of Yukawa couplings [13]. We may start with the base where the bulk mass terms are diagonalized, since the bulk mass terms are written in the form of Hermitian matrix, which may be diagonalized by suitable unitary transformations, keeping the kinetic and gauge interaction terms of fermions invariant [11,12]. Even in this base, however, the brane-localized mass terms still have off-diagonal elements in the flavor base in general. Namely, the fact that two types of fermion mass terms cannot be diagonalized simultaneously leads to physical flavor mixing.

[1]For the case of gravity-gauge-Higgs unification, see [3].

Once the flavor mixings are realized, it will be important to discuss flavor-changing neutral current (FCNC) processes, which have been playing a crucial role for checking the viability of various new physics models, as is seen in the case of supersymmetry model. This issue was first discussed in [14] in the context of extra dimensions. Since our model reduces to the SM at low energies, there is no FCNC processes at the tree level with respect to the zero-mode fields. However, it turns out that the exchange of nonzero Kaluza-Klein (KK) modes of gauge bosons causes FCNC at the tree level, though the rates of FCNC are suppressed by the inverse powers of the compactification scale (''decoupling'') [11,12]. The reason is the following. The gauge couplings of nonzero KK modes of gauge boson, whose mode functions are y-dependent, to zero-mode fermions are no longer universal since the overlap integral of mode function of fermion and KK gauge boson depends on the bulk mass M different from flavor by flavor in general.

In the previous papers, as typical processes of FCNC, we have calculated the $K^0 - \bar{K}^0$ mixing and the $D^0 - \bar{D}^0$ mixing amplitude at the tree level via nonzero KK gluon exchange and obtained the lower bounds for the compactification scale as the predictions of our model [11,12]. Interestingly, the obtained lower bounds of $\mathcal{O}(10)$ TeV were much milder than what we naively expect, assuming that the amplitude is simply suppressed by the inverse powers of the compactification scale, say $\mathcal{O}(10^3)$ TeV. We pointed out the presence of suppression mechanism of the FCNC processes, which is operative for light fermions in the GHU model. In the analysis, we focused on the simplified two-generation scheme in order to estimate the mass difference and the lower bound on the compactification scale.

On the other hand, this suppression mechanism in the third generation containing top and bottom quarks does not work so strongly by the absence of bulk masses, as we will discuss in the main text. Then it is expected that the dangerous large FCNC containing the third generation such as $B^0 - \bar{B}^0$ mixing arises and more stringent constraints will be obtained. Thus it would be more desirable to discuss the FCNC process in the three-generation scheme.

In this paper, we discuss flavor mixings in the three-generation model and we especially consider the typical FCNC processes, i.e., $B_d^0 - \bar{B}_d^0$ mixing and $B_s^0 - \bar{B}_s^0$ mixing, which is caused by the mixing between down and bottom quarks or strange and bottom quarks.

We will calculate the dominant contribution to the $B_d^0 - \bar{B}_d^0$ mixing and the $B_s^0 - \bar{B}_s^0$ mixing at the tree level by the nonzero KK gluon exchange. The rate of the FCNC processes is suppressed by the small mixings between the third generation and lighter generations. Comparing the prediction of our model with the data, the lower bound on the compactification scale is obtained.

This paper is organized as follows. After introducing our model in the next section, we summarize in Sec. III how the flavor mixing is realized in the context of the gauge-Higgs unification, which was clarified and described in detail in our previous paper [11,12]. In Sec. IV, as an application of the flavor mixing discussed in Sec. III, we calculate the mass difference of neutral B-mesons caused by the $B_d^0 - \bar{B}_d^0$ mixing and the $B_s^0 - \bar{B}_s^0$ mixing via nonzero KK gluon exchange at the tree level. We also obtain the lower bound for the compactification scale by comparing the obtained result with the experimental data. Our conclusion is given in Sec. V.

II. THE MODEL

The model we consider in this paper is a five-dimensional (5D) $SU(3)_{\rm color} \otimes SU(3) \otimes U'(1)$ GHU model compactified on an orbifold S^1/Z_2 with a radius R of S^1. The three-generation model is basically obtained by extending our previous model, but top-quark mass cannot be incorporating as it stands. It is known that the fermion masses have an upper bound in GHU,

$$m_q \leq \sqrt{n} M_W \qquad (M_W\text{: } W\text{-boson mass}), \tag{2.1}$$

where n is the number of indices of the representation the fermion belongs to [15]. Up-type quarks in our model belong to the totally symmetric tensor representation of $SU(3)$, i.e., $n = 2$, in our two-generation model. Thus, we should modify our model to obtain the correct top mass $m_t \sim 2M_W$. Obviously, the simplest choice would be a 4-rank tensor representation. The representations of rank 4 of $SU(3)$ are known to be $\overline{\mathbf{15}}$, $\overline{\mathbf{24}}$ and $\mathbf{27}$ [16]. We modify our model by using the smallest representation, $\overline{\mathbf{15}}$. Although a small gap still remains between top and twice of W-boson masses, it is attributed to the quantum correction of top Yukawa coupling. Focusing on the quark sector, we introduce three generations of bulk fermion in the $\mathbf{3}$, two generations of them in the $\bar{\mathbf{6}}$, and one generation of bulk fermion in the $\overline{\mathbf{15}}$ dimensional representations of $SU(3)$ gauge group,

$$\psi^i(\mathbf{3}) = \psi^i(\mathbf{3}, \mathbf{3}, 0) = Q_3^i \oplus d^i \quad (i = 1, 2, 3), \tag{2.2a}$$

$$\psi^i(\bar{\mathbf{6}}) = \psi^i(\mathbf{3}, \bar{\mathbf{6}}, 0) = \Sigma_6^i \oplus Q_6^i \oplus u^i \quad (i = 1, 2), \tag{2.2b}$$

$$\psi(\overline{\mathbf{15}}) = \psi(\mathbf{3}, \overline{\mathbf{15}}, -2/3) = \Theta \oplus \Delta \oplus \Sigma_{15} \oplus Q_{15} \oplus t, \tag{2.2c}$$

where all of the fermions are decomposed into those in the representations of $SU(2)$ subgroup of $SU(3)$ gauge group. Each representation or charge is denoted in the round bracket and they correspond to ($SU(3)_{\rm color}$, $SU(3)$, $U'(1)$), respectively. An extra $U'(1)$ is required for $\psi(\overline{\mathbf{15}})$ to fix the hypercharges. These sets of fermions contain ordinary quarks of the SM in the zero-mode sector, i.e., Q_3^i and Q_6^i ($i = 1$, 2) corresponding to the first two-generation quark doublets, $Q_3^{i=3}$ and Q_{15} corresponding to the third-generation quark doublet, and d^i ($i = 1$, 2, 3), u^i ($i = 1$, 2), t corresponding to three-generation down-type quark singlets, the first two-generation up-type quark singlets, top-quark singlet, respectively. $\psi^i(\bar{\mathbf{6}})$ have $SU(2)$

triplet exotic states Σ_6^i and $\psi(\overline{\mathbf{15}})$ also does $SU(2)$ quintet, quartet, triplet exotic states Θ, Δ, and Σ_{15}.

The bulk Lagrangian is given by

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}\mathrm{Tr}(F_{MN}F^{MN}) - \frac{1}{4}B_{MN}B^{MN} \\ &-\frac{1}{2}\mathrm{Tr}(G_{MN}G^{MN}) + \bar{\psi}^i(\mathbf{3})\{i\not{D}_3 - M^i\epsilon(y)\}\psi^i(\mathbf{3}) \\ &+ \bar{\psi}^{i=3}(\mathbf{3})i\not{D}_3\psi^{i=3}(\mathbf{3}) + \bar{\psi}^i(\bar{\mathbf{6}})\{i\not{D}_6 - M^i\epsilon(y)\}\psi^i(\bar{\mathbf{6}}) \\ &+ \bar{\psi}(\overline{\mathbf{15}})i\not{D}'_{15}\psi(\overline{\mathbf{15}}), \end{aligned} \tag{2.3}$$

where the gauge-kinetic terms for $SU(3)$, $U'(1)$, $SU(3)_{\mathrm{color}}$ and the covariant derivatives are

$$F_{MN} = \partial_M A_N - \partial_N A_M - ig[A_M, A_N], \tag{2.4a}$$
$$B_{MN} = \partial_M B_N - \partial_N B_M, \tag{2.4b}$$
$$G_{MN} = \partial_M G_N - \partial_N G_M - ig_s[G_M, G_N], \tag{2.4c}$$
$$\not{D} = \Gamma^M(\partial_M - igA_M - ig_sG_M), \tag{2.4d}$$
$$\not{D}' = \Gamma^M(\partial_M - igA_M - ig'B_M - ig_sG_M). \tag{2.4e}$$

The gauge fields A_M and G_M are written in a matrix form, e.g., $A_M = A_M^a\frac{\lambda^a}{2}$ in terms of Gell-Mann matrices λ^a. It should be understood that A_M in the covariant derivative $D_M = \partial_M - igA_M - ig_sG_M$ acts properly depending on the representations of the fermions. $M, N = 0, 1, 2, 3, 5$ denotes indices of the bulk space-time. The five-dimensional gamma matrices are given by $\Gamma^M = (\gamma^\mu, i\gamma^5)$ ($\mu = 0, 1, 2, 3$). g, g' and g_s are 5D gauge coupling constants of $SU(3)$, $U'(1)$ and $SU(3)_{\mathrm{color}}$, respectively. M^i ($i = 1, 2$) are generation-dependent bulk mass parameters of the first two generations of fermion accompanied by the sign function $\epsilon(y)$. For the third generation, the bulk mass parameter should be taken to be zero to reproduce top-quark mass.

The periodic boundary condition is imposed along S^1 and Z_2 parity assignments are taken for gauge fields as

$$A_\mu(-y) = PA_\mu(y)P^{-1}, \quad A_y(-y) = -PA_y(y)P^{-1}, \tag{2.5a}$$
$$G_\mu(-y) = G_\mu(y), \quad G_y(-y) = -G_y(y), \tag{2.5b}$$
$$B_\mu(-y) = B_\mu(y), \quad B_y(-y) = -B_y(y) \tag{2.5c}$$

where the orbifolding matrix is defined as $P = \mathrm{diag}(-, -, +)$ and operated in the same way at the fixed points $y = 0$, πR. We can see that the gauge symmetry $SU(3)$ is explicitly broken to $SU(2)\times U(1)$ by the boundary conditions. The gauge fields with Z_2 odd parity and even parity are expanded by use of mode functions,

$$S_n(y) = \frac{1}{\sqrt{\pi R}}\sin\frac{n}{R}y, \qquad C_n(y) = \frac{1}{\sqrt{2^{\delta_{n0}}\pi R}}\cos\frac{n}{R}y, \tag{2.6}$$

respectively. Then the gauge bosons are expanded in terms of the above mode functions S_n and C_n; for example, the Z_2 even gauge fields (gluon) are extracted as

$$\begin{aligned} G_\mu(x,y) &= \sum_{n=0}^{\infty} G_\mu^{(n)}(x)C_n(y), \\ G_y(x,y) &= \sum_{n=1}^{\infty} G_y^{(n)}(x)S_n(y). \end{aligned} \tag{2.7}$$

The Z_2 parities of fermions are assigned for each component of the representations as follows:

$$\begin{aligned} \psi^i(\mathbf{3}) &= \{Q_{3L}^i(+,+) + Q_{3R}^i(-,-)\}\oplus\{d_L^i(-,-) + d_R^i(+,+)\} \qquad (i = 1, 2, 3), \\ \psi^i(\bar{\mathbf{6}}) &= \{\Sigma_{6L}^i(-,-) + \Sigma_{6R}^i(+,+)\}\oplus\{Q_{6L}^i(+,+) + Q_{6R}^i(-,-)\}\oplus\{u_L^i(-,-) + u_R^i(+,+)\} \qquad (i = 1, 2), \\ \psi(\overline{\mathbf{15}}) &= \{\Theta_L(-,-) + \Theta_R(+,+)\}\oplus\{\Delta_L(+,+) + \Delta_R(-,-)\}\oplus\{\Sigma_{15L}(-,-) + \Sigma_{15R}(+,+)\}\oplus\{Q_{15L}(+,+) \\ &\quad + Q_{15R}(-,-)\}\oplus\{t_L(-,-) + t_R(+,+)\}. \end{aligned}$$

The signs in the round bracket stand for eigenvalues of Z_2 parities at two fixed points $y = 0$, πR and *vice versa*. Thus a chiral theory is realized in the zero-mode sector by Z_2 orbifolding. Similar to the gauge bosons, the five-dimensional Z_2 even (odd) fermion χ^i (ϕ^i) can be expanded as

$$\chi^i(x,y) = \chi_L^{i(0)}(x)f_L^i(y) + \sum_{n=1}^{\infty}\{\chi_L^{i(n)}(x)f_L^{i(n)}(y) + \chi_R^{(n)}(x)S_n(y)\}, \tag{2.8}$$

$$\phi^i(x,y) = \phi_R^{i(0)}(x)f_R^i(y) + \sum_{n=1}^{\infty}\{\phi_R^{i(n)}(x)f_R^{i(n)}(y) + \phi_L^{i(n)}(x)S_n(y)\}, \tag{2.9}$$

where the mode functions[2] which are given in [9], are

$$\begin{aligned} f_L^i(y) &= \sqrt{\frac{M^i}{1 - e^{-2\pi RM^i}}}\,e^{-M^i|y|}, \\ f_R^i(y) &= \sqrt{\frac{M^i}{e^{2\pi RM^i} - 1}}\,e^{M^i|y|}, \end{aligned} \tag{2.10}$$

[2],For the $\psi(\overline{\mathbf{15}})$ case the corresponding mode functions are easily obtained by $M^i \to 0$.

$$f_L^{i(n)}(y) = \frac{n}{Rm_n^i}\left\{C_n - \frac{RM^i}{n}\epsilon(y)S_n\right\}, \qquad f_R^{i(n)}(y) = \frac{n}{Rm_n^i}\left\{C_n + \frac{RM^i}{n}\epsilon(y)S_n\right\}, \tag{2.11}$$

where $m_n^i = \sqrt{(M^i)^2 + (n/R)^2}$.

Here we will focus on the zero-mode sector necessary for the argument of flavor mixing. The zero-mode sector of each component of $\psi^i(\mathbf{3})$, $\psi^i(\bar{\mathbf{6}})$ and $\psi(\overline{\mathbf{15}})$ are written in the following way:

$$Q_3^i = Q_{3L}^i f_L^i(y), \qquad d^i = d_R^i f_R^i(y) \qquad (i = 1, 2, 3), \tag{2.12a}$$

$$\Sigma_6^i = \Sigma_{6R}^i f_R^i(y), \qquad Q_6^i = Q_{6L}^i f_L^i(y), \qquad u^i = u_R^i f_R^i(y) \qquad (i = 1, 2) \tag{2.12b}$$

$$\Theta = \Theta_R f_R(y), \qquad \Delta = \Delta_L f_L(y), \qquad \Sigma_{15} = \Sigma_{15R} f_R(y), \qquad Q_{15} = Q_{15L} f_L(y), \qquad t = t_R f_R(y). \tag{2.12c}$$

u_R^i, d_R^i correspond to the up- and down-type right-handed quark, except for the top quark, and t_R corresponds to the right-handed top quark. We notice that there are two left-handed quark doublets Q_{3L} and $Q_{6L}(Q_{15L})$ per generation in the zero-mode sector, which are massless before electroweak symmetry breaking. In the one-generation case, for instance, one of two independent linear combinations of these doublets should correspond to the quark doublet in the SM, but the other one should be regarded as an exotic state. Moreover, having an exotic fermion Σ_{6R}, Σ_{15R}, Δ_L and Θ_R, we therefore introduce brane-localized four-dimensional Weyl spinors to form $SU(2) \times U(1)$ invariant brane-localized Dirac mass terms in order to remove these exotic massless fermions from the low-energy effective theory [13,17].[3]

$$\mathcal{L}_{\rm BLM} = \mathcal{L}_{\rm BLM}^{Q} + \mathcal{L}_{\rm BLM}^{\Sigma_6} + \mathcal{L}_{\rm BLM}^{\Sigma_{15}} + \mathcal{L}_{\rm BLM}^{\Delta} + \mathcal{L}_{\rm BLM}^{\Theta}, \tag{2.13}$$

where, for the first two generations,

$$\mathcal{L}_{\rm BLM}^{\Sigma_6} = \int_{-\pi R}^{\pi R} dy \sqrt{2\pi R} m_{\rm BLM}^{\Sigma_6} \delta(y - \pi R) \times \bar{\Sigma}_{6R}^i(x, y) \Sigma_{6L}^i(x) + ({\rm H.c.}), \tag{2.14a}$$

and for the third generation,

$$\mathcal{L}_{\rm BLM}^{\Sigma_{15}} = \int_{-\pi R}^{\pi R} dy \sqrt{2\pi R} m_{\rm BLM}^{\Sigma_{15}} \delta(y - \pi R) \times \bar{\Sigma}_{15R}(x, y) \Sigma_{15L}(x) + ({\rm H.c.}), \tag{2.14b}$$

$$\mathcal{L}_{\rm BLM}^{\Delta} = \int_{-\pi R}^{\pi R} dy \sqrt{2\pi R} m_{\rm BLM}^{\Delta} \delta(y) \bar{\Delta}_L(x, y) \Delta_R(x) + ({\rm H.c.}), \tag{2.14c}$$

$$\mathcal{L}_{\rm BLM}^{\Theta} = \int_{-\pi R}^{\pi R} dy \sqrt{2\pi R} m_{\rm BLM}^{\Theta} \delta(y - \pi R) \times \bar{\Theta}_R(x, y) \Theta_L(x) + ({\rm H.c.}) \tag{2.14d}$$

and for three generations $i = 1, 2, 3$

$$\mathcal{L}_{\rm BLM}^{Q} = \int_{-\pi R}^{\pi R} dy \sqrt{2\pi R} \delta(y) \bar{Q}_R^i(x) \{\eta_{ij} Q_{3L}^j(x, y) + \lambda_{ij} Q_L^j(x, y)\} + ({\rm H.c.}), \tag{2.14e}$$

where

$$Q_L(x, y) = [\, Q_{6L}^1(x, y) \quad Q_{6L}^2(x, y) \quad Q_{15L}(x, y)\,]^T. \tag{2.15}$$

Q_R, $\Sigma_{6,15L}$, Δ_R and Θ_L are the brane-localized Weyl fermions of doublet, triplet, quartet, and quintet of $SU(2)$ respectively. The 3×3 matrices η_{ij}, λ_{ij} and $m_{\rm BLM}$s are mass parameters. These brane-localized mass terms are introduced at opposite fixed points such that Q_R, Δ_R ($\Sigma_{6,15L}$, Θ_L) couples to $Q_{3,6,15L}$, Δ_L ($\Sigma_{6,15R}$, Θ_R) localized on the brane at $y = 0 (y = \pi R)$. Let us note that the matrices η_{ij}, λ_{ij} can be nondiagonal, which are the source of the flavor mixing [11–13].

III. FLAVOR MIXING

In the previous section we worked in the base where fermion bulk mass terms are written in a diagonal matrix in the generation space. Then Yukawa couplings as the gauge interaction of A_y are completely diagonalized in the generation space. Thus flavor mixing does not happen in the bulk and the brane-localized mass terms for the doublets Q_{3L} and $Q_{6L}(Q_{15L})$ are expected to lead to the flavor mixing. We now discuss how the flavor mixing is realized in this model.

First, we identify the SM quark doublet by diagonalizing the relevant brane-localized mass term,

[3]At tree level, we assume that there are no brane-localized kinetic terms for gauge fields. However, they are generated by quantum corrections in general. Even if we take them into account, our results are not affected, since it has nothing to do with flavor sector, although we have to note that 4D effective gauge coupling should be defined as the sum of the bulk gauge coupling and brane gauge coupling, i.e., $1/g_{4D}^2 = \pi R/g_{5D}^2 + 1/g_{\rm brane}^2$.

$$\int_{-\pi R}^{\pi R} dy\sqrt{2\pi R}\delta(y)\bar{Q}_R(x)[\,\eta \quad \lambda\,]\begin{bmatrix} Q_{3L}(x,y) \\ [3pt]Q_L(x,y)\end{bmatrix} \supset \sqrt{2\pi R}\bar{Q}_R(x)[\,\eta f_L(0) \quad \lambda f_L(0)\,]\begin{bmatrix} Q_{3L}(x) \\ [3pt]Q_L(x)\end{bmatrix}$$
$$= \sqrt{2\pi R}\bar{Q}'_R(x)[\,m_{\rm diag} \quad \mathbf{0}_{3\times 3}\,]\begin{bmatrix} Q_{\rm HL}(x) \\ Q_{\rm SML}(x)\end{bmatrix}, \tag{3.1}$$

where

$$\begin{bmatrix} U_1 & U_3 \\ U_2 & U_4 \end{bmatrix}\begin{bmatrix} Q_{\rm HL}(x) \\ Q_{\rm SML}(x)\end{bmatrix} = \begin{bmatrix} Q_{3L}(x) \\ Q_L(x)\end{bmatrix}, \qquad U^{\bar{Q}}Q_R(x) = Q'_R(x), \tag{3.2a}$$

$$U^{\bar{Q}}[\,\eta f_L(0) \quad \lambda f_L(0)\,]\begin{bmatrix} U_1 & U_3 \\ U_2 & U_4 \end{bmatrix} = [\,m_{\rm diag} \quad \mathbf{0}_{3\times 3}\,]. \tag{3.2b}$$

In Eq. (3.2), $\eta f_L(0)$ is an abbreviation of a 3×3 matrix whose (i, j) element is given by $\eta_{ij}f_L^j(0)$, for instance. U_3, U_4 are 3×3 matrices satisfying the unitarity condition

$$U_3^\dagger U_3 + U_4^\dagger U_4 = \mathbf{1}_{3\times 3}, \tag{3.3}$$

which indicates how the quark doublets of the SM are contained in each of $Q_{3L}(x)$ and $Q_{6,15L}(x)$ and compose a 6×6 unitary matrix together with U_1, U_2, which diagonalizes the brane-localized mass matrix. The eigenstate $Q_{\rm H}$ becomes massive and decouples from the low-energy processes, while $Q_{\rm SM}$ remains massless at this stage and is identified with the SM quark doublet. After this identification of the SM doublet, Yukawa couplings are read off from the higher-dimensional gauge interaction of A_y, whose zero mode is the Higgs field $H(x)$,

$$-\frac{g_4}{2}\{\langle H^\dagger\rangle \bar{d}_R^i(x)I_{RL}^{i(00)}U_3^{ij}Q_{\rm SML}^j(x) + \langle H^t\rangle i\sigma^2\bar{u}_R^i(x)(WI_{RL}^{(00)})^iU_4^{ij}Q_{\rm SML}^j(x)\} + {\rm H.c.}, \tag{3.4}$$

where $g_4 \equiv \frac{g}{\sqrt{2\pi R}}$, the matrix W indicates the factor $\sqrt{n}$ in (2.1),

$$W \equiv {\rm diag}(\sqrt{2}, \sqrt{2}, 2), \tag{3.5}$$

and $I_{RL}^{(00)}$ is an overlap integral of mode functions of fermions with matrix elements $(I_{RL}^{(00)})_{ij} = \delta_{ij}I_{RL}^{i(00)}$,

$$I_{RL}^{i(00)} = \int_{-\pi R}^{\pi R} dy f_L^i f_R^i = \begin{cases} \frac{\pi RM^i}{\sinh(\pi RM^i)} & (i = 1, 2) \\ 1 & (i = 3)\end{cases}, \tag{3.6}$$

which behaves as $2\pi RM^i e^{-\pi RM^i}$ for $\pi RM^i \gg 1$, thus realizing the hierarchical small quark masses without fine-tuning of M^i. We thus know that the matrices of Yukawa coupling $\frac{g_4}{2}Y_u$ and $\frac{g_4}{2}Y_d$ are given as

$$\frac{g_4}{2}Y_u = \frac{g_4}{2}WI_{RL}^{(00)}U_4, \qquad \frac{g_4}{2}Y_d = \frac{g_4}{2}I_{RL}^{(00)}U_3. \tag{3.7}$$

These matrices are diagonalized by bi-unitary transformations as in the SM and Cabibbo-Kobayashi-Maskawa (CKM) matrix is defined in a usual way [18],

$$\begin{aligned} \hat{Y}_d &= {\rm diag}(\hat{m}_d, \cdots) = V_{dR}^\dagger Y_d V_{dL} \\ \hat{Y}_u &= {\rm diag}(\hat{m}_u, \cdots) = V_{uR}^\dagger WY_u V_{uL}, \\ V_{\rm CKM} &\equiv V_{dL}^\dagger V_{uL}, \end{aligned} \tag{3.8}$$

where all the quark masses are normalized by the W-boson mass as $\hat{m}_f = \frac{m_f}{M_W}$. A remarkable point is that the Yukawa couplings $\frac{g_4}{2}Y_u$ and $\frac{g_4}{2}Y_d$ are related through the unitarity condition Eq. (3.3); on the contrary those are completely independent in the SM.

For an illustrative purpose to confirm the mechanism of flavor mixing, we will see how the realistic quark masses and mixing are reproduced. Here we leave aside *CP* violation, since the issue discussed in this paper is independent of it and assume that U_3, U_4 are real. Let us notice that 3×3 matrices $U_{3,4}$ can be parametrized because of (3.3) without loss of generality as

$$\begin{aligned} U_4 &= R_u\begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix}, \\ U_3 &= R_d\begin{bmatrix} \sqrt{1-a_1^2} & 0 & 0 \\ 0 & \sqrt{1-a_2^2} & 0 \\ 0 & 0 & \sqrt{1-a_3^2} \end{bmatrix}, \end{aligned} \tag{3.9}$$

where R_u and R_d are arbitrary 3×3 rotation matrices parametrized as

$$R_u = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_2' & \sin\theta_2' \\ 0 & -\sin\theta_2' & \cos\theta_2' \end{bmatrix} \begin{bmatrix} \cos\theta_3' & 0 & \sin\theta_3' \\ 0 & 1 & 0 \\ -\sin\theta_3' & 0 & \cos\theta_3' \end{bmatrix} \begin{bmatrix} \cos\theta_1' & -\sin\theta_1' & 0 \\ \sin\theta_1' & \cos\theta_1' & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{3.10a}$$

$$R_d = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_2 & \sin\theta_2 \\ 0 & -\sin\theta_2 & \cos\theta_2 \end{bmatrix} \begin{bmatrix} \cos\theta_3 & 0 & \sin\theta_3 \\ 0 & 1 & 0 \\ -\sin\theta_3 & 0 & \cos\theta_3 \end{bmatrix} \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{3.10b}$$

Actually the most general forms of U_3 and U_4 have a common orthogonal matrix multiplied from the right, which can, however, be however by suitable unitary transformations among the members of $Q_{\mathrm{SML}}(x)$.

Now physical observables $\hat{m}_u$, $\hat{m}_c$, $\hat{m}_t$, $\hat{m}_d$, $\hat{m}_s$, $\hat{m}_b$ and the angles of the CKM matrix are expressed in terms of a_i, $b_i(\equiv I_{RL}^{i(00)})$ and six rotation angles in R_u and R_d. Note that our theory has two free parameters which cannot be determined by the observables since nine physical observables are written in terms of 11 parameters.

As we have discussed in the previous paper [12], if the large mixings between the 1–3 and 2–3 generations are introduced, then the top-quark mass decreases from 160 GeV $\sim 2M_W$. Thus, we expect that the mixing angles between the third generation and the first two generations are considered to be small to keep $m_t \sim 2M_W$. Also, the relation between the masses of top and bottom quark $m_t^2 + (2m_b)^2 = (2M_W)^2$ for the $M^3 = 0$ holds[4] and we must choose $a_3 \sim 1$. It implies that the rotation angles θ_2', θ_3', θ_2, θ_3 and parameter $\sqrt{1-a_3^2}$ should be small, and also other six parameters a_1, a_2, b_1, b_2 and θ_1', θ_1 should take values close to those of the two-generation model [12].

Actually, for the case of $R_u = \mathbf{1}_{3\times 3}$, where the up-type quark mixings vanish, this case gives almost the most stringent lower bound from $K^0 - \bar{K}^0$ mixing; for example, these parameters are numerically found as

$$a_1^2 \approx 0.1023 \quad b_1^2 \approx 4.355\times 10^{-9} \quad \sin\theta_1 \approx -2.587\times 10^{-2}$$
$$a_2^2 \approx 0.9887 \quad b_2^2 \approx 1.302\times 10^{-4}, \quad \sin\theta_2 \approx 2.224\times 10^{-2}$$
$$a_3^2 \approx 0.9966, \quad \sin\theta_3 \approx 2.112\times 10^{-4}. \tag{3.11}$$

Also, for the another case of $R_d = \mathbf{1}_{3\times 3}$ where the down-type quark mixings vanish, these parameters are numerically found as

$$a_1^2 \approx 0.0650 \quad b_1^2 \approx 3.973\times 10^{-9} \quad \sin\theta_1' \approx 0.6704$$
$$a_2^2 \approx 0.9931 \quad b_2^2 \approx 2.235\times 10^{-4}, \quad \sin\theta_2' \approx -3.936\times 10^{-2}$$
$$a_3^2 \approx 0.9966 \quad \sin\theta_3' \approx 1.773\times 10^{-2}. \tag{3.12}$$

These two results show that the mixing angles θ_2, θ_3, θ_2', $\theta_3' \ll 1$, which is completely consistent with the above argument.

IV. $B^0 - \bar{B}^0$ MIXING

In this section, we apply the results of the previous section to representative FCNC processes, $B_d^0 - \bar{B}_d^0$ mixing and $B_s^0 - \bar{B}_s^0$ mixing responsible for the mass difference of two neutral B mesons.[5]

We focus on the FCNC processes of zero-mode down-type quarks due to gauge boson exchange at the tree level. First let us consider the processes with the exchange of zero-mode gauge bosons. If such types of diagrams exist with a sizable magnitude, it will easily spoil the viability of the model.

Concerning the Z-boson exchange, it is in principle possible for the tree-level FCNC to occur. Since the mode function of the zero-mode gauge boson is y-independent, the overlap integral of mode functions is generation-independent. Thus the gauge coupling of zero-mode gauge boson depends on only the relevant quantum numbers, such as the third component of weak isospin I_3. Therefore, the condition proposed by Glashow-Weinberg [20] to guarantee natural flavor conservation for the theories of 4D space-time is relevant.

Although there are right-handed down-type quarks belonging to different representations in our model, for example, the $SU(2)$ singlet d_R in $\psi(\mathbf{3})$ and one of components of the triplet Σ_R in $\psi(\bar{\mathbf{6}})$ or $\psi(\overline{\mathbf{15}})$, these are known to have the same quantum number $I_3 = 0$, and thus the Glashow-Weinberg condition is satisfied in this sector [11]. However, the quintet Θ_R in $\psi(\overline{\mathbf{15}})$ also contains the right-handed down-type quark, and this has the different quantum number $I_3 = 1$ from that of d_R^i belonging to $\psi(\mathbf{3})$.

What is worse, the quartet Δ_L in $\psi(\overline{\mathbf{15}})$ contains left-handed down-type quark with the different quantum number $I_3 = \frac{1}{2}$ from that of d_L^i belonging to the doublet Q_L in $\psi(\mathbf{3})$, $\psi(\bar{\mathbf{6}})$ or $\psi(\overline{\mathbf{15}})$ with the quantum number $I_3 = -\frac{1}{2}$. Thus, the condition of Glashow-Weinberg is not satisfied in the down-type quark sector and FCNC process due to the exchange of the zero-mode Z-boson arises at the tree

[4] As has been discussed in [16], it is expected that the difference between the top-quark mass and tree-level prediction $m_t \sim 2M_W$ is compensated by the QCD correction.

[5] For the studies of $B^0 - \bar{B}^0$ mixing in other new physics models, see for instance [19].

level.[6] However, the quintet Θ_R (quartet Δ_L) is an exotic fermion and acquires large $SU(2)$ invariant brane mass. Thus the mixing between d_R^i (d_L^i) and Θ_R (Δ_L) is inversely suppressed by the power of $m_{\rm BLM}$ and the FCNC vertex of Z-boson can be safely neglected. We may say that the condition of Glashow-Weinberg is satisfied in a good approximation in the processes via the zero-mode gauge boson exchange.

One may worry that Z' gauge boson exchange gives rise to FCNC processes, since an extra $U(1)$ gauge symmetry is indispensable for getting a realistic Weinberg angle. Note that the extra $U(1)$ gauge symmetry is explicitly broken by an anomaly and the gauge boson of the extra $U(1)$ gauge symmetry acquires a mass of the cutoff-scale order. In our model, the cutoff scale is a 5D Planck scale, which is larger than the intermediate scale 10^{13} GeV. Therefore, the FCNC effects by Z' gauge boson exchange can be safely neglected, comparing to the process by nonzero KK gluon exchanges, which is considered later.

Hence, the remaining possibility is the process via the exchange of nonzero KK gauge bosons. In this case, the mode functions of KK gauge bosons are y-dependent and their couplings to fermions are no longer universal because of nondegenerate bulk masses, even if the condition of Glashow-Weinberg is met.

Therefore, such progresses lead to FCNC at the tree level. In our previous papers [11,12], we have calculated $K^0 - \bar{K}^0$ mixing and $D^0 - \bar{D}^0$ mixing via the nonzero KK gluon exchange at the tree level and obtained a lower bound of the compactification scale as the prediction of our model. Along the same line of the argument as in our previous papers, we here study $B_d^0 - \bar{B}_d^0$ and $B_s^0 - \bar{B}_s^0$ mixings in the down-type quark sector by the nonzero KK gluon exchange at the tree level as the dominant contribution to these FCNC processes.

For such purpose, let us derive the four-dimensional effective QCD interaction vertices for the zero modes of down-type quarks relevant for our calculation,

$$
\begin{aligned}
\mathcal{L}_{\rm s} \supset & \frac{g_{\rm s}}{2\sqrt{2\pi R}} G_\mu^a (\bar{d}_R^i \lambda^a \gamma^\mu d_R^i + \bar{d}_L^i \lambda^a \gamma^\mu d_L^i) \\
& + \frac{g_{\rm s}}{2} G_\mu^{a(n)} \bar{d}_R^i \lambda^a \gamma^\mu d_R^j (V_{dR}^\dagger I_{RR}^{(0n0)} V_{dR})_{ij} \\
& + \frac{g_{\rm s}}{2} G_\mu^{a(n)} \bar{d}_L^i \lambda^a \gamma^\mu d_L^j (-1)^n \{V_{dL}^\dagger (U_3^\dagger I_{RR}^{(0n0)} U_3 \\
& + U_4^\dagger I_{RR}^{(0n0)} U_4) V_{dL}\}_{ij},
\end{aligned}
\tag{4.1}
$$

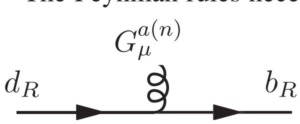
where $I_{RR}^{i(0n0)}$ and $I_{LL}^{i(0n0)}$ are overlap integrals relevant for gauge interaction,

$$
I_{RR}^{i(0n0)} = \frac{1}{\sqrt{\pi R}} \int_{-\pi R}^{\pi R} dy (f_R^i)^2 \cos\frac{n}{R} y = \frac{1}{\sqrt{\pi R}} \frac{(2RM^i)^2}{(2RM^i)^2 + n^2} \frac{(-1)^n e^{2\pi RM^i} - 1}{e^{2\pi RM^i} - 1}, \tag{4.2a}
$$

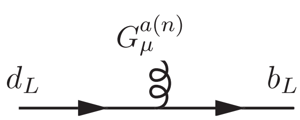
$$
I_{LL}^{i(0n0)} = I_{RR}^{i(0n0)}|_{M^i \to -M^i} = (-1)^n I_{RR}^{i(0n0)}, \tag{4.2b}
$$

since the chirality exchange corresponds to the exchange of two fixed points. We can see from (4.1) that the FCNC appears in the couplings of nonzero KK gluons due to the fact that $I_{RR}^{(0n0)}$ is not proportional to the unit matrix in the generation space, while the coupling of the zero-mode gluon is flavor-conserving, as we expected.

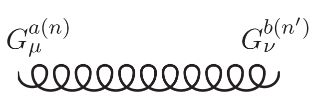
The Feynman rules necessary for the calculation of $B_d^0 - \bar{B}_d^0$ mixing can be read off from (4.1).



$$
= \frac{g_{\rm s}}{2} \left(V_{dR}^\dagger I_{RR}^{(0n0)} V_{dR} \right)_{31} \lambda^a \gamma^\mu R \,, \tag{4.3a}
$$



$$
= \frac{g_{\rm s}}{2} (-1)^n \left\{ V_{dL}^\dagger \left(U_3^\dagger I_{RR}^{(0n0)} U_3 + U_4^\dagger I_{RR}^{(0n0)} U_4 \right) V_{dL} \right\}_{31} \lambda^a \gamma^\mu L \,, \tag{4.3b}
$$

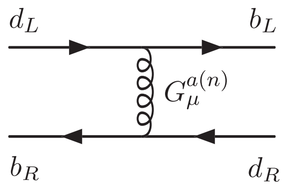


$$
= \delta_{nn'} \delta_{ab} \frac{\eta^{\mu\nu}}{k^2 - M_n^2} \qquad \left(\text{'t Hooft-Feynman gauge} \right) . \tag{4.3c}
$$

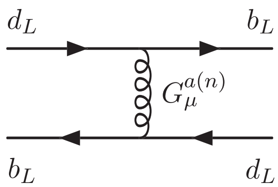
[6]The FCNC due to the exchanges of zero-mode photon and gluon trivially vanish because the fermions of our interest have the same electric charge and color.

Those for $B_s^0 - \bar{B}_s^0$ mixing are easily obtained by the replacements $d \leftrightarrow s$ and $31 \leftrightarrow 32$ in the matrix element of the vertices. The nonzero KK gluon exchange diagrams providing the dominant contribution to the process of $B_d^0 - \bar{B}_d^0$ and $B_s^0 - \bar{B}_s^0$ mixing are depicted in Fig. 1.[7]

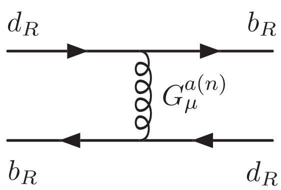
By noting the fact $k^2 \ll (\frac{n}{R})^2$ for $n \neq 0$ being the mass of n-th KK gluon and k^μ being internal momentum, the contributions from each type diagram of the $B_d^0 - \bar{B}_d^0$ mixing in Fig. 1 are written in the form of effective four-Fermi Lagrangian obtained by use of Feynman rules listed above,



$$\sim -\sum_{n=1}^{\infty} \frac{g_s^2}{4}\frac{1}{M_c^2}\left(\bar{b}_L\lambda^a\gamma^\mu d_L\right)\left(\bar{b}_R\lambda^a\gamma_\mu d_R\right) \times \frac{(-1)^n}{n^2}\left\{V_{dL}^\dagger\left(U_3^\dagger I_{RR}^{(0n0)}U_3 + U_4^\dagger I_{RR}^{(0n0)}U_4\right)V_{dL}\right\}_{31}\left(V_{dR}^\dagger I_{RR}^{(0n0)}V_{dR}\right)_{31}, \tag{4.4a}$$



$$\sim -\sum_{n=1}^{\infty} \frac{g_s^2}{4}\frac{1}{M_c^2}\left(\bar{b}_L\lambda^a\gamma^\mu d_L\right)\left(\bar{b}_L\lambda^a\gamma_\mu d_L\right) \times \frac{1}{n^2}\left\{V_{dL}^\dagger\left(U_3^\dagger I_{RR}^{(0n0)}U_3 + U_4^\dagger I_{RR}^{(0n0)}U_4\right)V_{dL}\right\}_{31}^2, \tag{4.4b}$$



$$\sim -\sum_{n=1}^{\infty} \frac{g_s^2}{4}\frac{1}{M_c^2}\frac{1}{n^2}\left(\bar{b}_R\lambda^a\gamma^\mu d_R\right)\left(\bar{b}_R\lambda^a\gamma_\mu d_R\right)\left(V_{dR}^\dagger I_{RR}^{(0n0)}V_{dR}\right)_{31}^2. \tag{4.4c}$$

Similarly, those for $B_s^0 - \bar{B}_s^0$ mixing are obtained by the replacements $d \leftrightarrow s$ and $31 \leftrightarrow 32$ in the matrix element of the vertices. The sum over the integer n is convergent and the coefficients of the effective Lagrangian (4.4) for the $B_d^0 - \bar{B}_d^0$ mixing and (4.4) after the replacements of $d \leftrightarrow s$ and $31 \leftrightarrow 32$ for the $B_s^0 - \bar{B}_s^0$ mixing are suppressed by the compactification scale as $1/M_c^2 = R^2$.

One may wonder about the exchange of an extra space component of gluon, $G_y^{a(n)}$. However, we found that such contribution is relatively suppressed by small masses of external quarks $m_q(m_q = m_d, m_s, m_b)$, as we have mentioned before [11]. Let us note that the zero mode of $G_y^{a(n)}$ ($n = 0$) is "modded out" by orbifolding and nonzero KK modes of $G_y^{a(n)}$ ($n \neq 0$) are absorbed as the longitudinal components of massive gluons $G_\mu^{a(n)}$ through Higgs-like mechanism. In the unitarity gauge, the contribution of such longitudinal components are taken into account by adding to the propagator Eq. (4.3c) a piece proportional to $\frac{k_\mu k_\nu}{M_n^2}$, where k_μ is the momentum transfer. By use of equations of motion for external quarks, its contribution to the amplitude is relatively suppressed by a factor $\frac{m_q^2}{M_n^2} = \mathcal{O}(m_q^2 R^2)$ and we can safely neglect the contribution of $G_y^{a(n)}$ exchange.

Comparing the results with the experimental data, we can estimate a lower bound on the compactification scale. The most general effective Hamiltonian for $\Delta B = 2$ processes due to some "new physics" at a high scale $\Lambda_{\rm NP} \gg M_W$ can be written as follows:

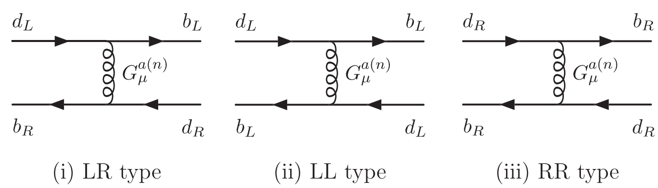


FIG. 1. The diagrams of $B_d^0 - \bar{B}_d^0$ mixing via KK gluon exchange. Those of $B_s^0 - \bar{B}_s^0$ mixing via KK gluon exchange are obtained by the replacements $d \leftrightarrow s$.

[7] In this calculation, the contributions of radiative corrections of QCD, which are known as the operator mixings or anomalous dimensions, are ignored, since the mass scale of B meson is sufficiently larger than the $\Lambda_{\rm QCD}$. In fact, these effects change only $\mathcal{O}(10\%)$ of our results.

$$\mathcal{H}_{\rm eff}^{\Delta B=2} = \frac{1}{\Lambda_{\rm NP}^2}\left(\sum_{i=1}^{5} z_i^q Q_i^q + \sum_{i=1}^{3} \tilde{z}_i^q \tilde{Q}_i^q\right) \tag{4.5}$$

where the 4-Fermi operators relevant for $B_d^0 - \bar{B}_d^0$ mixing are given as

$$Q_1^d = \bar{d}_L^\alpha \gamma_\mu b_L^\alpha \bar{d}_L^\beta \gamma^\mu b_L^\beta, \quad Q_2^d = \bar{d}_R^\alpha b_L^\alpha \bar{d}_R^\beta b_L^\beta,$$
$$Q_3^d = \bar{d}_R^\alpha b_L^\beta \bar{d}_R^\beta b_L^\alpha, \quad Q_4^d = \bar{d}_R^\alpha b_L^\alpha \bar{d}_L^\beta b_R^\beta, \quad Q_5^d = \bar{d}_R^\alpha b_L^\beta \bar{d}_L^\beta b_R^\alpha, \tag{4.6a}$$

and for $B_s^0 - \bar{B}_s^0$ mixing,

$$Q_1^s = \bar{s}_L^\alpha \gamma_\mu b_L^\alpha \bar{s}_L^\beta \gamma^\mu b_L^\beta, \qquad Q_2^s = \bar{s}_R^\alpha b_L^\alpha \bar{s}_R^\beta b_L^\beta,$$
$$Q_3^s = \bar{s}_R^\alpha b_L^\beta \bar{s}_R^\beta b_L^\alpha, \qquad Q_4^s = \bar{s}_R^\alpha b_L^\alpha \bar{s}_L^\beta b_R^\beta, \tag{4.6b}$$
$$Q_5^s = \bar{s}_R^\alpha b_L^\beta \bar{s}_L^\beta b_R^\alpha.$$

Indices α, β stand for the color degrees of freedom. The operators $\tilde{Q}_{1,2,3}$ are obtained from the $Q_{1,2,3}$ by the chirality exchange $L \leftrightarrow R$. Since the SM contribution is poorly known, we can get the constraint on the new physics directly from the experimental data, assuming that there is no accidental cancellation between the contributions of the SM and new physics. If we assume one of these possible operators gives dominant contribution to the mixing, each coefficient is independently constrained as follows, with the same constraints for $\tilde{z}_i^q$ as those for z_i^q $(i = 1, 2, 3)$ [21],

$$\begin{aligned}
|z_1^d| &\leq 2.3\times 10^{-5}\left(\frac{\Lambda_{\rm NP}}{1\ {\rm TeV}}\right)^2,\\
|z_1^s| &\leq 1.1\times 10^{-3}\left(\frac{\Lambda_{\rm NP}}{1\ {\rm TeV}}\right)^2,\\
|z_2^d| &\leq 7.2\times 10^{-7}\left(\frac{\Lambda_{\rm NP}}{1\ {\rm TeV}}\right)^2,\\
|z_2^s| &\leq 5.6\times 10^{-5}\left(\frac{\Lambda_{\rm NP}}{1\ {\rm TeV}}\right)^2,\\
|z_3^d| &\leq 2.8\times 10^{-6}\left(\frac{\Lambda_{\rm NP}}{1\ {\rm TeV}}\right)^2,\\
|z_3^s| &\leq 2.1\times 10^{-4}\left(\frac{\Lambda_{\rm NP}}{1\ {\rm TeV}}\right)^2,\\
|z_4^d| &\leq 2.1\times 10^{-7}\left(\frac{\Lambda_{\rm NP}}{1\ {\rm TeV}}\right)^2,\\
|z_4^s| &\leq 1.6\times 10^{-5}\left(\frac{\Lambda_{\rm NP}}{1\ {\rm TeV}}\right)^2,\\
|z_5^d| &\leq 6.0\times 10^{-7}\left(\frac{\Lambda_{\rm NP}}{1\ {\rm TeV}}\right)^2,\\
|z_5^s| &\leq 4.5\times 10^{-5}\left(\frac{\Lambda_{\rm NP}}{1\ {\rm TeV}}\right)^2.
\end{aligned} \tag{4.7}$$

where the new physics scale $\Lambda_{\rm NP}$ is regarded as the compactification scale in our case. All we have to do is to represent (4.4) and its replacements $d \leftrightarrow s$ and $31 \leftrightarrow 32$ of (4.4) by use of (4.6) and to utilize these constraints (4.7).

We can rewrite each type effective Lagrangian for $B_d^0 - \bar{B}_d^0$ mixing (4.4) in terms of effective Hamiltonian by using the Fierz transformation and the completeness condition for Gell-Mann matrices

$$\mathcal{H}_{{\rm eff},LL}^{\Delta B=2} = \frac{z_1^d Q_1^d}{R^{-2}}, \qquad \mathcal{H}_{{\rm eff},RR}^{\Delta B=2} = \frac{\tilde{z}_1^d \tilde{Q}_1^d}{R^{-2}},$$
$$\mathcal{H}_{{\rm eff},LR}^{\Delta B=2} = \frac{z_4^d Q_4^d + z_5^d Q_5^d}{R^{-2}}, \tag{4.8}$$

where

$$z_1^d = \frac{8\pi\alpha_{\rm s}}{3}\pi R \sum_{n=1}^{\infty} \frac{1}{n^2}\{V_{dL}^\dagger (U_3^\dagger I_{RR}^{(0n0)} U_3 + U_4^\dagger I_{RR}^{(0n0)} U_4) V_{dL}\}_{31}^2, \tag{4.9a}$$

$$\tilde{z}_1^d = \frac{8\pi\alpha_{\rm s}}{3}\pi R \sum_{n=1}^{\infty} \frac{1}{n^2}(V_{dR}^\dagger I_{RR}^{(0n0)} V_{dR})_{31}^2, \tag{4.9b}$$

$$z_4^d = -8\pi\alpha_{\rm s}\pi R \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}\{V_{dL}^\dagger (U_3^\dagger I_{RR}^{(0n0)} U_3 + U_4^\dagger I_{RR}^{(0n0)} U_4) V_{dL}\}_{31} (V_{dR}^\dagger I_{RR}^{(0n0)} V_{dR})_{31}, \tag{4.9c}$$

$$z_5^d = \frac{8}{3}\pi\alpha_{\rm s}\pi R \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}\{V_{dL}^\dagger (U_3^\dagger I_{RR}^{(0n0)} U_3 + U_4^\dagger I_{RR}^{(0n0)} U_4) V_{dL}\}_{31} (V_{dR}^\dagger I_{RR}^{(0n0)} V_{dR})_{31}. \tag{4.9d}$$

We note that there exist only the operators $Q_{1,4,5}$ or $\tilde{Q}_1$, even though the QCD corrections are considered. More precisely, in this model, the operators $Q_{2,3}$ or $\tilde{Q}_{2,3}$, which are generated by the pseudoscalar type diagrams accompanied with the extra component of the gluon $G_y^{a(n)}$ certainly exist; however, they are strongly suppressed, as was discussed before, and we can ignore them.

The four-dimensional $\alpha_{\rm s}$ is defined by

$$\alpha_{\rm s} = \frac{(g_{\rm s}^{4D})^2}{4\pi} = \frac{1}{2\pi R}\frac{g_{\rm s}^2}{4\pi}. \tag{4.10}$$

The constant $\alpha_{\rm s}$ should be estimated at the scale $\mu_b = m_b = 4.6$ GeV, where the $\Delta B = 2$ processes are actually measured [21]. So we have to take into account the renormalization group effect from the weak scale down to μ_b,

$$\alpha_{\rm s}^{-1}(m_b) = \alpha_{\rm s}^{-1}(M_Z) - \frac{23}{6\pi}\ln\frac{M_Z}{m_b} \to \alpha_{\rm s}(m_b) \approx 0.207, \tag{4.11}$$

where $\alpha_{\rm s}(M_Z) \approx 0.1184$ has been put [22].

Similarly, each type effective Hamiltonian for $B_s^0 - \bar{B}_s^0$ mixing are, respectively, rewritten by replacements $d \leftrightarrow s$ and $31 \leftrightarrow 32$ of (4.8) and (4.9).

Combining these results, we obtain the lower bounds for the compactification scale from the constraint (4.7). First let us assume that only one of the three types of diagrams (LL, RR, LR) gives dominant contribution to the mixing. Then we get lower bound on the compactification scale by use of the upper bound on the relevant coefficients z_1^q, $\tilde{z}_1^q$ and z_4^q given in (4.7) in the unit of TeV

$$\begin{aligned}
\mathrm{LL:}\ \frac{1}{R} &\gtrsim \sqrt{\frac{|z_1^d|}{2.3\times 10^{-5}}}\,[\mathrm{TeV}]\\
\mathrm{LL:}\ \frac{1}{R} &\gtrsim \sqrt{\frac{|z_1^s|}{1.1\times 10^{-3}}}\,[\mathrm{TeV}]\\
\mathrm{RR:}\ \frac{1}{R} &\gtrsim \sqrt{\frac{|\tilde{z}_1^d|}{2.3\times 10^{-5}}}\,[\mathrm{TeV}]\\
\mathrm{RR:}\ \frac{1}{R} &\gtrsim \sqrt{\frac{|\tilde{z}_1^s|}{1.1\times 10^{-3}}}\,[\mathrm{TeV}]\\
\mathrm{LR:}\ \frac{1}{R} &\gtrsim \sqrt{\frac{|z_4^d|}{2.1\times 10^{-7}}}\,[\mathrm{TeV}]\\
\mathrm{LR:}\ \frac{1}{R} &\gtrsim \sqrt{\frac{|z_4^s|}{1.6\times 10^{-5}}}\,[\mathrm{TeV}].
\end{aligned} \tag{4.12}$$

Let us note that LR-type diagrams yield both of Q_4^q and Q_5^q operators, as is seen in (4.8). We can, however, safely ignore the contribution of Q_5^q to the mixing, because the coefficients of the operator (4.9d) are smaller than that of Q_4^q and also because the magnitude of the hadronic matrix element of Q_4^q is known to be greater than that of Q_5^q, as the constraint for z_4^q is more severe that that for Q_5^q in (4.7). This is why we used the constraint for z_4^q alone to get the lower bound for the case of LR-type diagrams.

Since there is no bulk mass of third generation in this model, the "GIM-like" suppression mechanism from the large bulk masses, which acts much more severely on the contribution of the LR-type diagram [12], does not occur. Thus the contribution of the LR-type diagram is not expected to be smaller than those of the LL and RR diagram in general. Actually, for the case of $R_u = \mathbf{1}_{3\times 3}$ in (3.9), which gives almost the most stringent lower bound from $K^0 - \bar{K}^0$ mixing, the LR-type contribution is dominant for $B_s^0 - \bar{B}_s^0$ mixing while the LL type contribution is dominant for $B_d^0 - \bar{B}_d^0$ mixing

$$R^{-1} \gtrsim 1.71\ [\mathrm{TeV}] \qquad \text{for } B_d^0 - \bar{B}_d^0 \text{ mixing,} \tag{4.13a}$$
$$R^{-1} \gtrsim 2.54\ [\mathrm{TeV}] \qquad \text{for } B_s^0 - \bar{B}_s^0 \text{ mixing.} \tag{4.13b}$$

In the second case, $R_d = \mathbf{1}_{3\times 3}$, the contributions from the LR- and RR-type diagram vanish. This is because the down-type Yukawa coupling becomes diagonal: $V_{dL} = V_{dR} = \mathbf{1}_{3\times 3}$, namely, the mixings in the down-quark sector disappear. Note, however, that the lower bound obtained from the LL-type contribution, which does not vanish even though $V_{dL} = \mathbf{1}_{3\times 3}$. Actually, we obtain the lower bound on R^{-1} for $R_d = \mathbf{1}_{3\times 3}$;

$$R^{-1} \gtrsim 0.92\ [\mathrm{TeV}] \qquad \text{for } B_d^0 - \bar{B}_d^0 \text{ mixing,} \tag{4.14a}$$
$$R^{-1} \gtrsim 1.79\ [\mathrm{TeV}] \qquad \text{for } B_s^0 - \bar{B}_s^0 \text{ mixing.} \tag{4.14b}$$

This is because V_{uL} relevant for up-type quark mixing also contributes to the left-handed FCNC current. Namely, because of the mixing between Q_{3L} and Q_{6L} (Q_{15L}), U_4 also contributes to the FCNC vertex (4.3b) Thus even in the case of $V_{dL} = \mathbf{1}_{3\times 3}$ we get a meaningful lower bound on M_c

A comment is given. The obtained lower bounds are smaller than what we naively expect, assuming that the tree-level diagram relevant for the FCNC process is simply suppressed by $1/M_c^2$ [21],

$$M_c \gtrsim \mathcal{O}(10^3)\ [\mathrm{TeV}] \qquad \text{for } B_d^0 - \bar{B}_d^0 \text{ mixing,} \tag{4.15a}$$
$$M_c \gtrsim \mathcal{O}(10^2)\ [\mathrm{TeV}] \qquad \text{for } B_s^0 - \bar{B}_s^0 \text{ mixing,} \tag{4.15b}$$

which is much more stringent than the lower bound we obtained, in spite of the absence of the suppression by the large bulk masses. The obtained lower bounds also are milder than those from $K^0 - \bar{K}^0$ and $D^0 - \bar{D}^0$ mixings. This apparent discrepancy may be attributed to the very small mixing between the third generation and the first two generations.

V. SUMMARY

In this paper, we have discussed the $B_d^0 - \bar{B}_d^0$ and $B_s^0 - \bar{B}_s^0$ mixing in the framework of five-dimensional $SU(3)_{\mathrm{color}} \otimes SU(3) \otimes U'(1)$ gauge-Higgs unification scenario. In this model, several representations of $SU(3)$ are introduced in order to reproduce the quark sector of the standard model. Especially, the top quark t is embedded in $\overline{\mathbf{15}}$ to realize its mass $m_t \sim 2M_W$. And the flavor mixings appear as a breaking of flavor symmetry by the nondegenerate bulk mass terms of quarks. However, if the large flavor mixings between the 1–3 and 2–3 generations exist, the top-quark mass m_t in this model becomes smaller so that such mixings should be small. Then, it gives a natural explanation that top-quark mass is so large compared with those of other quarks and the smallness of the 1–3 and 2–3 generation, simultaneously.

In our previous studies [11,12], we pointed out that tree-level FCNCs appear and found a kind of suppression mechanism for light quarks what is called the "GIM-like mechanism." Thus, the neutral meson mixings for the light quarks, namely $K^0 - \bar{K}^0$ or $D^0 - \bar{D}^0$ mixing are strongly suppressed.

In the main text, we have calculated the contributions to $B_d^0 - \bar{B}_d^0$ and $B_s^0 - \bar{B}_s^0$ mixings by the nonzero KK gluon exchange at the tree level in the light of the recent progress in the measurements of $B^0 - \bar{B}^0$ mixing. For the processes with respect to the third generation, the "GIM-like" suppression mechanism, which is operative for the light first two-generation quarks, does not work since their bulk mass has to be vanished to realize top-quark mass. Therefore, we can anticipate large FCNC effects to arise and we are likely to obtain strong constraints for B-physics. The prediction of our model is that the lower bounds of compactification scale have been found to be of order $\mathcal{O}$(TeV), which is milder than those obtained from our study of $K^0 - \bar{K}^0$ and $D^0 - \bar{D}^0$ mixings in our previous paper [11,12] and from a naive expectation ($\sim$ 1000 TeV) where the dimension-six operator is simply suppressed by $1/M_c^2$ in spite of the absence of the GIM-like suppression by the large bulk masses. This is because the smallness of the mixings between 1–3 and 2–3 generations, i.e., θ_2, θ_3, θ_2', $\theta_3' \ll 1$. In our model, they should be small to reproduce the realistic top-quark mass $\sim 2M_W$, and then the induced $\Delta B = 2$ effective Hamiltonian are strongly suppressed. Thus the lower bound of compactification scale becomes small.

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