Production of two $c\bar{c}$ pairs in double-parton scattering

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We discuss production of two pairs of $c\bar{c}$ within a simple formalism of double-parton scattering. Surprisingly very large cross sections, comparable to single-parton scattering single $c\bar{c}$ pair production contribution, are predicted for LHC energies. Both the total inclusive cross section as a function of energy and differential distributions in rapidity and transverse momentum of charm quark/antiquark as well as some correlation distributions for $\sqrt{s} = 7$ TeV are shown. We have presented results for commonly used factorized Ansatz and with QCD-evolved double-parton distributions as discussed recently in the literature. The difference found between results of the naive and QCD refined approaches is found to be rather small. We discuss perspectives on how to identify the double-scattering contribution. The region of the phase space when cc, $\bar{c}\,\bar{c}$, $c\bar{c}$ are produced with a large rapidity interval between them is potentially very promising. We have compared the results of the double- and single-scattering $c\bar{c}c\bar{c}$ pair production mechanisms. The contribution of the latter mechanism is much smaller. A good signature of the $c\bar{c}c\bar{c}$ production is e.g. a measurement of two identical D^0D^0 or $\bar{D}^0\bar{D}^0$ mesons in one event. Predictions for ATLAS, CMS, ALICE, and LHCb are presented. Other options of possible measurements are discussed.

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I. INTRODUCTION AND FORMALISM

It is commonly believed that gluon-gluon fusion is the dominant mechanism of heavy Q or heavy $\bar{Q}$ production at high energies. Then in leading-order (LO) approximation the differential cross section for the single-parton scattering (SPS) production of heavy quark and heavy antiquark pair reads:

$$\frac{d\sigma}{dy_1 dy_2 d^2p_t} = \frac{1}{16\pi^2\hat{s}^2} x_1 g(x_1, \mu^2) x_2 g(x_2, \mu^2) \overline{|\mathcal{M}_{gg\to Q\bar{Q}}|^2}, \tag{1.1}$$

where longitudinal momentum fractions can be calculated from kinematical variables of final quark and antiquark as $x_1 = \frac{m_t}{\sqrt{s}}(\exp(y_1) + \exp(y_2))$, $x_2 = \frac{m_t}{\sqrt{s}}(\exp(-y_1) + \exp(-y_2))$ with y's being quark (antiquark) rapidities and m_t being a quark (antiquark) transverse mass. The leading-order matrix elements squared $\overline{|\mathcal{M}_{gg\to Q\bar{Q}}|^2}$ can be found e.g. in [1].

We have limited here to gluon-gluon fusion only. The quark-antiquark annihilation plays some role only close to the kinematical threshold and/or large rapidities. At Tevatron and LHC the quark-antiquark annihilation is practically negligible. The next-to-leading-order approach was developed already some time ago [2]. In general, the higher-order corrections do not change most of single-particle observables leading to a rough renormalization of the cross section by the so-called K factor (see e.g. [3] and references therein). The K factor for $c\bar{c}$ production is not too big when comparing leading-order results with leading-order gluon distributions and next-to-leading-order results with next-to-leading-order gluon distribution.

In the present paper we wish to estimate for the first time the contribution of double-parton scatterings (DPS) to $(c\bar{c})(c\bar{c})$ production. The mechanism of double-parton scattering production of two pairs of heavy quark and heavy antiquark is shown in Fig. 1 together with corresponding mechanism of single-scattering production.

The double-parton scattering has been recognized and discussed already in the seventies and eighties [4–12]. The activity stopped when it had been realized that the DPS contribution was negligible at the small energies. Several estimates of the cross section for different processes have been presented in recent years [13–22]. The theory of the double-parton scattering is quickly developing (see e.g. [23–30]) which is partly driven by experiments at the LHC.

In the present analysis we wish to concentrate on the production of the $(c\bar{c})(c\bar{c})$ four-parton final state which has not been carefully discussed so far, but, as will be shown here, is particularly interesting especially in the context of experiments being carried out at the LHC and/or high-energy atmospheric and cosmogenic neutrinos (antineutrinos).

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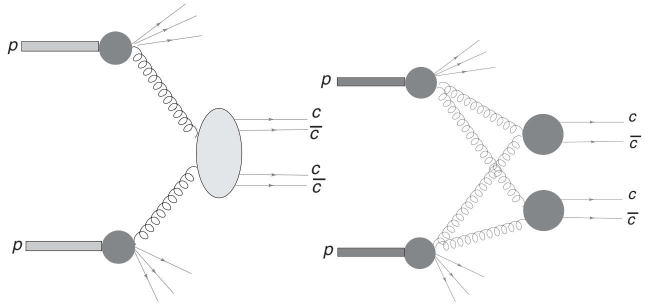

FIG. 1. SPS (left) and DPS (right) mechanisms of $(c\bar{c})(c\bar{c})$ production.

Because of its intermediate mass the charm quark production is close to the border of applicability of perturbative QCD. However, the perturbative QCD approach was shown by different authors to give a reasonable description of the inclusive charm data for different reactions such as photon-photon, photon-proton, and proton-proton [31–33]. The not too big scales for $c\bar{c}$ production lead to relatively large uncertainties, partially discussed in this paper (see also [34,35]), much larger than for beauty or top quarks/antiquarks. If the effects at low transverse momenta of c or $\bar{c}$ are not fully perturbative for the $(c\bar{c})(c\bar{c})$ production, then an extra cut on transverse momenta $p_t > p_{t,\min}$ of quarks or mesons can be imposed in addition. In practice this is the case when lower cut restrictions on meson or lepton transverse momenta are imposed by experimental setups.

The double-parton scattering formalism in the simplest form assumes two single-parton scatterings. Then in a simple probabilistic picture the cross section for double-parton scattering can be written as

$$\sigma^{\rm DPS}(pp \to c\bar{c}c\bar{c}X) = \frac{1}{2\sigma_{\rm eff}} \sigma^{\rm SPS}(pp \to c\bar{c}X_1) \cdot \sigma^{\rm SPS}(pp \to c\bar{c}X_2). \quad (1.2)$$

This formula assumes that the two subprocesses are not correlated and do not interfere. At low energies one has to include parton momentum conservation i.e. extra limitations: $x_1 + x_3 < 1$ and $x_2 + x_4 < 1$, where x_1 and x_3 are longitudinal momentum fractions of gluons emitted from one proton and x_2 and x_4 their counterparts for gluons emitted from the second proton. The "second" emission must take into account that some momentum was used up in the "first" parton collision. This effect is important at large quark or antiquark rapidities. Experimental data [36] provide an estimate of $\sigma_{\rm eff}$ in the denominator of formula (1.2). In our analysis we take a rather conservative value of $\sigma_{\rm eff} = 15$ mb.

The simple formula (1.2) can be generalized to include differential distributions. Again in leading-order approximation differential distribution can be written as

$$\frac{d\sigma}{dy_1 dy_2 d^2p_{1t} dy_3 dy_4 d^2p_{2t}} = \frac{1}{2\sigma_{\rm eff}} \frac{d\sigma}{dy_1 dy_2 d^2p_{1t}} \cdot \frac{d\sigma}{dy_3 dy_4 d^2p_{2t}}, \quad (1.3)$$

which by construction reproduces the formula for integrated cross section (1.2). This cross section is formally differential in 8 dimensions but can be easily reduced to 7 dimensions noting that physics of unpolarized scattering cannot depend on azimuthal angle of the pair or on azimuthal angle of one of the produced c ($\bar{c}$) quark (antiquark). The differential distributions for each single-scattering step can be written in terms of collinear gluon distributions with longitudinal momentum fractions x_1, x_2, x_3, and x_4 expressed in terms of rapidities y_1, y_2, y_3, y_4 and transverse momenta of quark (or antiquark) for each step (in the LO approximation identical for quark and antiquark).

A more general formula for the cross section can be written formally in terms of double-parton distributions (dPDF), e.g. F_{gg}, F_{qq}, etc. In the case of heavy quark (antiquark) production at high energies:

$$d\sigma^{\rm DPS} = \frac{1}{2\sigma_{\rm eff}} F_{gg}(x_1, x_3, \mu_1^2, \mu_2^2) F_{gg}(x_2, x_4, \mu_1^2, \mu_2^2) \times d\sigma_{gg\to c\bar{c}}(x_1, x_2, \mu_1^2) d\sigma_{gg\to c\bar{c}}(x_3, x_4, \mu_2^2) \times dx_1 dx_2 dx_3 dx_4. \quad (1.4)$$

It is physically motivated to write the double-parton distributions rather in the impact parameter space $F_{gg}(x_1, x_2, b) = g(x_1)g(x_2)F(b)$, where g are usual conventional parton distributions and $F(b)$ is an overlap of the matter distribution in the transverse plane where b is a distance between both gluons [37]. The effective cross section in (1.2) is then $1/\sigma_{\rm eff} = \int d^2b F^2(b)$ and in this approximation is energy independent.

The double-parton distributions in Eq. (1.4) are generally unknown. Usually one assumes a factorized form and expresses them via standard distributions for SPS. Even if factorization is valid at some scale, QCD evolution may lead to a factorization breaking. For some time the evolution was known only when the scale of both scatterings is the same [23,24,26] i.e. for heavy object, like double gauge boson production. Recently the evolution of dPDF was discussed also in the case of different scales [38]. This scheme was in fact implemented in practical applications in Ref. [26].

For double $c\bar{c}$ production this is not the case and has not been discussed so far in the literature. In the present study we shall apply both the factorized model commonly used in the literature [see Eq. (1.3)] and the approach using double-parton distributions [see Eq. (1.4)] fulfilling relevant evolution equations [26,38]. In explicit calculations presented below we use leading-order collinear gluon distributions (GRV94 [39], CTEQ6 [40], GJR08 [41], MSTW08 [42]) when applying simple factorized Ansatz and GS09 [26] double-gluon distribution in the QCD corrected formula.

Although in the present paper we concentrate on the double-parton contribution to the $c\bar{c}c\bar{c}$ final state, we shall also present some results for single-parton scattering contribution discussed for the first time very recently [43]. The relevant calculation is much more difficult technically as it requires explicit calculation of the $2 \to 4$ contributions on the parton level.

In practice rather mesons or nonphotonic electrons are measured and not quarks or antiquarks. In the present exploratory calculation we shall also show some selected results for charmed mesons. A good signature of the $c\bar{c}c\bar{c}$ final state is a production of two mesons, both containing c quark or two mesons both containing $\bar{c}$ antiquark. Here, as an example, we shall consider production of $D^0 D^0$ and $\bar{D}^0 \bar{D}^0$ meson pairs in one physical event. The measurement of D^0 or $\bar{D}^0$ is probably the easiest experimentally. The hadronization of c quarks ($\bar{c}$ antiquarks) to the charmed mesons will be done in terms of phenomenological hadronization functions as described in [34,35].

II. RESULTS AND OUTLOOK

A. Quarks and antiquarks

In Fig. 2 we compare cross sections for the single $c\bar{c}$ pair production as well as for single-parton and double-parton scattering $c\bar{c}c\bar{c}$ production as a function of proton-proton center-of-mass energy. In the left panel we present uncertainties due to the choice of gluon distributions and in the right panel those due to the choice of renormalization and factorization scale. At low energies the conventional single $c\bar{c}$ pair production dominates. The cross section for DPS production of the $c\bar{c}c\bar{c}$ system is more than 2 orders of magnitude smaller than that for single $c\bar{c}$ production. At high energy the situation reverses. For reference we show the proton-proton total cross section as a function of energy as parametrizes in Ref. [44]. At low energy the $c\bar{c}$ or $c\bar{c}c\bar{c}$ cross sections are much smaller than the total cross section. At higher energies the contributions dangerously approach the expected total cross section.[1] This shows that inclusion of unitarity effect and/or saturation of parton distributions may be necessary. The effect of saturation in $c\bar{c}$ production has been included e.g. in Refs. [45–47] but not checked versus experimental data. Presence of double-parton scattering changes the situation. The double-parton scattering is therefore potentially a very important ingredient in the context of high-energy neutrino production in the atmosphere [47–49] or of cosmogenic origin [50]. We leave this rather difficult issue for future studies in which the LHC charm data must be included. At LHC energies the cross section for both terms becomes comparable.[2] This is a completely new situation when the double-parton scattering gives a huge contribution to inclusive charm production.

In Figs. 3 and 4, we present single c ($\bar{c}$) distributions. Within approximations made in this paper the distributions are identical in shape to single-pair production distributions. This means that the double-scattering contribution produces naturally an extra center-of-mass energy dependent K factor to be contrasted with an approximately energy-independent K factor due to next-to-leading-order corrections. One can see a strong dependence on the factorization and renormalization scales which at high energies produce almost factor 5 uncertainties and precludes a more precise estimation. A better estimate could be done when LHC charm data are published and the theoretical distributions are somewhat adjusted to experimental data.

So far we have discussed only single-particle spectra of c or $\bar{c}$ (rapidity, transverse momentum distributions) which due to factorization and renormalization scale dependence do not provide a clear test of the existence of double-parton scattering contributions. A more stringent test could be performed by studying correlation observables. In particular, correlations between c and $\bar{c}$ are very interesting even without double-parton scattering terms [33]. In Fig. 5 we show distribution in the difference of c and $\bar{c}$ rapidities $y_{\mathrm{diff}} = y_c - y_{\bar{c}}$ (left panel) as well as in the $c\bar{c}$ invariant mass $M_{c\bar{c}}$ (right panel). We show both terms: when $c\bar{c}$ are emitted in the same parton scattering ($c_1\bar{c}_2$ or $c_3\bar{c}_4$) and when they are emitted from different parton scatterings ($c_1\bar{c}_4$ or $c_2\bar{c}_3$). In the latter case we observe a long tail for a large rapidity difference as well as at large invariant masses of $c\bar{c}$.

In particular, cc (or $\bar{c}\,\bar{c}$) should be predominantly produced from two different parton scatterings which opens a possibility to study the double-scattering processes. Of course, a small amount may come from single-parton scattering production of $c\bar{c}c\bar{c}$ discussed by one of us very recently [43].

In Fig. 6 we present distribution in the transverse momentum of the $c\bar{c}$ (or cc or $\bar{c}\,\bar{c}$) pair $|\vec{p}_{\perp c\bar{c}}|$, where $\vec{p}_{\perp c\bar{c}} = \vec{p}_{\perp c} + \vec{p}_{\perp \bar{c}}$. For comparison this is a Dirac delta function in the leading-order approximation to single-pair $c\bar{c}$ production. In contrast, the double-parton scattering mechanism provides a broad distribution extending to large transverse momenta. Next-to-leading-order corrections obviously destroy the δ-like leading-order correlation but the corresponding spectra are not as hard as those for DPS. We believe that similar distributions for $D\bar{D}$ or/and e^+e^- or $\mu^+\mu^-$ pairs would be a useful observables to identify the DPS contributions but this requires real Monte Carlo simulations, including actual limitations of experimental

[1] New experiments at LHC will provide new input for parametrizations of the total cross section.

[2] If the inclusive cross section for c or $\bar{c}$ is shown, the cross section should be multiplied by a factor of 2 — c or two $\bar{c}$ in each event.

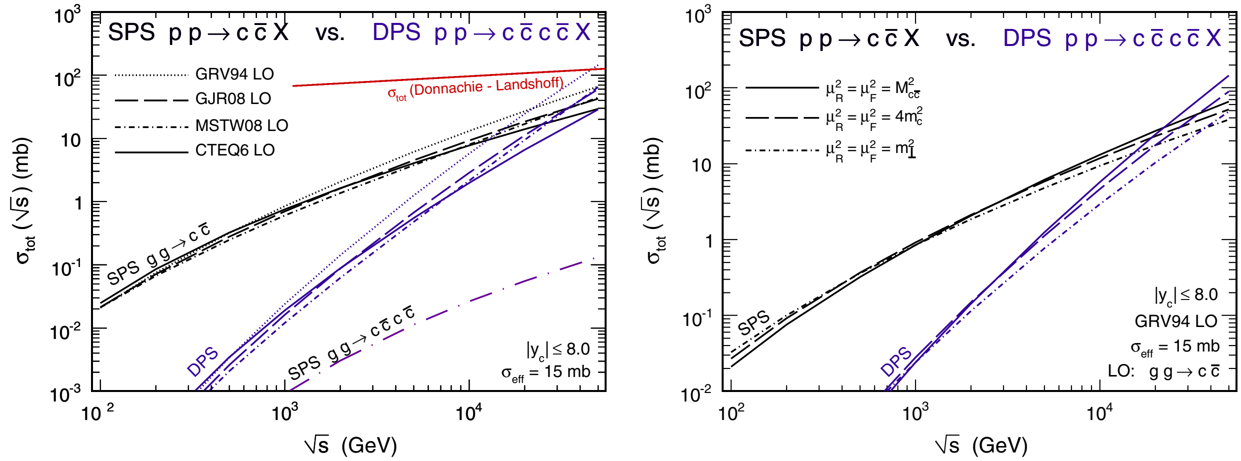

FIG. 2 (color online). Total LO cross section for single $c\bar{c}$ pair and SPS and DPS $c\bar{c}c\bar{c}$ production as a function of center-of-mass energy (left panel) and uncertainties due to the choice of (factorization, renormalization) scales (right panel). We show in addition a parametrization of the total cross section in the left panel. The cross section for $c\bar{c}c\bar{c}$ should be multiplied in addition by a factor 2 in the case when all c ($\bar{c}$) are counted.

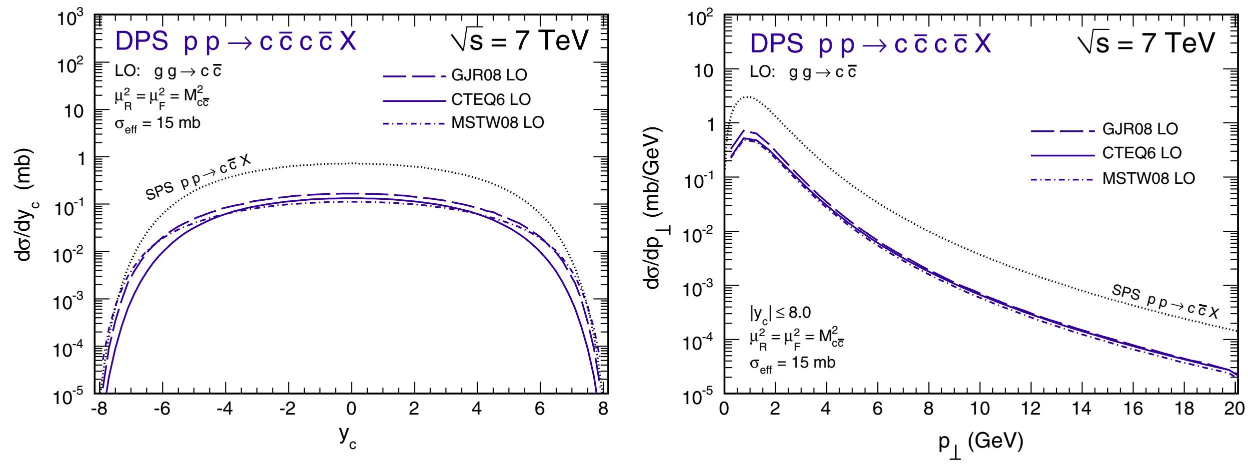

FIG. 3 (color online). Distribution in rapidity (left panel) and transverse momentum (right panel) of c or $\bar{c}$ quarks at $\sqrt{s} = 7$ TeV. Cross section for DPS should be multiplied in addition by a factor 2 in the case when all c ($\bar{c}$) are counted.

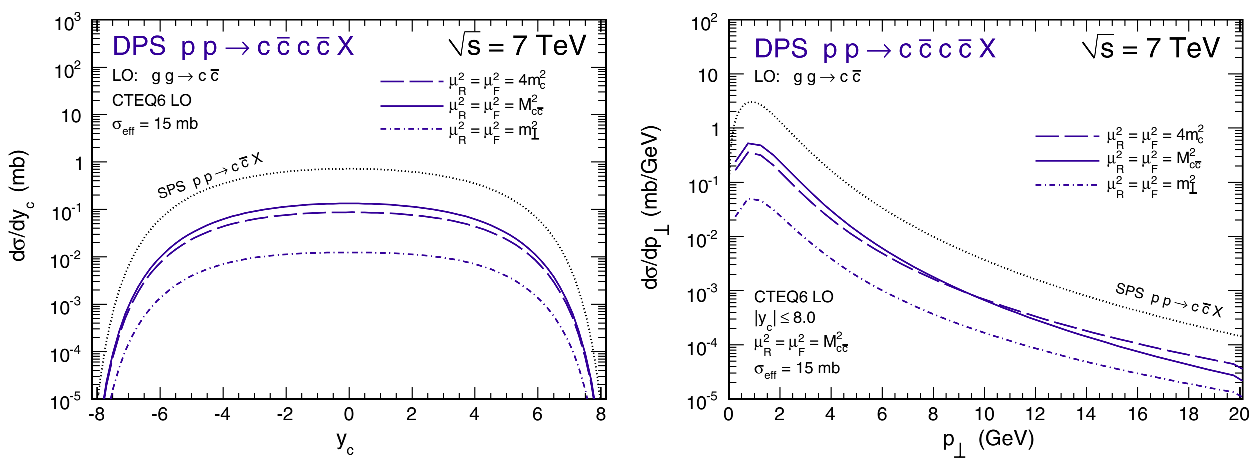

FIG. 4 (color online). Uncertainties related to renormalization and factorization scales choice for distributions in rapidity (left panel) and transverse momentum (right panel) of c or $\bar{c}$ quarks at $\sqrt{s} = 7$ TeV. Cross section for DPS should be multiplied in addition by a factor 2 in the case when all c ($\bar{c}$) are counted.

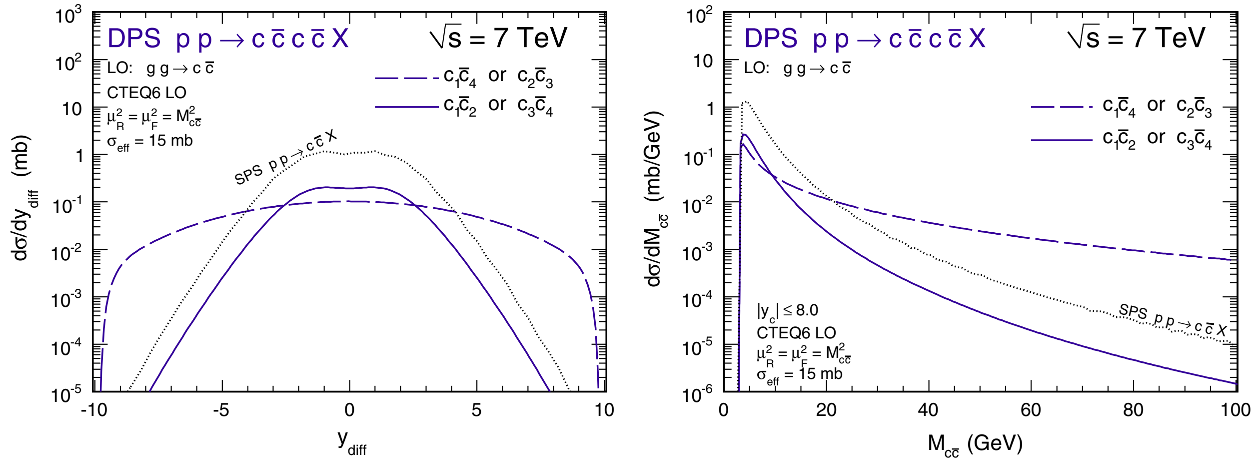

FIG. 5 (color online). Distribution in rapidity difference (left panel) and in invariant mass of the $c\bar{c}$ pair (right panel) at $\sqrt{s} = 7$ TeV.

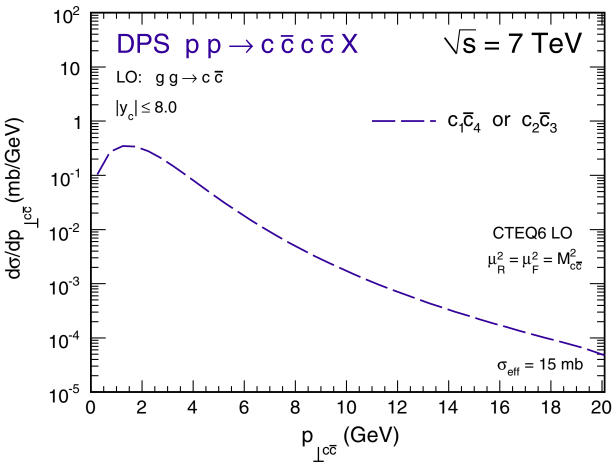

FIG. 6 (color online). Distribution in transverse momentum of $c\bar{c}$ pairs from the different parton scatterings at $\sqrt{s} = 7$ TeV.

apparatus. Correlations between outgoing nonphotonic electrons has been studied at much lower Relativistic Heavy Ion Collider energy in Ref. [35].

In the present approach we have calculated cross section in a simple leading-order approach. A better approximation would be to include multiple gluon emissions. This can be done e.g. in soft gluon resummation or in the k_t-factorization approach. For example, the second approach does not lead to dramatic changes in neither distribution in rapidity nor of distributions in transverse momentum of c ($\bar{c}$) (see e.g. [33]) compared to the collinear approach. It could, however, change distributions in the transverse momentum of $c\bar{c}$ or in the azimuthal angle between c and $\bar{c}$ [33]. This will be discussed in detail elsewhere.

Our predictions using the simple factorized model, especially when the DPS cross section is large, should be taken with caution as here potentially some corrections could be important. We believe, however, that our simple estimate can be used e.g. to calculate the cross section for semihard mesons that are being measured e.g. by the LHCb Collaboration. A future comparison would be very useful to shed new light on the approach as well as on our understanding of DPS.

Up to now we have presented results for the simple factorized Ansatz. In Fig. 7 we compare the results of the simple factorized Ansatz and those for double-parton distributions with QCD evolution [26]. The effect of the QCD evolution of dPDFs is relatively small and can be safely neglected taking into account all other uncertainties, in particular, those due to the choice of the factorization and renormalization scale.

Production of two $c\bar{c}$ pairs in the leading-order approximation is only a first step in trying to identify DPS contribution. In the next step we are planning next-to-leading-order calculation of the same process as well as calculation in the k_t-factorization approach.

B. Mesons and outlook

The distributions of quarks and/or antiquarks cannot be directly measured. These are rather mesons (or baryons) that are measured experimentally. The ALICE Collaboration can measure charmed mesons in a relatively narrow interval of (pseudo)rapidity $-0.9 < \eta_D < 0.9$ but in broad range of transverse momenta. In principle, all main collaborations at the LHC can measure charmed mesons. The ATLAS and CMS detectors can measure charmed meson in the pseudorapidity region of $-2.5 < \eta_D < 2.5$. This means that the pseudorapidity difference up to 5 units is possible using only the main detectors. For the identification of double-scattering terms rather large rapidity differences between cc or $\bar{c}\,\bar{c}$ (i.e. for instance large (pseudo) rapidity differences between $D^0 D^0$ or $\bar{D}^0 \bar{D}^0$) would be particularly useful. We suggest the following cuts in order to concentrate on the DPS contribution: $\eta_1 \in (-2.5, -2.0)$ and $\eta_2 \in (2.0, 2.5)$, where η_1, η_2 are pseudorapidities of one and second D^0 or one and second $\bar{D}^0$.

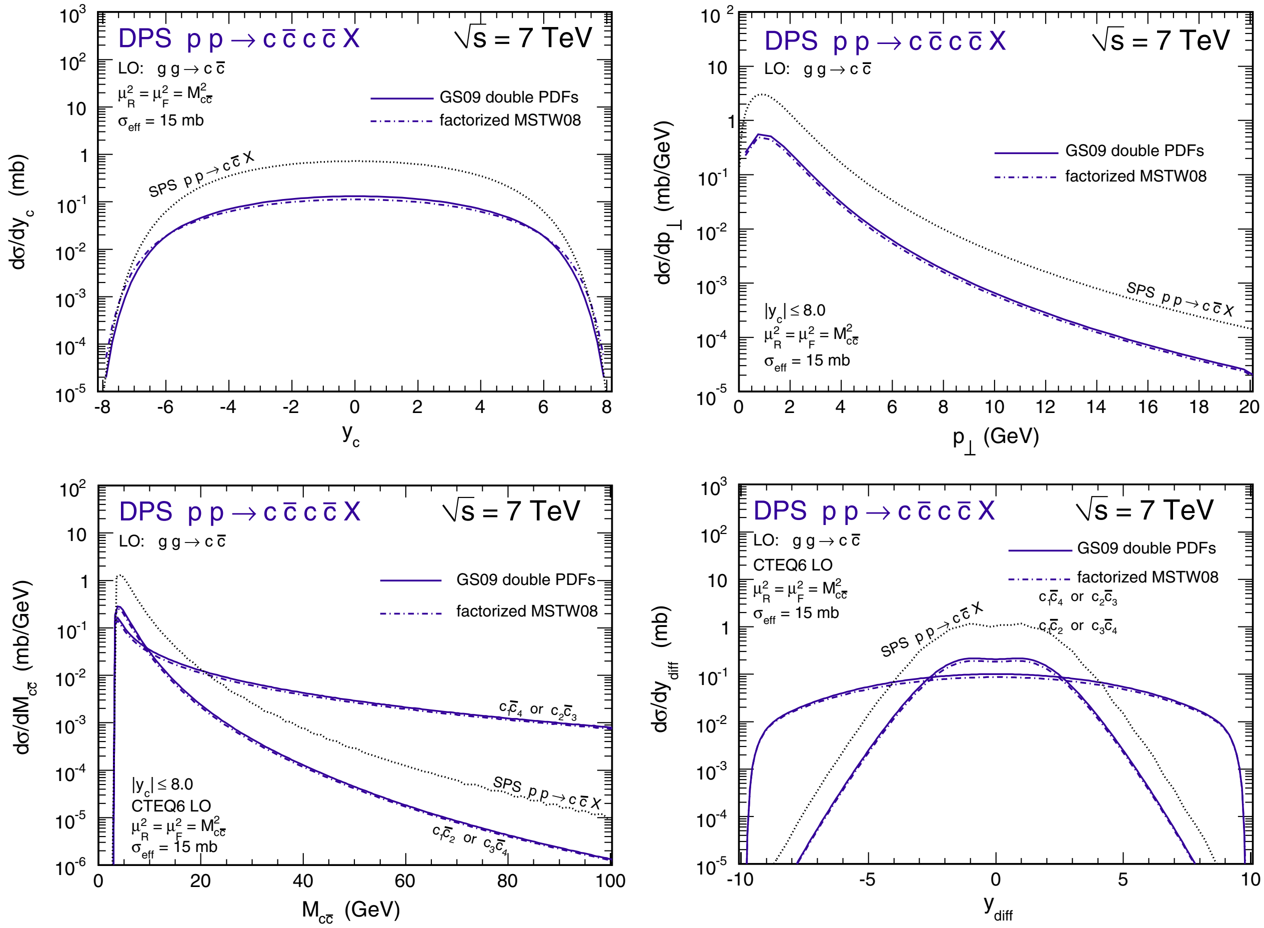

FIG. 7 (color online). Comparison of the results for the simple factorized Ansatz and for the DPDF approach with QCD evolution. All distributions are shown for $\sqrt{s} = 7$ TeV.

Usually single-particle distributions of mesons are studied in the literature. The correlation of mesons was studied e.g. in Ref. [35]. In our case here we have to store kinematical information about both mesons coming from hadronization of quarks (antiquarks) from two different parton scatterings. We have written a computer code that hadronizes both c quarks and/or both $\bar{c}$ antiquarks produced in two different double-parton scatterings. It is physically motivated to assume independent hadronization of both charmed quarks or antiquarks. The D^0 or $\bar{D}^0$ mesons are produced more often than other charmed mesons. The relevant probabilities are $P(c \to D^0) = P(\bar{c} \to \bar{D}^0) \approx 0.56$.

The main detectors (ATLAS or CMS) can measure only mesons with transverse momenta $p_t > 0.5$–1 GeV. In contrast, ALICE can measure mesons also at very low transverse momenta ($p_t > 0.1$ GeV). In Table I we show the cross section for different lower cuts on meson transverse momenta. For ALICE (last column) we show also results when an extra cut on the transverse momentum of the meson pair $p_{t,D^0D^0} > 4$ GeV is imposed in order to increase the purity of the double-parton mechanism. The cross section of the order of a fraction of μb is obtained which could be easily measured.

One can observe that the cross section strongly depends on the lower cut on meson transverse momenta. For ALICE kinematics and $p_{t,\min} = 4$ GeV the cross section is 6.2 nb. The LHCb experiments measures only forward emitted particles. For their acceptance ($2 < y < 4$ and 3 GeV $< p_t <$ 12 GeV) the relevant cross section is $\sigma_{D^0D^0} + \sigma_{\bar{D}^0\bar{D}^0} = 51.8$ nb. However, we do not expect that the DPS contribution dominates here.

TABLE I. The DPS cross section $(\sigma_{D^0D^0} + \sigma_{\bar{D}^0\bar{D}^0})/2$ in mb for the production of one meson in $\eta_1 \in (-2.5, 2.0)$ and the second meson in $\eta_2 \in (2.0, 2.5)$ (ATLAS, CMS), second column, and for η_1, $\eta_2 \in (-0.9, 0.9)$ (ALICE), third column, for different lower cuts on both mesons transverse momenta.

$p_{t,\min}$ (GeV)	ATLAS or CMS	ALICE	ALICE p_{t,D^0D^0} >4 GeV
0.0	2.59×10^{-3}	0.66×10^{-2}	0.58×10^{-3}
1.0	1.47×10^{-4}	2.48×10^{-3}	0.41×10^{-3}
2.0	0.32×10^{-5}	2.93×10^{-4}	1.54×10^{-4}
3.0	2.55×10^{-7}	0.35×10^{-4}	2.46×10^{-5}
4.0	2.33×10^{-8}	0.62×10^{-5}	0.49×10^{-5}

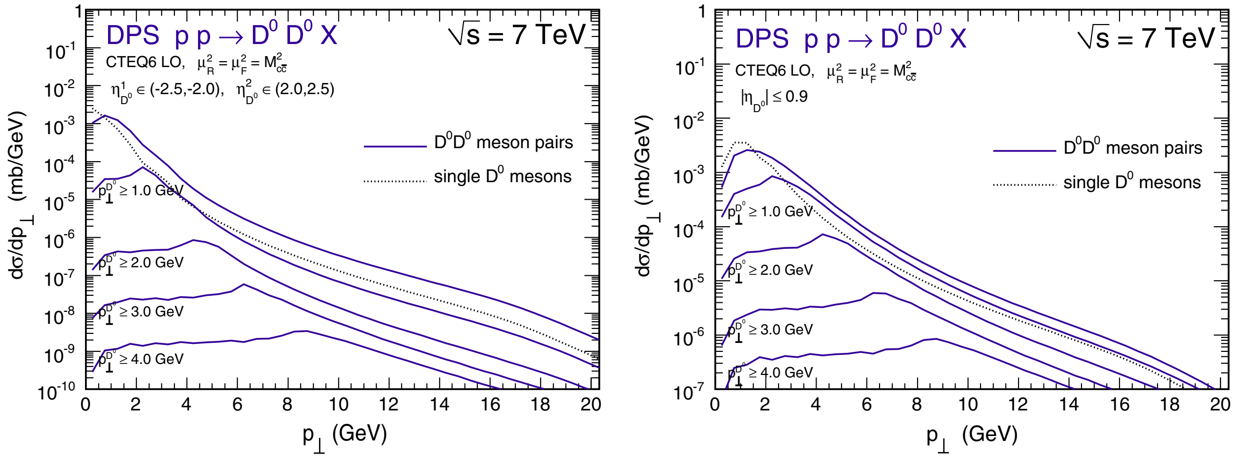

FIG. 8 (color online). Transverse momentum distribution of the $D^0 D^0$ (or $\bar{D}^0 \bar{D}^0$) pairs for the rapidity interval from Table I: ATLAS or CMS (left) and ALICE (right) for different cuts on transverse momenta of each meson in the pair. The distribution in transverse momentum of single D^0 is shown for comparison (dashed line). All distributions are shown for $\sqrt{s} = 7$ TeV.

The cross sections estimated here are subjected to uncertainties due to the choice of factorization and renormalization scales. Typical uncertainties of the order of factor 2 can be expected. Since leading-order cross sections are calculated, in reality the cross sections should be rather larger than estimated here.

Above we have discussed that the sum of transverse momenta of two c (or two $\bar{c}$) has a long tail. This is of course not an observable. In Fig. 8 we show instead the distribution in the transverse momentum of the $D^0 D^0$ pair (or $\bar{D}^0 \bar{D}^0$ pair) for the rapidity interval from Table I. These distributions have a slope comparable to that for the single D^0 (or $\bar{D}^0$) transverse momentum distribution which is shown for comparison.

Since a measurement of the cases with a large rapidity interval between mesons is not an easy, one could also try a measurement of electrons/positrons or μ^+/μ^-. The ALICE forward muon spectrometer [51] covers the pseudorapidity interval $2.5 < \eta < 4$ which when combined with the central detector means pseudorapidity differences up to 5. This is expected to be a region of phase space where double-parton scattering contribution would most probably dominate over the single-parton scattering contribution. This will be a topic of a forthcoming analysis. Next-to-leading-order corrections are not expected to give a major contribution at large pseudorapidity differences and/or large invariant masses of $\mu^+\mu^-$ but this must be verified in the future. The CMS detector is devoted especially to measurements of muons. The lower transverse momentum threshold is however rather high, the smallest being about 1.5 GeV at $\eta = \pm 2$–2.4 which may be interesting for double-parton scattering searches. This requires special dedicated Monte Carlo studies.

We expect that semileptonic decays are the main source for semihard muons or electrons. Furthermore, this contribution can, in principle, be separated experimentally by taking into account that the secondary vertices are shifted with respect to the primary ones. This should allow a separation of the semileptonic "signal" from other possible sources of dilepton continuum like Drell-Yan processes for instance.

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