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$B \rightarrow D^* \tau \bar{\nu}_{\tau}$ sensitivity to new physics

Svjetlana Fajfer,^{1,2} Jernej F. Kamenik,^{1,2} and Ivan Nišandžić¹

¹J. Stefan Institute, Jamova 39, P. O. Box 3000, 1001 Ljubljana, Slovenia

²Department of Physics, University of Ljubljana, Jadranska 19, 1000 Ljubljana, Slovenia

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B physics has played a prominent role in investigations of new physics effects at low-energies. Presently, the largest discrepancy between a standard model prediction and experimental measurements appears in the branching ratio of the charged current mediated $B \rightarrow \tau \bar{\nu}_{\tau}$ decay, where the large τ mass lifts the helicity suppression arising in leptonic *B* decays. Less significant systematic deviations are also observed in the semileptonic $B \rightarrow D^{(*)} \tau \bar{\nu}_{\tau}$ rates. Because of the rich spin structure of the final state, the decay mode $B \rightarrow D^* \tau \bar{\nu}_{\tau}$ offers a number of tests of such possible standard model deviations. We investigate the most general set of lowest dimensional effective operators leading to helicity suppressed modifications of $b \rightarrow c$ (semi)leptonic transitions. We explore such contributions to the $B \rightarrow D^* \tau \bar{\nu}_{\tau}$ decay amplitudes by determining the differential decay rate, longitudinal D^* polarization fraction, $D^* - \tau$ opening angle asymmetry and the τ helicity asymmetry. We identify the size of possible new physics contributions to these observables constrained by the present $B \rightarrow D^{(*)} \tau \bar{\nu}_{\tau}$ rate measurements and find significant modifications are still possible in all of them. In particular, the opening angle asymmetry can still deviate by as much as 80%.

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I. INTRODUCTION

Several recent experimental results in *B* physics have significantly constrained the possibility of large New Physics (NP) effects in rare *B* processes. In particular, new *CP* violating effects in the $B_s \rightarrow J/\psi\phi$ decay are already constrained to be of the order of the Standard Model (SM) expectations [1]. Similarly, the recent LHCb bound on Br($B_s \rightarrow \mu^+ \mu^-$) [2] already rules out any significant enhancement over the SM prediction in this decay.

On the other hand, existing measurements of the branching ratio for the charged current mediated $B \rightarrow$ $\tau \bar{\nu}_{\tau}$ process yield results which are systematically higher than the SM expectations [3] and the current world average for this lepton helicity suppressed decay rate is a factor of 2 above the SM predicted value [4]. B meson decays with τ leptons in the final state offer possibilities of significant NP contributions not present in processes with light leptons. Namely, the large tau mass can uplift the helicity suppression of certain (semi)leptonic decay amplitudes which are unobservable in decays with light leptons in the final state. NP models with extended ElectroWeak (EW) symmetry breaking sectors—the Two Higgs Doublet Models (THDMs) being the canonical examples-often predict enhancements in such helicity suppressed amplitudes. Existing studies of the $B \rightarrow D \tau \bar{\nu}_{\tau}$ decay [5-8] have already shown how such NP effects can be overconstrained, and how additional kinematical observables in the three-body decay offer tests of the underlying short distance contributions not possible in the twobody leptonic mode.

In this respect, the $B \rightarrow D^* \tau \bar{\nu}_{\tau}$ decay having two detectable particles of nonzero spin in the final state (D^*, τ) offers the opportunity of an even more complete investigation of the structure of possible NP contributions to $b \rightarrow$ $c\tau\bar{\nu}_{\tau}$ transitions [5]. The experimental reconstruction of the D^* in the $D\pi$ final state allows to obtain the helicity structure of this state directly. Similarly, the τ lepton helicity can be inferred from its decays to $\pi \nu_{\tau}$ final states [6,7]. This means that a number of experimental observables sensitive to possible NP effects, can be introduced. In the present study, we explore several such observables, like the differential distribution over the lepton invariant mass, the longitudinal D^* branching fraction, a $D^* - \tau$ opening angle asymmetry, as well as the τ helicity fractions. To this end we employ a model-independent effective field theory approach and identify NP contributions, which naturally predict helicity suppressed contributions in (semi)leptonic B meson decays while preserving the well established SM form of charged lepton currents (c.f. [9] for a recent related discussion).

The paper is organized as follows: in Sec. II we investigate NP-inducing helicity suppressed contributions to $b \rightarrow c \ell \bar{\nu}_{\ell}$ within the effective theory approach and evaluate existing constraints coming from the $B \rightarrow D \tau \bar{\nu}_{\tau}$ rate measurements. In Sec. III we focus on the various kinematical distributions and spin observables accessible in the $B \rightarrow D^* \tau \bar{\nu}_{\tau}$ mode and estimate their sensitivity to such NP, while the explicit derivation of the relevant polarized differential rates and evaluation of the corresponding hadronic matrix elements is relegated to the Appendixes. Finally, we conclude in Sec. IV.

II. NEW PHYSICS IN $b \rightarrow c \tau \bar{\nu}_{\tau}$

Following [9], we consider the effective weak Hamiltonian, relevant for $b \rightarrow c \ell \bar{\nu}_{\ell}$ transitions in the presence of NP contributing only to charged current interactions of quarks, while manifestly preserving the well-tested universal V - A structure of leptonic charged currents

$$\mathcal{H}_{\text{eff}} = \frac{4G_F V_{cb}}{\sqrt{2}} J_{bc,\mu} \sum_{\ell=e,\mu,\tau} (\bar{\ell} \gamma^{\mu} P_L \nu_{\ell}) + \text{H.c.}, \quad (1)$$

where $P_{L,R} \equiv (1 \mp \gamma_5)/2$, while J_{bc}^{μ} is the effective $b \rightarrow c$ charged current. In particular, we are interested in NP contributions, which lead to charged lepton helicity suppression in $B \rightarrow D^{(*)} \ell \bar{\nu}_{\ell}$, and are thus inaccessible in semileptonic decays to light leptons [9]. In general, this is the case if the NP contributions to J_{bc}^{μ} can be written as a total derivative of a scalar operator. In the effective field theory expansion, the most relevant (lowest dimensional) contributions to J_{bc}^{μ} are then

$$J_{bc}^{\mu} = \bar{c}\gamma^{\mu}P_{L}b + g_{SL}i\partial^{\mu}(\bar{c}P_{L}b) + g_{SR}i\partial^{\mu}(\bar{c}P_{R}b), \quad (2)$$

where the first term corresponds to the SM charged current, while $g_{SL,SR}$ are dimensionful NP couplings. If the NP contributions are associated with a high NP scale $\Lambda_{NP} \gg$ v_{EW} , then $g_{SL,SR} \sim v_{EW}/\Lambda_{NP}^2$, where v_{EW} is the SM EW condensate. A particular and well-known realization of such NP contributions is the THDM type II where only g_{SR} receives a significant contribution. It is of the form $g_{SR} \sim -m_b \tan^2 \beta/m_{H^+}^2$ [10], where $\tan\beta$ is the ratio of the two EW condensates in the model, and m_{H^+} is the mass of the physical charged Higgs boson. Further NP contributions to J_{bc}^{μ} relevant for helicity suppressed decays can be obtained via insertions of the ∂^2 operator and are thus necessarily suppressed by at least two additional powers of Λ_{NP}^{-1} .

In specific models one may relate NP effects in $b \rightarrow c$ transitions to other sectors, i.e. $b, s \rightarrow u$ $(B \rightarrow \tau \bar{\nu}_{\tau}, K \rightarrow \mu \bar{\nu}_{\mu})$ or $c \rightarrow d, s$ $(D_{(s)} \rightarrow \ell \bar{\nu}_{\ell})$, c.f. [11,12], resulting in a more constrained parameter space. In the present study we will however not assume any underlying flavor structure and focus exclusively on observables in the $b \rightarrow c$ sector. Before exploring such NP effects in various kinematical distributions of the $B \rightarrow D^* \tau \bar{\nu}_{\tau}$, we need to consider existing constraints coming from the measurement of the $B \rightarrow D \tau \bar{\nu}_{\tau}$ decay rate. In particular it turns out that most hadronic and SM parametric uncertainties cancel in the ratio between the tau and light lepton branching ratios [8,13], i.e.

$$R = \frac{\operatorname{Br}(B \to D\tau \bar{\nu}_{\tau})}{\operatorname{Br}(B \to De \bar{\nu}_{e})}.$$
(3)

This ratio can already be predicted with considerable precision in the SM, and present estimates using either Lattice results with $R_{\text{SM}}^{\text{Latt.}} = 0.296(16)$ (updated value based on [8] using the recent precise form factor shape determination from [14]) or heavy quark expansion with $R_{\text{SM}}^{\text{HQET}} =$ 0.302(15) [7] agree within the stated errors. NP of the form in (2) results in a modification of the *R* ratio between the tau and light lepton rates

$$R/R_{\rm SM} = 1 + 1.5 \operatorname{Re}[m_{\tau}(g_{SR} + g_{SL})] + 1.0|m_{\tau}(g_{SR} + g_{SL})|^2, \qquad (4)$$

where we have again updated the expression in [8] using the form factor shape determination from [14], and the $\overline{\text{MS}}$ values at the m_B scale have been used for the bottom and charm quark masses. Comparing these expressions with the experimentally determined values [15]

$$Br(B^+ \to \bar{D}^0 \tau^+ \nu_{\tau})_{exp} = (0.77 \pm 0.25)\%,$$

$$Br(B^+ \to \bar{D}^0 \ell^+ \nu_{\ell})_{exp} = (2.23 \pm 0.11)\%, \text{ for } \ell = e, \mu,$$
(5)

one can obtain constraints¹ in the complex plane of $(g_{SR} + g_{SL})$ as shown in the left-hand plot in Fig. 1.

For completeness, we provide SM predictions for the branching fractions by using the experimentally measured decay rates to light leptons, i.e. assuming no NP in those modes. Using inputs from [14,15] we obtain

$$Br(B^+ \to \bar{D}^0 \tau^+ \nu_{\tau})_{SM} = (0.66 \pm 0.05)\%,$$

$$Br(B^0 \to D^- \tau^+ \nu_{\tau})_{SM} = (0.64 \pm 0.05)\%.$$
(6)

III. $B \rightarrow D^* \tau \nu_{\tau}$ DIFFERENTIAL DECAY RATES

We consider the decay of a *B* meson to a polarized D^* (of helicity +, - or 0), a τ lepton of a given helicity ($\lambda_{\tau} = \pm 1/2$), and $\bar{\nu}_{\tau}$ (with helicity $\lambda_{\nu} = 1/2$) as mediated by the effective Hamiltonian of the form in Eq. (1). The relevant kinematical variables describing the three-body decay are $q^2 \equiv (p_B - p_{D^*})^2$, where p_{B,D^*} are the *B* and D^* momenta, respectively, and the angle θ between the D^* and τ three-momenta in the $\tau - \bar{\nu}_{\tau}$ rest-frame. The detailed derivation of the polarized double differential rates is given in the Appendix A with the final result in Eq. (A14).

We focus first on the decay distributions in absence of tau helicity information. Summing over both tau helicities λ_{τ} in (A14), we obtain

$$\frac{d^{2}\Gamma_{\tau}}{dq^{2}d\cos\theta} = \frac{G_{F}^{2}|V_{cb}|^{2}|\mathbf{p}|q^{2}}{256\pi^{3}m_{B}^{2}} \left(1 - \frac{m_{\tau}^{2}}{q^{2}}\right)^{2} \times \left[(1 - \cos\theta)^{2}|H_{++}|^{2} + (1 + \cos\theta)^{2}|H_{--}|^{2} + 2\sin^{2}\theta|H_{00}|^{2} + \frac{m_{\tau}^{2}}{q^{2}}((\sin^{2}\theta(|H_{++}|^{2} + |H_{--}|^{2}) + 2|H_{0t} - H_{00}\cos\theta|^{2})],$$
(7)

¹The related neutral *B* decay modes have also been measured [15] in both *D* and D^* final states, however with less significance compared to the charged *B* modes resulting in less stringent bounds.



FIG. 1 (color online). The 68% (in darker green shade) and 95% (in lighter yellow shade) C.L. allowed regions in the complex plane of NP parameters appearing in the effective $b \rightarrow c$ charged current in Eq. (2). Shown are fixed combinations (the orthogonal combinations are set to zero): $g_{SR} + g_{SL}$ (left-hand plot) bounded by $B \rightarrow D\tau \bar{\nu}_{\tau}$, $g_{SR} - g_{SL}$ (center plot) constrained only by $B \rightarrow D^*\tau \bar{\nu}_{\tau}$, and g_{SL} (right-hand plot) contributing to both modes. The best fit NP benchmark point in the later case is marked with the symbol *. All the shown constraints have been obtained from the ratios $R^{(*)}$.

where $|\mathbf{p}|$ is defined in Eq. (A2), and H_{mn} are the relevant $(q^2 \text{ dependent})$ helicity amplitudes, defined in Appendix B. Performing integration over $d \cos \theta$ in (7), we obtain

$$\frac{d\Gamma_{\tau}}{dq^2} = \frac{G_F^2 |V_{cb}|^2 |\mathbf{p}| q^2}{96\pi^3 m_B^2} \left(1 - \frac{m_{\tau}^2}{q^2}\right)^2 \left[(|H_{++}|^2 + |H_{--}|^2 + |H_{00}|^2) \left(1 + \frac{m_{\tau}^2}{2q^2}\right) + \frac{3}{2} \frac{m_{\tau}^2}{q^2} |H_{0t}|^2 \right], \quad (8)$$

in agreement with the well-known result [16,17]. The presence of NP quark charged currents defined in (2) only affects the H_{0t} helicity amplitude and can be encoded compactly as

$$H_{0t} = H_{0t}^{\rm SM} \bigg[1 + (g_{SR} - g_{SL}) \frac{q^2}{m_b + m_c} \bigg].$$
(9)

In the numerical evaluation of such NP effects we use the $\overline{\text{MS}}$ values for the bottom and charm quark masses at the m_b scale. The task of extracting information on NP from the differential decay rates thus reduces to obtaining sensitivity to the H_{0t} helicity amplitude.

We start with the $B \rightarrow D^* \tau \bar{\nu}_{\tau}$ branching fractions by integrating (8) over q^2 . These are also the only already measured observables in these modes. As in the case of the $B \rightarrow D \tau \bar{\nu}_{\tau}$ decay, most theoretical uncertainties, related to the evaluation of the hadronic form factors defined in Appendix B are significantly reduced if one normalizes the $B \rightarrow D^* \tau \bar{\nu}_{\tau}$ rates to the modes with the light charged leptons in the final state [13]—one considers the ratio

$$R^* \equiv \frac{\operatorname{Br}(B \to D^* \tau \bar{\nu}_{\tau})}{\operatorname{Br}(B \to D^* e \bar{\nu}_e)}.$$
(10)

In this way we obtain

$$R^* = R^*_{\rm SM} \{1 + 0.12 \, {\rm Re}[m_\tau (g_{SR} - g_{SL})] + 0.05 |m_\tau (g_{SR} - g_{SL})|^2 \},$$
(11)

where we find for the SM prediction (using the recent precise experimental extraction [18] of the relevant form factor ratios)²

$$R_{\rm SM}^* = 0.252(3). \tag{12}$$

The stated hadronic uncertainty is dominated by the estimate of higher-order perturbative and power corrections to the heavy quark limit of the A_0/A_1 form factor ratio which presently cannot be extracted directly from data (see Appendix B). At this level of precision, EM corrections affecting $B \rightarrow D^* e \bar{\nu}_e$ and $B \rightarrow D^* \tau \bar{\nu}_\tau$ differently could become important [19] but the related uncertainty due to such effects depends on the particular experimental setup and would require a dedicated study beyond the scope of the present paper. The above expressions are to be compared with the experimentally determined branching fractions [15]

$$Br(B^+ \to \bar{D}^{*0}\tau^+\nu_{\tau})_{exp} = (2.1 \pm 0.4)\%,$$

$$Br(B^+ \to \bar{D}^{*0}\ell^+\nu_{\ell})_{exp} = (5.68 \pm 0.19)\%, \text{ for } \ell = e, \mu.$$
(13)

From this we can again obtain constraints in the complex plane of $(g_{SL} - g_{SR})$ as shown in the central plot in Fig. 1. We observe that while certainly being complementary to the $B \rightarrow D\tau \bar{\nu}_{\tau}$ mode, NP contributions to the integrated $B \rightarrow D^* \tau \bar{\nu}_{\tau}$ branching fraction are much more diluted.

We also note that at present, the experimental measurements are systematically above SM predictions in $B \rightarrow D^{(*)}\tau \bar{\nu}_{\tau}$ decays. This is clearly demonstrated in the special case, where we set $g_{SR} = 0$ and study the combined constraints from both decay modes in the

²The value is obtained by averaging over the B^{\pm} and B^{0} modes, for which in absence of EM corrections, R_{SM}^{*} differs by less than 0.001.



FIG. 2 (color online). The differential ratios $R^*(q^2)$ (left-hand plot) and $R_L^*(q^2)$ (right-hand plot) as functions of q^2 . The black dashed curves are the SM predictions, while the (bright yellow) dotted lines denote predictions for the NP benchmark point (see text for details). The 95% C.L. allowed regions (due to existing constraints from the $B \rightarrow D^{(*)}\tau\bar{\nu}_{\tau}$ branching ratio measurements) for fixed NP parameter combinations (the orthogonal combinations being set to zero) $g_{SR} - g_{SL}$ and g_{SL} are shown in lighter (green) and darker (red) shades, respectively. The g_{SR} case is almost degenerate with g_{SL} and is therefore not shown.

right-hand plot in Fig. 1. We observe that such a fit mildly prefers a non-SM solution with $g_{SL} \simeq -0.9 \text{ GeV}^{-1}$ (combining *R* and *R*^{*} into a single χ^2 fit, we obtain $\chi^2 = 0.3$ in the best fit point compared to $\chi^2 = 2.9$ in the SM). In the following we will use this benchmark point to evaluate the discriminating power of the various $B \rightarrow D^* \tau \bar{\nu}_{\tau}$ observables.

Again for completeness, we provide SM predictions for the $B \rightarrow D^* \tau \bar{\nu}_{\tau}$ branching fractions by normalizing the value of R^*_{SM} to the experimentally measured decay rates to light leptons, i.e. assuming no NP in those modes. Using inputs from [15,18] we obtain

$$Br(B^+ \to \bar{D}^{*0}\tau^+\nu_{\tau})_{SM} = (1.43 \pm 0.05)\%,$$

$$Br(B^0 \to D^{*-}\tau^+\nu_{\tau})_{SM} = (1.29 \pm 0.06)\%,$$
(14)

in agreement with previous estimates [13].

Let us next explore the influence of NP on the ratio of singly differential decay rates with tau and light leptons in the final state

$$R^{*}(q^{2}) = \frac{d\Gamma_{\tau}/dq^{2}}{d\Gamma_{\ell}/dq^{2}} = \left(1 - \frac{m_{\tau}^{2}}{q^{2}}\right)^{2} \left[\left(1 + \frac{m_{\tau}^{2}}{2q^{2}}\right) + \frac{3}{2}\frac{m_{\tau}^{2}}{q^{2}}\frac{|H_{0t}|^{2}}{|H_{++}|^{2} + |H_{--}|^{2} + |H_{00}|^{2}}\right].$$
 (15)

Here, $d\Gamma_{\ell}/dq^2$ is the differential decay rate to a light charged lepton, where helicity suppressed effects are negligible. We note in passing that due to the mass of the τ lepton, the endpoints of the q^2 spectrum differ for $B \rightarrow D^* \tau \nu$ and $B \rightarrow D^* \ell \nu$. This feature affects the definition of q^2 -dependent observables $R^*(q^2)$ and $R_L^*(q^2)$ (defined below), in particular, they vanish below $q_{\min,\tau}^2 = m_{\tau}^2$. Similarly, for R^* and R_L^* which are integrated over q^2 , the integration limits differ between the numerator and denominator.

Taking into account present constraints on the NP contributing in linear combinations g_{SL} and $g_{SR} - g_{SL}$ we obtain the 95% C.L. allowed bands in the left-hand plot in Fig. 2. We observe that significant effects in $R^*(q^2)$

(and consequently R^*) are still possible, especially if NP contributions are aligned with the $g_{SR} - g_{SL}$ direction—if they appear in the form of a pseudoscalar density operator to which $B \rightarrow D\tau \bar{\nu}_{\tau}$ has no sensitivity.

A. Longitudinal D^* polarization and the opening angle asymmetry

Since NP of the form (2) only contributes to longitudinally polarized D^* (D_L^*) in the final state, an increased sensitivity can be expected by using information on the polarization of the D^* , which can be inferred from the angular distributions of its decay products (i.e. $D\pi$). In (8) only H_{00} and H_{0t} contribute D_L^* 's, leading to a prediction for the longitudinal rate, again normalized to the light lepton mode

$$R_L^* \equiv \frac{\text{Br}(B \to D_L^* \tau \bar{\nu}_\tau)}{\text{Br}(B \to D^* e \bar{\nu}_\tau)}$$

= 0.115(2){1 + 0.27 Re[$m_\tau (g_{SR} - g_{SL})$]
+ 0.10| $m_\tau (g_{SR} - g_{SL})$ |²}, (16)

where we have also given the estimated hadronic uncertainty of the SM prediction. In addition to this inclusive observable, one can also study the singly differential longitudinal rate ratio $R_L^*(q^2)$ defined analogously to $R^*(q^2)$ in Eq. (15). The presently allowed ranges for this observable are shown in the right-hand plot in Fig. 2. Compared to $R^*(q^2)$ this observable clearly exhibits an increased sensitivity to NP contributions.

While the D^* polarization information can be extracted directly from the angular distribution in (7), this requires experimental fits to two-dimensional decay distributions. In order to avoid such challenges, we propose a simple angular (opening angle) asymmetry defined as the difference between partial rates where the angle θ between the D^* and τ three-momenta in the $\tau - \bar{\nu}_{\tau}$ rest-frame is greater or smaller than $\pi/2$

$$A_{\theta}(q^{2}) = \frac{\int_{-1}^{0} d\cos\theta (d^{2}\Gamma_{\tau}/dq^{2}d\cos\theta) - \int_{0}^{1} d\cos\theta (d^{2}\Gamma_{\tau}/dq^{2}d\cos\theta)}{d\Gamma_{\tau}/dq^{2}}$$

$$= \frac{3}{4} \frac{|H_{++}|^{2} - |H_{--}|^{2} + 2\frac{m_{\tau}^{2}}{q^{2}}\operatorname{Re}(H_{00}H_{0t})}{[(|H_{++}|^{2} + |H_{--}|^{2} + |H_{00}|^{2})(1 + \frac{m_{\tau}^{2}}{2q^{2}}) + \frac{3}{2}\frac{m_{\tau}^{2}}{q^{2}}|H_{0t}|^{2}]}.$$
(17)

In the decay modes with light leptons, this asymmetry (A^{ℓ}_{θ}) can be used to probe for the presence of right-handed $b \rightarrow c$ currents, since these contribute with opposite sign to $H_{\pm\pm}$ relative to the SM. In the tau modes, it is sensitive only to the real part of NP $g_{SL} - g_{SR}$ contributions and thus provides complementary information compared to the total rate (or R^*). The presently allowed ranges for the A_{θ} asymmetry are shown in the left-hand plot in Fig. 3. We observe that significant deviations from the SM prediction in this observable are still allowed. Also note that in the SM this observable exhibits a zero crossing at $q_0^2 \simeq 5.6 \text{ GeV}^2$, while this is not necessarily the case in presence of NP. On the other hand, the inclusive asymmetry A_{θ} integrated over q^2 is very small in the SM with $A_{\theta,\text{SM}} = -6.0(8)\%$; for our NP benchmark point we obtain $A_{\theta,\text{NP}} = 3.4\%$, but even values as low as -30% are still allowed.

B. Using τ helicity

It has been pointed out recently [7], that the spin of the tau lepton originating from semileptonic *B* decays can be inferred using the distinctive tau decay patterns. Therefore it is beneficial to consider the $B \rightarrow D^* \tau \bar{\nu}_{\tau}$ decays with taus in a given helicity state ($\lambda_{\tau} = \pm 1/2$). In particular, assuming the standard V - A structure of the leptonic charged current entering the relevant effective weak Hamiltonian (1), the $\lambda_{\tau} = 1/2$ helicity final states are suppressed by the tau lepton mass. Using the derivation of the polarized differential decay rates in Appendix A we obtain

$$\frac{d\Gamma_{\tau}}{dq^{2}} (\lambda_{\tau} = -1/2) = \frac{G_{F}^{2} |V_{cb}|^{2} |\mathbf{p}| q^{2}}{96 \pi^{3} m_{B}^{2}} \left(1 - \frac{m_{\tau}^{2}}{q^{2}}\right)^{2} \times (H_{--}^{2} + H_{++}^{2} + H_{00}^{2}), \\
\frac{d\Gamma_{\tau}}{dq^{2}} (\lambda_{\tau} = 1/2) = \frac{G_{F}^{2} |V_{cb}|^{2} |\mathbf{p}| q^{2}}{96 \pi^{3} m_{B}^{2}} \left(1 - \frac{m_{\tau}^{2}}{q^{2}}\right)^{2} \times \frac{m_{\tau}^{2}}{2q^{2}} (H_{--}^{2} + H_{++}^{2} + H_{00}^{2} + 3H_{0t}^{2}).$$
(18)

Again we can define a useful tau spin asymmetry

$$A_{\lambda}(q^2) = \frac{d\Gamma_{\tau}/dq^2(\lambda_{\tau} = -1/2) - d\Gamma_{\tau}/dq^2(\lambda_{\tau} = 1/2)}{d\Gamma_{\tau}/dq^2},$$
(19)

which has the explicit form

 $A_{\lambda}(q^2)$

$$=1 - \frac{6|H_{0t}|^2 m_{\tau}^2}{(2q^2 + m_{\tau}^2)(|H_{--}|^2 + |H_{00}|^2 + |H_{++}|^2) + 3|H_{0t}|^2 m_{\tau}^2}$$
(20)

The presently allowed ranges for this asymmetry are shown in the right-hand plot in Fig. 3. We observe that also in this observable significant deviations from SM predictions can be expected. Even in the inclusive asymmetry, integrated over q^2 , where the SM predicts $A_{\lambda,\text{SM}} = 0.829(15)$, our NP benchmark point yields $A_{\lambda,\text{NP}} = 0.36$, while even slightly negative values are still possible.



FIG. 3 (color online). The differential asymmetries $A_{\theta}(q^2)$ (left-hand plot) and $A_{\lambda}(q^2)$ (right-hand plot) as functions of q^2 . The black dashed curves are the SM predictions, while the (bright yellow) dotted lines denote predictions for the NP benchmark point (see text for details). The 95% C.L. allowed regions (due to existing constraints from the $B \rightarrow D^{(*)}\tau\bar{\nu}_{\tau}$ branching ratio measurements) for fixed NP parameter combinations (the orthogonal combinations being set to zero) $g_{SR} - g_{SL}$ and g_{SL} are shown in lighter (green) and darker (red) shades, respectively. The g_{SR} case is almost degenerate with g_{SL} and is therefore not shown.

IV. CONCLUSIONS

Within the effective field theory approach we have studied the most general lowest-dimensional contributions to helicity suppressed (semi)leptonic $b \rightarrow c$ transitions and found that a precise study of the exclusive decay mode $B \rightarrow D^* \tau \bar{\nu}_{\tau}$ could clarify the possible existence of such non-SM physics.

Most hadronic inputs entering the theoretical predictions for the $B \rightarrow D^* \tau \bar{\nu}_{\tau}$ rates can be substantially reduced by normalizing to the $B \rightarrow D^* e \bar{\nu}_e$ mode. Using the helicity amplitude formalism we have investigated the presence of τ mass suppressed helicity amplitudes not observable in $B \rightarrow D^* \ell \bar{\nu}_\ell$ with $\ell = e, \mu$. We have estimated these contributions using heavy quark expansion including leading perturbative and power corrections and derived precise predictions for the $B \rightarrow D^* \tau \bar{\nu}_{\tau}$ branching fractions in the SM. In order to further refine these estimates, Lattice QCD results for the form factor ratio A_0/A_1 (defined in the Appendix B) would be most valuable.

The $B \rightarrow D^* \tau \bar{\nu}_{\tau}$ mode has an unique sensitivity to the pseudoscalar density operator which does not contribute to the $B \rightarrow D \tau \nu_{\tau}$ decay mode, while the opposite is true for the scalar density operator. Consequently, the precise experimental study of both $B \rightarrow D^{(*)} \tau \bar{\nu}_{\tau}$ decay modes can be extremely useful in constraining these kinds of beyond SM physics, especially, since present branching fraction measurements of all these helicity suppressed semileptonic modes are systematically above SM predictions.

Because of the rich spin structure of the $B \rightarrow D^* \tau \bar{\nu}_{\tau}$ final state one can introduce new observables such as the longitudinal polarization fraction of the D^* (R_L^*), the $D^* - \tau$



FIG. 4 (color online). Regions allowed by potential future 10% relative precision measurement of R_L^* (shaded in gray and bounded by dot-dashed lines), 10% precision determination of A_{λ} (shaded in green and bounded by dashed lines), and 5% precision measurement of A_{θ} (shaded in red and bounded by dotted lines) in the complex plane of NP parameter g_{SL} appearing in the effective $b \rightarrow c$ charged current in Eq. (2) (g_{SR} is set to zero identically). All observables are assumed to be SM-like. The present $B \rightarrow D^{(*)} \tau \bar{\nu}_{\tau}$ decay rates' best fit NP benchmark point is marked with the symbol *.

opening angle asymmetry (A_{θ}) , and the τ helicity asymmetry try (A_{λ}) . The discriminating power of these observables is demonstrated in Fig. 4 where we show the constraints that such possible future measurements would impose on the NP parameter space for a specific case of g_{SR} , for which present $B \rightarrow D^{(*)} \tau \bar{\nu}_{\tau}$ rate measurements prefer values away from zero (SM). We see that $\mathcal{O}(10\%)$ precision measurements of R_L^* , A_θ , A_λ could critically probe such effects. We have also determined the differential ratios R^* , R_L^* and asymmetries A_{θ}, A_{λ} as a function of the leptons' invariant mass squared. All observables still allow for significant modifications of the corresponding SM predictions and could thus help to disentangle the short-distance contributions to $B \rightarrow D^{(*)} \tau \bar{\nu}_{\tau}$ rates. In term, our study points out the importance of precision measurements of $B \rightarrow D^* \tau \bar{\nu}_{\tau}$ at the LHCb experiment, as well as at the future Super B factories.

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APPENDIX A: KINEMATICS AND HELICITY STRUCTURE OF THE $B \rightarrow D^* \tau \bar{\nu}_{\tau}$

Following [16] we consider the kinematics of the decay of a B meson to a polarized D^* together with a charged lepton-antineutrino pair. As throughout the paper, we will assume that the decay is mediated by interactions of the form (1), i.e. that the leptons are produced via the standard left-handed charged current as in the SM, while NP could modify quark charged currents. However, generalization of our results to nonstandard forms of the leptonic charged currents is straightforward. We denote the momenta of B, D^*, ℓ, ν with $p_B, p_{D^*}, k_\ell, k_\nu$, respectively, while $q \equiv p_B - p_B$ $p_{D^*} = k_{\ell} + k_{\nu}$. We also introduce the angles θ between the D^* and τ three-momenta in the $\tau - \bar{\nu}_{\tau}$ rest-frame, as well as χ , between the plane of the charged lepton and antineutrino momenta, and the decay plane of the D^* . Helicity basis vectors of the D^* (vector) meson are denoted as ε^{α} , while assuming standard lepton interactions, we can use $\tilde{\varepsilon}^{\mu}$ for the four basis vectors describing the total helicity of the charged lepton-neutrino system. In the rest-frame of the *B* meson with z axis along the trajectory of the D^* , a suitable basis for the lepton pair helicities is

$$\begin{split} \tilde{\varepsilon}_{\mu}(\pm) &= \frac{1}{\sqrt{2}}(0, \pm 1, -i, 0), \\ \tilde{\varepsilon}_{\mu}(0) &= \frac{1}{\sqrt{q^2}}(|\mathbf{p}|, 0, 0, -q_0), \\ \tilde{\varepsilon}_{\mu}(t) &= \frac{1}{\sqrt{q^2}}(q_0, 0, 0, -|\mathbf{p}|), \end{split}$$
(A1)

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where $q_0 = (m_B^2 - m_{D^*}^2 + q^2)/2m_B$ and

$$|\mathbf{p}| = \frac{\lambda^{1/2}(m_B^2, m_{D^*}^2, q^2)}{2m_B},$$
 (A2)

with $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + bc + ca)$. They satisfy the following normalization and completeness relations:

$$\tilde{\varepsilon}^*_{\mu}(m)\tilde{\varepsilon}^{\mu}(m') = g_{mm'}, \quad \text{for } (m, m' = t, \pm, 0), \quad (A3)$$

$$\sum_{m,m'} \tilde{\varepsilon}_{\mu}(m) \tilde{\varepsilon}_{\nu}^*(m') g_{mm'} = g_{\mu\nu}.$$
 (A4)

Similarly a convenient helicity basis for the D^* is

$$\varepsilon_{\alpha}(\pm) = \mp \frac{1}{\sqrt{2}}(0, 1, \pm i, 0),$$

$$\varepsilon_{\alpha}(0) = \frac{1}{m_{D^{*}}}(|\mathbf{p}|, 0, 0, E_{D^{*}}),$$
(A5)

where $E_{D^*} = (m_B^2 + m_{D^*}^2 - q^2)/2m_B$ is the energy of D^* in the *B* rest-frame. These basis vectors satisfy the following normalization:

$$\varepsilon^*_{\alpha}(m)\varepsilon^{\alpha}(m') = -\delta_{mm'},$$
 (A6)

and completeness relation

$$\sum_{mm'} \varepsilon_{\alpha}(m) \varepsilon_{\beta}(m') \delta_{mm'} = -g_{\alpha\beta} + \frac{p_{D^*\alpha} p_{D^*\beta}}{m_{D^*}^2}.$$
 (A7)

We can now introduce helicity amplitudes, $H_{\pm\pm}$, H_{00} and H_{0t} describing the decay of a pseudoscalar meson into the three helicity states of a vector meson and the four helicity states of the leptonic pair

$$H_{mm}(q^2) = \tilde{\varepsilon}(m)^{\mu*} H_{\mu}(m), \quad \text{for } m = 0, \pm, H_{0t}(q^2) = \tilde{\varepsilon}(m = t)^{\mu*} H_{\mu}(n = 0).$$
(A8)

Here, $H_{\mu}(m)$ is a corresponding hadronic matrix element, and *m*, *n* denote helicity projections of the D^* meson and the leptonic pair in the *B* rest-frame.

If mediated by the effective Hamiltonian of the form (1) for arbitrary quark charged current J_{bc}^{μ} , the $B \rightarrow D^* \ell \nu_{\ell}$ triply differential decay rate can be written as

$$\frac{d^2 \Gamma_{\ell}}{dq^2 d \cos\theta d\chi} = \frac{G_F^2 |V_{cb}|^2}{(2\pi)^4} \frac{|\mathbf{p}|}{2m_B^2} \left(1 - \frac{m_{\ell}^2}{q^2}\right) L_{\mu\nu} H^{\mu\nu}, \quad (A9)$$

where $L_{\mu\nu}$, $H_{\mu\nu}$ are the leptonic and hadronic current tensors. Using completeness relations of the helicity basis vectors we can rewrite $L_{\mu\nu}H^{\mu\nu}$ as

$$L_{\mu\nu}H^{\mu\nu} = L_{\mu'\nu'}g^{\mu'\mu}g^{\nu'\nu}H_{\mu\nu} = \sum_{mm',nn'} (L_{\mu'\nu'}\tilde{\varepsilon}^{\mu'}(m)\tilde{\varepsilon}^{\nu'}(n)g_{mm'}g_{nn'}) \times (\tilde{\varepsilon}^{\mu*}(m')\tilde{\varepsilon}^{\nu}(n')H_{\mu\nu}).$$
(A10)

Following [20,21] we can expand the leptonic tensor in terms of a complete set of Wigner's d^J functions, reducing $L_{\mu\nu}H^{\mu\nu}$ to the following compact form:

$$L_{\mu\nu}H^{\mu\nu} = \frac{1}{8} \sum_{\lambda_{\ell},\lambda_{D^{*}},\lambda_{\ell\nu},\lambda_{\ell\nu}',J,J'} (-1)^{J+J'} |h_{(\lambda_{\ell},\lambda_{\nu})}|^{2} \\ \times \delta_{\lambda_{D^{*}}\lambda_{\ell\nu}} \delta_{\lambda_{D^{*}}\lambda_{\ell\nu}'} d^{J}_{\lambda_{\ell\nu},\lambda_{\ell}-1/2}(\theta) \\ \times d^{J}_{\lambda_{\ell\nu}',\lambda_{\ell}-1/2}(\theta) H_{\lambda_{D^{*}}\lambda_{\ell\nu}} H^{*}_{\lambda_{D^{*}}\lambda_{\ell\nu}'},$$
(A11)

where *J* and *J'* run over 1 and 0. In term, the lepton helicity amplitudes, $h_{(\lambda_{\ell},\lambda_{\nu})}$ for a left-handed weak current are given by

$$h_{(\lambda_{\ell},\lambda_{\nu})} = \frac{1}{2} \bar{u}_{\ell}(\lambda_{\ell}) \gamma^{\mu} (1 - \gamma^5) \upsilon_{\nu}(\lambda_{\nu}) \tilde{\epsilon}_{\mu}(\lambda_{\ell\nu}), \quad (A12)$$

where for massless right-handed antineutrinos $\lambda_{\nu} = 1/2$ and $\lambda_{\ell\nu} = \lambda_{\ell} - \lambda_{\nu}$ in the $\ell\nu$ center of mass frame by angular momentum conservation. It follows that the two nonvanishing $|h_{(\lambda_{\ell},\lambda_{\nu})}|^2$ contributions are

$$|h_{-1/2,1/2}|^2 = 2(q^2 - m_\ell^2)$$
 and
 $|h_{1/2,1/2}|^2 = 2\frac{m_\ell^2}{2q^2}(q^2 - m_\ell^2).$ (A13)

Finally, using the standard [15] convention for Wigner's d-functions and performing the trivial integration over χ we obtain

$$\frac{d^{2}\Gamma_{\ell}}{dq^{2}d\cos\theta}(\lambda_{\ell} = -1/2) = \frac{G_{F}^{2}|V_{cb}|^{2}|\mathbf{p}|q^{2}}{256\pi^{3}m_{B}^{2}} \left(1 - \frac{m_{\ell}^{2}}{q^{2}}\right)^{2} \\ \times \left[(1 - \cos\theta)^{2}H_{++}^{2} + (1 + \cos\theta)^{2}H_{--}^{2} + 2\sin^{2}\theta H_{00}^{2}\right], \\ \frac{d^{2}\Gamma_{\ell}}{dq^{2}d\cos\theta}(\lambda_{\ell} = 1/2) = \frac{G_{F}^{2}|V_{cb}|^{2}|\mathbf{p}|q^{2}}{256\pi^{3}m_{B}^{2}} \left(1 - \frac{m_{\ell}^{2}}{q^{2}}\right)^{2}\frac{m_{\ell}^{2}}{q^{2}} \\ \times \left[(\sin^{2}\theta(H_{++}^{2} + H_{--}^{2}) + 2(H_{0t} - H_{00}\cos\theta)^{2}\right], \quad (A14)$$

from which Eqs. (7), (8), and (18) can be easily derived via summation over λ_{ℓ} and/or integration over $\cos\theta$.

APPENDIX B: HELICITY AMPLITUDES AND HADRONIC MATRIX ELEMENTS

In the SM (in presence of only the first term in (2)), the helicity amplitudes H_{mn} can be written as

$$H_{\pm\pm}^{\rm SM}(q^2) = (m_B + m_{D^*})A_1(q^2) \mp \frac{2m_B}{m_B + m_{D^*}} |\mathbf{p}| V(q^2),$$

$$H_{00}^{\rm SM}(q^2) = \frac{1}{2m_{D^*}\sqrt{q^2}} \Big[(m_B^2 - m_{D^*}^2 - q^2)(m_B + m_{D^*})A_1(q^2) - \frac{4m_B^2 |\mathbf{p}|^2}{m_B + m_{D^*}} A_2(q^2) \Big],$$

$$H_{0t}^{\rm SM}(q^2) = \frac{2m_B |\mathbf{p}|}{\sqrt{q^2}} A_0(q^2).$$
 (B1)

where the form factors parametrizing the relevant hadronic matrix elements are defined as

$$\langle D^*(p_{D^*}, \epsilon_{\alpha}) | \bar{c} \gamma_{\mu} b | B(p_B) \rangle = \frac{2iV(q^2)}{m_B + m_{D^*}} \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} p_B^{\alpha} p_{D^*}^{\beta},$$
(B2a)

$$\langle D^{*}(p_{D^{*}}, \epsilon_{\alpha}) | \bar{c} \gamma_{\mu} \gamma_{5} b | B(p_{B}) \rangle = 2m_{D^{*}} A_{0}(q^{2}) \frac{\epsilon^{*} \cdot q}{q^{2}} q_{\mu}$$

$$+ (m_{B} + m_{D^{*}}) A_{1}(q^{2}) \left(\epsilon_{\mu}^{*} - \frac{\epsilon^{*} \cdot q}{q^{2}} q_{\mu} \right)$$

$$- A_{2}(q^{2}) \frac{\epsilon^{*} \cdot q}{m_{B} + m_{D^{*}}} \left((p_{B} + p_{D^{*}})_{\mu} - \frac{m_{B}^{2} - m_{D^{*}}^{2}}{q^{2}} q_{\mu} \right)$$

$$(B2b)$$

In presence of NP of the form (2), one needs to evaluate two additional matrix elements given by

$$\langle D^*(p_{D^*}, \epsilon_{\alpha}) | \bar{c}b | B(p_B) \rangle = 0,$$
 (B3a)

$$\langle D^*(p_{D^*}, \boldsymbol{\epsilon}_{\alpha}) | \bar{c} \gamma_5 b | B(p_B) \rangle = \frac{1}{m_b + m_c} q^{\mu} \langle D^* | \bar{c} \gamma_{\mu} \gamma_5 b | \bar{B}^0 \rangle$$
$$= \frac{2m_{D^*}}{m_b + m_c} A_0(q^2) \boldsymbol{\epsilon}^* \cdot q. \quad (B3b)$$

The final effect of such contributions to the differential rates can be encoded into the H_{0t} helicity amplitude as given in Eq. (9).

We can use further information on the form factors given by precise differential decay-rate measurements in $B \rightarrow D^* \ell \bar{\nu}_{\ell}$ [18], as well as their perturbatively computable properties and relations in the heavy quark limit for the *b*, *c* quarks [22,23]. In this limit it is customary to employ a new kinematical variable

$$w \equiv v_B \cdot v_{D^*} = \frac{m_B^2 + m_{D^*}^2 - q^2}{2m_B m_{D^*}},$$
 (B4)

with v_B^{μ} , and $v_{D^*}^{\mu}$ being the four-velocities of the *B* and D^* meson, respectively. One can then define an universal form factor

$$h_{A_1}(w) = A_1(q^2) \frac{1}{R_{D^*}} \frac{2}{w+1},$$
 (B5)

and ratios R_1 , R_2 , and R_0 in terms of which

$$A_{0}(q^{2}) = \frac{R_{0}(w)}{R_{D^{*}}} h_{A_{1}}(w),$$

$$A_{2}(q^{2}) = \frac{R_{2}(w)}{R_{D^{*}}} h_{A_{1}}(w),$$

$$V(q^{2}) = \frac{R_{1}(w)}{R_{D^{*}}} h_{A_{1}}(w),$$
(B6)

where $R_{D^*} = 2\sqrt{m_B m_{D^*}}/(m_B + m_{D^*})$. The *w* dependence of these quantities in the heavy quark limit reads

$$h_{A_1}(w) = h_{A_1}(1)[1 - 8\rho^2 z + (53\rho^2 - 15)z^2 - (231\rho^2 - 91)z^3],$$

$$R_1(w) = R_1(1) - 0.12(w - 1) + 0.05(w - 1)^2,$$

$$R_2(w) = R_2(1) + 0.11(w - 1) - 0.06(w - 1)^2,$$

$$R_0(w) = R_0(1) - 0.11(w - 1) + 0.01(w - 1)^2,$$
 (B7)

where $z = (\sqrt{w+1} - \sqrt{2})/(\sqrt{w+1} + \sqrt{2})$. The first three expressions can be found in [24], while we have derived the fourth using the results of [24]. Above relations contain free parameters $h_{A_1}(1)$, $R_1(1)$, $R_2(1)$, ρ^2 , which can be extracted from the well-measured $B \rightarrow D^* \ell \bar{\nu}_{\ell}$ decay distributions. In our numerical evaluation of the $B \rightarrow$ $D^* \tau \bar{\nu}_{\tau}$ differential decay rates we employ the results of a recent Belle analysis [18]. The virtue of this approach is that most of the associated hadronic uncertainties actually cancel in ratios of decay rates to tau versus light leptons, as previously demonstrated for the case of $B \rightarrow D \tau \bar{\nu}_{\tau}$ [8].

In addition to these inputs, the $B \to D^* \tau \bar{\nu}_{\tau}$ rate also depends on $R_0(1)$, which cannot be extracted from $B \to D^* \ell \bar{\nu}_{\ell}$ studies, since it only appears in the helicity suppressed amplitude H_{0t} . In the exact heavy quark limit $[R_0(1)]_{\text{HQET}} = 1$. Leading order perturbative (in α_s) and power $(1/m_{b,c})$ corrections are known [22,23] for the linear combination

$$R_3(1) = \frac{R_2(1)(1-r) + r[R_0(1)(1+r) - 2]}{(1-r)^2} = 0.97, \quad (B8)$$

where $r = m_{D^*}/m_B$.

The same calculation predicts $R_2(1) = 0.80$, yielding eventually $R_0(1) = 1.22$. Experimentally [18] however, $R_2(1) = 0.864(25)$ and inserting this value into Eq. (B8) yields our final result $R_0(1) = 1.14$. In all our numerical calculations we use directly Eq. (B8) and conservatively assign a 10% uncertainty to this value accounting for higher-order corrections. In the future, a more reliable and precise determination of $R_0(1)$ [or equivalently $R_3(1)$] could be obtained on the lattice, similarly as has already been done for the helicity suppressed $B \rightarrow D$ matrix elements [25]. $B \rightarrow D^* \tau \bar{\nu}_{\tau}$ SENSITIVITY TO ...

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