Neutrino properties in $E_6 \times SU(2)_F$ supersymmetric GUT with spontaneous *CP* violation

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We examined the neutrino sector in $E_6 \times SU(2)_F$ supersymmetric grand unified theory with spontaneous *CP* violation. At first glance, the discrete symmetry, which is introduced in order to solve the supersymmetric *CP* problem, constrains the allowed operators too strongly for the neutrino sector to be consistent with the experimental data, i.e., the μ neutrino becomes massless as commented in the previous paper. We showed that this issue can be solved if some operators are taken into account. And we saw that such operators do not play an important role in studying quark and charged lepton sectors. The predictions on the neutrino masses and mixings are the same as the E_6 models, which are consistent with various experiments on the neutrino oscillations.

I. INTRODUCTION

Weak scale supersymmetry (SUSY) is one of the most promising candidates for physics beyond the standard model (SM) [1]. The minimal supersymmetry extension of the standard model (MSSM) has several attractive features. It, for instance, provides a solution to the gauge hierarchy problem and a dark matter candidate as the lightest supersymmetric particle (LSP). Moreover, supersymmetric grand unified theory (SUSY GUT) is strongly motivated by the remarkable coincidence of three SM gauge coupling constants around 10^{16} GeV.

Grand unified theory can also explain the origin of hierarchical structures of masses and mixing angles in the SM particles by simply assuming that the **10**'s of $SU(5)$ induce stronger hierarchical Yukawa structure than the **5̄**'s. However, the SUSY GUT scenario is generically suffering from various theoretical and phenomenological difficulties. For instance,

(1) SUSY flavor problem: if generic soft SUSY breaking terms are introduced, flavor changing neutral current (FCNC) processes exceeds the experimental constraints [2].

(2) SUSY *CP* problem: if generic soft SUSY breaking terms are introduced, *CP* violating observables such as electric dipole moments exceeds the experimental constraints [2].

(3) μ problem: supersymmetric Higgs mass μ must be the same order of the soft SUSY breaking scale though it is a SUSY parameter.

(4) The doublet-triplet splitting (DTS) problem and proton decay: there must be huge mass separation between doublet Higgs (weak scale) and triplet Higgs (above GUT scale) in order to make proton's lifetime longer than experimental limit. Moreover,

recent proton decay constraint suggests that triplet Higgs mass should naively be larger than 10^{18} GeV, but it generically spoils the success of the gauge coupling unification.

(5) Unrealistic GUT relation: generically, unification of quark and leptons in GUT tends to result in unrealistic Yukawa relations which are inconsistent with the observed masses and mixings of quark and leptons.

E_6 unification is quite attractive because it can naturally induce the feature that the **10**'s of $SU(5)$ induce stronger hierarchical Yukawa structure than the **5̄**'s, which plays an important role in obtaining realistic Yukawa hierarchies in $SU(5)$ GUT. Moreover, if we introduce a family symmetry, $SU(3)_F(SU(2)_F)$, all three generation quark and leptons can be unified into a single (two) multiplet(s), and after breaking the family symmetry and the E_6 symmetry, realistic quark and lepton masses and mixings can be obtained. Furthermore, the third generation **10** of $SU(5)$ can have different sfermion masses from the other sfermion masses, because it is from the third generation **27** of E_6. It is remarkable that such effective SUSY sfermion mass spectrum [3] can satisfy the LHC constraints with lighter stop, which is important to keep the naturalness in SUSY models.

Unfortunately, such effective SUSY type sfermion mass spectrum is generically suffering from the new type of SUSY *CP* problem, in which chromo electric dipole moment (CEDM) of up quark becomes too large to satisfy the experimental constraints. The 1–3 mixings of up-type squark masses become too large and have a generically $\mathcal{O}(1)$ complex phase after diagonalizing the complex Yukawa couplings if the stop mass is different from the other two up-type squarks.

Recently, a scenario that solves this problem as well as the old type of SUSY *CP* problem by introducing spontaneous *CP* violation has been proposed [4]. The essential point is that the discrete symmetry, which is introduced in

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order to solve the old type of SUSY *CP* problem, leads to real up-type Yukawa couplings, which can solve the new type of SUSY *CP* problem and to complex down-type Yukawa couplings which give $\mathcal{O}(1)$ the Kobayashi-Maskawa (KM) [5] phase. The model has remarkable features, for example, the doublet-triplet splitting is naturally realized by introducing the anomalous $U(1)_A$ gauge symmetry, predictive Yukawa matrices of up quarks, down quarks and charged leptons are obtained because of the reduced number of the $\mathcal{O}(1)$ parameters, smaller Cabibbo-Kobayashi-Maskawa (CKM) [6] matrix element $V_{ub} \sim \lambda^4$, where $\lambda \sim 0.22$ is the Cabibbo mixing, is obtained.

In this paper, we examine the neutrino sector in the above model with the spontaneous *CP* violation, since this has not been studied in detail in the previous paper. At first glance, the discrete symmetry, which is introduced in order to solve the SUSY *CP* problem, constrains the allowed operators too strongly for the neutrino sector to be consistent with the experimental data, i.e., the μ neutrino becomes massless as commented in the previous paper. We will show that this issue on the neutrino sector can be solved if some operators, which are not important in studying quark and charged lepton sectors, are taken into account. And we clarify the reason why such operators play an important role only in the neutrino sector. The predictions on the neutrino masses and mixings are the same as the E_6 models, i.e., $\sin\theta_{12} \sim \sin\theta_{23} \sim \lambda^{0.5}$, $\sin\theta_{13} \sim \lambda$, $\Delta m_{12}^2/\Delta m_{23}^2 \sim \lambda^2$, etc., which are consistent with various experiments on the neutrino oscillations. Since the operators are restricted by the discrete symmetry, the predictions on the $\mathcal{O}(1)$ coefficients are expected. However, we are not able to find any simple predictions between the $\mathcal{O}(1)$ coefficients because there are a lot of operators for the right-handed neutrino masses.

As the result, we can obtain an attractive model in which
(1) realistic quark and lepton masses and mixings are obtained,
(2) all the *CP* phases in the model, the KM phase, *CP* phases in neutrino sector and *CP* phases in sfermion sectors, are given from one phase which is obtained by the spontaneous *CP* violation,
(3) as the result, SUSY *CP* problem is solved,
(4) the effective SUSY type sfermion mass spectrum is predicted.

The paper proceeds as follows: in Sec. II, we briefly review the basic features of $E_6 \times SU(2)_F \times U(1)_A$ SUSY GUT with spontaneous *CP* violation. In Sec. III, we calculate neutrino masses and Maki-Nakagawa-Sakata (MNS) [7] matrix explicitly, after specifying massless $\bar{\mathbf{5}}$ modes. We see that a specific type of higher-dimensional operators is essential for the realistic neutrino masses. We also show that the above higher-dimensional operators only affect the neutrino sector. The last section is devoted to summary and discussion.

II. BASIC FEATURES OF $E_6 \times SU(2)_F \times U(1)_A$ SUSY GUT WITH SPONTANEOUS *CP* VIOLATION

A. E_6 unification and $SU(2)_F$ family symmetry

First of all, **27** is the fundamental representation for the E_6 group. In terms of $E_6 \supset SO(10) \times U(1)_{V'}$ (and $[SO(10) \supset SU(5) \times U(1)_V]$) it is decomposed as

$$\mathbf{27} = \mathbf{16}_1[\mathbf{10}_1 + \bar{\mathbf{5}}_{-3} + \mathbf{1}_5] + \mathbf{10}_{-2}[\mathbf{5}_{-2} + \bar{\mathbf{5}}'_2] + \mathbf{1}'_4[\mathbf{1}'_0], \quad (1)$$

where acutes are used to distinguish different $\bar{\mathbf{5}}(\mathbf{1})$'s. Note that each **27** contains two $\bar{\mathbf{5}}$'s (and 1's) of $SU(5)$. This nature is essential for realizing different Yukawa structures of up-type quarks, down-type quarks, charged leptons, and neutrinos from a single hierarchical structure of an E_6 invariant Yukawa couplings [8–10].

In order to break the E_6 gauge group into the SM gauge group G_{SM}, we introduce three types of Higgs fields. The first one is the **27** representation Higgs H. We also introduce the $\overline{\mathbf{27}}$ representation Higgs $\bar{H}$ in order to satisfy the D-flatness condition of the E_6 gauge interaction. H, $\bar{H}$ obtain vacuum expectation values (VEVs)

$$\langle H \rangle = \langle \mathbf{1}_H \rangle \neq 0, \quad (2)$$

and they break E_6 into $SO(10)$. The second one is an adjoint Higgs A, which belongs to the **78** representation. It gets Dimopoulos-Wilczek (DW) [11] VEV proportional to $U(1)_{B-L}$ direction

$$\langle A \rangle = \langle \mathbf{45}_A \rangle \propto U(1)_{B-L}, \quad (3)$$

and it breaks $SO(10)$ into $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. The third type include C and $\bar{C}$, and they are also **27** and $\overline{\mathbf{27}}$ representations. They acquire VEVs

$$\langle C \rangle = \langle \mathbf{16}_C[\ni \mathbf{1}_C] \rangle \neq 0, \quad (4)$$

and they break $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ into G_{SM}. The superpotential which gives superheavy masses for $(\mathbf{5}, \bar{\mathbf{5}})$ and $(\mathbf{5}, \bar{\mathbf{5}}')$ pairs is

$$W = Y_{ij}^H \Psi_i \Psi_j H + Y_{ij}^C \Psi_i \Psi_j C, \quad (5)$$

where $\Psi_i(i = 1, 2, 3)$ are matter fields in family i. After H and C acquire VEVs, 3 degrees of freedom among $\bar{\mathbf{5}}_i$ and $\bar{\mathbf{5}}'_i$ get superheavy masses, while the remaining three are massless at the GUT scale. We assume the Yukawa hierarchies

$$Y_{ij}^H \sim Y_{ij}^C \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \quad (6)$$

and ratio of two VEVs

$$\frac{\langle C \rangle}{\langle H \rangle} \sim \lambda^{0.5} \quad (7)$$

up to $\mathcal{O}(1)$ coefficients. These assumption can be realized proper charge assignment of anomalous $U(1)_A$ symmetry and the breaking effect of the $SU(2)_F$ family symmetry, which we explain later, respectively.[1] Under these assumptions, three massless modes are mainly $(\bar{\mathbf{5}}_1, \bar{\mathbf{5}}'_1, \bar{\mathbf{5}}_2)$. Note that third generation does not appear at the leading order. So we end up following different Yukawa hierarchies

$$
Y_d \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix},
$$

$$
Y_d \sim Y_e^T \sim \begin{pmatrix} \lambda^6 & \lambda^{5.5} & \lambda^5 \\ \lambda^5 & \lambda^{4.5} & \lambda^4 \\ \lambda^3 & \lambda^{2.5} & \lambda^2 \end{pmatrix}, \tag{8}
$$

$$
M_\nu \sim \lambda^n \begin{pmatrix} \lambda^7 & \lambda^{6.5} & \lambda^6 \\ \lambda^{6.5} & \lambda^6 & \lambda^{5.5} \\ \lambda^6 & \lambda^{5.5} & \lambda^5 \end{pmatrix} \frac{\langle H_u \rangle^2}{\langle \bar{H} \rangle^2} \Lambda,
$$

when the MSSM Higgses H_u and H_d are included in $\mathbf{10}_H$ and $\mathbf{16}_C$. Here n is a number, and we used the relation $\langle \bar{C} \rangle / \langle \bar{H} \rangle \sim \lambda^{0.5}$.

In order to realize the Yukawa hierarchy between the $\mathcal{O}(1)$ top quark and the other quarks naturally, we employ the $SU(2)_F$ family gauge symmetry. We treat first two generation fields as the doublet $\Psi_a (a = 1, 2)$, whereas the third generation fields Ψ_3 and all Higgs fields are treated as singlets under $SU(2)_F$. Here the index a can be raised or lowered by the antisymmetric symbols ϵ^{ab} and ϵ_{ab}. We also introduce flavon fields F_a and $\bar{F}^a$, which are singlets under E_6 and doublet and antidoublet under $SU(2)_F$, respectively. VEVs of F_a and $\bar{F}^a$ break $SU(2)_F$, then this effect generates the hierarchical Yukawa structure in (6). Since massless $\bar{\mathbf{5}}$'s do not contain the third generation at the leading order in E_6 GUT, both left-handed and right-handed components of the top quark belong to the $SU(2)_F$ singlet, while all other quarks and charged leptons are (partially or completely) $SU(2)_F$ doublets.

$SU(2)_F$ family symmetry is also useful for solving the SUSY FCNC problem. Suppose that the soft SUSY breaking terms are mediated to the visible sector above the scale where E_6 and $SU(2)_F$ symmetries are respected, such as in gravity mediation. Then the symmetry guarantees sfermion masses which degenerate at the leading order except for $\mathbf{10}_3$ [10]:

$$
\tilde{m}_{\mathbf{10}}^2 = \begin{pmatrix} m_0^2 & 0 & 0 \\ 0 & m_0^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix}, \qquad \tilde{m}_{\bar{\mathbf{5}}}^2 = \begin{pmatrix} m_0^2 & 0 & 0 \\ 0 & m_0^2 & 0 \\ 0 & 0 & m_0^2 \end{pmatrix}, \tag{9}
$$

where $\tilde{m}_{\mathbf{10}}$ and $\tilde{m}_{\bar{\mathbf{5}}}$ are squared sfermion mass matrices at the GUT scale in the $\mathbf{10}$ and $\bar{\mathbf{5}}$ fields of $SU(5)$, respectively. For sfermions in $\mathbf{10}$, FCNC observables provide stringent constraints mainly on the first two generations, while constraints for $\bar{\mathbf{5}}$'s are stringent for all three generations. This is because the mixing angles of the $\mathbf{10}$ fields are small, while those of the $\bar{\mathbf{5}}$ fields are large. Therefore, in this model, FCNC constraints can be evaded by raising m_0.[2] Moreover, the weak scale is not destabilized as long as m_3 is around the weak scale. So this type of sfermion masses ("effective SUSY" or "*modified* universality") [10,13–16] solves the SUSY FCNC problem without spoiling naturalness. We also comment that LHC constraints to this type of sfermion masses are much weaker than to the constrained MSSM type of sfermion masses [17].

Unfortunately, there is a tension between these modified universal sfermion masses and complex Yukawa couplings, which are important to obtain the nonvanishing KM phase. In the basis in which Yukawa matrices are diagonal, sfermion mass matrices become

$$
\Delta_{\mathbf{10}} \equiv V_{\mathbf{10}}^\dagger \tilde{m}_{\mathbf{10}}^2 V_{\mathbf{10}} \sim \tilde{m}_{10}^2 + (m_3^2 - m_0^2) \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix},
$$

$$
\Delta_{\bar{\mathbf{5}}} \equiv V_{\bar{\mathbf{5}}}^\dagger \tilde{m}_{\bar{\mathbf{5}}}^2 V_{\bar{\mathbf{5}}} = \tilde{m}_{\bar{\mathbf{5}}}^2, \tag{10}
$$

where $V_{\mathbf{10}}$ and $V_{\bar{\mathbf{5}}}$ are the diagonalizing unitary matrices for the $\mathbf{10}$ and $\bar{\mathbf{5}}$ fields of $SU(5)$, respectively, and we take the CKM like matrix as $V_{\mathbf{10}}$. Since the mass insertion matrices $\delta_{\mathbf{10}} \equiv \Delta_{\mathbf{10}}/m_0^2$ do not vanish in the limit $m_0 \to \infty$, the CEDM of up quark from this SUSY contribution is not decoupled. The constraints for these parameters by mercury (neutron) become [15,18–20]

$$
\text{Im}\left[(\delta_{u_L})_{13}(\delta_{u_R})_{31}\right] \lesssim 3 \times 10^{-7} (9 \times 10^{-7}) \left(\frac{m_3}{500 \text{ GeV}}\right)^2, \tag{11}
$$

which are much smaller than the prediction $\lambda^6 \sim 10^{-4}$. This is a serious problem on the "effective SUSY" or "modified universality," which we call the new type of SUSY CP problem.

B. Anomalous $U(1)_A$ symmetry

An anomalous $U(1)_A$ symmetry [21,22] is introduced in order to solve the doublet-triplet splitting problem and μ problem, and to provide the origin of the hierarchical Yukawa structures [23]. This is a gauge symmetry whose anomalies are canceled by the Green-Schwarz mechanism [24]. The theory possesses the Fayet-Iliopoulos term ξ^2, and we assume its magnitude as $\xi = \lambda \Lambda$. Here Λ is the cutoff scale of the theory and we set $\Lambda = 1$. Let us denote

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[1]Strictly speaking, (7) is not true in the $U(1)_A$ framework. The correct relation is $\lambda^c \langle C \rangle / \lambda^h \langle H \rangle \sim \lambda^{0.5}$, where c and h are $U(1)_A$ charges of C and H, respectively.

[2]There is upper limit of m_0 comes from charge and color breaking effect. For detail discussion, see [12].

the gauge symmetries of the theory except for $U(1)_A$ as G_a. Consider a theory consisting of all the G_a and $U(1)_A$ invariant terms, including nonrenormalizable operators. Then it is shown in [23] that the theory has a supersymmetric vacuum where all the fields that are negatively charged under $U(1)_A$ get VEVs in the following way: [3]

$$\langle Z_i^+ \rangle = 0 \quad (z_i^+ > 0) \quad \langle Z_i^- \rangle \sim \lambda^{-z_i} \quad (z_i^- < 0). \quad (12)$$

Here Z_i is G_a singlet field whose $U(1)_A$ charge is z_i. This argument can be extended to the case where Z_i is composite operator that is made by G_a nonsinglet fields. For example, in the $Z^- = \bar{X}X$ case, $\langle \bar{X}X \rangle \sim \lambda^{-(x+\bar{x})}$ leads to $|\langle X \rangle| = |\langle \bar{X} \rangle| \sim \lambda^{-(x+\bar{x})/2}$, once the D-flatness condition of G_a is taken account. Note that the above results can be obtained under the natural assumption that all the interactions, which are allowed by the symmetry of the theory, are introduced with $\mathcal{O}(1)$ coefficients.

It is important to mention that a term whose total $U(1)_A$ charge is negative does not appear at the $U(1)_A$ breaking vacuum. The reason is that this type of term should originally be accompanied by at least one positively charged field but its VEV is always vanishing according to (12) [SUSY-zero mechanism] [23,25]. A $U(1)_A$ symmetry and its specific SUSY vacuum are applied in several aspects of phenomenological model building. For example, an appropriate $U(1)_A$ charge assignment for the Higgs sector can ensure the DTS via the DW mechanism [23], and the SUSY-zero mechanism can be applied to solve the μ problem [26].

For the following arguments, we will briefly review the points on the solution for the μ problem. Since the MSSM Higgses, H_u and H_d, have negative $U(1)_A$ charges, the mass term is forbidden by the SUSY-zero mechanism. With the positively charged singlet field S, the term SH_uH_d can be allowed, but $\langle S \rangle = 0$ in the SUSY limit. However, if SUSY is broken at the weak scale Λ_W, the VEV of S becomes nonvanishing and the order of the weak scale. This results in the SUSY Higgs mass with $\mathcal{O}(\Lambda_W)$. The b parameter also becomes the weak scale.

C. Spontaneous *CP* violation and discrete symmetry

As we have seen, one of the attractive features of the $E_6 \times SU(2)_F$ SUSY GUT is the modified universal sfermion mass (9), which evades FCNC constraints while maintaining naturalness. However, such type of mass spectrum is generically suffering from the CEDM constraint of the up quark [20]. Fortunately, we can evade this constraint by introducing a spontaneous *CP* violation (SCPV) [27,28]. By using SCPV, we can realize the real up-type Yukawa couplings together with the KM phase.

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[3]From now on, each superfield is denoted by an uppercase letter, whereas the corresponding lowercase letter indicates an associated $U(1)_A$ charge. The consistency of (12) requires the number of positively charged fields to be larger than that of negatively charged fields by one.

Let us introduce the $E_6 \times SU(2)_F$ singlet field $S(s > 0)$. Then we obtain the following superpotential made of flavon fields F_a, $\bar{F}^a$ and S:

$$W = \lambda^s S \left[\sum_{n=0}^{n_f} c_n \lambda^{(f+\bar{f})n} (\bar{F}F)^n \right]. \quad (13)$$

Here c_n are real $\mathcal{O}(1)$ coefficients and n_f is the number where the SUSY-zero mechanism truncates the sum. When $n_f \geq 2$, the F-flatness condition with respect to S leads to complex VEV $\langle \bar{F}F \rangle$ and then *CP* symmetry can be spontaneously broken. Using $SU(2)_F$ gauge symmetry and its D-flatness condition, we can take the case where only $\langle F \rangle$ is complex without loss of generality:

$$\langle F_a \rangle \sim \begin{pmatrix} 0 \\ e^{i\rho} \lambda^{-(f+\bar{f})/2} \end{pmatrix}, \qquad \langle \bar{F}^a \rangle \sim \begin{pmatrix} 0 \\ \lambda^{-(f+\bar{f})/2} \end{pmatrix}. \quad (14)$$

Unfortunately, the SCPV affects μ generation in the $U(1)_A$ framework and leads to an unwanted outcome [4]

$$\mathrm{Arg}\,[\mu b^*] = \mathcal{O}(1), \quad (15)$$

if $S\bar{F}F$ exists. This rephasing invariant phase induces electric dipole moments of quarks and leptons, and give rise to the SUSY *CP* problem again. In order to evade such unwanted relation, we introduce an additional discrete symmetry Z_6, which forbids $S\bar{F}F$. Then we can avoid this old type of SUSY *CP* problem. Moreover, this symmetry results in the real up-type Yukawa matrix, which solves the new type of SUSY *CP* problem, i.e., the diagonalizing unitary matrices become real and therefore the mass insertion matrices for the up-type quark become real, if the MSSM Higgs H_u is included in $\mathbf{10}_H$. This symmetry constrains the model and reduces the number of $\mathcal{O}(1)$ parameters. As a by-product, the up-quark Yukawa coupling y_u and the CKM matrix element V_{ub} become closer to experimental values $y_u \sim \lambda^8$, $V_{ub} \sim \lambda^4$ than the expectation of E_6 GUT $y_u \sim \lambda^6$, $V_{ub} \sim \lambda^3$ [4].

It is also reported in [4] that the discrete symmetry makes one of the neutrinos massless. This result seems to be unrealistic, but we will show that this issue can be solved by introducing some operators, which have not been considered in the previous paper.

III. CALCULATION OF NEUTRINO MASS AND MNS MATRIX

A. Field contents and Yukawa couplings

First, we summarize the field content of the model and its representations under the E_6, $SU(2)_F$, $U(1)_A$ and Z_6 symmetries.

We introduce the following fields, which are listed in Table I. All matter fields Ψ belong to **27** representation of E_6. We make its first two generations as a doublet $\Psi_a(a = 1, 2)$ and the third generation as a singlet Ψ_3 of the $SU(2)_F$, respectively. The $SU(2)_F$ and *CP* are simultaneously broken by the VEVs of the flavon fields F_a and

TABLE I.   Field contents and charge assignment under $E_6 \times SU(2)_F \times U(1)_A \times Z_6$.

	Ψ_a	Ψ_3	F_a	$\bar{F}^a$	H	$\bar{H}$	C	$\bar{C}$	C'	$\bar{C}'$
E_6	**27**	**27**	**1**	**1**	**27**	$\overline{\mathbf{27}}$	**27**	$\overline{\mathbf{27}}$	**27**	$\overline{\mathbf{27}}$
$SU(2)_F$	**2**	**1**	**2**	$\bar{\mathbf{2}}$	**1**	**1**	**1**	**1**	**1**	**1**
$U(1)_A$	4	$\frac{3}{2}$	$-\frac{3}{2}$	$-\frac{5}{2}$	-3	1	-4	-1	7	9
Z_6	0	0	1	0	0	0	5	3	3	3

	A	A'	Z_0	Z_2	Z_3	Z_4	S	S'	S''
E_6	**78**	**78**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
$SU(2)_F$	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
$U(1)_A$	-1	5	-1	-3	-2	-5	9	8	5
Z_6	3	3	0	2	3	4	0	4	4

$\bar{F}^a$. All the other fields are singlets under $SU(2)_F$.[4] H is the field whose VEV $\langle 1_H \rangle \neq 0$ breaks E_6 into $SO(10)$, and $\bar{H}$ is introduced to maintain the D-flatness condition. The VEV of A breaks $SO(10)$ into $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. C is the field whose VEV breaks $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ into G_{SM}, and $\bar{C}$ is also introduced to maintain the corresponding D-flatness condition. Basically, the F-flatness conditions of the positively charged fields, A', C', $\bar{C}'$ etc., determine the VEVs of the negatively charged fields. For example, the F-flatness conditions with respect to A' make A acquire DW-type VEVs for the $SO(10)$ adjoint component to solve the DTS problem. The alignment between the VEVs of A and C, $\bar{C}$ are realized and at the same time, the pseudo Nambu-Goldstone modes become heavy [29]. In Table I, $U(1)_A$ charges are assigned so that the DTS and appropriate Yukawa hierarchies are realized. Also, Z_6 charges are determined so that the SUSY CP problem is evaded.

S, S', and S'' are introduced to realize the SCPV and μ generation. The relevant superpotential are

$$W_S = \lambda^s S + \lambda^{s+3z_2} SZ_2^3 + \lambda^{s+3h} SH^3, \quad (16)$$

$$W_{S'} = \lambda^{s'+z_2} S'Z_2 + \lambda^{s'+\bar{c}+c} S'\bar{C}C + \lambda^{s'+2\bar{f}+2f} S'(\bar{F}F)^2, \quad (17)$$

$$W_{S''} = \lambda^{s''+z_2} S''Z_2 + \lambda^{s''+\bar{c}+c} S''\bar{C}C. \quad (18)$$

First, we obtain the VEV $\langle Z_2 \rangle$ from the F-flatness condition of W_S with respect to S. Since the one of the three solutions is real, we assume $\langle Z_2 \rangle$ is real. Second, the F-flatness condition of $W_{S''}$ determines the real VEV $\langle \bar{C}C \rangle$.[5] Third, the F-flatness condition of $W_{S'}$ leads the complex VEV $\langle \bar{F}F \rangle$, which breaks the $SU(2)_F$ and CP spontaneously. Note that the operator H^3 gives the SM Higgs mass after developing the VEV of H if the SM Higgses are included in $\mathbf{10_H}$. Then, after breaking SUSY, W_S generates the μ and b terms. Since H^3 does not couple to S' and S'', μ and b are real.

Let us examine the mass matrices of quarks and leptons in this model. Under the charge assignment of Table I, the following interactions between matter and Higgs fields are allowed:

$$Y^H: \begin{pmatrix} 0 & d\Psi^a(A, Z_3, \bar{H}H)\Psi_a & 0 \\ d\Psi^a(A, Z_3, \bar{H}H)\Psi_a & c\lambda^{2(\psi_a+\bar{f})}\bar{F}^a\Psi_a\bar{F}^b\Psi_b & b\lambda^{\psi_a+\psi_3+\bar{f}}\bar{F}^a\Psi_a\Psi_3 \\ 0 & b\lambda^{\psi_a+\psi_3+\bar{f}}\Psi_3\bar{F}^a\Psi_a & a\lambda^{2\psi_3}\Psi_3\Psi_3 \end{pmatrix}\lambda^h H, \quad (19)$$

$$Y^C: \begin{pmatrix} 0 & f'\lambda^{2\psi_a+f+\bar{f}}F^a\Psi_a\bar{F}^b\Psi_b & g'\lambda^{\psi_a+\psi_3+f}F^a\Psi_a\Psi_3 \\ f'\lambda^{2\psi_a+f+\bar{f}}\bar{F}^a\Psi_a F^b\Psi_b & 0 & 0 \\ g'\lambda^{\psi_a+\psi_3+f}\Psi_3 F^a\Psi_a & 0 & 0 \end{pmatrix}\lambda^c C. \quad (20)$$

Here we explicitly write in all $\mathcal{O}(1)$ coefficients, a, b, c, d, f', and g'. Note that all the $\mathcal{O}(1)$ coefficients are assumed to be real because of the original CP symmetry.

In (19), the structure of the (1, 2) and (2, 1) elements is a little bit complicate, so we should consider this point carefully. $\Psi^a(A, Z_3, \bar{H}H)\Psi_a$ consist of several types of terms.

Clearly, some additional field(s) for the (1, 2) and (2, 1) elements are needed because $\Psi^a\Psi_a = \epsilon^{ab}\Psi_a\Psi_b = 0$, where $\epsilon^{12} = -\epsilon^{21} = 1$, $\epsilon^{11} = \epsilon^{22} = 0$. The negatively $U(1)_A$ charged fields which can have nonvanishing VEVs

[4]To cancel the Witten's anomaly, an odd number of additional doublets of $SU(2)_F$ are required.

[5]We must forbid the term $S^{(\prime)}\bar{F}F\bar{C}C$, which lead the complex VEV $\langle \bar{C}C \rangle$. If we can choose the basis in which $\langle C \rangle$ is real and $\langle \bar{C} \rangle$ has an opposite phase to $\langle F \rangle$, the up-type Yukawa can keep real. However, it changes the phase of the down-type Yukawa into removable, and the KM phase cannot be realized.

can be a candidate for the additional fields. There are two possibilities. One is using the adjoint Higgs A: $\Psi^a A Z_3 \Psi_a$, $\Psi^a A^2 \Psi_a$, etc. Since the VEV of A breaks $SO(10)$ into $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, the contributions of these terms are different for the different components of Ψ_a under $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. Because the VEV of A is proportional to the B-L charge, there is no contribution for the field which has vanishing B-L charge. The other possibility, which has not been considered in the previous paper, is using $\bar{H}H$: $\Psi^a \bar{H}H \Psi_a$, etc. Since the VEV of H breaks E_6 into $SO(10)$, it is useful to write down the contributions with the representations of $SO(10)$. For example, $\epsilon^{ab} \mathbf{10}_{\Psi_a} \mathbf{1}_{\Psi_b}$ and $\epsilon^{ab} \mathbf{16}_{\Psi_a} \mathbf{10}_{\Psi_b}$. Note that the terms $\epsilon^{ab} \mathbf{10}_{\Psi_a} \mathbf{10}_{\Psi_b}$ and $\epsilon^{ab} \mathbf{16}_{\Psi_a} \mathbf{16}_{\Psi_b}$ are vanishing and give no contribution. Here we use a conventions for the component fields of $SO(10)$ of a field X as

$$X = \mathbf{27}_X = \mathbf{16}_X + \mathbf{10}_X + \mathbf{1}'_X, \qquad (21)$$

$$\mathbf{16}_X = Q_X + U_X + E_X + D_X + L_X + N_X, \qquad (22)$$

$$\mathbf{10}_X = H_X^C + (H_u)_X + \bar{H}_X^C + (H_d)_X. \qquad (23)$$

Then, we can write down the concrete components of the interactions $\Psi^a(A, Z_3, \bar{H}H)\Psi_a$ as

$$d\Psi^a \langle (A, Z_3, \bar{H}H) \rangle \Psi_a H \ni d_5 \lambda^5 \epsilon^{ab} H_{\Psi_a}^C \bar{H}_{\Psi_b}^C \mathbf{1}'_H,$$

$$-\tfrac{1}{2} d_q \lambda^5 \epsilon^{ab} Q_{\Psi_a} U_{\Psi_b} (H_u)_H, \quad -\tfrac{1}{2} d_q \lambda^5 \epsilon^{ab} Q_{\Psi_a} D_{\Psi_b} (H_d)_H,$$

$$-\tfrac{3}{2} d_l \lambda^5 \epsilon^{ab} L_{\Psi_a} E_{\Psi_b} (H_d)_H, \quad h \lambda^5 \epsilon^{ab} L_{\Psi_a} N_{\Psi_b} (H_u)_H, \qquad (24)$$

where d_5, d_q, d_l, h are the $\mathcal{O}(1)$ coefficients that are different from each other generically. As we will discuss later, the h term is very important for neutrino masses.

B. Massless modes

Once we write down the Yukawa couplings, we can calculate the massless combinations of $\bar{\mathbf{5}}_i$ and $\bar{\mathbf{5}}'_i$. We mainly follow the procedure given in [30]. First, we fix $\mathcal{O}(1)$ coefficient of two VEVs:

$$\frac{\lambda^c \langle C \rangle}{\lambda^h \langle H \rangle} \equiv x \lambda^{0.5}, \qquad (25)$$

where x is a real $\mathcal{O}(1)$ coefficient. Then, after the Higgs and flavon fields acquire VEVs, (19) and (20) induce following mass matrix for $\mathbf{5}_i$ and $(\bar{\mathbf{5}}'_i, \bar{\mathbf{5}}_i)$:

$$\begin{pmatrix} 0 & \alpha d_5 \lambda^5 & 0 & 0 & f e^{i\rho} \lambda^{5.5} & g e^{i\rho} \lambda^{3.5} \\ -\alpha d_5 \lambda^5 & c\lambda^4 & b\lambda^2 & f e^{i\rho} \lambda^{5.5} & 0 & 0 \\ 0 & b\lambda^2 & a & g e^{i\rho} \lambda^{3.5} & 0 & 0 \end{pmatrix}$$

$$\equiv (M_1 M_2). \qquad (26)$$

Here, we reparametrize $\mathcal{O}(1)$ coefficients as $f \equiv xf'$ and $g \equiv xg'$. Each power of λ is determined by the corresponding $U(1)_A$ charge. It is important to note that $\alpha = 1$ for the colored Higgs components (H^C, $\bar{H}^{\bar{C}}$) of $\mathbf{5}$ and $\bar{\mathbf{5}}'$, and $\alpha = 0$ for the doublet Higgs components (H_u, H_d) of those, since the (1, 2) and (2, 1) elements of (26) originate from the B-L conserving VEV of A.

In order to find the massless combinations, let us diagonalize the 3×6 matrix (26) as follows:

$$V^\dagger (M_1 M_2) \begin{pmatrix} U_{10}^H & U_{10}^0 \\ U_{16}^H & U_{16}^0 \end{pmatrix} = (M_H^{\text{diag}} 0). \qquad (27)$$

Here V is a 3×3 unitary matrix and

$$U \equiv \begin{pmatrix} U_{10}^H & U_{10}^0 \\ U_{16}^H & U_{16}^0 \end{pmatrix} \qquad (28)$$

is a 6×6 unitary matrix, and they rotate $\mathbf{5}_i$ and $(\bar{\mathbf{5}}'_i, \bar{\mathbf{5}}_i)$ ($i = 1, 2, 3$) into their mass eigenstates, respectively. Our task is finding matrices U_{10}^0 and U_{16}^0 which are related by

$$M_1 U_{10}^0 + M_2 U_{16}^0 = 0. \qquad (29)$$

According to the calculation in Appendix A, they become

$$U_{10}^0 = \begin{pmatrix} -\frac{a\alpha d_5(bg-af)}{(ac-b^2)^2}\lambda^{2.5}e^{i\rho} & 1 & \mathcal{O}(\lambda^{5.5}) \\ \frac{bg-af}{ac-b^2}\lambda^{1.5}e^{i\rho} & \frac{a\alpha d_5}{ac-b^2}\lambda & \mathcal{O}(\lambda^{4.5}) \\ -\left(\frac{g}{a}+\frac{b}{a}\frac{bg-af}{ac-b^2}\right)\lambda^{3.5}e^{i\rho} & -\frac{b\alpha d_5}{ac-b^2}\lambda^3 & \mathcal{O}(\lambda^{6.5}) \end{pmatrix}, \qquad (30)$$

$$U_{16}^0 = \begin{pmatrix} 1 & 0 & 0 \\ \mathcal{O}(\lambda^6) & 0 & 1 \\ -\frac{bg-af}{ac-b^2}\frac{\alpha d_5}{g}\lambda^3 & -\frac{\alpha d_5^2}{ac-b^2}\frac{a}{g}\lambda^{2.5}e^{-i\rho} & -\frac{f}{g}\lambda^2 \end{pmatrix}. \qquad (31)$$

The calculation is taken at the leading order, but $(U_{10}^0)_{13,23,33}$ and $(U_{16}^0)_{21}$ are calculated at the next-leading order because the corresponding leading terms are cancelled. Therefore, the massless modes of $\bar{\mathbf{5}}$ can be written as

$$\bar{\mathbf{5}}_i^0 \equiv (U_{10}^{0\dagger})_{ij}\bar{\mathbf{5}}'_j + (U_{16}^{0\dagger})_{ij}\bar{\mathbf{5}}_j = \begin{pmatrix} \bar{\mathbf{5}}_1 + \cdots \\ \bar{\mathbf{5}}'_1 + \cdots \\ \bar{\mathbf{5}}_2 + \cdots \end{pmatrix}. \qquad (32)$$

In order to calculate neutrino mass, we should also specify the MSSM Higgs doublets H_u and H_d. They are the combinations which remain massless at the GUT scale. According to the Appendix A in [4], they are

$$H_u \subset \mathbf{5}_H \qquad (33)$$

$$H_d \subset \bar{\mathbf{5}}'_H + \beta_H e^{-i\rho}\lambda^{0.5}\bar{\mathbf{5}}_C. \qquad (34)$$

Here β_H is a real $\mathcal{O}(1)$ coefficient and the phase $e^{-i\rho}$ comes from VEVs of the F and Z_4.[6] As mentioned in [4], the second term in (34) is important to ensure the KM phase and make all charged leptons massive.

C. Neutrino mass

Now we calculate neutrino masses in our model. In this subsection we explicitly write the cutoff scale Λ. First, we compute the Dirac neutrino mass matrix. Since we have 6 right-handed neutrino, $\mathbf{1}_i$ and $\mathbf{1}'_i$, from the interactions $\mathbf{10}_i \mathbf{1}'_j \mathbf{10}_H$ and $\mathbf{16}_i \mathbf{16}_j \mathbf{10}_H$, the 3×6 Dirac neutrino Yukawa matrix Y_{ν_D} is given by

$$Y_{\nu_D} = (Y_{\nu_D 1'} Y_{\nu_D 1}) = (U_{10}^{0T}(Y_{1'}^H)^T U_{16}^{0T}(Y_1^H)^T), \quad (35)$$

where

$$Y_{1'}^H = \begin{pmatrix} 0 & h\lambda^5 & 0 \\ -h\lambda^5 & c\lambda^4 & b\lambda^2 \\ 0 & b\lambda^2 & a \end{pmatrix}, \quad (36)$$

$$Y_1^H = \begin{pmatrix} 0 & -\frac{3}{2}d_l\lambda^5 & 0 \\ \frac{3}{2}d_l\lambda^5 & c\lambda^4 & b\lambda^2 \\ 0 & b\lambda^2 & a \end{pmatrix}.$$

It is also important that $\alpha = 0$ in the mixing matrices (30) and (31). Using these facts, we can express the Dirac mass as follows:

$$Y_{\nu_D} = \begin{pmatrix} h\frac{bg-af}{ac-b^2}\lambda^{6.5}e^{i\rho} & -f\lambda^{5.5}e^{i\rho} & -g\lambda^{3.5}e^{i\rho} & 0 & -\frac{3}{2}d_l\lambda^5 & 0 \\ 0 & -h\lambda^5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{3}{2}d_l\lambda^5 & \left(c - \frac{bf}{g}\right)\lambda^4 & \frac{bg-af}{g}\lambda^2 \end{pmatrix}. \quad (37)$$

Note that the second generation would be massless without h. The contribution from the term $\Psi^a \bar{H} H \Psi_a H$ is essential for obtaining the realistic neutrino sector.

Second, we introduce the 6×6 Majorana right-handed neutrino mass matrix. Since each **27** has two 1's, the Majorana mass has following three types of contribution:

$$W = \frac{(Y_{1'1'})_{ij}}{\Lambda} \Psi_i \Psi_j \bar{H}\bar{H} + \frac{(Y_{1'1})_{ij}}{\Lambda} \Psi_i \Psi_j \bar{H}\bar{C} + \frac{(Y_{11})_{ij}}{\Lambda} \Psi_i \Psi_j \bar{C}\bar{C}$$

$$\rightarrow \frac{(Y_{1'1'})_{ij}}{\Lambda} \mathbf{1}'_i \mathbf{1}'_j \langle \mathbf{1}'_{\bar{H}}\rangle \langle \mathbf{1}'_{\bar{H}}\rangle + \frac{(Y_{1'1})_{ij}}{\Lambda} \mathbf{1}'_i \mathbf{1}'_j \langle \mathbf{1}'_{\bar{H}}\rangle \langle \mathbf{1}_{\bar{C}}\rangle + \frac{(Y_{11})_{ij}}{\Lambda} \mathbf{1}_i \mathbf{1}_j \langle \mathbf{1}_{\bar{C}}\rangle \langle \mathbf{1}_{\bar{C}}\rangle. \quad (38)$$

Then the Majorana mass term is 6×6 matrix

$$(\mathbf{1}'_i \mathbf{1}_i)(Y_{N_R^c})_{ij}\begin{pmatrix} \mathbf{1}'_j \\ \mathbf{1}_j \end{pmatrix}\frac{\langle\bar{H}\rangle^2}{\Lambda}, \quad Y_{N_R} = \begin{pmatrix} Y_{1'1'} & Y_{1'1} \\ Y_{1'1}^T & Y_{11} \end{pmatrix}. \quad (39)$$

Here, we parametrize $Y_{N_R}^{-1}$ with $\mathcal{O}(1)$ coefficients as

$$Y_{N_R}^{-1} = \begin{pmatrix} N_{11}\lambda^{-13} & N_{12}\lambda^{-12} & N_{13}\lambda^{-10} & N_{14}\lambda^{-12.5} & N_{15}\lambda^{-11.5} & N_{16}\lambda^{-9.5} \\ N_{12}\lambda^{-12} & N_{22}\lambda^{-11} & N_{23}\lambda^{-9} & N_{24}\lambda^{-11.5} & N_{25}\lambda^{-10.5} & N_{26}\lambda^{-8.5} \\ N_{13}\lambda^{-10} & N_{23}\lambda^{-9} & N_{33}\lambda^{-7} & N_{34}\lambda^{-9.5} & N_{35}\lambda^{-8.5} & N_{36}\lambda^{-6.5} \\ N_{14}\lambda^{-12.5} & N_{24}\lambda^{-11.5} & N_{34}\lambda^{-9.5} & N_{44}\lambda^{-12} & N_{45}\lambda^{-11} & N_{46}\lambda^{-9} \\ N_{15}\lambda^{-11.5} & N_{25}\lambda^{-10.5} & N_{35}\lambda^{-8.5} & N_{45}\lambda^{-11} & N_{55}\lambda^{-10} & N_{56}\lambda^{-8} \\ N_{16}\lambda^{-9.5} & N_{26}\lambda^{-8.5} & N_{36}\lambda^{-6.5} & N_{46}\lambda^{-9} & N_{56}\lambda^{-8} & N_{66}\lambda^{-6} \end{pmatrix}, \quad (40)$$

where N_{ij} are $\mathcal{O}(1)$ complex parameters.

Finally, we can calculate the neutrino mass using the seesaw mechanism. The result is

$$M_\nu \equiv Y_\nu \frac{\langle H_u\rangle^2}{\Lambda}, \quad Y_\nu = Y_{\nu_D} Y_{N_R}^{-1} Y_{\nu_D}^T = \begin{pmatrix} y_{11}\lambda^{-1} & y_{12}\lambda^{-1.5} & y_{13}\lambda^{-2} \\ y_{12}\lambda^{-1.5} & y_{22}\lambda^{-2} & y_{23}\lambda^{-2.5} \\ y_{13}\lambda^{-2} & y_{23}\lambda^{-2.5} & y_{33}\lambda^{-3} \end{pmatrix}, \quad (41)$$

where y_{ij} are complicate combinations of $\mathcal{O}(1)$ coefficient:

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[6]If we choose $z_2 = -1$ (or $z_2 = -2$), then the term $\bar{C}'Z_2^5 C$ (or $\bar{C}'Z_2^2 C$) changes the phase in (34) into arbitrary. For a detailed discussion of the Higgs sector, see Appendix A in [4].

$$y_{11} = N_{11}\left(h\frac{bg-af}{ac-b^2}\right)^2 e^{2i\rho} + N_{22}f^2 e^{2i\rho} + N_{33}g^2 e^{2i\rho} + N_{55}\left(-\frac{3}{2}d_l\right)^2 - 2N_{12}fh\frac{bg-af}{ac-b^2}e^{2i\rho} - 2N_{13}gh\frac{bg-af}{ac-b^2}e^{2i\rho}$$
$$- 2N_{15}h\left(-\frac{3}{2}d_l\right)\frac{bg-af}{ac-b^2} + 2N_{23}fge^{2i\rho} - 2N_{25}f\left(-\frac{3}{2}d_l\right)e^{i\rho} - 2N_{35}g\left(-\frac{3}{2}d_l\right)e^{i\rho}, \tag{42}$$

$$y_{12} = -h\left[N_{12}h\frac{bg-af}{ac-b^2}e^{i\rho} - N_{22}fe^{i\rho} - N_{23}ge^{i\rho} + N_{25}\left(-\frac{3}{2}d_l\right)\right], \tag{43}$$

$$y_{13} = h\frac{bg-af}{ac-b^2}e^{i\rho}\left[N_{14}\left(\frac{3}{2}d_l\right) + N_{15}\left(\frac{ac-b^2}{a} + \frac{bg-af}{b}\frac{b}{a}\right) + N_{16}\frac{bg-af}{g}\right] - fe^{i\rho}\left[N_{24}\left(\frac{3}{2}d_l\right)\right.$$
$$+ N_{25}\left(\frac{ac-b^2}{a} + \frac{bg-af}{b}\frac{b}{a}\right) + N_{26}\frac{bg-af}{g}\right] - ge^{i\rho}\left[N_{34}\left(\frac{3}{2}d_l\right) + N_{35}\left(\frac{ac-b^2}{a} + \frac{bg-af}{b}\frac{b}{a}\right)\right.$$
$$+ N_{36}\frac{bg-af}{g}\right] - \frac{3}{2}d_l\left[N_{45}\left(\frac{3}{2}d_l\right) + N_{55}\left(\frac{ac-b^2}{a} + \frac{bg-af}{b}\frac{b}{a}\right) + N_{56}\frac{bg-af}{g}\right], \tag{44}$$

$$y_{22} = h^2 N_{22}, \tag{45}$$

$$y_{23} = -h\left[N_{24}\left(\frac{3}{2}d_l\right) + N_{25}\left(\frac{ac-b^2}{a} + \frac{bg-af}{b}\frac{b}{a}\right) + N_{26}\frac{bg-af}{g}\right], \tag{46}$$

$$y_{33} = N_{44}\left(\frac{3}{2}d_l\right)^2 + N_{55}\left(\frac{ac-b^2}{a} + \frac{bg-af}{b}\frac{b}{a}\right)^2 + 2N_{46}\left(\frac{3}{2}d_l\right)\frac{bg-af}{g} + 2N_{56}\frac{bg-af}{g}\left(\frac{ac-b^2}{a} + \frac{bg-af}{b}\frac{b}{a}\right). \tag{47}$$

These masses give $\Delta m_{12}^2/\Delta m_{23}^2 \sim \lambda^2 = 4.8 \times 10^{-2}$, which is consistent with the experimental facts $\Delta m_{12}^2/\Delta m_{23}^2 \simeq 3.1 \times 10^{-2}$ [31] up to the combination of $\mathcal{O}(1)$ coefficients.

D. MNS matrix

We can also calculate the MNS matrix in our model. First, the charged lepton Yukawa matrix is given [4] as

$$Y_e = \begin{pmatrix} \left[\frac{bg-af}{ac-b^2}\left(f' + \frac{bg'}{a}\right) - \frac{gg'}{a}\right]\beta_H e^{2i\delta}\lambda^6 & \frac{3}{2}d_l\lambda^5 & 0 \\ 0 & \beta_H f' e^{i\delta}\lambda^{4.5} & \beta_H g' e^{i\delta}\lambda^{2.5} \\ -\frac{3}{2}d_l\lambda^5 & \left(\frac{ac-b^2}{a} + \frac{bg-af}{g}\frac{b}{a}\right)\lambda^4 & \frac{bg-af}{g}\lambda^2 \end{pmatrix}. \tag{48}$$

Then we can calculate the MNS matrix. Following the diagonal procedure given in [32], it is computed, at the leading order,

$$V_{\text{MNS}} = \begin{pmatrix} 1 & v_{12}\lambda^{0.5} & v_{13}\lambda \\ -v_{12}^*\lambda^{0.5} & 1 & v_{23}\lambda^{0.5} \\ (v_{12}^*v_{23}^* - v_{13}^*)\lambda & -v_{23}^*\lambda^{0.5} & 1 \end{pmatrix}, \tag{49}$$

where v_{12}, v_{13}, and v_{23} are written as

$$v_{12} = \frac{\frac{3}{2}d_l}{\beta_H}\frac{1}{1 - gg'\frac{bf+c}{bg-af}} - \frac{y_{12}y_{33} - y_{13}y_{23}}{y_{22}y_{33} - y_{23}^2}, \tag{50}$$

$$v_{13} = -\frac{y_{13}}{y_{33}} - v_{23}^*\frac{y_{12}y_{33} - y_{13}y_{23}}{y_{22}y_{33} - y_{23}^2}, \tag{51}$$

$$v_{23} = \frac{\beta_H gg'}{bg - af} - \frac{y_{23}}{y_{33}}. \tag{52}$$

This result can be compared with experimental values. The model says $\tan\theta_{12} = V_{12}/V_{11} \sim \lambda^{0.5} = 0.47$, $\tan\theta_{23} = V_{23}/V_{33} \sim \lambda^{0.5} = 0.47$ up to the combination of $\mathcal{O}(1)$ coefficients. These are consistent with experiment $\tan\theta_{12} \simeq 0.68$, $\tan\theta_{23} \gtrsim 0.747$ [31]. It is also interesting that recent T2K result [33] $0.087 < \sin\theta_{13} < 0.275$ (assuming $\delta_{CP} = 0$) supports our result $\sin\theta_{13} = |V_{13}| \sim \lambda = 0.22$, up to the combination of $\mathcal{O}(1)$ coefficients.

E. The role of the $\Psi\bar{H}H\Psi H$ terms

As we have seen, the terms $\Psi^a \bar{H} H \Psi_a H$ played an important role for the nonvanishing second generation neutrino mass. Here we show that these types of higher-dimensional operators do not affect other structures of this model, for example, the quark masses and charged lepton masses.

First, we classify the $SO(10)$ invariant Yukawa interaction derived from the E_6 invariant Yukawa term $\Psi_i \Psi_j H$ (or $\Psi_i \Psi_j C$):

$$27_i 27_j 27_{H,C} = 16_i 16_j 10_{H,C} + 10_i 10_j 1_{H,C} + 16_i 10_j 16_{H,C}$$
$$+ 10_i 1_j 10_{H,C}. \tag{53}$$

Note that these terms have common $\mathcal{O}(1)$ coefficients, and all terms are symmetric between i and j, although at the $SO(10)$ level, the coefficients of the terms $16_i 10_j 16_{H,C}$ and $10_i 1_j 10_{H,C}$ do not have to be taken as symmetric.

This property changes if we include higher-dimensional operators like $\Psi_i \bar{H} H \Psi_j H$ or $\Psi_i \bar{H} H \Psi_j C$. Each term in the right-hand side of (53) receives extra contribution, which generically differs from each other. Therefore, they cannot have common $\mathcal{O}(1)$ coefficients, and the number of parameters increases.

Fortunately, in our model, only the (1, 2) and (2, 1) elements of Y^H have room for including $\bar{H}H$, and in the other Yukawa elements, SUSY zero forbids such higher-dimensional operators. Since these elements are antisymmetric due to $SU(2)_F$, the extra contributions $16_i 16_j 10_H$ and $10_i 10_j 1_H$ vanish. Therefore, the higher-dimensional operator including $\bar{H}H$ only affects the Dirac neutrino mass through $10_i 1_j 10_H$, while the quark sector and charged lepton sector is unchanged.

According to the previous paper [4], the model has characteristic predictions $V_{ub} \sim \lambda^4$ and $V_{cb} y_b = y_c$ at the GUT scale. We will show in Appendix B that these predictions are not spoiled even if the model generically contains such higher-dimensional operators. Sufficient conditions for these predictions are $\langle A \rangle \propto Q_{B-L}$ and the (2, 2), (2, 3), (3, 2), and (3, 3) components of Y^C are vanishing. Such conditions are satisfied in more general E_6 GUTs with family symmetry. The former condition should be satisfied to solve the doublet-triplet splitting problem, and the latter can be satisfied by the SUSY-zero mechanism if the anomalous $U(1)_A$ charge of C is smaller than that of H. The discrete symmetry for solving the

SUSY CP problem is not necessary. Therefore, these two predictions are rather general ones. The first prediction $V_{ub} \sim \lambda^4$ has been tested by B-factory experiments, and the second prediction $V_{cb} y_b = y_c$ can be tested in future experiments by measuring the $\tan\beta \equiv \langle H_u \rangle / \langle H_d \rangle$.

IV. SUMMARY AND DISCUSSION

In this paper we conducted a detail calculation of the neutrino sector in the $E_6 \times SU(2)_F$ SUSY GUT with spontaneous CP violation. Originally, the E_6 GUT could explain the realistic neutrino sector as well as quark and charged lepton sector. However, the previous paper reported the problem of the neutrino mass because of the new discrete symmetry, which is introduced in order to solve the SUSY CP problem. We computed the neutrino masses and the MNS matrix explicitly and found that the term like $\Psi_i \bar{H} H \Psi_j H$, which were not considered in the previous paper, is important to make the second generation massive. These higher-dimensional operators only contribute in the neutrino sector, so the structures of quarks and charged leptons, which were calculated in the previous paper, are kept valid.

Our result reproduces the experimental value well. $\Delta m_{12}^2 / \Delta m_{23}^2$, $\tan\theta_{12}$, $\tan\theta_{23}$, and $\sin\theta_{13}$ are all consistent with experimental values up to the combination of $\mathcal{O}(1)$ coefficients. Combining the previous paper and our result, the concrete model reproducing realistic masses and mixings for all the SM fermions is constructed. Note that even if the model has the "modified universal sfermion mass spectrum," we have no SUSY CP problem.

In this model, because of the strong constraint of the Yukawa structure, there are characteristic predictions $V_{ub} \sim \lambda^4$ and $V_{cb} y_b = y_c$ at the GUT scale [4]. These predictions are valid even if we generically include the higher-dimensional operators $\Psi_i \bar{H} H \Psi_j H$ and $\Psi_i \bar{H} H \Psi_j C$. Unfortunately, we could not obtain any characteristic prediction in the neutrino sector because of the huge number of parameters in the Majorana neutrino mass terms.

ACKNOWLEDGMENTS

APPENDIX A: DIAGONALIZATION PROCEDURE OF THE SUPERHEAVY YUKAWA MATRIX

In this appendix, we derive the expression for the two matrices U_{10}^0 (30) and U_{16}^0 (31) which describe the massless $\bar{5}_i^0$ as the combination of $\bar{5}_i$ and $\bar{5}_i'$ as (32). All calculations are performed at the leading order.

First, we show the strategy of finding U which diagonalizes the superheavy Yukawa matrix as (27). In the following, we assume that M_1 has rank 3, and therefore we have an inverse matrix M_1^{-1}. Then we calculate $M_1^{-1}M_2$ and parametrize it as

$$M_1^{-1}M_2 \equiv C \equiv \begin{pmatrix} c_{11}\lambda^{0.5} & c_{12}\lambda^{-0.5} & c_{13}\lambda^{-2.5} \\ c_{21}\lambda^{1.5} & c_{22}\lambda^{0.5} & c_{23}\lambda^{-1.5} \\ c_{31}\lambda^{3.5} & c_{32}\lambda^{2.5} & c_{33}\lambda^{0.5} \end{pmatrix}. \quad (A1)$$

Here c_{ij} is the combination of $\mathcal{O}(1)$ coefficients. After expressing U_{10}^0 and U_{16}^0 in terms of c_{ij}, we will substitute concrete parameters in them. Then we rotate $M_1^{-1}(M_1 M_2) = (1_{3\times3}C)$ as

$$(1_{3\times3}C)U' = (C'0_{3\times3}), \quad (A2)$$

where U' is a 6×6 unitary matrix

$$U' \equiv \begin{pmatrix} U_1 & U_3 \\ U_2 & U_4 \end{pmatrix} \quad (A3)$$

and C' is a 3×3 matrix. If we find unitary matrices V and V' which diagonalize $M_1 C'$ as

$$V^\dagger M_1 C' V' = M_H^{\mathrm{diag}}, \quad (A4)$$

then we can express the matrix U as

$$U \equiv \begin{pmatrix} U_{10}^H & U_{10}^0 \\ U_{16}^H & U_{16}^0 \end{pmatrix} = U' \begin{pmatrix} V' & 0 \\ 0 & T \end{pmatrix} = \begin{pmatrix} U_1 V' & U_3 T \\ U_2 V' & U_4 T \end{pmatrix}. \quad (A5)$$

Here T is a unitary matrix which corresponds to the degree of freedom of the rotation for the three 0's keeping with (27).

Since we are only interested in U_{10}^0 and U_{16}^0 in this paper, our task is to find the matrix U'. After tedious calculation, we can find C' and U' as

$$(1_{3\times3}C)U' = \begin{pmatrix} c_{31}\lambda^{-2.5} & 0 & 0 & 0 & 0 & 0 \\ c_{32}\lambda^{-1.5} & 1 & 0 & 0 & 0 & 0 \\ c_{33}\lambda^{0.5} & \left(c_{22}-\frac{c_{12}c_{23}}{c_{13}}\right)\left(c_{32}-\frac{c_{12}c_{33}}{c_{13}}\right)\lambda^3 & 1 & 0 & 0 & 0 \end{pmatrix}, \quad (A6)$$

$$U' = \begin{pmatrix} \frac{1}{c_{13}}\lambda^{2.5} & -\frac{c_{23}}{c_{13}}\lambda & -\frac{c_{33}}{c_{13}}\lambda^3 & \frac{c_{23}}{c_{13}}\left(c_{21}-\frac{c_{11}c_{23}}{c_{13}}\right)\lambda^{2.5} & 1 & 0 \\ 0 & 1 & -\left(c_{22}-\frac{c_{12}c_{23}}{c_{13}}\right)(c_{32}-)\lambda^3 & -\left(c_{21}-\frac{c_{11}c_{23}}{c_{13}}\right)\lambda^{1.5} & \frac{c_{23}}{c_{13}}\lambda & -\left(c_{22}-\frac{c_{12}c_{23}}{c_{13}}\right)\lambda^{0.5} \\ 0 & 0 & 1 & -\left(c_{31}-\frac{c_{11}c_{33}}{c_{13}}\right)\lambda^{3.5} & \frac{c_{33}}{c_{13}}\lambda^3 & -\left(c_{32}-\frac{c_{12}c_{33}}{c_{13}}\right)\lambda^{2.5} \\ \frac{c_{11}}{c_{13}}\lambda^3 & \left(c_{21}-\frac{c_{11}c_{23}}{c_{13}}\right)\lambda^{1.5} & \left(c_{31}-\frac{c_{11}c_{33}}{c_{13}}\right)\lambda^{3.5} & 1 & 0 & 0 \\ \frac{c_{12}}{c_{13}}\lambda^2 & \left(c_{22}-\frac{c_{12}c_{23}}{c_{13}}\right)\lambda^{0.5} & \left(c_{32}-\frac{c_{12}c_{33}}{c_{13}}\right)\lambda^{2.5} & -\left(c_{21}-\frac{c_{11}c_{23}}{c_{13}}\right) \times \left(c_{22}-\frac{c_{12}c_{23}}{c_{13}}\right)\lambda^2 & \frac{c_{23}}{c_{13}}\left(c_{22}-\frac{c_{12}c_{23}}{c_{13}}\right)\lambda^{1.5} & 1 \\ 1 & -\frac{c_{12}}{c_{13}}\left(c_{22}-\frac{c_{12}c_{23}}{c_{13}}\right)\lambda^{2.5} & -\frac{c_{12}}{c_{13}}\left(c_{32}-\frac{c_{12}c_{33}}{c_{13}}\right)\lambda^{4.5} & -\frac{c_{11}}{c_{13}}\lambda^3 & -\frac{1}{c_{13}}\lambda^{2.5} & -\frac{c_{12}}{c_{13}}\lambda^2 \end{pmatrix} \quad (A7)$$

We choose a basis which $(U_{16}^0)_{12}$, $(U_{16}^0)_{13}$ and $(U_{16}^0)_{22}$ are zero by the rotation

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{c_{23}}{c_{13}}\left(c_{22}-\frac{c_{12}c_{23}}{c_{13}}\right)\lambda^{1.5} \\ 0 & -\frac{c_{23}}{c_{13}}\left(c_{22}-\frac{c_{12}c_{23}}{c_{13}}\right)\lambda^{1.5} & 1 \end{pmatrix}. \quad (A8)$$

After that, we can get U_{10}^0 and U_{16}^0 in terms of c_{ij} as

$$U_{10}^0 = \begin{pmatrix} \frac{c_{23}}{c_{13}}\left(c_{21}-\frac{c_{11}c_{23}}{c_{13}}\right)\lambda^{2.5} & 1 & \frac{c_{23}}{c_{13}}\left(c_{22}-\frac{c_{12}c_{23}}{c_{13}}\right)\lambda^{1.5} \\ -\left(c_{21}-\frac{c_{11}c_{23}}{c_{13}}\right)\lambda^{1.5} & \frac{c_{23}}{c_{13}}\lambda & -\left(c_{22}-\frac{c_{12}c_{23}}{c_{13}}\right)\lambda^{0.5} \\ -\left(c_{31}-\frac{c_{11}c_{33}}{c_{13}}\right)\lambda^{3.5} & -\frac{c_{33}}{c_{13}}\lambda^3 & \left(c_{32}-\frac{c_{12}c_{33}}{c_{13}}\right)\lambda^{2.5} \end{pmatrix},$$

$$(A9)$$

$$U_{16}^0 = \begin{pmatrix} 1 & 0 & 0 \\ -\left(c_{21}-\frac{c_{11}c_{23}}{c_{13}}\right)\left(c_{22}-\frac{c_{12}c_{23}}{c_{13}}\right)\lambda^2 & 0 & 1 \\ -\frac{c_{11}}{c_{13}}\lambda^3 & -\frac{1}{c_{13}}\lambda^{2.5} & -\frac{c_{12}}{c_{13}}\lambda^2 \end{pmatrix}.$$

$$(A10)$$

We can check these matrices satisfy the relation (29) at the leading order. Finally, we obtain (30) and (31) by substituting the concrete expressions in c_{ij}:

$$c_{11} = \frac{bg-af}{a\alpha d_5}, \quad c_{12} = \frac{ac-b^2}{\alpha d_5^2}\frac{f}{a}, \quad c_{13} = \frac{ac-b^2}{\alpha d_5^2}\frac{g}{a}$$

$$(A11)$$

$$c_{21} = 0, \quad c_{22} = \frac{f}{\alpha d_5}, \quad c_{23} = \frac{g}{\alpha d_5} \quad (A12)$$

$$c_{31} = \frac{g}{a}, \quad c_{32} = -\frac{bf}{a\alpha d_5}, \quad c_{33} = -\frac{bg}{a\alpha d_5}. \quad (A13)$$

APPENDIX B: THE CONDITION OF THE PREDICTIONS

In Sec. III, we obtained the realistic neutrino sector by introducing the higher-dimensional operator $\Psi^a\bar{H}H\Psi_a H$, and saw that such operators cannot be written in other elements of Y^H and Y^C because of the SUSY-zero mechanism. In this appendix, we will clarify the conditions of the characteristic predictions $V_{ub} \sim \lambda^4$ and $V_{cb}y_b = y_c$, and show that the conditions can be satisfied even if we generically include the higher-dimensional operators $\Psi_i\bar{H}H\Psi_j H$ and $\Psi_i\bar{H}H\Psi_j C$.

In order to see the essence of the predictions, let us choose the basis where the up-type Yukawa matrix Y_u is diagonal. In this basis, V_{ub} is simply written only by the down-type Yukawa matrix as $V_{ub} \sim (Y_d)_{13}/(Y_d)_{33}$. The relation $V_{cb}y_b = y_c$ is also simplified as $(Y_d)_{23} = (Y_u)_{22}$. Therefore, we focus on the quantities $(Y_d)_{13}$ and $(Y_d)_{23}$.

Let us determine the third generation of the massless $\bar{\mathbf{5}}$ combination $\bar{\mathbf{5}}_3^0$, which is mainly $\bar{\mathbf{5}}_2$, in this basis. Since we can choose the basis where $\bar{\mathbf{5}}_3^0$ does not contain $\bar{\mathbf{5}}_1$ and $\bar{\mathbf{5}}_1'$, we parametrize

$$\bar{\mathbf{5}}_3^0 \equiv \frac{1}{\sqrt{1+|\alpha|^2+|\beta|^2+|\gamma|^2}}(\bar{\mathbf{5}}_2 + \alpha\bar{\mathbf{5}}_2' + \beta\bar{\mathbf{5}}_3' + \gamma\bar{\mathbf{5}}_3).$$

$$(B1)$$

As we saw in Sec. III B, the massless modes are determined by diagonalizing the 3×6 mass matrix $(M_1 M_2)$. Both Y_u and M_1 is obtained from Y^H, but here we consider general case where these two matrices cannot be diagonalized simultaneously. In this case, M_1 is the form

$$M_1 \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}\begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}\begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

$$\sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}. \quad (B2)$$

On the other hand, M_2 is obtained from Y^C. Again, we consider general case

$$M_2 \sim \lambda^{0.5}\begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}\begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}\begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

$$\sim \begin{pmatrix} \lambda^{6.5} & \lambda^{5.5} & \lambda^{3.5} \\ \lambda^{5.5} & \lambda^{4.5} & \lambda^{2.5} \\ \lambda^{3.5} & \lambda^{2.5} & \lambda^{0.5} \end{pmatrix}. \quad (B3)$$

Then we can obtain the massless combination of $\bar{\mathbf{5}}_3^0$ by solving

$$(M_1 M_2)\begin{pmatrix} 0 \\ \alpha \\ \beta \\ 0 \\ 1 \\ \gamma \end{pmatrix} = 0. \quad (B4)$$

The solution becomes generically

$$\alpha \sim \lambda^{0.5}, \quad \beta \sim \lambda^{2.5}, \quad \gamma \sim \lambda^2. \quad (B5)$$

Let us evaluate $(Y_d)_{13}$ and $(Y_d)_{23}$ to obtain the conditions for $V_{ub} \sim \lambda^4$ and $V_{cb}y_b = y_c$. The down-type Yukawa matrix Y_d consists of two types of interaction: one is the interaction between **10** and $\bar{\mathbf{5}}$ via Y_{down}^H, the other is between **10** and $\bar{\mathbf{5}}'$ via $\lambda^{0.5}Y^C$. Here we denote the matrix Y^H appearing in the down-type quark Yukawa matrix as Y_{down}^H. Y^C is written in this basis as

$$Y^C \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \begin{pmatrix} \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^3 & \lambda \\ \lambda^2 & \lambda & \lambda^{-1} \end{pmatrix} \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

$$\sim \begin{pmatrix} \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^3 & \lambda \\ \lambda^2 & \lambda & \lambda^{-1} \end{pmatrix}. \tag{B6}$$

For example, the leading contributions for $(Y_d)_{13}$ are $(Y^H_{\text{down}})_{12}$, $\gamma(Y^H_{\text{down}})_{13}$, $\alpha\lambda^{0.5}(Y^C)_{12}$, and $\beta\lambda^{0.5}(Y^C)_{13}$. They are of order λ^5, and therefore V_{ub} becomes $\mathcal{O}(\lambda^3)$. In order to obtain $V_{ub} \sim \lambda^4$, all the leading contributions must vanish. The first two contributions vanish if $\langle A \rangle$ is proportional to $Q_{B\text{-}L}$, which plays an important role in solving the doublet-triplet splitting problem. This is because Y^H_{down} is the same as Y_u and therefore the off-diagonal elements of Y^H_{down} are vanishing in the basis in which Y_u is diagonal. Another important point is that the $U(1)_A$ charge of C is smaller than that of H by 1. This nature forbids the (2, 2), (2, 3), (3, 2) and (3, 3) elements (before rotating the basis) of Y^C by the SUSY-zero mechanism.[7] Therefore, after rotating, Y^C becomes

$$Y^C \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \begin{pmatrix} 0 & \lambda^4 & \lambda^2 \\ \lambda^4 & 0 & 0 \\ \lambda^2 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

$$\sim \begin{pmatrix} \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^5 & \lambda^3 \\ \lambda^2 & \lambda^3 & \lambda^5 \end{pmatrix}. \tag{B7}$$

This changes the determination of massless $\bar{\mathbf{5}}^0_3$ composition (B5) into

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[7] In our model, the (1,1) entry of Y^C is also forbidden by the discrete symmetry.

$$\alpha \sim \lambda^{2.5}, \qquad \beta \sim \lambda^{4,5}, \qquad \gamma \sim \lambda^2, \tag{B8}$$

so the terms containing α and β become smaller. The above two points are the essential points for $V_{ub} = 0$ at the leading order.

Next, we see $(Y_d)_{23}$. The main contributions are from $\gamma(Y^H_{\text{down}})_{23}$, $(Y^H_{\text{down}})_{22}$, $\alpha\lambda^{0.5}(Y^C)_{22}$, and $\beta\lambda^{0.5}(Y^C)_{23}$, but the first contribution is vanishing if $\langle A \rangle$ is proportional to Q_{B-L}.[8] The second term is nothing but $(Y_u)_{22}$, so the relation $V_{cb}y_b = y_c$ is realized when α and β become smaller due to the SUSY-zero mechanism as noted above.

How does above discussion changed if we generically include the higher-dimensional operators $\Psi_i \bar{H} H \Psi_j H$ and $\Psi_i \bar{H} H \Psi_j H$? First, Y_u and Y^H_{down} are derived from same expression $16_i 16_j 10_H$ at the $SO(10)$ level, therefore simultaneous diagonalizability of Y_u and Y^H_{down} is not affected by the higher-dimensional operators. Second, the form of Y^C with the SUSY-zero mechanism (B7) does not change because adding $\bar{H} H$ ($\bar{h} + h < 0$) cannot revive the terms which were forbidden by the SUSY-zero mechanism. Therefore, the characteristic predictions $V_{ub} \sim \lambda^4$ and $V_{cb}y_b = y_c$ are affected whether or not we include such higher-dimensional operators.

What becomes obvious in this appendix is that the conditions for $V_{ub} \sim \lambda^4$ and $V_{cb}y_b = y_c$ can be satisfied in the more general E_6 GUTs with family symmetry. The discrete symmetry for solving the SUSY CP problem is not necessary. One prediction $V_{ub} \sim \lambda^4$ has been already tested by the B-factory experiments. The other prediction $V_{cb}y_b = y_c$ at the GUT scale means that comparably small $\tan\beta \equiv \langle H_u \rangle / \langle H_d \rangle$, which can be tested in future experiments.

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[8] If the adjoint Higgs A couples only to the (1,2) [and (2,1)] components as in the model we are considering, then the condition that $\langle A \rangle$ is proportional to Q_{B-L} is not necessary, because Y^H_{down} is almost Y_u and therefore $\gamma(Y^H_{\text{down}})_{23}$ becomes very small in the basis in which Y_u is diagonal.

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